In search of the "lost capital". A theory for valuation, investment decisions, performance measurement

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Abstract

This paper presents a theoretical framework for valuation, investment decisions, and performance measurement based on a nonstandard theory of residual income. It is derived from the notion of "unrecovered" capital, which is here named "lost" capital because it represents the capital foregone by the investors. Its theoretical strength and meaningfulness is shown by deriving it from four main perspectives: financial, microeconomic, axiomatic, accounting. Implications for asset valuation, capital budgeting and performance measurement are investigated. In particular: an aggregation property is shown, which makes the simple average residual income play a major role in valuation; a dual relation between the standard theory and the lost-capital theory is proved, clarifying the way periodic performance is computed in the two paradigms and the rationale for measuring performance with either paradigm; the average accounting rate of return is shown to be more reliable than the internal rate of return as a capital budgeting criterion, and maximization of the average residual income is shown to be equivalent to maximization of Net Present Value (NPV). Two metrics are also presented: one enjoys the nice property of robust goal congruence irrespective of the sign of the cash flows; the other one enjoys periodic consistency in the sense of Egginton (1995). The results obtained suggest that this theory might prove useful for real-life applications in firm valuation, capital budgeting decision-making, ex ante and ex post performance measurement, incentive compensation. A numerical example illustrates the implementation of the paradigm to the EVA model and the Edwards-Bell-Ohlson model.

Keywords. Residual income, valuation, capital budgeting, performance measurement, lost capital, accounting rate, average, Economic Value Added.
JEL codes. M41, G11, G12, G31, M21, M52, D46.

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1 Introduction

Corporate finance and accounting find a common terrain in the study of the notion of residual income, also called excess profit or abnormal earnings. Residual income is formally computed as the difference between the actual income and the counterfactual income investors would receive if they invested their funds at the opportunity cost of capital. Coined by the General Electric Company, the term first appears in the literature in Solomons (1965, p. 63), although the same concept, differently labeled, was studied even earlier (e.g. Preinreich, 1936, 1938; Edwards and Bell, 1961, Bodenhorn, 1964). The contributions of Peasnell (1981, 1982), Peccati (1987, 1989, 1991) and Ohlson (1989, 1995) have caused a renewed interest in this notion among corporate finance and accounting scholars, with particular regard to firm valuation, performance measurement, incentive compensation (value-based management). A large number of theoretical and applied studies have appeared in both applied finance and accounting, (e.g. Stewart, 1991; Ohlson, 1995; Feltham and Ohlson, 1995; Rappaport, 1998; Lundholm and O'Keefe, 2001; Young and O'Byrne, 2001; Martin, Petty and Rich, 2003; Weaver and Weston, 2003; O'Byrne and Young, 2006), and a large number of textbooks and professional publications in corporate finance, managerial finance and accounting directly deal with the topic (e.g. Brealey and Myers, 2000; Copeland, Koller and Murrin, 2000; Palepu, Healey and Bernard, 2000; Grinblatt and Titman, 2002; Revsine, Collins and Johnson, 2005; Arnold, 2005).

It is well-known that there is a lifespan consistency of residual income (RI) with Net Present Value (NPV): the sum of the discounted residual incomes generated by the project (firm) equals the project's NPV (e.g. Peasnell, 1982; Peccati, 1989; Martin and Petty, 2000; Vélez-Pareja and Tham, 2003). A line of research in accounting finance and corporate finance is devoted to exploiting this property for valuation purposes; it investigates the relations existing between residual income and firm valuation and studies the opportunity of replacing cash flows with residual incomes in the computation of the market value of a firm (e.g. Peasnell, 1981, 1982; Ohlson, 1989, 1995; Penman, 1992; O'Hanlon and Peasnell; 2002; Brief, 2007; Schüler and Krotter, 2008). Residual income is periodic in nature and this makes it a good candidate for performance measurement. The literature on performance measurement is opulent and is particularly aimed at providing appropriate performance measures and at devising compensation plans capable of aligning shareholders' interests and managers' interests (e.g. Solomons, 1965; Peccati, 1991; Gallo and Peccati, 1993; Egginton, 1995; Reichelstein, 1997; Rogerson, 1997; Pfeiffer, 2000; Pfeiffer and Schneider, 2007; Schultze and Weiler, 2008).

This paper focusses on the very *notion* of residual income, aiming at exploring an alternative theory of residual income, previously introduced by Magni (2000, 2001, 2004, 2005). It is here labelled *lost-capital* theory, because its essential feature is the consideration of the capital lost (i.e., foregone) by the investors. The purpose of this work is just to show how it formally relates to the standard theory.

In order to show the theoretical strength of the new paradigm, this paper presents it in four different ways, related to four different perspectives: (i) a financial perspective, which generates the lost-capital residual income from arbitrage theory; (ii) a microeconomic derivation, which focusses on the economic agent's wealth; (iii) a mathematical perspective using an axiomatic approach; (iii) an accounting derivation of the paradigm via two alternative depreciation schedules. This should sufficiently underline the multifaceted theoretical significance of the residual income, its sound economic meaning, and its formal robustness. The usefulness of the theory is shown in three main areas:

1. asset valuation: residual incomes aggregate in a value sense, as opposed to the standard paradigm where residual incomes aggregate in a cash-flow sense. This enables one to compute the firm's market value leaving out any consideration about timing, which makes the lost-capital paradigm a good candidate for firm valuation in real-life applications. The role of the average RI is particularly underlined

2. capital budgeting: a decision rule based on the average accounting rate of return (ARR) is shown to be superior to the internal-rate-of-return (IRR) rule: no problems of existence or uniqueness arise and, contrary to the IRR, the rule is equivalent to the NPV rule. The rule may be reframed in terms of average RI: the latter is shown to be a perfect substitute of the NPV so that maximization of the NPV may be replaced by maximization of average RI, possibly time-scaled for projects with different life

3. performance measurement: interpretation is given to the different measurement process of the two theories and, in particular, it is highlighted that the lost-capital theory takes account of the fact that choice affects not only the return rate, but also the capital invested. The use of the lost-capital residual income for compensating managers implies that shareholders are willing to reward management on the basis of the real alternative scenario that would occur if the firm were managed in a value-neutral way. In other words, the capital charge is a comprehensive one: both return rate and capital are different from what they would be if the investors chose not to undertake the project. This is revealed by an interesting dual relation, according to which the two theories are mutually generative. Furthermore, Fernández's (2002) *Created Shareholder Value* is transformed into the corresponding lost-capital metric. The latter is a goal-congruent metric, which is more general than Grinyer's (1985, 1987) Earned Economic Income, because it is not affected by change in sign of the cash flows. A metric here named *maintainable* RI is shown to be periodically consistent in the sense of Egginton (1995). This might prove useful in performance evaluations given that these metrics directly tie performance to value creation.

Throughout the paper it is assumed that an economic activity f (firm, project) is undertaken at time 0, which generates the cash-flow vector $\vec{f}=(f_1, f_2, \dots, f_n)$, $f_t \in \mathbb{R}$ where f_t is the cash flow received by the owners of the asset at time t. The initial investment is $f_0 > 0$ and f_n is inclusive of the liquidation value. The setting is therefore a classical one (with no managerial flexibility)

Cash flows may be thought of as certain or certainty equivalents of random cash flows, which implies that the discount rate is the risk-free rate. Alternatively, the reader may regard cash flow as expected values: this is most common in corporate finance (e.g. Brealey and Myers, 2000; Fernández, 2002; Damodaran, 2005, 2006), accounting (e.g. Peasnell, 1981, 1982; O'Hanlon and Peasnell, 2002; Brief, 2007) and value-based management (Arnold and Davies, 2000; Martin and Petty, 2000; Young and O'Byrne, 2001). In the latter case, the cost of capital is a required rate of return taking account of the risk of the enterprise. The numerical example in the Appendix is consistent with the latter interpretation[1] Furthermore, there is no opening accounting error (as is usual in capital budgeting), that is, the book value at time 0 coincides with f_0, and the theoretical analysis holds either in a proprietary approach (equity value is to be computed) and an entity approach (firm value is to be computed); thus, the reader may equivalently view the cash-flow vector $\vec{f}$ as a vector of equity cash flows or as a vector of free cash flows. In the numerical example we use three amongst the most common discounted-cash-flow techniques to reach the equity value: (i) equity-cash-flow discounting at the cost of equity, (i) free-cash-flow discounting at the weighted average cost of capital, (iii) adjusted present value method (see Myers, 1974; Brealey and Myers, 2000; Damodaran, 2005, 2006; Fernández, 2002; Copeland, Koller and Murrin, 2000).

The paper is structured as follows. Section 2 shows important relations between accounting rates and book values and interprets accounting rates as internal return rates of one-period projects composing the economic activity under consideration. It also supplies the classical definition of residual income as currently in use among finance scholars and accounting scholars. Section 3 is a theoretical presentation of the new paradigm from four different points of view: they are conventionally labelled: (i) *financial* (owing to the arbitrage argument used), (ii) *microeconomic* (owing to the focus on the economic agent's wealth and its evolution through time) (iii) *mathematical* (given that an axiomatic approach is followed), (iv) *accounting* (the residual income is obtained as a difference between depreciation charges). Section 4 draws attention to an aggregation result whereby time is inessential in valuation: only the sum of residual incomes is of concern for computing market values. In section 5 an important profitability index is drawn from the lost-capital framework: the Chisini mean of average of accounting rates is shown to be more general and reliable than the IRR, and compatible with the NPV. The *time-scaled* residual income is then introduced, whose maximization is equivalent to NPV maximization. It is also shown that the impact of income on value is given by the unit

[1] A discussion on the relation between cost of capital and cash flows is beyond the scope of the paper. A well-written analysis of the methods to exogenously extract a cost of capital is Armitage's (2005) book. For various perspectives on the cost of capital, see Tuttle and Litzenberger (1968), Hamada (1972), Rubinstein (1973), Fama (1977), Lewellen (1977), Weston and Chen (1980), Haley (1984), Stark (1986), Copeland and Weston (1988), Ohlson (1995), O'Hanlon and Steele (1997), Ruback (2002); Ogier, Rugman and Spicer (2004), Bøssaerts and Odegaard (2006), Damodaran (2006), Morana (2007), Magni (2009).

price of a zero-coupon bond (or of an equivalent-risk asset). Section 6 focusses on periodic performance and the relations between the two paradigms. In particular, a dual relation is shown, according to which standard residual income may be viewed as a function of lost-capital residual income and viceversa. Furthermore, it shows that the lost-capital companion of Fernández's (2002) Created Shareholder Value is aligned in sign with the Net Present Value, that is, *robust goal congruence* holds (see Mohnen and Bareket, 2007), which implies that this metric might be particularly interesting for incentive compensation. Whatever the asset base, the average RI (properly time-scaled if projects have different life), is periodically consistent in the sense of Egginton (1995) and may be obtained as a residual income where the assets base is specified so that the average surplus of book value over lost capital is constant through time. Some concluding remarks end the paper. In the Appendix the conversion process from standard metric to lost-capital metrics is illustrated for two metrics: the Economic Value Added (Stewart, 1991) and the Edwards-Bell-Ohlson (Edwards and Bell, 1961; Ohlson, 1995) model. A final illustrative example is also presented.

Main notational conventions are collected in Table 0 at the end of the paper.

2 The standard theory

Consider the cash-flow stream $\vec{f}$ released by asset f (project or firm) and received by the owners of the asset. Let x_t, $t = 1, 2, \dots, n$ be the profit and b_t the book value.[2] The symbol b_n represents book value *after* the firm has been liquidated, so b_n=0. We assume, unless otherwise specified, that the average book value $\overline{b} := \sum_{t=1}^{n} b_{t-1}/n$ is positive. A fundamental accounting identity is

$$x_t = f_t - b_{t-1} + b_t \qquad t = 1, 2, \dots, n \tag{1}$$

which is often called *clean surplus relation* (see Brief and Peasnell, 1996). Letting a_t be the accounting rate of return, $a_t = x_t/b_{t-1}$, clean surplus may be rewritten as

$$a_t = \frac{f_t + b_t}{b_{t-1}} - 1 \qquad t = 1, 2, \dots, n. \tag{2}$$

which is well-defined as long as $b_{t-1} \neq 0$. Equation (2) is highly significant, as is now illustrated. Consider the vectors $e_t = (\vec{0}_{t-1}, 1, \vec{0}_{n-t}) \in \mathbb{R}^n$, $t, 1, 2, \dots, n$ where $\vec{0}^k$ is the null vector in $\mathbb{R}^k$; consider also the vectors $\vec{f}_t = -b_{t-1}e_t + (f_t + b_t)e_{t+1} \in \mathbb{R}^n$, $t = 1, 2, \dots, n$. They are interpretable as one-period projects: the investors invest capital b_{t-1} at time $t-1$ and receive the cash flow f_t along with the end-of-period value b_t at time t. We have

$$\vec{f} = \vec{f}_1 + \vec{f}_2 + \dots \vec{f}_n. \tag{3}$$

Using the clean surplus relation recursively, one easily finds, after some manipulations,

$$b_0 = \sum_{t=1}^{n} \frac{f_t}{\prod_{k=1}^{t}(1 + a_k)}. \tag{4}$$

[2]Depending on the perspective, b_t is the equity book value or the firm book value (equity+liabilities).

This means that the vector of accounting rates $\vec{a} = (a_1, a_2, \ldots, a_n)$ is an *internal* discount function. This fact is known in the accounting literature: it has been shown, among others, by Kay (1976), Peasnell (1982), and Brief and Lawson (1992). However, the straightforward link of this internal discount function with the notion of *internal return vector* introduced by Weingartner (1966) is not appreciated. An internal return vector is a vector $\vec{r} = (r_1, r_2, \ldots, r_n)$ of return rates such that

$$f_0 = \sum_{t=1}^{n} \frac{f_t}{\prod_{k=1}^{t}(1 + r_k)}. \tag{5}$$

The particular case where $\vec{r} = (r, r, \ldots, r)$ is just the *internal rate of return*. Thus, the notion of internal return vector just generalizes the IRR notion. The link between the internal discount function $\vec{a}$ and the internal return vector $\vec{r}$ should now be evident from eqs. (4) and (5): if $f_0 = b_0$, the vector $\vec{a}$ is an internal return vector. Reminding that there is no opening accounting error, we have the following

Proposition 1. *The accounting rate of return is a one-period IRR, and the internal discount function generated by the accounting rates is an internal return vector. Also, an IRR is a constant accounting rate that leads to a zero-NPV project .*

The above proposition allows us to assert that the accounting rate of profit is itself an internal rate of return. Owing to eqs. (2) and (3), the economic activity f may be ideally interpreted as a portfolio of n consecutive one-period projects $\vec{f_t}$, each of which has an internal rate of return (IRR) equal to a_t, $t = 1, 2, \ldots n$ (see also Gronchi, 1984, and Manca, 1989, on the splitting up of cash-flow streams). The relation of the (constant) IRR with the accounting rates has been studied in depth during the last decades. It is widely known in the literature that it is not possible to obtain the IRR as a meaningful average of accounting rates:

$$r \neq \frac{\sum_{t=1}^{n} a_t b_{t-1}}{\sum_{t=1}^{n} b_{t-1}}. \tag{6}$$

Just because of this fact, the accounting rates are often regarded less significant than the IRR and the above average is considered unhelpful for analysis and decision-making. However, the average of accounting rates do lead to the IRR if book values are replaced by their present values computed at IRR:

$$r = \frac{\sum_{t=1}^{n} a_t \frac{b_{t-1}}{(1+r)^{-t}}}{\sum_{t=1}^{n} \frac{b_{t-1}}{(1+r)^{-t}}} \tag{7}$$

(see Peasnell, 1982; Franks and Hodges, 1984; Peccati, 1989, 1991; Brief and Lawson, 1992).[3]

Remark 1. It is worth noting that the definition of accounting rate of profit enables one to rewrite the clean surplus relation as

$$b_t = b_{t-1}(1 + a_t) - f_t \tag{8}$$

[3] Note that circularity arises in this relation.

(see Peasnell, 1982, p. 108). The above relation coincides with the recursion formula used in financial and actuarial mathematics for computing the balance (residual debt) in a loan contract (Kellison, 1991; Castagnoli and Peccati, 2002; Promislow, 2006; Werner and Sotskov, 2006), where b_0 is the amount borrowed, $a_t b_{t-1}$ represents interest and f_t is the installment. This fact enables one to interpret f as a loan contract whereby shareholders lend the firm the amount b_0 and receive the installment f_t at time t. In this view, b_t is the residual debt the firm owes the shareholders. The idea of capital as a residual debt is not new: "The corporation *owes* the capital, it does not *own* it. The shareholders own it" (Fetter, 1937, p. 9); and the corresponding idea of profit as representing shareholders' interest is also sometimes acknowledged: "the profit is equal to interest on the capital value existing at the beginning of the period" (Hansen, 1972, p. 15). The same idea is at the core of Anthony's (1975) notion of profit.

The standard definition of residual income, universally accepted in accounting and finance, is computed as a difference between two profits: the actual profit x_t and the counterfactual profit that shareholders would (have) obtain(ed) if they (had) invested b_0 in an economic activity whose period rate of return is i_t, also known as *cost of capital*:

$$x_t^{\mathcal{S}} = x_t - i_t b_{t-1} \tag{9}$$

($\mathcal{S}$:=standard). Note that three elements are into play: profit, book value, cost of capital. The product $i_t b_{t-1}$ is also known as *capital charge*. From the general framework of (9) different metrics are generated, grounded on different notions of capital employed (asset side, equity side, economic, accounting, etc.), of cash flows employed (free cash flow, equity cash flow, capital cash flow[4]), of internal discount function employed (ROA, RONA, ROE, etc.).

As anticipated, the clean surplus relation implies a lifespan consistency with the NPV:

$$\text{NPV} = \sum_{t=1}^{n} \frac{x_t^{\mathcal{S}}}{\prod_{k=1}^{t}(1+i_k)} \tag{10}$$

which holds for any book value depreciation.

3 The *lost-capital* theory

This section presents a different way of representing the foregone return (the capital charge), and therefore a different way of interpreting the notion of residual income. It has been originally introduced and investigated in Magni (2000, 2005, 2006). This section shows that it is possible to derive this notion from four different (but logically equivalent) arguments: an arbitrage-based argument; an axiomatic approach; an economic argument focussed on the investor's wealth; an accounting argument involving alternative depreciation schedules.

[4]For the notion of capital cash flow, see Ruback (2002) and Fernández (2002).

3.1 The financial derivation

Suppose p is a portfolio traded in the market which replicates the cash-flow vector $\vec{f}$=$(f_1, f_2, \ldots, f_n)$. Let $F(s,t) = \prod_{k=s+1}^{t}(1+i_k)$ represent the yield term structure, so that $F(0,t)^{-1}$ is the unit price of a zero-coupon bond expiring at t.[5] The market value of p is $p_0 = \sum_{t=1}^{n} f_t F(0,t)^{-1}$. If $p_0 \neq b_0$ (i.e. NPV$\neq$0) the investor may exploit arbitrage opportunities. For example, assuming (with no loss of generality) $p_0 > b_0$, investors may invest in f, take a short position in p and reinvest the arbitrage gain $(p_0 - b_0)$ in portfolio p. The resulting net cash flow will be zero at each date, and investors will receive a net final cash flow Γ, such that $\Gamma = (p_0 - b_0)F(0,n)$=NPV$\cdot F(0,n)$ (see Table 1). The latter is the accumulated NPV (sometimes called "excess return" or "net final value"). Let us now measure the periodic gain released by this strategy. Note that the long and short positions in p may be netted out to result in a net short position (see Table 2).

Let

$$b_t^* = b_{t-1}^*(1+i_t) - f_t \tag{11}$$

be the value of the short position: the amount $x_t = a_t \cdot b_{t-1}$ is the profit from the long position, the amount $x_t^* = i_t \cdot b_{t-1}^*$ is interest paid on short position and represents the cost paid for undertaking the arbitrage strategy. The latter also represents the income that shareholders would have earned if they had invested in portfolio p rather than in firm (project) f. It is then interpretable as a "lost" capital (the same capital is named "unrecovered" by O'Hanlon and Peasnell, 2002). The periodic gain is given by the difference of interest on long and short positions:

$$x_t^{\mathcal{L}} = x_t - x_t^* = x_t - i_t \cdot b_{t-1}^* \tag{12}$$

($\mathcal{L}$:=lost-capital). We may also rewrite the latter as

$$x_t^{\mathcal{L}} = b_{t-1}(a_t - i_t^*) \tag{13}$$

where $i_t^* := i_t b_{t-1}^*/b_{t-1}$. The spread $(a_t - i_t^*)$ measures the period margin per unit of capital invested. Noting that $b_n^* = f_0 F(0,n) - \sum_{t=1}^{n-1} f_t F(t,T)$ and using the equalities $x_t = b_{t-1} - b_t + f_t$ and $b_t^* - b_{t-1}^* = i_t b_{t-1}^* - f_t$, one finds that the sequence $\{x_t^{\mathcal{L}}\}_1^n$ of periodic gains decomposes Γ:

$$x_1^{\mathcal{L}} + x_2^{\mathcal{L}} + \ldots + x_n^{\mathcal{L}} = \Gamma = \text{NPV} \cdot F(0,n). \tag{14}$$

3.2 The (micro)economic derivation

Consider an economic agent who currently invests funds in an asset yielding profit at a period rate equal to i_t, and let W_0 be his wealth at time 0. Suppose he has the opportunity of withdrawing the amount f_0 $(=b_0)$

[5] If cash flows are seen as expected values, one only needs consider twin securities instead of zero-coupon bonds, with i_t being the one-period expected return rate of the twin security.

from the asset and investing it in an economic activity, denoted by f. If the investor's choice is to keep his funds in the asset, his wealth evolves according to the recursive equation

$$W_t(\vec{\imath}) = W_{t-1}(\vec{\imath})(1 + i_t) \tag{15}$$

where $W_t(\vec{\imath}) := W_t(i_1, i_2, \ldots, i_t)$ so that $W_t(\vec{\imath}) = W_0 F(0, t)$. If, instead, he chooses to invest in f, he periodically receives the amount f_t at time t, which he may reinvest in the asset;[6] in this case, the investor's wealth is composed of activity f and the asset, and the investor's wealth amounts to

$$W_t(\vec{b}, \vec{f}, \vec{\imath}) = b_t + \Big(W_{t-1}(\vec{b}, \vec{f}, \vec{\imath}) - b_{t-1}\Big)(1 + i_t) + f_t \tag{16}$$

where we set $W_t(\vec{b}, \vec{f}, \vec{\imath}) := W_t(b_1, \ldots, b_t, f_0, f_1, f_2, \ldots f_t, i_1, \ldots, i_t)$. Solving eq. (16) one finds

$$W_t(\vec{b}, \vec{f}, \vec{\imath}) = b_t + (W_0 - f_0)\, F(0, t) + \sum_{k=1}^{t} f_k F(k, t).$$

This implies that wealth increase, in the latter case, is

$$W_t(\vec{b}, \vec{f}, \vec{\imath}) - W_{t-1}(\vec{b}, \vec{f}, \vec{\imath}) = x_t + i_t \Bigg((W_0 - f_0)\, F(0, t-1) + \sum_{k=1}^{t-1} f_k F(k, t-1) \Bigg),$$

whereas wealth increase in the opposite case (i.e., leaving funds in the asset) is

$$W_t(\vec{\imath}) - W_{t-1}(\vec{\imath}) = i_t W_0 F(0, t-1).$$

Therefore, the excess increase in wealth is given by the difference of the alternative wealth increases:

$$\begin{aligned} \text{excess wealth increase in period } (t-1, t) &= \big(W_t(x_t, \vec{\imath}) - W_{t-1}(x_t, \vec{\imath})\big) - \big(W_t(\vec{\imath}) - W_{t-1}(\vec{\imath})\big) \\ &= x_t - i_t f_0 F(0, t-1) + i_t \sum_{k=1}^{t-1} f_k F(k, t-1). \end{aligned} \tag{17}$$

But

$$i_t f_0 F(0, t-1) - i_t \sum_{k=1}^{t-1} f_k F(k, t-1) = i_t \cdot b^*_{t-1} = i^*_t \cdot b_{t-1},$$

so that eq. (17) becomes

$$\text{excess wealth increase in period } (t-1, t) = x_t - i^*_t \cdot b_{t-1} = x^{\mathcal{L}}_t. \tag{18}$$

It is worth noting that we have found $x^{\mathcal{L}}_t$ by making use of two alternative hypotheses about the evolution of the investor's wealth, namely the two dynamic systems in eq. (15) and eq. (16).

[6] Note that this is just the standard assumption of the NPV rule.

We may ideally part the investor's wealth into two assets in both cases:

$$W_{t-1}(\vec{b},\vec{f},\vec{\iota}) = \overbrace{b_{t-1}}^{\text{economic activity with return rate } a_t} + \overbrace{\big(W_{t-1}(\vec{b},\vec{f},\vec{\iota}) - b_{t-1})\big)}^{\text{asset with return rate } i_t} \tag{19}$$

$$W_{t-1}(\vec{\iota}) = \overbrace{\big(W_{t-1}(\vec{\iota}) - W_{t-1}(\vec{b},\vec{f},\vec{\iota}) + b_{t-1}\big)}^{\text{asset with return rate } i_t} + \overbrace{\big(W_{t-1}(\vec{b},\vec{f},\vec{\iota}) - b_{t-1}\big)}^{\text{asset with return rate } i_t}. \tag{20}$$

The differential return between the two alternatives is not dependent on the second addend, which is shared by both alternatives; it may therefore be dismissed and, applying the corresponding rates of return to the first addends, one finds

$$\text{excess wealth increase in period } (t-1,t) = a_t \cdot b_{t-1} - i_t \cdot (W_{t-1}(\vec{\iota}) - W_{t-1}(\vec{b},\vec{f},\vec{\iota}) + b_{t-1}).$$

It is easy to see that

$$W_{t-1}(\vec{\iota}) - W_{t-1}(\vec{b},\vec{f},\vec{\iota}) = b_{t-1} - b^*_{t-1}, \tag{21}$$

so one finds back

$$\text{excess wealth increase in period } (t-1,t) = x_t b_{t-1} - i_t \cdot b^*_{t-1} = x_t b_{t-1} - i^*_t \cdot b_{t-1} = x^{\mathcal{L}}_t.$$

3.3 The axiomatic derivation

This section derives both the standard ($\mathcal{S}$) and the lost-capital ($\mathcal{L}$) residual income by a simple axiomatic approach. We begin by giving a most general definition of residual income.

Definition 1. Residual income is income in excess of a capital charge $C_t \in \mathbb{R}$: that is, $\mathrm{RI}_t = x_t - C_t$.

Let RI_t denote residual income in the period from $t-1$ to t. To prevent the above definition to be excessively lax and thus unhelpful, a first natural requirement is that RI_t be linked to the notion of NPV. As a most general property, we require that some discounting process of all residual incomes should lead to the NPV.
Property 1. (npv-consistency) There exists a vector $\vec{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_n) \in \mathbb{N}^n$ such that

$$\sum_{t=1}^{n} \frac{\mathrm{RI}_t}{(1+i_1)(1+i_2)\cdots(1+i_{\sigma_t})} = \mathrm{NPV}. \tag{22}$$

Now, referring to section 3.2 above, it is worth noting that the investor's wealth increase generated in the span $[0,t]$ is given by $[W_t(\vec{b},\vec{f},\vec{\iota}) - W_0]$ if investors undertake firm f, and by $[W_t(\vec{\iota}) - W_0]$ if they invest funds at the opportunity cost of capital i_t. The corresponding excess wealth increase generated in the span $[0,t]$ is then

$$I_{0,t} = \big[W_t(\vec{b},\vec{f},\vec{\iota}) - W_0\big] - \big[W_t(\vec{\iota}) - W_0\big].$$

Thus, a second, rather natural, condition is that the sum of all past t residual incomes should equal the investor's excess wealth increase $I_{0.t}$. In formal terms, additive coherence is required:

Axiom 1. (Additive coherence) The sum of the first t residual incomes is equal to excess wealth increase generated in the span $[0,t]$:

$$\sum_{j=1}^{t} \mathrm{RI}_j = I_{0,t} \qquad \text{for all} \quad t = 1,2,\dots,n. \tag{23}$$

Proposition 2. *Definition 1 and Axiom 1 imply that the capital charge is $C_t = i_t^* \cdot b_{t-1}$. The corresponding residual income is npv-consistent, with $\vec{\sigma} = (n,n,\dots,n)$.*

Proof. Definition 1 is formally represented as $\mathrm{RI}_t = x_t - C_t$, and Axiom 1 implies $\mathrm{RI}_t = I_{0,t} - I_{0,t-1}$. Thus, $x_t - C_t = I_{0,t} - I_{0,t-1}$. But $I_{0,t} - I_{0,t-1} = b_t - b_t^* - b_{t-1} + b_{t-1}^*$. Reminding that $x_t - f_t = b_t - b_{t-1}$ and $i_t \cdot b_{t-1}^* - f_t = b_t^* - b_{t-1}^*$, one gets to $C_t = i_t \cdot b_{t-1}^* = i_t^* \cdot b_{t-1}$. Property 1 is fulfilled by picking $\vec{\sigma} = (n,n,,\dots,n)$, given that

$$\sum_{t=1}^{n} \frac{\mathrm{RI}_t}{(1+i_1)\cdots(1+i_n)} = \frac{\sum_{t=1}^{n}(x_t - i_t^* b_{t-1})}{(1+i_1)\cdots(1+i_n)} = \frac{\mathrm{NPV}\cdot F(0,n)}{(1+i_1)\cdots(1+i_n)} = \mathrm{NPV}$$

(see equation (14)). □

Proposition 2 shows that, given the general framework of Definition 1, the $\mathcal{L}$ residual income is generated if additive coherence is required. Note that Axiom 1 requires residual income to be aggregated in a value sense. If, instead, aggregation is required in a cash-flow sense, the $\mathcal{S}$ paradigm is generated, as is now shown.

Axiom 1'. (Adjusted additive coherence) The capitalised sum of the first t residual incomes is equal to excess wealth increase generated in the first t periods:

$$\sum_{j=1}^{t} \mathrm{RI}_j \cdot F(j,t) = I_{0,t} \qquad \text{for all} \quad t = 1,2,\dots,n. \tag{24}$$

Proposition 3. *Definition 1 and Axiom 1' imply that the capital charge is $C_t = i_t b_{t-1}$. The corresponding residual income is npv-consistent, with $\vec{\sigma} = (1,2,\dots,n)$.*

Proof. Definition 1 implies $\mathrm{RI}_t = x_t - C_t$ and Axiom 1' implies $\mathrm{RI}_t = I_{0,t} - (1+i_t)I_{0,t-1}$. Thus, $x_t - C_t = I_{0,t} - (1+i_t)I_{0,t-1}$. Using the equalities $I_{0,t} - I_{0,t-1} = b_t - b_t^* - b_{t-1} + b_{t-1}^*$ and $b_t^* = b_{t-1}^*(1+i_t) - f_t$ one gets to $C_t = i_t b_{t-1}$; npv-consistency derives from clean surplus by choosing $\vec{\sigma} = (1,2,\dots,n)$. □

The $\mathcal{S}$ residual income and the $\mathcal{L}$ residual income are then particular cases of a general residual-income framework individuated by Definition 1 and Property 1 (see Table 3).

3.4 The accounting derivation

In an important work on residual income, Egginton (1995) investigates seven different ways of calculating a depreciation charge: annuity depreciation, IRR depreciation, equivalent replacement cost depreciation, depreciation of maintainable RI, lease charge, straight line depreciation, and Adjusted RI. For each depreciation schedule, the author computes the corresponding residual income, such that $x_t^{\mathcal{S}} = f_t - \mathrm{Dep}_t(b_{t-1}, b_t) - i_t b_{t-1}$, where $\text{Dep}_t(b_{t-1}, b_t) := b_{t-1} - b_t$. The Adjusted RI, which is actually identical to Anthony's (1975) notion of profit,[7] has the particular feature that $\text{Dep}_t(b_{t-1}, b_t) = b_{t-1} - b_t = f_t - i_t b_{t-1}$ (see Egginton, 1995, eq. (9) at p. 210). But this is just the recurrence equation for the lost capital (see eq. (11) above). In other words, Egginton implicitly chooses $b_t = b_t^* = b_0 F(0,t) - \sum_{j=1}^{t-1} f_j F(j, t-1)$, so that $\text{Dep}_t(b_{t-1}, b_t) = \text{Dep}_t(b_{t-1}^*, b_t^*)$. This means that the Adjusted RI is computed as $x_t^{\mathcal{S}} = f_t - \text{Dep}_t(b_{t-1}^*, b_t^*) - i_t b_{t-1}^*$. Note that the capital charge of the Adjusted RI is just the capital charge of the lost-capital theory $C_t = i_t b_{t-1}^*$. Now, if we subtract *any* depreciation charge from the depreciation charge of the Adjusted RI we obtain the $\mathcal{L}$ residual-income framework:

$$\text{Dep}_t(b_{t-1}^*, b_t^*) - \text{Dep}_t(b_{t-1}, b_t) = (b_{t-1}^* - b_t^*) - (b_{t-1} - b_t) = (f_t - i_t b_{t-1}^*) - (f_t - a_t b_{t-1}) = x_t - i_t b_{t-1}^* = x_t^{\mathcal{L}}.$$

The accounting meaning of the $\mathcal{L}$ theory in terms of depreciation is now enlightening. The depreciation for Adjusted RI serves as a benchmark to reflect the market-determined decline in the asset's value. If the asset's decline in value determined by the market is greater than the decline in value determined by the accounting policy, then performance is positive.

It is worth noting that the Adjusted RI is the only RI metric that the two theories share. Indeed, $x_t^{\mathcal{L}} = x_t^{\mathcal{S}}$ for all $t = 1, 2, \ldots, n$ if and only if the two capital charges coincide: $i_t b_{t-1}^* = i_t b_{t-1}$ for all $t = 1, 2, \ldots, n$. This implies $b_{t-1} = b_{t-1}^*$ for all $t = 1, 2, \ldots, n$, which means that the residual income is just the Adjusted RI. Therefore, the Adjusted RI is, at the same time, a standard RI and a lost-capital RI. Therefore, the depreciation charge of Egginton's Adjusted RI plays a prominent role in the $\mathcal{L}$ theory. We finally highlight the fact that the capital charge $i_t^* b_{t-1}$ of the $\mathcal{L}$ theory is equal to the sum of the project's cash flow at time t and the depreciation of the Adjusted RI: $i_t^* b_{t-1} = f_t + \text{Dep}_t(b_{t-1}^*, b_t^*)$.

4 Implications for valuation

Residual income has been used for firm and project valuation long since: Carsberg (1966) testifies of discounting procedures involving excess profits rather than cash flows: among others, the author emphasizes Leake's (1921) contribution to valuation of Goodwill, obtained by discounting the surplus of profit over a normal return on capital. In later years, Preinreich (1936, 1938) hints at the capital value obtained as the sum of book values plus the discounted excess profits. The formal link between DCF valuation and residual income is made more explicit by Lücke (1955), Edey (1957) and Edwards and Bell (1961). Bodenhorn (1964) acknowledges that the sum of discounted residual incomes (which he calls "pure earnings") is equal to the NPV *regardless* of the depreciation pattern. In recent years, Peasnell (1981, 1982), Peccati (1987, 1989), Ohlson (1989, 1995), Gallo and Peccati (1993) adopt a more formal treatment.

As seen, the $\mathcal{L}$ residual income is npv-consistent as required by Property 1, but it is worth underlining

[7] See also Tomkins, 1973.

that such a consistency is independent of the asset base. Using $x_t = f_t + (b_t - b_{t-1})$, one may write

$$\text{NPV} = F(0,n)^{-1}\sum_{t=1}^{n}[x_t - i_t b^*_{t-1}] = F(0,n)^{-1}[\sum_{t=1}^{n} f_t - f_0 - \sum_{t=1}^{n} i_t b^*_{t-1}].$$

Therefore, the discounted sum of the $\mathcal{L}$ residual incomes is a constant function with respect to book values:

$$\frac{\partial}{\partial b_1}\text{NPV} = \frac{\partial}{\partial b_2}\text{NPV} = \ldots = \frac{\partial}{\partial b_n}\text{NPV} = 0$$

for all $b_t \in \mathbb{R}$.

The independence from book values makes $\mathcal{L}$ residual income an appropriate valuation tool; however, the two theories lead to the firm's market value with opposite procedures: theory $\mathcal{S}$ requires a *discount-then-sum* mechanism, while theory $\mathcal{L}$ requires a *sum-then-discount* approach. That is,

$$\frac{x_1^{\mathcal{S}}}{F(0,1)} + \frac{x_2^{\mathcal{S}}}{F(0,2)} + \ldots + \frac{x_n^{\mathcal{S}}}{F(0,n)} = \text{NPV}$$

whereas

$$\text{NPV} = (x_1^{\mathcal{L}} + x_2^{\mathcal{L}} + \ldots + x_n^{\mathcal{L}})\frac{1}{F(0,n)}. \tag{25}$$

Thus, in the $\mathcal{S}$ theories RIs are computationally treated as *cash flows*, whereas in the $\mathcal{L}$ theory RIs are treated as *values*: they are summed as values referred to time n, and their aggregation determines the accumulated NPV; once this value is discounted back to time 0, the net present value is obtained. The $\mathcal{L}$ paradigm then provides a powerful result of income aggregation: the grand total residual income (i.e., the grand total income minus the grand total capital charge) exactly matches the accumulated NPV. This reflects what Penman calls the "aggregation property of accounting" (Penman, 1992, p. 237). Implications for valuation are summarised in the following

Proposition 4. *Consider any sequence $\vec{k} = (k_1, k_2 \ldots, k_n) \in \mathbb{R}^n$ such that*

$$\sum_{t=1}^{n} k_t = \sum_{t=1}^{n} x_t^{\mathcal{L}}. \tag{26}$$

Then, the market value of the firm is given by

$$v_0 = b_0 + (k_1 + k_2 + \ldots + k_n)\frac{1}{F(0,n)}. \tag{27}$$

Proof. Straightforward from the assumption, eq. (26) and the equality v_0=NPV+b_0. □

This result implies that the $\mathcal{L}$ paradigm tends to offset errors in valuation: one does not have to worry about forecasting each and every residual income and imputing it to the correct period, because only the grand total counts.

In particular, we have the following relevant case:

Corollary 1. *Let* $\vec{k} = (k, k, \ldots, k)$ *be a sequence of residual incomes fulfilling condition* (26)*. Then,*

$$v_0 = b_0 + nk \cdot [F(0,n)]^{-1}. \tag{28}$$

It is worth noting that the simple arithmetic mean of residual incomes $\overline{x}^{\mathcal{L}} = \sum_{t=1}^{n} x_t^{\mathcal{L}}/n$ satisfies the assumptions of Corollary 1, which implies

$$v_0 = b_0 + n\overline{x}^{\mathcal{L}} \cdot [F(0,n)]^{-1}. \tag{29}$$

Therefore, we have proved the following important

Proposition 5. *The value of a firm is a linear affine function of the simple arithmetic mean of* $\mathcal{L}$ *residual incomes.*

A practical consequence is that NPV may be calculated with no recourse to cash flows: one only needs forecast the average RI, or, equivalently, the average income and the average capital charge. Given the considerable amount of historic accounting data available to the investors, it may be easier, in some cases, to determine the average RI than each and every cash flow. Graham, Dodd, and Cottle's (1962) words fit particularly well in this context:

> The most important single factor determining a stock's value is now held to be the *indicated average future earning power*, i.e., the estimated average earnings for a future span of years. Intrinsic value would then be found by first forecasting this earning power and then multiplying that prediction by an appropriate 'capitalization factor'" Graham, Dodd, and Cottle (1962, p. 28).

Equation (29) puts the above qualitative statement on a solid quantitative footing: once adjusted the average earnings with the capital charge, they are multiplied by the proper capitalization factor, which is, $n\,[F(0,n)]^{-1}$. Hence, the $\mathcal{L}$ theory seems to be a reliable tool for making project and firm evaluation.

5 Implications for capital budgeting

The shortcomings of using ARRs in place of economic rates of return has been the focus of several decades of academic research (e.g. Harcourt, 1965; Solomon, 1966; Kay, 1976; Peasnell, 1982; Brief and Lawson, 1992). Contrary to the IRR and the NPV, accounting measures are usually considered of little help for making capital budgeting decisions, because "it is widely presumed in the accounting and economic literatures that, for the most part in practice, ARRs are artifacts without economic significance" (Peasnell, 1982, p. 368) and the idea of comparing accounting rates of return with the cost of capital is "clearly like comparing apples with oranges" (Rappaport, 1986, p. 31). Likewise, neither income maximization nor residual income maximization is equivalent to NPV maximization (but see Anctil, 1996; Anctil, Jordan and Mukherji, 1998), which implies that accounting measures may not be used for project selection.

Opposing this view, this section shows that the $\mathcal{L}$ theory enables one to give a significant interpretation of the (weighted) average of accounting rates and that maximization of a simple average residual income is equivalent to maximization of NPV. As we have seen, the NPV is obtained as

$$\mathrm{NPV} = F(0,n)^{-1}\sum_{t=1}^{n} x_t^{\mathcal{L}} = F(0,n)^{-1}\sum_{t=1}^{n}(x_t - i_t^* \cdot b_{t-1}) = F(0,n)^{-1}\sum_{t=1}^{n}(a_t - i_t^*)b_{t-1}.$$

Now, applying the notion of Chisini mean (see Chisini, 1929; Graziani and Veronese, 2009), we search for a constant rate $\overline{a}$ such that

$$F(0,n)^{-1}\sum_{t=1}^{n}(a_t - i_t^*)b_{t-1} = F(0,n)^{-1}\sum_{t=1}^{n}(\overline{a} - i_t^*)b_{t-1}. \tag{30}$$

One finds

$$\overline{a} = \frac{\sum_{t=1}^{n} a_t b_{t-1}}{\sum_{t=1}^{n} b_{t-1}}. \tag{31}$$

Unlike the IRR, its existence and uniqueness is guaranteed owing to the linearity of the equations, it is not circular and does not depend on costs of capital.

Now we prove that this Chisini mean may replace the IRR for accept/reject decisions:

Proposition 6. *Project f is worth undertaking if and only if the average accounting rate is greater than the average comprehensive cost of capital:*

$$\overline{a} > \overline{\imath}^* \tag{32}$$

where $\overline{\imath}^* := \frac{\sum_{t=1}^{n} i_t^* b_{t-1}}{\sum_{t=1}^{n} b_{t-1}}$.

Proof. Just consider that $\overline{a} > \overline{\imath}^*$ if and only if $\sum_{t=1}^{n} b_{t-1}(\overline{a} - \overline{\imath}^*) = \sum_{t=1}^{n} b_{t-1}(a_t - i_t^*) > 0$, which is equivalent to NPV> 0.[8] □

Note that $\overline{a}$ essentially represents the average income per unit of capital invested and $(\overline{a} - \overline{\imath}^*)$ essentially measures the average RI per unit of capital invested. Eq. (32) states that a project is profitable if such a residual income is positive. Let $g := \overline{a} - \overline{\imath}^*$. We have $g = g(b_1, b_2, \ldots b_{N-1})$. It is easy to see that $\frac{\partial}{\partial b_t} g(b_1, b_2, \ldots b_{n-1})$ is not identically zero for all $t = 1, 2, \ldots, n$ and for all $b_t \in \mathbb{R}$. This means that the per-unit average RI changes if book value changes. However, for all $t = 1, 2, \ldots, n$ and for all $b_t \in \mathbb{R}$, either $g(b_1, b_2, \ldots b_{n-1}) > 0$ or $g(b_1, b_2, \ldots b_{n-1}) < 0$. This stems from the fact that $g(b_1, b_2, \ldots b_{N-1}) = \frac{\sum_{t=1}^{n}(x_t - i_t b_{t-1}^*)}{\sum_{t=1}^{n} b_{t-1}} = \frac{-f_0 + \sum_{t=1}^{n}(f_t - i_t b_{t-1}^*)}{\sum_{t=1}^{n} b_{t-1}}$. The denominator is positive by assumption, so the sign of g depends on the numerator, which is a constant. Hence, the ARR rule above stated is robust under changes in the depreciation pattern: it holds for *any* book value depreciation.

Evidently, this rule is more reliable that the IRR rule, given that the latter is not necessarily compatible with the NPV rule.[9] The shortcomings of the IRR rule for ranking projects are also well-known. The IRR

[8] If $\sum_{t=1}^{n} b_{t-1} < 0$, the ARR rule still holds with the sign reversed.

[9] The IRR rule may be incompatible with the NPV rule even if the IRR is unique: this occurs whenever the NPV graph lies below the horizontal axis for all rates except in one point, where the graph is tangent to the horizontal axis.

rule suggests to undertake the project with the highest IRR or, equivalently, the project with the highest margin $r-i$. By contrast, the ARR rule suggests to undertake the project with the highest margin $\overline{a}-\overline{\iota}^*$. This is equivalent to NPV maximization. To show it, we only note that the average ARR is invariant under changes in book value if the average book value $\overline{b}$ remains unchanged. Given that one may always choose depreciation patterns such that the average book values of the projects coincide (even if they have different life), we have the following

Proposition 7. *Consider a set of projects j whose length is n_j, $j=2,\dots,K$. If book value depreciations are chosen so that the sum of book values coincide for all projects, maximization of the margin $\overline{a}-\overline{\iota}^*$ is equivalent to NPV maximization.*

Proof. The equality $\sum_{t=1}^{n_j} b_{j,t-1} = \sum_{t=1}^{n_k} b_{k,t-1}$ for $j,k=1,2,\dots,K$ implies that the problem $\max_{1\leq j\leq K}$ NPV$=\max_{1\leq j\leq K}\sum_{t=1}^{n_j} b_{j,t-1}(\overline{a}_j-\overline{\iota}_j^*)$ is equivalent to $\max_{1\leq j\leq K}(\overline{a}_j-\overline{\iota}_j^*)$. □

Practically, one may for example consider the outlay f_0^j of project j, and consider the following depreciation schedules: $b_{k,1}=b_{j,0}-b_{k,0}$, $b_{k,t}=0$ for $t>1$ and for all $k=1,2,\dots,K$. This implies that the total book value for all projects is $\sum_{t=1}^{n_k} b_{k,t}=b_{j,0}$ for all $k=1,2,\dots,K$. Then, the corresponding margins are computed and the projects are correctly ranked.[10]

Not only is the sign of $g(\cdot)$ invariant under changes in book values; it is easy to show that the average residual income $\overline{x}^{\mathcal{L}}$ is independent of book values, because we may rewrite it as $\overline{x}^{\mathcal{L}}=(-f_0+\sum_{t=1}^{n}(f_t-i_t b_{t-1}^*))/n$, where book values b_t do not appear. This striking result means that the simple arithmetic mean of RIs may replace the NPV for capital budgeting valuation and decision. In particular, considering that NPV $=(nF(0,n)^{-1})\cdot\overline{x}^{\mathcal{L}}$ we have, for K projects of equal life, $\max_{1\leq j\leq K}\text{NPV}_j=\max_{1\leq j\leq K}\overline{x}_j^{\mathcal{L}}$, where the subscript j refers to project j=$1,2,\dots,K$. This means that the (average) RI ranking is equivalent to the NPV ranking.

The $\mathcal{L}$ arithmetic mean of RI is then a perfect substitute of the NPV when decision makers deal with projects of equal life, because it correctly signals value creation. Evidently, this result does not hold in the $\mathcal{S}$ theory. As a simple counterexample, consider n=5, $\vec{f}=(260,460,220,80,290)$, f_0=1000, $i_t=0.1$ for all t. We have NPV=16.53 >0 and the sequence of residual incomes is $(60,170,-150,-60,-40)$ in the $\mathcal{S}$ paradigm and $(60,176,-126,-49,-34)$ in the $\mathcal{L}$ paradigm. The simple arithmetic means are $\overline{x}^{\mathcal{S}}=-4<0$ and $\overline{x}^{\mathcal{L}}=5.4>0$ respectively. The $\mathcal{S}$ paradigm erroneously signals value destruction.

More generally, consider project j, $j=1,2,\dots,K$, and let n_j be its length. Denoting with $Z:=\max_{1\leq j\leq K} n_j$ the maximum length, we may scale the project's length by considering the ratio n_j/Z, and construct the *time-scaled* residual income $\alpha_j\overline{x}_j^{\mathcal{L}}$, where $\alpha_j=(n_j/Z)F(n_j,Z)$. In this way, all projects may be

[10] It is easily seen that the classical rule proposed by Teichroew, Robichek and Montalbano (1965) for accept/reject decision is only is only a particular case of the ARR rule where the book value depreciation is chosen such that $b_t=b_{t-1}(1+a_t)-f_t$, where $a_t=i$ if $b_{t-1}<0$ or $a_t=k$, $k>-1$, if $b_{t-1}>0$. But the rule proposed by these authors is not compatible with the NPV for project ranking.

considered of the same length (=Z), and maximization of the time-scaled RI is equivalent of maximization of NPV, given that $\text{NPV}_1 > \text{NPV}_2$ if and only if $\alpha_1 \overline{x}_1^{\mathcal{L}} > \alpha_2 \overline{x}_2^{\mathcal{L}}$. The ranking of projects may thus be grounded on the average RI or on its time-scaled version. We have then the following

Proposition 8. *Maximization of average RI (or time-scaled RI) is equivalent to NPV maximization.*

The above proposition says that maximization of the average residual income is equivalent to maximization of NPV even for unequal-life projects, provided the average RIs is adjusted to take account of the different lifespan.

Remark 2. The time-scaled RI is a constant residual income which is scaled in order to account for the project life. Viceversa, the average RI may be defined as the accumulated NPV per unit of length:

$$\overline{x}^{\mathcal{L}} = \frac{\text{NPV}}{n} F(0, n).$$

Because $\overline{x}^{\mathcal{L}} = \sum_{t=1}^{n} b_{t-1}(\overline{a} - \overline{\imath})/n$, the relation between NPV and accounting rates is significant:

$$\overline{a} = \overline{\imath} + \lambda \tag{33}$$

with $\lambda := \frac{\text{NPV} \cdot F(0,n)}{\sum_{t=1}^{n} b_{t-1}}$. Thus, the accounting rate is the sum of the average comprehensive cost of capital and the ratio of accumulated NPV to the grand total capital invested. Hence, the ARR is decomposed into two parts: the first one represents interest foregone, the second one represents the accumulated NPV per unit of total capital invested. And the latter is just the average residual income per unit of capital invested: $\lambda = \overline{x}_t^{\mathcal{L}} / \overline{b}$ where $\overline{b} := \sum_{t=1}^{n} b_{t-1}/n$ is the average capital invested in a period.

Remark 3. An interesting relation between income and asset prices may be provided. Note that the value of any t-period asset may be written as a function of incomes:

$$v_0^t(x_1, x_2, \ldots, x_t) = b_0 + F(0, t)^{-1} \Big(\sum_{j=1}^{t} x_j - \sum_{j=1}^{t} i_j^* b_{j-1}\Big).$$

Hence, $\partial v_0^t / \partial x_k = F(0, t)^{-1}$ for all $k = 1, 2, \ldots, t$ and all $(x_1, x_2, \ldots, x_t) \in \mathbb{R}^t$. This means that the impact of income on an asset is market-determined and, in particular, the market value of an additional euro available at time t is equal to the increase in value due to a unit income increase occurred in *any* period. In other words, to increase an investor's wealth by a euro at time t is financially equivalent to increasing income by a euro in the interval $[0, t]$. We stress that the equivalence holds no matter when the income increase occurs: it may be distributed in various ways over the span $[0, t]$. This enables one to write the value of any n-year asset as a linear combination:

$$v_0^n = f_1 \frac{\partial v_0^1}{\partial x_{k_1}} + f_2 \frac{\partial v_0^2}{\partial x_{k_2}} + \ldots f_n \frac{\partial v_0^n}{\partial x_{k_n}} \tag{34}$$

where k_j, is an arbitrary natural number between 1 and j, $j = 1, 2, \ldots, n$. In other words, the derivatives of values with respect to income represent a system of unit prices: in case i_t is the risk-free rate, they describe the yield term structure.

6 Implications for performance measurement

6.1 A dual relation

Since Solomons's (1965) classical book, the notion of residual income has often been advocated as a measure of performance and as a tool for incentive compensation. The literature has grown dramatically since. Among many others, a special mention should be devoted to Rogerson's (1997) contribution regarding incentive compensation: the author copes with the situation where the principal delegates decisions on investment level to the agent who is better informed about the investment opportunities. The agent is assumed to be impatient and aims at maximizing a utility function which depends on RI via a reward contract that linearly links residual income to wages. Assuming positive operating cash flows governed by a specified stochastic path (of which only the distributional parameters are known to the principal), the author shows that there is a unique allocation rule (and thus a unique depreciation schedule), called the "Relative Marginal Benefit" rule, which is optimal in the sense that it maximizes both the principal's expected NPV and the manager's utility function. Reichelstein's (1997) paper shows that residual income in combination with Relative Marginal Benefit allocation rule is the unique linear performance metric that achieves strong goal congruence in this context (see also Bromwich and Walker, 1998). Under the same information structure of Rogerson (1997) and Reichelstein (1997), Mohnen (2003) and Mohnen and Bareket (2007) show that the Relative Marginal Benefit allocation rule is not optimal if exogenous capital constraints (or mutually exclusive projects) are introduced in the decision problem. Other significant contributions in this vein are Mohnen (2003), Mohnen and Bareket (2007), Pfeiffer and Velthuis (2009), Baldenius, Dutta, and Reichelstein (2006). Baldenius and Reichelstein (2005) examine efficient inventory management from an incentive and control perspective; Schultze and Weiler (2008) devise a bonus bank system where an internal market is created; the quitting manager may sell the bonus bank to the entering manager. The authors show that if the purchase price for the bonus bank is computed with the Nash bargaining solution, the quitting manager will choose the optimal investment level and will have no incentive to overstate value creation in his reporting. Grinyer and Walker (1990) and Stark (2000) take a dynamic perspective on investment decision-making: they focus on real-option frameworks where there is some flexibility for subsequent decisions; the authors find that a residual income-type performance measure can be designed which supports optimal investment and disinvestment decisions. Friedl (2007) analyses residual income as a performance measure for investments in flexible manufacturing systems showing the occurrence of underinvestment if residual income is used in a standard way, and providing some adjustment to achieve goal congruence. He also shows that, under the assumption of an existing waiting option, investment will be undertaken too early, unless proper adjustment is made to guarantee goal congruence (see also Antle, Bogetoft and Stark, 2001, 2007; Arya and Glover, 2001; Friedl, 2005). In applied corporate finance, the quest for an appropriate performance measure has triggered the popularization of many metrics, especially in the value-based management literature (see Stewart, 1991;

Madden, 1999; Martin and Petty, 2000; Young and O'Byrne, 2001; Fernández, 2002; Martin, Petty and Rich, 2003; Fabozzi and Grant, 2000).

This section aims at illustrating the formal relations between the $\mathcal{S}$ residual income and the $\mathcal{L}$ residual income. This analysis may contribute to a better understanding of the way the $\mathcal{L}$ residual income works and hopefully arouse interest among management accounting scholars for possible use in incentive compensation as well as ex-post (and ex-ante) performance measurement.

We then ask: if performance is measured by the $\mathcal{L}$ paradigm instead of the $\mathcal{S}$ paradigm, what is the discrepancy? Will the measure be greater or smaller? Will the two paradigms signal positive and negative performance in the same periods? The following proposition provides some hints.

Proposition 9. *The spread between $\mathcal{L}$ residual income and S residual income is given by the compounded value of past standard residual income*

$$x_t^{\mathcal{L}} - x_t^{\mathcal{S}} = i_t \sum_{k=1}^{t-1} x_k^{\mathcal{S}} F(k, t-1) \qquad t \geq 1 \tag{35}$$

where we set $\sum_{k=1}^{0} f_k F(k, 0) := 0$.

Proof. Since $b_t^* = b_0 F(0, t-1) - \sum_{k=1}^{t-1} f_k F(k, t-1)$ and $f_k = b_{k-1}(1 + a_k) - b_k$, we have

$$b_{t-1}^* = b_0 F(0, t-1) - \sum_{k=1}^{t-1} \left[b_{k-1}(1 + a_k) - b_k\right] F(k, t-1).$$

Upon rearranging terms, we find

$$b_{t-1} - b_{t-1}^* = \sum_{k=1}^{t-1} b_{k-1}(a_k - i_k) F(k, t-1) \tag{36}$$

$$= \sum_{k=1}^{t-1} x_k^{\mathcal{S}} F(k, t-1). \tag{37}$$

Consequently, $i_t \sum_{k=1}^{t-1} x_k^{\mathcal{S}} F(k, t-1) = i_t(b_{t-1} - b_{t-1}^*)$, so that the thesis is proved, given that $x_t^{\mathcal{L}} - x_t^{\mathcal{S}} = i_t(b_{t-1} - b_{t-1}^*)$. $\square$

The term $i_t(b_{t-1} - b_{t-1}^*)$ reveals the formal nature of the conceptual difference between the two paradigms. It represents the interest on the excess capital invested $(b_{t-1} - b_{t-1}^*)$: as seen, the $\mathcal{L}$ paradigm is concerned not only with the interest rate that could have been exploited by the investor, but also with the capital to which that interest rate could be applied. Thus, while $a_t > i_t$ signals positive performance in the $\mathcal{S}$ paradigm, because it implies $x_t^{\mathcal{S}} > 0$ (as long as book value is positive), the capital lost by the investor may be greater than the actual capital invested (i.e $b_{t-1}^* > b_{t-1}$), so that the $\mathcal{L}$ excess profit may signal a smaller performance with respect to the $\mathcal{S}$ paradigm's: the interest that could have been yielded by the surplus of capital may be so great as to offset the positive effect of the ARR: whenever $0 < x_t^{\mathcal{S}} < i_t[b_{t-1}^* - b_{t-1}]$, one gets $x_t^{\mathcal{L}} < 0 < x_t^{\mathcal{S}}$, which informs that a negative performance is measured by the $\mathcal{L}$ paradigm. The additional component may symmetrically act as a sort of insurance bonus: if $a_t < i_t$, performance may still be regarded positive in the $\mathcal{L}$ paradigm if $b^*_{t-1} < b_{t-1}$, which means that past performance has been so positive that the actual capital invested is greater than the capital lost by investors, and that the fact that the accounting rate is smaller than the cost of capital is more than compensated by the greater basis to which the accounting rates is applied: $a_t b_{t-1} > i_t b^*_{t-1}$.

To signal positive performance, the ARR must be greater than the comprehensive cost of capital i^*_t by an additional term: we have

$$i^*_t = i_t + i_t \frac{b^*_{t-1} - b_{t-1}}{b_{t-1}}$$

The second addend in the right-hand side is the product of the cost of capital and the relative increase (decrease) in capital due to acceptance of the project. For example, suppose i_t=0.1, b_{t-1}=80, b^*_{t-1}=100; then, if project had been rejected, the capital invested would be higher than the the actual capital employed; in particular, it would be higher by a 25%=(100−80)/80. This means that investors could have invested a 25% more capital than they actually invest, and they could have earned a 10% on that 25%, so that an additional 2.5% would accrue to them. Therefore, for a positive performance to occur, the ARR must be greater than 10%; in particular, the threshold level is i^*=12.5%=10%+2.5%. In general, the required cutoff rate i^*_t may be greater, equal or smaller than the cost of capital i_t. The latter case occurs whenever the additional-interest component is negative, which means that the actual capital b_{t-1} exceeds the lost capital b^*_{t-1} and therefore the investor forego (not a return but) a cost. To summarise: the $\mathcal{S}$ residual income tells us that, if the accounting rate a_t is greater than the cost of capital i_t, then a positive performance occurs; however, if a_t is greater than i_t but, at the same time, the basis to which a_t is applied is different (smaller or greater), then the final effect cannot be *a priori* established: return rate and capital are both fundamental elements to take account of in the capital charge.

The following proposition shows that either paradigm can be generated by the other.

Proposition 10. *Theory $\mathcal{S}$ and theory $\mathcal{L}$ are mutually generative. In particular,*

$$x^{\mathcal{L}}_t = x^{\mathcal{S}}_t + i_t \sum_{k=1}^{t-1} x^{\mathcal{S}}_k F(k, t-1) \qquad t \geq 1 \tag{38}$$

and

$$x^{\mathcal{S}}_t = x^{\mathcal{L}}_t - i_t \sum_{k=1}^{t-1} x^{\mathcal{L}}_k \qquad t \geq 1 \tag{39}$$

where we set $\sum_{k=1}^{0} x^{\mathcal{S}}_k F(k, t-1) = \sum_{k=1}^{0} x^{\mathcal{L}}_k := 0$.

Proof. Equation (38) is just eq. (35). To prove eq. (39) one just has to prove that

$$\sum_{k=1}^{t-1} x^{\mathcal{L}}_k = \sum_{k=1}^{t-1} x^{\mathcal{S}}_k F(k, t-1).$$

Noting that $x_1^{\mathcal{S}} = x_1^{\mathcal{L}}$, the latter equality is derived by induction. $\square$

Corollary 2. *The surplus of capital $b_{t-1} - b_{t-1}^*$ invested in the t-period is either a function of past $\mathcal{S}$ residual incomes and a function of past $\mathcal{L}$ residual incomes:*

$$b_{t-1} - b_{t-1}^* = \sum_{k=1}^{t-1} x_k^{\mathcal{S}} F(k, t-1) \tag{40}$$

$$b_{t-1} - b_{t-1}^* = \sum_{k=1}^{t-1} x_k^{\mathcal{L}} \tag{41}$$

Proof. Use $\sum_{k=1}^{t-1} x_k^{\mathcal{L}} = \sum_{k=1}^{t-1} x_k^{\mathcal{S}} F(k, t-1)$ and eq. (37). $\square$

Both paradigms may then be interpreted as providing performance measures that depend on the past performance measures of the alternative paradigms; this fact hints at a dual theory of residual income. For example, form the point of view of a standard-looking evaluator the $\mathcal{L}$ theory may be interpreted as an accumulation system of standard residual incomes. Positive (negative) performances will positively (negatively) reverberate in the following periods, so tending to increase (decrease) $x_t^{\mathcal{L}}$ with respect to $x_t^{\mathcal{S}}$. If performance is good in one year according to the $\mathcal{S}$ theory, next-year $\mathcal{L}$ residual income will be positively affected regardless of whether a_t is greater or smaller than i_t. For example, if it should happen that $a_t < i_t$ in some period, then, although $x_t^{\mathcal{S}} < 0$, the $\mathcal{L}$ residual income benefits from the second addend of eq. (38), which acts as an insurance bonus. If, instead, $a_t > i_t$, the insurance part become an additional return. Evidently, the additional term works well if $b_{t-1}^* < b_{t-1}$. But this just depends on the past performances. If it occurs that $b_{t-1}^* > b_{t-1}$, the additional term is negative, which tends to lower residual income even if $a_t > i_t$. Again, this depends on the past performances. Symmetrically, the $\mathcal{S}$ paradigm is obtained as the current $\mathcal{L}$ residual income minus a charge given by the past $\mathcal{L}$ residual incomes, and positive (negative) lost-capital past performances negatively (positively) reverberate on current $\mathcal{S}$ residual incomes.

Remark 4. In terms of management compensation, the efficacy of the $\mathcal{L}$ paradigm as opposed to the $\mathcal{S}$ paradigm also depends on the type of compensation plan selected. For example there are at least three ways of using a metric: the historical use, according to which the manager's bonus is a share of the residual income:

$$\text{bonus} = \alpha\%\,\text{RI};$$

an $\alpha\beta$ compensation plan, according to which bonus is tied to residual income variation:

$$\text{bonus} = \alpha\%\ \text{RI} + \beta\%\ \Delta\ \text{RI};$$

and the excess residual-income improvement plan, according to which the expected residual-income improvement (EI) plays a major role:

$$\text{bonus} = \text{target bonus} + \beta\% \ (\Delta \ \text{RI} - EI)$$

(see Young and O'Byrne, 2001). For positive-residual-income companies using either the historical plan or an $\alpha\beta$ plan, we can say that the manager's bonuses computed with the lost-capital paradigm are greater than the ones computed in the standard paradigm, because in the former both RI and Δ RI are greater than the corresponding ones in the latter (proof is straightforward using eqs. (38) and (39)). However, things are complicated by the fact that comparisons may be made along two dimensions: the type of metric selected and the paradigm chosen. That is, a metric in a paradigm may be compared with the same metric in the alternative paradigm, or with an alternative metric in the same paradigm, or with an alternative metric in the alternative paradigm. Having two paradigms and a wide set of metrics it may be the case that a metric in one paradigm is more incentive than a different metric in the alternative paradigm.

Remark 5. Compensating managers with the $\mathcal{S}$ residual income boils down to forgetting that choice affects capital. To invest funds at a determined rate of return makes capital change in time. This implies in turn that managers' compensation is not *entirely* tied to the alternative return stemming from the choice of investing at the rate i_t. An example may be of some help. Two firms, A and B, are incorporated with 10000 euros each and managers are compensated on the basis of the standard residual income. Firm A's managers use the amount to purchase a piece of land. The land is sold after three years at a price of 12947 and there is no intermediate cash flow. Suppose the book value is $b_0 = 10000$, b_1=10700, b_2 =11770. Firm B's managers purchase a piece of land in a different place and sell the land after three years at a price of 13310 (with no intermediate cash flow). Assume firm B's book values are $b_0 = 10000$, b_1=11000, b_2 =12100. Hence, incomes are 700, 1070, 1177 in firm A and 1000, 1100, 1210 in firm B. Assuming a cost of capital equal to 10% in all periods, firm B's residual incomes are zero in each period, because the firm just replicates a financial investment with a 10% return; in other words, managers of firm B behave in a value-neutral way. The RIs in firm A are zero in the second and third period, but in the first period RI is equal to $700 - 0.1 \cdot 10000 = -300$. The difference between the two firms lies in the first period performance: firm A's managers employ funds at a 7% (10700/10000$-$1), firm B's managers invests funds at 10% on the same capital (11000/10000$-$1). However, in the second period, while in both firms funds are employed at 10%, firm B's shareholders can benefit from investing a greater capital (11000>10700), which has been created thanks to a better performance in the first period. Firm A's shareholders then lose (i.e., forego) 300 euros capital with respect to the shareholders of firm B, and thus forego a 30 euros return (=0.1$\cdot$ 300) in the second period. This negative performance reverberates in the third period as well: firm A's shareholders lose 330 euros (=300+30) capital with respect to firm B's, and so they forego a 33 return (=0.1$\cdot$330). These figures (-30 and -33) are just the $\mathcal{L}$ residual incomes of firm A in the second and third year respectively. That is, contrary to the $\mathcal{S}$ residual income, the $\mathcal{L}$ theory ties (performance and) reward to the real alternative income that would have been generated in each period if funds were invested at the cost of capital. Shareholders of firm B then better off than shareholders of firm A not only in the first period, but in the second and third period as well. The use of the $\mathcal{L}$ paradigm in compensation plans means that managers are rewarded by taking account not only what the return rate would be, but also what the capital would be if they acted in a value-neutral way.

6.2 Goal-congruence and periodic consistency

If residual income is aligned in sign with the NPV in each period, then it is said to enjoy *goal congruence*; if, in addition, goal congruence is such that the RI ranking of projects provides in each period the same ranking as the NPV, then *robust goal congruence* holds (see Reichelstein, 1997; Mohnen and Bareket, 2007). In order to align managers' behaviors to shareholders' objectives, compensation should be tied to value creation, that is, to the NPV. A mystifying problem in value-based management is just that RI is not, in general, goal congruent. To circumvent the problem, a possible route is to make some adjustments to residual income itself or to devise compensation plans so as to tie residual income to value creation (Ehrbar, 1998; Stewart, 1991; O'Hanlon and Peasnell, 2000; Young and O'Byrne, 2001; Martin, Petty and Rich, 2003). Grinyer (1985, 1987) proposes an index labelled *Earned Economic Income*, which has the *goal congruence* property, given that it is aligned with the Net Present Value. This index is exactly equal to the above-mentioned Rogerson's (1997) metric. However, such a metric is equal in sign to the NPV only if the project's cash flows are all of the same sign (Martin, Petty and Rich, 2003; Peasnell, 1995; Grinyer, 1995).

Converting Fernández's (2002) Created Shareholder Value (CSV) into the corresponding lost-capital metric, one obtains a metric which is robustly goal congruent *irrespective* of the sign of the cash flows. The CSV belongs to the class of standard residual income models. It is computed by picking f_t=ECF$_t$, $b_{E,t}$:=E$_t$ for every $t{\geq}1$, and $i_t{=}k_{e_t}$. In other words, market values are chosen as the equity's book value (except at time 0, when the usual initial condition $b_{E,0}{:=}f_0$ holds). Given that $a_1{=}(\mathrm{E}_1 + f_1 - f_0)/f_0$ (see Fernández, 2002, p. 281), and (owing to the choice of market values as book values) $a_t{=}k_{e_t}$ for $t > 1$, the resulting residual income is

$$\mathrm{CSV}_1 = f_0(a_1 - k_{e_1}) = \mathrm{E}_1 + f_1 - f_0(1 + k_{e_1}) \tag{42}$$

and

$$\mathrm{CSV}_t = \mathrm{E}_{t-1}(k_{e_t} - k_{e_t}) = 0 \qquad t > 1. \tag{43}$$

In order to convert the standard CSV into its $\mathcal{L}$ companion, the capital charge $k_{e_t}b_{E,t-1}$ must be replaced by $k_{e_t}b^*_{E,t-1}$ so that residual income becomes

$$\mathcal{L}\text{-CSV}_1 = f_0(a_1 - k_{e_1}) = \mathrm{E}_1 + f_1 - f_0(1 + k_{e_1}) \tag{44}$$

and

$$\mathcal{L}\text{-CSV}_t = k_{e_t}(\mathrm{E}_{t-1} - b^*_{E,t-1}) \qquad t > 1. \tag{45}$$

It is noteworthy that

$$\mathcal{L}\text{-CSV}_1 = \big(\frac{\mathrm{E}_1 + f_1}{1 + k_{e_1}} - f_0\big)(1 + k_{e_1}) = \mathrm{NPV}(1 + k_{e_1}).$$

As for $t>1$, remind that

$$\mathrm{E}_t=\sum_{j=t+1}^{n}\frac{f_j}{F_{k_e}(t,j)},$$

where $F_{k_e}(t,j):=\prod_{h=t+1}^{j}(1+k_{e_h})$, and

$$b^*_{E,t}=\sum_{j=t+1}^{n}\frac{f_j}{F_{k_e}(t,j)}+b^*_{E,n}\frac{1}{F_{k_e}(t,n)}.$$

Also,

$$b^*_{E,n}=f_0F_{k_e}(0,n)-\sum_{j=1}^{n}f_jF_{k_e}(j,n)=-\mathrm{NPV}\cdot F_{k_e}(0,n).$$

Therefore,

$$\mathrm{E}_t-b^*_{E,t}=-b^*_{E,n}\frac{1}{F_{k_e}(t,n)}=\mathrm{NPV}\cdot F_{k_e}(0,t)$$

whence

$$\mathcal{L}\text{-CSV}_t=k_{e_t}(\mathrm{E}_{t-1}-b^*_{E,t-1})=k_{e_t}\cdot\mathrm{NPV}\cdot F_{k_e}(0,t-1).$$

This robust goal congruence holds, unlike Grinyer's proposal, for any sequence of cash flows, with no restraint on their sign.[11] Note also that the $\mathcal{L}$ companion of CSV measures the increase of Net Present Value period by period, because

$$k_{e_t}\cdot\mathrm{NPV}\cdot F_{k_e}(0,t-1)=\mathrm{NPV}\cdot F_{k_e}(0,t)-\mathrm{NPV}\cdot F_{k_e}(0,t-1).$$

Egginton (1995) invokes a notion of *periodic consistency* for RI to be a legitimate tool for performance appraisal and control. According to the author, a RI metric is said to enjoy periodic consistency if it fulfills two requirements: (A) ex ante RIs should reflect the NPV ranking between different projects, so that if project 1 has a higher NPV than project 2, the ex ante RIs of project 1 exceed those of project 2 in every period (i.e. robust goal congruence must hold); (B) the ex ante RI sequence should be constant or increasing, to prevent manager from adopting less profitable project with good early rewards. The author finds a (standard) RI that fulfills both requirements for projects of equal life. He calls it the *maintainable* RI. It is found by choosing an asset base so that residual income will be constant over years: solving $\sum_{t=1}^{n}N\cdot F(0,t)^{-1}=\sum_{t=1}^{n}f_tF(0,t)^{-1}-f_0$, the author finds $N=\big(\sum_{t=1}^{n}f_tF(0,t)^{-1}-f_0\big)/\sum_{t=1}^{n}F(0,t)^{-1}$ (Egginton, 1995, eq. (17)). Charging depreciation as $\mathrm{Dep}_t(b_{t-1},b_t)=f_t-N-ib_{t-1}$ the book value for each period is computed, and the resulting RI is $x^{\mathcal{S}}_t=f_t-(f_t-N-i_tb_{t-1})-ib_{t-1}=N$, where $N+i_tb_{t-1}$ represents income (Egginton, 1995, eqs. (18)-(19)). We may use the same approach and find that asset base that guarantees constant $\mathcal{L}$ residual incomes. Solving $\sum_{t=1}^{n}M\cdot F(0,n)^{-1}=\sum_{t=1}^{n}f_tF(0,t)^{-1}-f_0$ we find $M=(\sum_{t=1}^{n}f_tF(0,t)^{-1}-f_0)/(n\cdot F(0,n)^{-1})$. Charging depreciation as $\mathrm{Dep}_t(b^*_{t-1},b^*_t)=f_t-M-ib^*_{t-1}$, where $(M+ib^*_{t-1})$ is the income, one finds $x^{\mathcal{L}}_t=(M+i_tb^*_{t-1})-i_tb^*_{t-1}=M$. It is worth noting that the depreciation

[11] If an entity approach is taken, rather than a proprietary approach, then $\mathcal{L}$-CSV becomes Drukarczyk and Schüler's (2000) Net Economic Income.

charge selected is such that $b_t = b_{t-1} + M + ib^*_{t-1}$ which simply goes to $b_t = b^*_t + tM$ for all $t = 1, 2, \ldots, n$. Hence, M is the arithmetic mean of the surplus of capital $M = (b_t - b^*_t)/t$ for period. But Corollary 2 informs that, whatever the asset base, $b_t - b^*_t = \sum_{k=1}^{t-1} x^{\mathcal{L}}_k$ for all t. Picking t=n, we find

$$M = \frac{1}{n}\sum_{k=1}^{n} x^{\mathcal{L}}_k = \overline{x}^{\mathcal{L}}. \tag{46}$$

In other words, if the book value depreciation is such that the surplus of capital is constant, then the resulting RI is equal to the average residual income resulting from *any* book value depreciation. We name M this measure *maintainable* RI, in analogy with Egginton's. Obviously, if the $\mathcal{L}$ maintainable RI is scaled for time it coincides with the time-scaled RI previously introduced. By Proposition 8, this time-scaled RI fulfills both requirements (A) and (B), even for unequal-life projects. Actually, the reason is that, by scaling RIs, a bundle of projects may be compared in terms of residual income as if the projects' life were equal: it is as if projects gave their respective owners constant (maintainable) RIs for the same length of time.

7 Conclusions

This paper aims at providing a theoretical foundation for a new notion of residual income, whose features suggest a fruitful use in valuation, capital budgeting, performance measurement. Originally introduced with the name of *Systemic Value Added* (Magni, 2000, 2001, 2003), the new paradigm translates the notion of opportunity cost (capital charge) in a nonstandard way. The different capital charge derives from the fact that account is taken not only of the return rate foregone by the investors, but also of the capital foregone by the investors. In other words, if the investors invested in the alternative asset, they would own, at the beginning of each period, a different capital than the actual one. This capital would generate additional return at the opportunity cost of capital. By undertaking the project investors definitely lose this capital, which is then "unrecovered", as O'Hanlon and Peasnell (2002) put it.

This paper presents four theoretical frameworks that generate the paradigm: (i) an arbitrage-based perspective whereby the project's (firm's) cash-flow stream may be replicated by investing funds at the cost of capital; (ii) a microeconomic-based outlook, where the investors' wealth is seen to evolve through time depending on the course of action selected; (iii) an axiomatic approach where residual income is required to equal investors' excess wealth increase and be npv-consistent; (iv) an accounting approach based on two alternative book value depreciation charges, one of which is the depreciation charge of Egginton's (1995) Adjusted RI and the other is *any* depreciation. In these four perspectives the capital charge is given different (equivalent) meanings: it represents (i) interest on the short position of an arbitrage strategy, (ii) interest on the investor's alternative wealth, (iii) an additive-coherence-fulfilling opportunity cost, (iv) the sum of the project's cash flow and the depreciation for Adjusted RI.

Some important theoretical features are discussed alongside implications for valuation, capital budgeting, performance measurement:

- the lost-capital residual income enjoys an aggregation result: residual income are additively coherent in the sense that their sum equals the project's accumulated NPV. This implies that this paradigm tends to offset forecasting errors: single periods do not count, only the average residual income is relevant for valuation. Hence, to value an asset the fundamental step is to determine the future average residual income (simple arithmetic mean). This result gives a quantitative foundation to Graham, Dodd, and Cottle's (1962) words: "Intrinsic value would then be found by first forecasting this earning power and then multiplying that prediction by an appropriate 'capitalization factor'" (p. 28)

- unlike the standard theory, the new theory allows one to give a significant role to accounting rates. In particular, the weighted average of accounting rates, unanimously considered nonsignificant and unhelpful for decision-making, turns out to be a reliable indicator of profitability. This average, which is a Chisini (1929) mean, gives no problem of existence nor multiplicity and may well replace the IRR rule: a project is worth undertaking if and only if the average accounting rate is greater than the average comprehensive cost of capital, and the difference between the average ARR and the average comprehensive cost of capital provides the same ranking as the NPV ranking. The simple average of residual incomes may also be used for accept/reject decision and for ranking project of equal lives, because the NPV is a multiple of the average residual income, which implies maximization of NPV is equivalent of maximization of the average residual income. In case of unequal lives, it is possible to make use of the time-scaled RI. These results gives accounting as a scientific discipline a major role for capital budgeting decision-making

- periodic performance in the two theories differs in size and, possibly, in sign; the formal relations the two residual incomes bear are condensed in a dual relation, which shows that either theory can be generated by the other. Compensating managers with the new paradigm means that managers are rewarded taking account of the entire return that would accrue to shareholders if funds were invested at the cost of capital; that is, taking account that shareholders not only forego a return rate on the actual capital, but they also forego an additional capital on which the cost of capital could be applied. This implies that the new paradigm is a path-dependent residual income that keeps memory of the capital lost by the investors. Quantitatively, this implies that the lost-capital paradigm tends to amplify results with respect to the standard paradigm, both in positive and negative sense. For example, if the $\alpha\beta$ compensation plan is used (where bonus $=$ α% RI+β%Δ RI), the lost-capital paradigm is more incentive for positive-residual-income companies, because both residual income and its variations (Δ) are greater in the lost-capital paradigm than in the standard one

- particular metrics can be generated in the lost-capital paradigm that are goal congruent: adopting a proprietary approach, the lost-capital companion of Fernández's (2002) Created Shareholder Value is shown to enjoy robust goal congruence, irrespective of the sign of the cash flows; in this case, residual income does measure value creation. The average lost-capital income is shown to equal a maintainable RI with specified book value depreciation such that the surplus of capital per period is constant over time. The time-scaled RI (=maintainable RI) fulfills Egginton's (1995) requirements of periodic consistency.

The paper aims at attracting scholars' interests for further investigations, both in a theoretical sense and in an applicative sense. As for the latter, this work gives some specific clues for asset valuation and capital budgeting decisions, and investigates the source of differences in performance measurement. It does not give practical guides for incentive compensation, and future researches should be devoted to verifying whether and how the paradigm may be specifically used for devising compensation plans capable of coping with the principal-agent problem. It may well be the case that the search for a satisfying compensation plan will lead to an index based on multiple metrics, possibly involving the use of both paradigms. Other important situations may be coped with in the future, such as real options. It is widely known that the option value may be computed via stochastic dynamic programming as a generalised NPV (see Dixit and Pindyck, 1994): the procedure is formally equivalent to options pricing. Given the equivalence of NPV and the average lost-capital RI, interesting results may be expected if the lost-capital theory is used for valuing a real option.

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Appendix

Conversion is made by replacing the capital charge of the $\mathcal{S}$ theory with the comprehensive capital charge of the $\mathcal{L}$ residual income. For illustrative purposes, we focus on Stewart's (1991) Economic Value Added (EVA) and on the Edwards-Bell-Ohlson (EBO) model (Edwards and Bell, 1961; Ohlson, 1995).[12] The two metrics belong to the set of standard residual income models, and are complementary: EVA adopts an entity (claimholders) approach; EBO adopts a proprietary (shareholder) approach.

EVA

Assume that (i) the book value of the firm's assets $b_{A,t}$ is chosen as the capital invested outstanding capital, (ii) the free cash flows (FCF) are taken as the relevant cash flows (iii) the Return On Net Assets (RONA) is taken as the accounting rate of return, and (iv) the Weighted Average Cost of Capital (WACC) is taken as the opportunity cost of capital. Then, clean surplus becomes

$$b_{A,t} = b_{A,t-1} \cdot (1 + \text{RONA}_t) - \text{FCF}_t$$

for $t>0$, and $b_{A,0}:=f_0$. Reminding that $b_{A,t-1} \cdot \text{RONA}_t = \text{NOPAT}_t$, the standard performance measure becomes

$$x_t^{\mathcal{S}} = \text{NOPAT}_t - \text{WACC}_t \cdot b_{A,t-1}. \tag{47}$$

[12] Abusing notation, we will henceforth use the acronym EBO to refer to the corresponding residual income as well.

If, instead, theory $\mathcal{L}$ is applied, one gets

$$b^*_{A,t} = b^*_{A,t-1} \cdot (1 + \text{WACC}_t) - \text{FCF}_t$$

for $t>0$, with $b^*_{A,0}:=f_0$ and $b^*_{A,t}$ is the lost capital. Thus, the lost-capital metric is

$$x^{\mathcal{L}}_t = \text{NOPAT}_t - \text{WACC}_t \cdot b^*_{A,t-1}. \tag{48}$$

The metrics in eqs. (47) and (48) represent the original Economic Value Added and its lost-capital companion, respectively.

EBO

A different metric is generated when (i) the book value of equity $b_{E,t}$ is taken as the outstanding capital, (ii) the equity cash flows (ECF) are taken as the relevant cash flows, (iii) the Return On Equity (ROE) is taken as the periodic rate of return, and (iv) the cost of equity (k_e) is taken as the opportunity cost of capital. We have

$$b_{E,t} = b_{E,t-1} \cdot (1 + \text{ROE}_t) - \text{ECF}_t$$

for $t>0$, with $b_{E,0}:=f_0$. Therefore, reminding that $b_{E,t-1} \cdot \text{ROE}_t=\text{PAT}_t$, the standard measure becomes

$$x^{\mathcal{S}}_t = \text{PAT}_t - k_{e_t} \cdot b_{E,t-1}. \tag{49}$$

If one applies theory $\mathcal{L}$, one gets

$$b^*_{E,t} = b^*_{E,t-1} \cdot (1 + k_{e_t}) - \text{ECF}_t$$

for $t > 0$, with $b^*_{E,0} := f_0$. Thus, the lost-capital measure results in

$$x^{\mathcal{L}}_t = \text{PAT}_t - k_{e_t} \cdot b^*_{E,t-1}. \tag{50}$$

The metrics in eqs. (49) and (50) represent EBO as originally conceived and its lost-capital companion, respectively (see Table 5).

We apply the two paradigms to a firm created to undertake a project that requires an initial investment of 13 800, of which 12 000 are spent in fixed assets and 1 800 in working capital requirements. Straight-line depreciation is assumed for the fixed assets. It is also assumed that the required return on assets is 12% and the book value of debt equals the market value of debt (i.e. debt rate=required return to debt). Other input data are collected in Table 6; Table 7 gives the firm's accounting statements and the resulting cash flows, and Table 8 focusses on equity and firm valuation. The market value of equity is first found by using three different discounted-cash-flow methods: the Adjusted Present Value (APV) method, introduced by Myers (1974), the ECF-k_e method (equity approach), and the FCF-WACC method (entity approach). Logically, they all give the same result (see Fernández, 2002).

Afterwards, a residual-income perspective is used to obtain the market value: Tables 9-13 show the application of the two paradigms to the EVA model and the EBO model. Obviously, both residual income paradigms supply the same market values as the discounted-cash-flow technique's and the same NPV. The average RI (=maintain-abale RI) is also computed for each case: it is positive in both equity and entity perspective (see Tables 9-10), consistently with the NPV. Value creation is also signalled by the average ARR, which is contrasted with the comprehensive cost of equity: the average ROE is 27.47%, which is greater than the average comprehensive cost of equity:

$(\sum_{t=1}^{5} k_{e_t} b^*_{E,t-1} / \sum_{t=1}^{5} b_{E,t-1}) = 10.86\%$. The difference between the two rates is 16.61; applying it to the total book value $\sum_{t=1}^{5} b_{E,t-1}$=25000 and discounting to time 0 one gets back the NPV.

The examples show a situation of positive EVAs and EBOs in each period. First of all, note that in the first period the two paradigms give the same answer, because the initial capital invested is the same ($b_0 = b^*_0$). In the next periods, the lost-capital metrics are constantly greater than the standard metrics. Also, the periodic variation in the lost-capital metrics are greater. For example, in Table 9 the standard EVA's variations are given by $(281, 282, 283, 286)$, the lost-capital EVA's variations are $(282, 313, 347, 376)$. In Table 10 we have, consistently, that the EBO's variations are $(296, 298, 306, 372)$ and $(302, 350, 427, 811)$, respectively.

As anticipated, the $\mathcal{L}$ residual income has an insurance component for negative situations. Suppose the fourth-year sales amount to 8 000 instead of 10 000 (Table 11), other things equal. Both paradigms report negative performance in the fourth year.[13] Yet, the lost-capital paradigm smoothes the negativeness, because it takes account of the fact that the past year's results were better, which implies that the lost capital at the beginning of the fourth year is smaller than the actual capital employed: $b_{A,3} > b^*_{A,3}$ and $b_{E,3} > b^*_{E,3}$. It is easy to see that if the fourth-year sales are equal to 8 600 instead of 10 000 (other things unvaried), the corresponding $\mathcal{S}$ metrics become negative, whereas the $\mathcal{L}$ metrics keep positive (Table 12). In this case, while the RONA (respectively, ROE) is indeed smaller than the WACC (respectively, k_e) in the fourth year, the bonus given by the additional amount $\text{WACC}_4 \cdot (b_{A,3} - b^*_{A,3})$=96 (respectively, $k_{e_4} \cdot (b_{E,3} - b^*_{E,3})$=185) is so high as to more than compensate the negative standard EVA (respectively, EBO): we have $16 = -80 + 96$, and $164 = -21 + 185$.

Evidently, the bonus may symmetrically act a penalty role if past performance is negative. For example, consider the case where in the third year sales amount to 8 000 (other things unvaried). This makes the third-year residual incomes negative for both paradigms (Table 13). Due to insurance bonus for positive past performances, the lost-capital residual incomes are less negative than the standard ones. Yet, the third-year negative performance penalizes the fourth-year performance, which is smaller than that reported by the standard residual incomes. Note that in the fifth year, performance recorded by the $\mathcal{L}$ paradigm is again higher than the standard one's, due to the renewed recent positive performance of the fourth year. In other words, as compared to the $\mathcal{S}$ metric, performance is amplified in negative and in positive sense (bonus and penalty roles).

If maintainable RI is used, performance is always positive, consistently with the sign of the NPV. This means that the surplus of capital invested per period is constant and equal to the maintainable RI (which is in turn equal to the average RI). Table 10 tells us that the $\mathcal{L}$ maintainable RI is 830.6: it is greater than 464.2, which is found in Table 11; this means that (profitability) and performance diminishes (this is obvious, given that Table 11 refers to the case of fourth-year sales equal to 8000.). Analogously, the case treated in Table 12 is halfway between the former two. Table 13 deals with the case where third-year sales are equal to $8000 < 10000$; the maintainable RI is 412.2, which means that the NPV will be smaller than the case described in Table 11 (fourth-year sales equal to 8000). This is intuitive: while the total sales over the time span [0,5] coincide, the distribution of income in Table 11 is more favourable; which implies that the NPV will be greater than Table 13's.

[13] The reader should not be discomforted by the fact that *each* period's residual income changes. If one period's sales change, the corresponding ECF and FCF change, so that the market value of equity is changed in every year, which implies that both k_e and WACC change in every year, which in turn induces a change in the capital charge of every period.

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Table 0. Main notational conventions

a_t, $\vec{a}$	accounting rate (scalar and vector)
$\overline{a}$	average accounting rate of return
ARR	accounting rate of return
b_t	book value
$\overline{b}$	simple arithmetic mean of book values
b_t^*	lost, unrecovered capital
$b_{A,t}$	total capital invested (book value)
$b_{A,t}^*$	total lost capital
$b_{E,t}$	equity (book value)
$b_{E,t}^*$	lost equity
$b_{j,t}$	book value of project j
C_t	capital charge
CSV	Created Shareholder Value
D	debt (market value=book value)
Dep_t	depreciation charge
DVTS	discounted value of tax shields
Δ	variation
E	equity (market value)
EBIT	Earnings Before Interest and Taxes
EBO	Edwards-Bell-Ohlson
ECF	Equity Cash Flow
EI	expected residual-income improvement
EVA	Economic Value Added
f	firm, project
f_t, $\vec{f}$	cash flow (scalar, vector)
$F(s,t)$	accumulation factor from s to t
FCF	Free Cash Flow
Γ	arbitrage gain at time n
i_t,	opportunity cost of capital
i_t^*	comprehensive cost of capital
$\bar{\imath}^*$	average comprehensive cost of capital
$I_{0,t}$	excess wealth increase generated in the span $[0,t]$
k_D	required return on debt (=debt rate)
k_e	cost of equity
k_U	required return on assets

(The Table is continued on the next page)

Table 0. (continued) Notational Conventions

$\mathcal{L}$	lost-capital
M	maintainable
n_j	project's j length
NOPAT	Net Operating Profit After Taxes
NPV	Net Present Value
PAT	Profit After Taxes
PBT	Profit Before Taxes
PV[A; B]	$\sum_{t=1}^{n} \frac{\mathrm{A}_t}{\prod_{k=1}^{t}(1+\mathrm{B}_k)}$
r, $\vec{r}$	internal rate of return, internal return vector
RI	residual income
$\mathcal{S}$	standard
ROA, RONA, ROE	Return On Assets, Return On Net Assets, Return On Equity
T	corporate tax rate
v_0	market value of project/firm f
v_0^t	market value of a t-period asset
V_U	value of the unlevered firm
W_0	investor's wealth at time 0
$W_t(\vec{\imath})$	investor's wealth at time t if economic activity f is undertaken
$W_t(\vec{b}, \vec{f}, \vec{\imath})$	investor's wealth at time t economic activity f is not undertaken
WACC	Weighted Average Cost of Capital
WCR	Working Capital Requirements
x_t	income
x_t^R	residual income (general)
$x_t^{\mathcal{S}}$	residual income (standard)
$x_t^{\mathcal{L}}$	residual income (lost capital)
$\overline{x}_t^{\mathcal{L}}$	simple arithmetic mean of $\mathcal{L}$ residual incomes

Table 1. Arbitrage strategy

	Cash flows				
Time	0	1	2	...	n
Investment on p	$-f_0$	f_1	f_2	...	f_n
Short position on p	p_0	$-f_1$	$-f_2$	...	$-f_n$
Long position on p	$-(p_0 - f_0)$	0	0	...	Γ
Total	0	0	0	...	Γ

Table 2. Arbitrage strategy: netting out positions on p

	Cash flows				
Time	0	1	2	$\cdots$	n
Investment in f	$-f_0$	f_1	f_2	$\cdots$	f_n
Net short position on p	f_0	$-f_1$	$-f_2$	$\cdots$	$-(f_n - \Gamma)$
Total	0	0	0	...	Γ

Table 3. The residual-income framework and the axiomatic approach

Residual income	Definition 1	Property 1	Axiom 1	Axiom 1'
General	✓	✓		
Lost-capital	✓	✓	✓	
Standard	✓	✓		✓

Table 4. The economic derivations of lost-capital residual income

financial	$\overbrace{\boxed{a_t b_{t-1}}}^{\text{return from long position}}$	$\overbrace{\boxed{i_t b^*_{t-1}}}^{\text{interest on short position (lost return)}}$
microeconomic	$\overbrace{\boxed{W_t(x_t, \vec{\iota}) - W_{t-1}(x_t, \vec{\iota})}}^{\text{wealth increase}}$	$\overbrace{\boxed{W_t(\vec{\iota}) - W_{t-1}(\vec{\iota})}}^{\text{lost wealth increase}}$
accounting	$\overbrace{\boxed{\text{Dep}_t(b^*_{t-1}, b^*_t)}}^{\text{depreciation of Adjusted RI}}$	$\overbrace{\boxed{\text{Dep}_t(b_{t-1}, b_t)}}^{\text{any depreciation}}$

Table 5. EVA and EBO variables in the two paradigms

	a_t	i_t	b_t	b_t^*	$\Longrightarrow$	capital charge
	$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$		$\Downarrow$
Standard Paradigm						
EVA	RONA	WACC	$b_{A,t}$		$\Longrightarrow$	$\text{WACC}_t \cdot b_{A,t-1}$
EBO	ROE	k_e	$b_{E,t}$		$\Longrightarrow$	$k_{e_t} \cdot b_{E,t-1}$
Lost-capital Paradigm						
EVA	RONA	WACC	$b^*_{A,t}$	$b^*_{A,t}$	$\Longrightarrow$	$\text{WACC}_t \cdot b^*_{A,t-1}$
EBO	ROE	k_e	$b_{E,t}$	$b^*_{E,t}$	$\Longrightarrow$	$k_{e_t} \cdot b^*_{A,t-1}$

Table 6. Input data

Investment	13 800	Depreciation rate	20%
Gross Fixed Assets	12 000	Corporate tax rate	33%
WCR	1 800	Required return on assets	12%
Sales	10 000	Debt rate	7%
Cost of Sales	3 670	Required return on debt (k_D)	7%
Gen. & Admin. Expenses	1 600		
Debt	4 000		

Table 7. Balance Sheet, Income Statement, Cash Flows

time	0	1	2	3	4	5
BALANCE SHEET						
Gross fixed assets	12 000	12 000	12 000	12 000	12 000	12 000
−cumulative depreciation	0	−2 400	−4 800	−7 200	−9 600	−12 000
Net fixed assets	12 000	9 600	7 200	4 800	2 400	0
WCR	1 800	1 800	1 800	1 800	1 800	0
NET ASSETS	13 800	11 400	9 000	6 600	4 200	0
Debt	4 000	4 000	4 000	4 000	4 000	0
Equity (book value)	9 800	7 400	5 000	2 600	200	0
NET WORTH & LIABILITIES	13 800	11 400	9 000	6 600	4 200	0
INCOME STATEMENT						
Sales		10 000	10 000	10 000	10 000	10 000
Cost of sales		3 670	3 670	3 670	3 670	3 670
Gen. & Adm. expenses		1 600	1 600	1 600	1 600	1 600
Depreciation		2 400	2 400	2 400	2 400	2 400
EBIT		2 330	2 330	2 330	2 330	2 330
Interest		280	280	280	280	280
PBT		2 050	2 050	2 050	2 050	2 050
Taxes		677	677	677	677	677
PAT		1 374	1 374	1 374	1 374	1 374
+Depreciation		2 400	2 400	2 400	2 400	2 400
+Δ Debt		0	0	0	0	−4 000
−Δ WCR		0	0	0	0	1 800
ECF		3 774	3 774	3 774	3 774	1 574
FCF[14]		3 961	3 961	3 961	3 961	5 761
ROE		14.02%	18.56%	27.47%	52.83%	686.75%
Average ROE	27.47%					

[14]FCF=ECF$-\Delta$D $+ k_D$D $\cdot (1-T)$.

Table 8. Valuation

time	0	1	2	3	4	5
k_U		12%	12%	12%	12%	12%
V_U=PV[FCF; k_U]	15 300	13 175	10 795	8 129	5 144	0
DVTS=PV[T·k_D·D; k_D][(a)]	379	313	242	167	86	0
v=V_U+DVTS	15 679	13 488	11 038	8 296	5 230	0
E=V_U+DVTS−D	**11 679**	9 488	7 038	4 296	1 230	0
k_e		13.550%	13.943%	14.670%	16.461%	27.907%
E=PV[ECF; k_e]	**11 679**	9 488	7 038	4 296	1 230	0
average (comprehensive) cost of equity 10.86%						
WACC		11.290%	11.199%	11.053%	10.786%	10.151%
v=PV[FCF; WACC]	15 679	13 488	11 038	8 296	5 230	0
E=v−D	**11 679**	9 488	7 038	4 296	1 230	0
NPV=E−b_E	1 879					

[(a)]We use k_D to discount tax shields. However, it is worth noting that there is a lively debate in the literature on which is the correct discount rate for discounting tax shields. While this issue is not relevant to this paper, the reader may be willing to turn to the contributions of Myers (1974), Tham and Vélez-Pareja (2001), Arzac and Glosten (2005), Fernández (2005), Cooper and Nyborg (2006). (To bypass the issue, the reader may well dismiss the first five rows of the Table and consider k_e as exogenously given).

Table 9. EVA in the two paradigms

time	0	1	2	3	4	5
NOPAT=EBIT·$(1-T)$		1 561	1 561	1 561	1 561	1 561
b_A=D+b_E	13 800	11 400	9 000	6 600	4 200	0
b^*_A (lost capital)	13 800	11 397	8 712	5 714	2 369	−3 151
Standard Paradigm						
capital charge (opportunity cost)		1 558	1 277	995	712	426
EVA		**3**	**284**	**566**	**849**	**1 135**
NPV (=discount and sum)	1 879					
E=b_E+NPV	11 679					
Lost-capital Paradigm						
capital charge (opportunity cost)		1 558	1 276	963	616	240
EVA		**3**	**285**	**598**	**945**	**1 321**
NPV (=sum and discount)	1 879					
E=b_E+NPV	11 679					
average RI (maintainable RI)		**603.4**	**603.4**	**603.4**	**603.4**	**603.4**

Table 10. EBO in the two paradigms

time	0	1	2	3	4	5
PAT		1 374	1 374	1 374	1 374	1 374
b_E	9 800	7 400	5 000	2 600	200	0
b_E^* (lost equity capital)	9 800	7 354	4 606	1 509	−2 017	−4 153
Standard Paradigm						
capital charge (opportunity cost)		1 328	1 032	733	428	56
EBO		**46**	**342**	**640**	**946**	**1 318**
NPV (=discount and sum)	1 879					
E=b_E+NPV	11 679					
Lost-capital Paradigm						
capital charge (opportunity cost)		1 328	1 025	676	248	−563
EBO		**46**	**348**	**698**	**1 125**	**1 936**
NPV (=sum and discount)	1 879					
E=b_E+NPV	11 679					
average RI (maintainable RI)		**830.6**	**830.6**	**830.6**	**830.6**	**830.6**

Table 11. Fourth-year sales equal to 8 000

year	1	2	3	4	5
EVA					
Standard Paradigm	**9**	**291**	**575**	**−477**	**1 135**
Lost-capital paradigm	**9**	**292**	**608**	**−381**	**1 188**
average RI (maintainable RI)	**343.2**	**343.2**	**343.2**	**343.2**	**343.2**
EBO					
Standard Paradigm	**34**	**326**	**616**	**−439**	**1 318**
Lost-capital paradigm	**34**	**330**	**671**	**−251**	**1 537**
average RI (maintainable RI)	**464.2**	**464.2**	**464.2**	**464.2**	**464.2**

Table 12. Fourth-year sales equal to 8 600

year	1	2	3	4	5
EVA					
Standard Paradigm	**7**	**289**	**573**	**−80**	**1 135**
Lost-capital paradigm	**7**	**290**	**605**	**16**	**1 228**
average RI (maintainable RI)	**429.2**	**429.2**	**429.2**	**429.2**	**429.2**
EBO					
Standard Paradigm	**37**	**331**	**624**	**−21**	**1 318**
Lost-capital paradigm	**37**	**336**	**680**	**164**	**1 658**
average RI (maintainable RI)	**575**	**575**	**575**	**575**	**575**

Table 13. Third-year sales equal to 8 000

year	1	2	3	4	5
EVA					
Standard Paradigm	**9**	**292**	**−763**	**849**	**1 135**
Lost-capital paradigm	**9**	**293**	**−730**	**803**	**1 173**
average RI (maintainable RI)	**309.6**	**309.6**	**309.6**	**309.6**	**309.6**
EBO					
Standard Paradigm	**32**	**323**	**−727**	**946**	**1 318**
Lost-capital paradigm	**32**	**328**	**−673**	**894**	**1 480**
average RI (maintainable RI)	**412.2**	**412.2**	**412.2**	**412.2**	**412.2**

Table 14. CSV in the two paradigms

time	0	1	2	3	4	5
outstanding capital	9 800	9 488	7 038	4 296	1 230	0
lost equity capital	9 800	7 354	4 606	1 509	−2 017	−4 153
Standard Paradigm						
CSV		**2134**	**0**	**0**	**0**	**0**
NPV (=discount and sum)	1 879					
E=b_E+NPV	11 679					
Lost-capital Paradigm						
CSV		**2 134**	**298**	**357**	**459**	**906**
NPV (=sum and discount)	1 879					
E=b_E+NPV	11 679					