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5. October 2006

Online at <http://mpra.ub.uni-muenchen.de/1742/>  
MPRA Paper No. 1742, posted 11. February 2007

# Repulsive Particle Swarm Method on Some Difficult Test Problems of Global Optimization

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**I. Introduction:** Optimization of non-convex (multi-modal) functions is the subject matter of research in global optimization. During the 1970's or before only little work was done in this field, but in the 1980's it attracted the attention of many researchers. Since then, a number of methods have been proposed to find the global optima of non-convex (multi-modal) problems of combinatorial as well as continuous types. Among these methods, genetic algorithms, simulated annealing, particle swarm, ants colony, tunneling, taboo search, etc. have been quite successful as well as popular.

It may be noted that no method can guarantee that it would surely find the global optimum of an arbitrary function in a finite number of attempts, howsoever large. There is one more point to be noted. A particular method might be quite effective in solving some (class of) problems, but it may cut a sorry figure at the others. Next, each of these methods operates with a number of parameters that may be changed at choice to make it more effective. This choice is often problem oriented. A particular choice may be extremely effective in a few cases, but it might be ineffective (or counterproductive) in certain other cases. Additionally, there is a relation of trade-off among those parameters. These features make all these methods a subject of trial and error exercises.

There is another feature of these methods (and the literature regarding them) that deserves a mention here. Each method of global optimization has quite many variants. The proponents of those variants introduce some changes into the original algorithm, test their variants on a few (popularly used) benchmark functions (often of too small dimensions) and haste to suggest that the proposed variant(s) performs better than the original (or other variants of the) method. There is no harm in introducing a variant of any method that functions well or better than the others. In a field of research, which is attractive as well as alive, this is expected and welcome. However, the observed tendency to test those variants on a couple of popular (and easy!) benchmark problems and push the method into the market does not augur well. The extant literature on the subject matter shows how some benchmark problems are in frequent use - Ackley, Griewank, Himmelblau, Levy, Michalewicz, Rastrigin, Rosenbrock, Schwefel and a couple of others, but much less frequently; so much so that some authors churn out 'literature' profusely with the test problems like Himmelblau's, Griewank's and Rastrigin's functions alone. *This is not to say that these test problems are simple or trivial.* Intended is only to point out that frequent use of these functions introduces a specific bias into the research efforts and keeps us away from many harder problems that characterize the challenging task of global optimization research.

**II. The Objectives:** The objectives of this paper are plain and simple: to test a particular variant of the (Repulsive) Particle Swarm method on some rather difficult problems. A

number of such problems are collected from the extant literature and a few of them are newly introduced. First, we introduce the Particle Swarm (PS) method of global optimization and its variant called the 'Repulsive Particle Swarm' (RPS) method. Then we endow the particles with some stronger local search abilities – much like tunneling – so that each particle can make a search in its neighborhood to optimize itself. Next, we introduce the test problems, the existing as well as the new ones. We also give plots of some of these functions to help appreciation of the optimization problem. Finally, we present the results of the optimization exercise. We append the (Fortran) computer program that we have developed and used in this exercise. *En passant* we may add that this program has been used to optimize a large (over 60) number of benchmark problems (see Mishra, 2006 (c), (d)).

**III. The Particle Swarm Method of Global Optimization:** As it is well known, the problems of the existence of global order, its integrity, stability, efficiency, etc. emerging at a collective level from the selfish nature of individual human beings have been long standing. The laws of development of institutions, formal or informal that characterize 'the settle habits of thinking and acting at a collective level', have been sought in this order. Thomas Hobbes, George Berkeley, David Hume, John Locke and Adam Smith visualized the global system arising out of individual selfish actions. In particular, Adam Smith (1759) postulated the role of *invisible hand* in establishing the harmony that led to the said global order. The neo-classical economists applied the tools of equilibrium analysis to show how this grand synthesis and order is established while each individual is rational and selfish. The postulate of perfect competition was felt to be a necessary one in demonstrating that. However, Alfred Marshall limited himself to partial equilibrium analysis and, thus, indirectly allowed for the role of the *invisible hand* (while general equilibrium economists - Leon Walras, Kenneth Arrow, Gerard Debreu and John Nash, etc - have held that the establishment of order can be explained by their approach). Yet, Thorstein Veblen (1898, 1899) never believed in the mechanistic view and pleaded for economics as an evolutionary science. Friedrich von Hayek (1944) believed in a similar philosophy and held that locally optimal decisions give rise to the global order and efficiency. Later, Herbert Simon (1982) postulated the '*bounded rationality*' hypothesis and argued that the hypothesis of perfect competition is not necessary for explaining the emergent harmony and order at the global level. Elsewhere, Ilya Prigogine (1984) demonstrated how 'order' emerges from 'chaos'.

The PS method is an instance of successful application of the philosophy of Simon's *bounded rationality* and decentralized decision-making to solve the global optimization problems (Simon, 1982; Bauer, 2002; Fleischer, 2005). It allows for *limited knowledge, memory, habit formation, social learning*, etc, not entertained before. In the animal world we observe that a swarm of birds or insects or a school of fish searches for food, protection, etc. in a very typical manner (Sumper, 2006). If one of the members of the swarm sees a desirable path to go, the rest of the swarm will follow quickly. Every member of the swarm searches for the best in its locality - learns from its own experience. Additionally, each member learns from the others, typically from the best performer among them. The PS method mimics this behavior (Wikipedia: [http://en.wikipedia.org/wiki/Particle\\_swarm\\_optimization](http://en.wikipedia.org/wiki/Particle_swarm_optimization)). Every individual of the swarm is considered as a particle in a multidimensional space that has a position and a

velocity. These particles fly through hyperspace and remember the best position that they have seen. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on these good positions. There are two main ways this communication is done: (i) “swarm best” that is known to all (ii) “local bests” are known in neighborhoods of particles. Updating of the position and velocity are done in each iteration as follows:

$$v_{i+1} = \omega v_i + c_1 r_1 (\hat{x}_i - x_i) + c_2 r_2 (\hat{x}_g - x_i)$$

$$x_{i+1} = x_i + v_{i+1}$$

where,

- $x$  is the position and  $v$  is the velocity of the individual particle. The subscripts  $i$  and  $i+1$  stand for the recent and the next (future) iterations, respectively.
- $\omega$  is the inertial constant. Good values are usually slightly less than 1.
- $c_1$  and  $c_2$  are constants that say how much the particle is directed towards good positions. Good values are usually right around 1.
- $r_1$  and  $r_2$  are random values in the range  $[0,1]$ .
- $\hat{x}$  is the best that the particle has seen.
- $\hat{x}_g$  is the global best seen by the swarm. This can be replaced by  $\hat{x}_L$ , the local best, if neighborhoods are being used.

The Repulsive Particle Swarm method of optimization is a variant of the classical Particle Swarm method invented by Eberhart and Kennedy (1995) (see Wikipedia, <http://en.wikipedia.org/wiki/RPSO>). It is particularly effective in finding out the global optimum in very complex search spaces (although it may be slower on certain types of optimization problems).

In the traditional RPS the future velocity,  $v_{i+1}$  of a particle at position with a recent velocity,  $v_i$ , and the position of the particle are calculated by:

$$v_{i+1} = \omega v_i + \alpha r_1 (\hat{x}_i - x_i) + \omega \beta r_2 (\hat{x}_{hi} - x_i) + \omega \gamma r_3 z$$

$$x_{i+1} = x_i + v_{i+1}$$

where,

- $x$  is the position and  $v$  is the velocity of the individual particle. The subscripts  $i$  and  $i+1$  stand for the recent and the next (future) iterations, respectively.
- $r_1, r_2, r_3$  are random numbers,  $\in [0,1]$
- $\omega$  is inertia weight,  $\in [0.01,0.7]$
- $\hat{x}$  is the best position of a particle
- $x_h$  is best position of a randomly chosen other particle from within the swarm
- $z$  is a random velocity vector
- $\alpha, \beta, \gamma$  are constants

Occasionally, when the process is caught in a local optimum, some *chaotic* perturbation in position as well as velocity of some particle(s) may be needed.

**IV. Additional Local Search by the Particles:** The traditional RPS gives little scope of local search to the particles. They are guided by their past experience and the communication received from the others in the swarm. We have modified the traditional RPS method by endowing stronger (wider) local search ability to each particle. Each particle flies in its local surrounding and searches for a better solution. The domain of its search is controlled by a new parameter (*nstep*). This local search has no preference to gradients in any direction and resembles closely to tunneling. This added exploration capability of the particles brings the RPS method closer to what we observe in real life. However, in some cases moderately wide search (*nstep*=9, say; see program) works better.

**V. Randomized Neighbourhood Topology:** Each particle learns from its ‘chosen’ inmates in the swarm. At the one extreme is to learn from the best performer in the entire swarm. This is how the particles in the original PS method learn. However, such learning is not natural. How can we expect the individuals to know as to the best performer and interact with all others in the swarm? We believe in limited interaction and limited knowledge that any individual can possess and acquire. So, our particles do not know the ‘best’ in the swarm. Nevertheless, they interact with some chosen inmates that belong to the swarm. Now, the issue is: how does the particle choose its inmates? One of the possibilities is that it chooses the inmates closer (at lesser distance) to it. But, since our particle explores the locality by itself, it is likely that it would not benefit much from the inmates closer to it. Other relevant topologies are : (the celebrated) *ring topology*, ring topology hybridized with random topology, *star topology*, *von Neumann topology*, etc.

Now, let us visualize the possibilities of choosing (a predetermined number of) inmates randomly from among the members of the swarm. This is much closer to reality in the human world. When we are exposed to the mass media, we experience this. Alternatively, we may visualize our particles visiting a public place (e.g. railway platform, church, etc) where it (he) meets people coming from different places. Here, geographical distance of an individual from the others is not important. Important is how the experiences of others are communicated to us. There are large many sources of such information, each one being selective in what it broadcasts and each of us selective in what we attend to and, therefore, receive. This selectiveness at both ends transcends the geographical boundaries and each one of us is practically exposed to randomized information. Of course, two individuals may have a few common sources of information. We have used these arguments in the scheme of dissemination of others’ experiences to each individual particle. Presently, we have assumed that each particle chooses a pre-assigned number of inmates (randomly) from among the members of the swarm. However, this number may be randomized to lie between two pre-assigned limits.

**VI. The Benchmark Functions:** It has already been mentioned that the RPS variant described above has been tested on over 60 box-bound benchmark functions. In a great majority of cases it has succeeded at locating the minimum of these functions. In this paper we propose to test the method on some new and some well known difficult problems.

**1. New function #1:** We introduce a 2-d problem with  $f_{\min}(-8.4666, -9.9988) = -0.18466$ .

$$f(x) = \left[ \left| \cos \sqrt{|x_1^2 + x_2^2|} \right|^{0.5} + (x_1 + x_2)/100 \right]; x_i \in [-10, 10]; i = 1, 2$$

**2. New function #2:** This is a variant of function #1, where  $\cos(\cdot)$  is replaced by  $\sin(\cdot)$ . This function has the optimum  $f_{\min}(-9.94112, -9.99952) = -0.199441$ . It is given as:

$$f(x) = \left[ \left| \sin \sqrt{|x_1^2 + x_2^2|} \right|^{0.5} + (x_1 + x_2)/100 \right]; x_i \in [-10, 10]; i = 1, 2$$

**3. New function #3:** In the domain  $x \in [-10, 10]$  with  $f_{\min}(-1.98682, -10) = -1.01983$ , this function is

$$f(x) = [ \{ (\sin(\cos(x_1) + \cos(x_2)))^2 - (\cos(\sin(x_1) + \sin(x_2)))^2 \} + x_1 ]^2 + 0.01(x_1 + x_2)$$

**4. New function #4:** In the domain  $x \in [-10, 10]$  with  $f_{\min}(2.8863, 1.82326) = -2.28395$ , this function is defined as

$$f(x) = -\ln \left[ [ \{ (\sin(\cos(x_1) + \cos(x_2)))^2 - (\cos(\sin(x_1) + \sin(x_2)))^2 \} + x_1 ]^2 + [(x_1 - 1)^2 + (x_2 - 1)^2] / 10 \right]$$

**5. New function #5 (Quintic function):** In the domain  $x \in [-10, 10]$  with  $f_{\min} = 0$  for  $x_i = -1$  or  $2$ ;  $i = 1, 2, \dots, m$  this function (with multiple global minima) is defined as

$$f(x) = \sum_{i=1}^m |x_i^5 - 3x_i^4 + 4x_i^3 + 2x_i^2 - 10x_i - 4| : x_i \in [-10, 10]; i = 1, 2, \dots, m$$

**6. New function #6 (Needle-eye function):** This function is m-dimensional ( $m \geq 1$ ) and defined with a small (say 0.0001) eye. If  $|x_i| \leq eye \forall i$  then  $f(x) = 1$ . Else

$$f(x) = \sum_{i=1}^m (100 + |x_i|) t_i ; t_i = 1 \text{ if } |x_i| > eye, 0 \text{ otherwise} .$$

Minimization of this function becomes more difficult with smaller eye and larger m (dimension).

**7. New function #7 (Zero-sum function):** Defined in the domain  $x \in [-10, 10]$  this function (in  $m \geq 2$ ) has  $f(x) = 0$  if  $\sum_{i=1}^m x_i = 0$ . Otherwise  $f(x) = 1 + \left( 10000 \left| \sum_{i=1}^m x_i \right| \right)^{0.5}$ . This function has innumerable many minima but it is extremely difficult to obtain any of them. Larger is the value of m (dimension), it becomes more difficult to optimize the function.

**8. Corana function:** On  $x_i \in [-1000, 1000]$ ;  $i = 1, 2, \dots, 4$  this four-variable function is defined as follows and has  $f_{\min}(0, 0, 0, 0) = 0$ .

$$f(x) = \begin{cases} \sum_{i=0}^4 0.15(z_i - 0.05 \operatorname{sgn}(z_i))^2 d_i \text{ if } |x_i - z_i| < 0.05 \\ \sum_{i=1}^4 d_i x_i^2 \text{ otherwise} \end{cases} \quad \left| \begin{array}{l} z_i = 0.2 |x_i / 0.2| + 0.49999 | \operatorname{sgn}(x_i) \\ d_i = 1, 1000, 10, 100 \end{array} \right.$$

**9. Modified RCOS function:** In the domain  $x_1 \in [-5, 10]$ ,  $x_2 \in [0, 15]$  this 2-variable function has  $f_{\min}(-3.196989, 12.52626) = -0.179891$ . It is specified as

$$f(x) = -1/(f_1 + f_2 + f_3 + e)$$

$$f_1 = a(x_2 - bx_1^2 + cx_1 - d)^2 ; f_2 = e(1 - g) \cos(x_1) \cos(x_2) ; f_3 = \log(x_1^2 + x_2^2 + 1)$$

where,  $g = 1/(8\pi)$ ;  $b = 5.1/(4\pi^2)$ ;  $c = 5/\pi$ ;  $a = 1$ ;  $d = 6$ ;  $e = 10$ .

**10. Freudenstein Roth function:** On  $x_i \in [-10, 10]$ ;  $i = 1, 2$  this 2-variable function is defined as follows and has  $f_{\min}(5, 4) = 0$ .

$$f(x) = [-13 + x_1 + ((5 - x_2)x_2 - 2)x_2]^2 + [-29 + x_1 + ((x_2 + 1)x_2 - 14)x_2]^2$$

**11. ANNs XOR function:** This function is in nine variables. It is defined as follows.

$$f(x) = f_1 + f_2 + f_3 + f_4$$

where,

$$f_1 = [1 + \exp\{-x_7 / (1 + e^{(-x_1 - x_2 - x_5)}) - x_8 / (1 + e^{(-x_3 - x_4 - x_6)}) - x_9\}]^{-2}$$

$$f_2 = [1 + \exp\{-x_7 / (1 + e^{(-x_5)}) - x_8 / (1 + e^{(-x_6)}) - x_9\}]^{-2}$$

$$f_3 = [1 - [1 + \exp\{-x_7 / (1 + e^{(-x_1 - x_5)}) - x_8 / (1 + e^{(-x_3 - x_6)}) - x_9\}]^{-1}]^2$$

$$f_4 = [1 - [1 + \exp\{-x_7 / (1 + e^{(-x_2 - x_5)}) - x_8 / (1 + e^{(-x_4 - x_6)}) - x_9\}]^{-1}]^2$$

It is very difficult to minimize this function We obtain (by RPS)  $f_{\min} = 0.95979$  for:

$$x = (0.99999, 0.99993, -0.89414, 0.99994, 0.55932, 0.99994, 0.99994, -0.99963, -0.08272).$$

**12. Perm function #1:** In the domain  $x \in [-4, 4]$ , the function has  $f_{\min} = 0$  for  $x = (1, 2, 3, 4)$ . It is specified as

$$f(x) = \sum_{k=1}^4 \left[ \sum_{i=1}^4 (i^k + \beta) \{(x_i / i)^k - 1\} \right]^2$$

The value of  $\beta$  ( $=50$ ) introduces difficulty to optimization. Smaller values of beta raise this difficulty further.

**13. Perm function #2:** In the domain  $x \in [-1, 1]$ , and for a given  $\beta$  ( $=10$ ), this m-variable function has  $f_{\min} = 0$  for  $x_i = (i)^{-1}$ :  $i = 1, 2, \dots, m$ . It is specified as

$$\sum_{k=1}^4 \left[ \sum_{i=1}^4 (i + \beta) \{(x_i)^k - (i)^{-k}\} \right]^2$$

Smaller values of beta raise difficulty in optimization.

**14. Power-sum function:** Defined on four variables in the domain  $x \in [0, 4]$ , this function has  $f_{\min} = 0$  for any permutation of  $x = (1, 2, 2, 3)$ . The function is defined as

$$f(x) = \sum_{k=1}^4 \left[ b_k - \sum_{i=1}^4 x_i^k \right]^2; \quad b_k = (8, 18, 44, 114) \text{ for } k = (1, 2, 3, 4) \text{ respectively.}$$

**15. Goldstein Price function:** On  $x_i \in [-10, 10]$ ;  $i = 1, 2$  this 2-variable function is defined as follows and has  $f_{\min}(0, -1) = 3$ .

$$f(x) = (f_1)(f_2)$$

where,

$$f_1 = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)]$$

$$f_2 = [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$$

**16. Bukin's functions:** Bukin's functions are almost fractal (with fine seesaw edges) in the surroundings of their minimal points. Due to this property, they are extremely difficult to optimize by any method of global (or local) optimization and find correct values of decision variables (i.e.  $x_i$  for  $i=1,2$ ). In the search domain  $x_1 \in [-15, -5]$ ,  $x_2 \in [-3, 3]$  the 6<sup>th</sup> Bukin's function is defined as follows.

$$f_6(x) = 100\sqrt{|x_2 - 0.01x_1^2|} + 0.01|x_1 + 10| \quad ; \quad f_{\min}(-10, 1) = 0$$

This account does not exhaust the list of difficult (benchmark) functions. Bukin's (others than specified above), Hougen's, Giunta's, Weierstrass's, Factorial, Decanomial, SeqP, AMGM, etc. (see the computer program) are prominent among such difficult ones.

Elsewhere (Mishra, 2006 (a) and (b)) also we faced difficult global optimization problems.

**VII. Performance of the RPS Method:** In table-1 we present the results of our optimization efforts of the selected test functions described in the preceding section. For sake of comparison, we have optimized those functions with the Genetic Algorithm (GA) of David Carroll or Simulated Annealing (SA) of William Goffe and the (proposed variant of) RPS. We have changed three parameters in Carroll's codes: *maxgen* in *ga.inp* file is set to 500; *nparam* is set to the required dimension in the same file for different functions; *parameter* (*indmax=1000,nchrmax=60,nparmax=10*) is set in the *params.f* file. Goffe's SA program is used as it is. On the other hand, for RPS we have used our own program (appended) with varying parameters. Note that instead of bringing them into the limits, we have placed heavy penalties whenever the arguments ( $x_i; i = 1, 2, \dots, m$ ) are out of bounds (see the computer program).

<b>Table-1. Results of Minimization of Specified Functions by GA and RPS Methods</b>				
Sl No.	Function	m	The Minimal Value Obtained	
			GA or SA	RPS/SA
1	New function (N#1)	2	0.41490 GA	-1.18887
2	New function (N#2)	2	0.00000 GA	-0.19938
3	New function (N#3)	2	0.58581 GA	-1.01983
4	New function (N#4)	2	-1.14419 GA	-2.28395
5	New function (N#5) (Quintic function)	2	0.00000 SA	0.00094 ??
6	New function (N#6) (Needle-eye function)	8	1 GA/SA	1
7	Zero-sum function (N#7)	8	0.00000 GA	1.04573
8	Corana function	4	0.00000 GA/SA	0.00000
9	Modified RCOS function	2	-0.03786 GA	-0.17989
10	Freundenstein Roth function	2	0.00000 SA	0.00000
11	ANNs XOR function	9	0.959759 SA	0.95979
12	Perm #1 function	4	16.00000 SA	0.00000
13	Perm #2 function	2	0.00011 (!) GA/SA	0.00000 (!)
14	Power-sum function	4	***** GA/SA	0.00000
15	Goldenstein Price function	2	***** GA/SA	3.00000
16	Bukin's 6 <sup>th</sup> function	2	0.00000 (??) GA/SA	0.00000 (??)

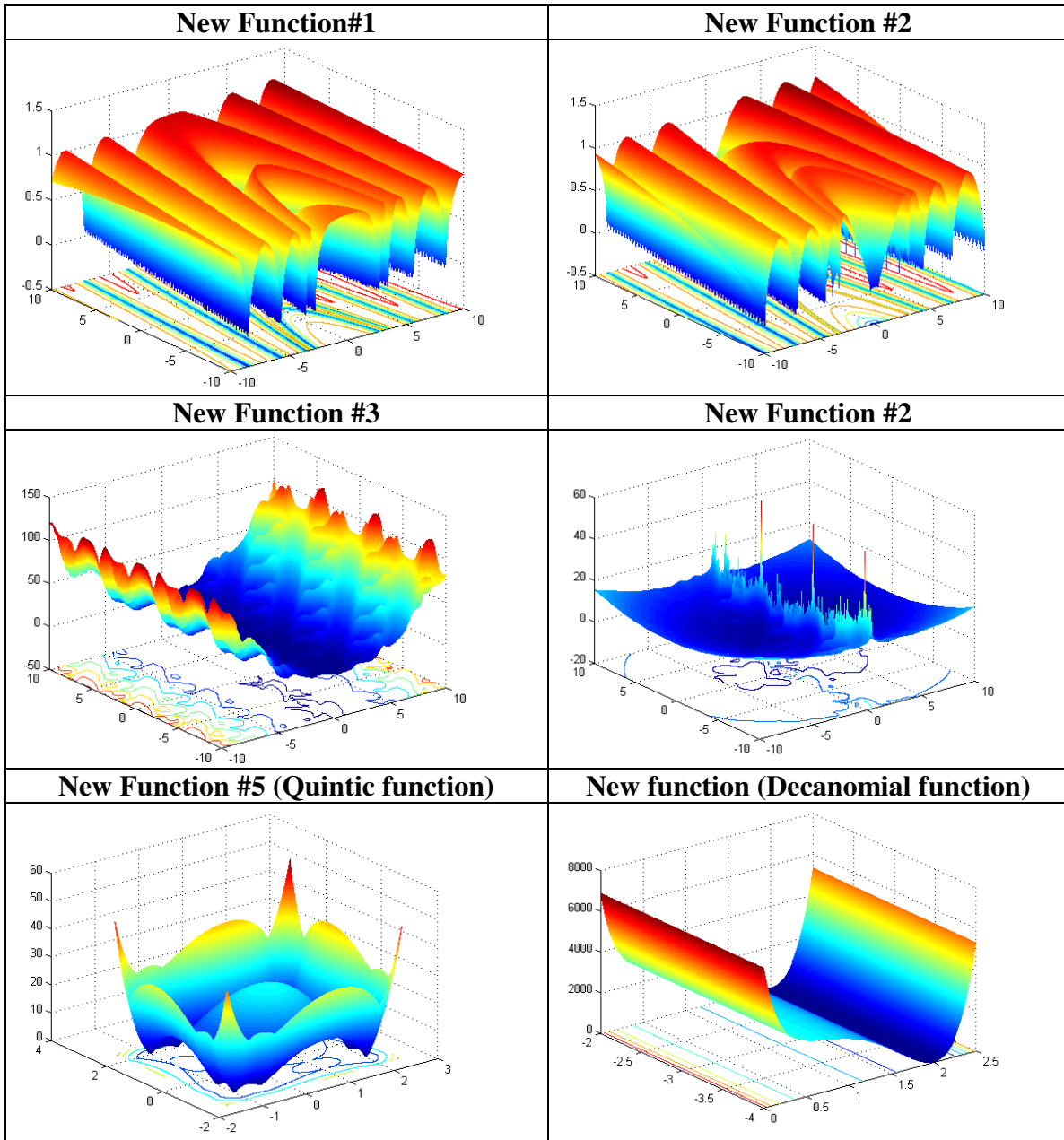
Note: ■ Shaded = Better results; ■ Shaded = Perform equally well; (??) Decision variables far from the correct ones; \*\*\*\*\* = Unstable; !! = Results degenerate fast as the dimension increases

The results indicate that none of the (three) method can assuredly find the optimum of an arbitrary function. In case of the Needle-eye and the Corana functions all three methods perform equally well while in case of Bukin's 6<sup>th</sup> function all of them yield the values of decision variables far away from the right ones. In case of zero-sum function, GA performs better than the RPS. In case of the Perm #2 function, all of the methods fail when the dimension grows larger. In several cases, GA falters or fails while RPS succeeds. In case of N#1 through N#5 and the ANNs XOR functions the RPS performs better than the Genetic algorithm, but comparably or worse than SA. In case of the quintic function SA outperforms RPS.



**VIII. Conclusion:** From what we have seen above, one may jump at the conclusion that the RPS performs better than the GA at least. But we would like not to do so. We would only conclude that none could assure a supremacy over the other(s). Each one faltered in some case; each one succeeded in some others.

It is needed that we find out some criteria to classify the problems that suit (or does not suit) a particular method. This classification will highlight the comparative advantages of using a particular method for dealing with a particular class of problems.



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1: C      PROGRAM TO FIND GLOBAL MINIMUM BY REPULSIVE PARTICLE SWARM METHOD
2: C      WRITTEN BY SK MISHRA, DEPT. OF ECONOMICS, NEHU, SHILLONG (INDIA)
3: C      -----
4: C      PARAMETER (N=100, NN=40, MX=100, NSTEP=15, ITRN=5000, NSIGMA=1, ITOP=3)
5: C      PARAMETER (NPRN=100) ! DISPLAYS RESULTS AT EVERY 100 TH ITERATION
6: C      PARAMETER (N=50, NN=25, MX=100, NSTEP=9, ITRN=10000, NSIGMA=1, ITOP=3)
7: C      PARAMETER (N=100, NN=15, MX=100, NSTEP=9, ITRN=10000, NSIGMA=1, ITOP=3)
8: C      IN CERTAIN CASES THE ONE OR THE OTHER SPECIFICATION WORKS BETTER
9: C      DIFFERENT SPECIFICATIONS OF PARAMETERS MAY SUIT DIFFERENT TYPES
10: C     OF FUNCTIONS OR DIMENSIONS - ONE HAS TO DO SOME TRIAL AND ERROR
11: C     -----
12: C     N = POPULATION SIZE. IN MOST OF THE CASES N=30 IS OK. ITS VALUE
13: C     MAY BE INCREASED TO 50 OR 100 TOO. THE PARAMETER NN IS THE SIZE OF
14: C     RANDOMLY CHOSEN NEIGHBOURS. 15 TO 25 (BUT SUFFICIENTLY LESS THAN
15: C     N) IS A GOOD CHOICE. MX IS THE MAXIMAL SIZE OF DECISION VARIABLES.
16: C     IN F(X1, X2, ..., XM) M SHOULD BE LESS THAN OR EQUAL TO MX. ITRN IS
17: C     THE NO. OF ITERATIONS. IT MAY DEPEND ON THE PROBLEM. 200 (AT LEAST)
18: C     TO 500 ITERATIONS MAY BE GOOD ENOUGH. BUT FOR FUNCTIONS LIKE
19: C     ROSENBROCKOR GRIEWANK OF LARGE SIZE (SAY M=30) IT IS NEEDED THAT
20: C     ITRN IS LARGE, SAY 5000 OR EVEN 10000.
21: C     SIGMA INTRODUCES PERTURBATION & HELPS THE SEARCH JUMP OUT OF LOCAL
22: C     OPTIMA. FOR EXAMPLE : RASTRIGIN FUNCTION OF DMENSION 30 OR LARGER
23: C     NSTEP DOES LOCAL SEARCH BY TUNNELLING AND WORKS WELL BETWEEN 5 AND
24: C     15, WHICH IS MUCH ON THE HIGHER SIDE.
25: C     ITOP <=1 (RING); ITOP=2 (RING AND RANDOM); ITOP=>3 (RANDOM)
26: C     NSIGMA=0 (NO CHAOTIC PERTURBATION); NSIGMA=1 (CHAOTIC PERTURBATION)
27: C     NOTE THAT NSIGMA=1 NEED NOT ALWAYS WORK BETTER (OR WORSE)
28: C     SUBROUTINE FUNC( ) DEFINES OR CALLS THE FUNCTION TO BE OPTIMIZED.
29: C     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
30: C     COMMON /RNDM/IU, IV
31: C     COMMON /KFF/KF, NFCALL
32: C     INTEGER IU, IV
33: C     CHARACTER *70 TIT
34: C     DIMENSION X(N, MX), V(N, MX), A(MX), VI(MX), TIT(50)
35: C     DIMENSION XX(N, MX), F(N), V1(MX), V2(MX), V3(MX), V4(MX), BST(MX)
36: C     A1 A2 AND A3 ARE CONSTANTS AND W IS THE INERTIA WEIGHT.
37: C     OCCASIONALLY, TINKERING WITH THESE VALUES, ESPECIALLY A3, MAY BE
38: C     NEEDED.
39: C     DATA A1, A2, A3, W, SIGMA /.5D00, .5D00, .0005D00, .5D00, 1.D-03/
40:
41: C     WRITE (*, *) '-----'
42: C     DATA TIT(1) /'KF=1 NEW FUNCTION(N#1) 2-VARIABLES M=2'/
43: C     DATA TIT(2) /'KF=2 NEW FUNCTION(N#2) 2-VARIABLES M=2'/
44: C     DATA TIT(3) /'KF=3 NEW FUNCTION(N#3) 2-VARIABLES M=2'/
45: C     DATA TIT(4) /'KF=4 NEW FUNCTION(N#4) 2-VARIABLES M=2'/
46: C     DATA TIT(5) /'KF=5 NEW QUINTIC FUNCTION M-VARIABLES M=?'/
47: C     DATA TIT(6) /'KF=6 NEW NEEDLE-EYE FUNCTION (N#6) M-VARIABLES M=?'/
48: C     DATA TIT(7) /'KF=7 NEW ZERO-SUM FUNCTION (N#7) M-VARIABLES M=?'/
49: C     DATA TIT(8) /'KF=8 CORANA FUNCTION 4-VARIABLES M=4'/
50: C     DATA TIT(9) /'KF=9 MODIFIED RCOS FUNCTION 2-VARIABLES M=2'/
51: C     DATA TIT(10) /'KF=10 FREUDENSTEIN ROTH FUNCTION 2-VARIABLES M=2'/
52: C     DATA TIT(11) /'KF=11 ANNS XOR FUNCTION 9-VARIABLES M=9'/
53: C     DATA TIT(12) /'KF=12 PERM FUNCTION #1 (SET BETA) 4-VARIABLES M=4'/
54: C     DATA TIT(13) /'KF=13 PERM FUNCTION #2 (SET BETA) M-VARIABLES M=?'/
55: C     DATA TIT(14) /'KF=14 POWER-SUM FUNCTION 4-VARIABLES M=4'/
56: C     DATA TIT(15) /'KF=15 GOLDSTEIN PRICE FUNCTION 2-VARIABLES M=2'/
57: C     DATA TIT(16) /'KF=16 BUKIN 6TH FUNCTION 2-VARIABLES M=2'/
58: C     DATA TIT(17) /'KF=17 NEW FUNCTION (N#8) 2-VARIABLES M=2'/
59: C     DATA TIT(18) /'KF=18 DEF'L CORRUG SPRING FUNCTION M-VARIABLES M=?'/
60: C     DATA TIT(19) /'KF=19 NEW FACTORIAL FUNCTION M-VARIABLES M=?'/
61: C     DATA TIT(20) /'KF=20 NEW DECANOMIAL FUNCTION 2-VARIABLES M=2'/
62: C     DATA TIT(21) /'KF=21 JUDGE FUNCTION 2-VARIABLES M=2'/
63: C     DATA TIT(22) /'KF=22 NEW DODECAL FUNCTION 3-VARIABLES M=3'/
64: C     DATA TIT(23) /'KF=23 NEW SUM-EQ-PROD FUNCTION 2-VARIABLES M=2'/
65: C     DATA TIT(24) /'KF=24 NEW AM-EQ-GM FUNCTION M-VARIABLES M=?'/
66: C     DATA TIT(25) /'KF=25 YAO-LIU FUNCTION#2 M-VARIABLES M=?'/
67: C     DATA TIT(26) /'KF=26 YAO-LIU FUNCTION#3 M-VARIABLES M=?'/

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68:      DATA TIT(27) / 'KF=27 YAO-LIU FUNCTION#4 M-VARIABLES M=?' /
69:      DATA TIT(28) / 'KF=28 YAO-LIU FUNCTION#6 M-VARIABLES M=?' /
70:      DATA TIT(29) / 'KF=29 YAO-LIU FUNCTION#7 M-VARIABLES M=?' /
71:      DATA TIT(30) / 'KF=30 YAO-LIU FUNCTION#12 M-VARIABLES M=?' /
72:      DATA TIT(31) / 'KF=31 YAO-LIU FUNCTION#13 M-VARIABLES M=?' /
73:      DATA TIT(32) / 'KF=32 YAO-LIU FUNCTION#14 2-VARIABLES M=2' /
74:      DATA TIT(33) / 'KF=33 YAO-LIU FUNCTION#15 4-VARIABLES M=4' /
75:      DATA TIT(34) / 'KF=34 LINEAR PROGRAMMING-I : 2-VARIABLES M=2' /
76:      DATA TIT(35) / 'KF=35 LINEAR PROGRAMMING-II : 3-VARIABLES M=3' /
77:      DATA TIT(36) / 'KF=36 TRIGON FUNCTION : M-VARIABLES M=?' /
78:      C -----
79:      DO I=1,36
80:      WRITE(*,*) TIT(I)
81:      ENDDO
82:      WRITE(*,*) '-----'
83:      WRITE(*,*) 'CHOOSE FUNCTION CODE [KF] AND NO. OF VARIABLES [M]'
84:      READ(*,*) KF,M
85:
86:      LCOUNT=0
87:      NFCALL=0
88:      WRITE(*,*) '4-DIGITS SEED FOR RANDOM NUMBER GENERATION'
89:      READ(*,*) IU
90:      DATA FMIN /1.0E30/
91:      C GENERATE N-SIZE POPULATION OF M-TUPLE PARAMETERS X(I,J) RANDOMLY
92:      DO I=1,N
93:          DO J=1,M
94:              CALL RANDOM(RAND)
95:              X(I,J)=(RAND-0.5D00)*10
96:      C WE GENERATE RANDOM(-5,5). HERE MULTIPLIER IS 10. TINKERING IN SOME
97:      C CASES MAY BE NEEDED
98:          ENDDO
99:          F(I)=1.0D30
100:      ENDDO
101:      C INITIALISE VELOCITIES V(I) FOR EACH INDIVIDUAL IN THE POPULATION
102:      DO I=1,N
103:          DO J=1,M
104:              CALL RANDOM(RAND)
105:              V(I,J)=(RAND-0.5D+00)
106:      C V(I,J)=RAND
107:          ENDDO
108:      ENDDO
109:      DO 100 ITER=1,ITRN
110:      C LET EACH INDIVIDUAL SEARCH FOR THE BEST IN ITS NEIGHBOURHOOD
111:          DO I=1,N
112:              DO J=1,M
113:                  A(J)=X(I,J)
114:                  VI(J)=V(I,J)
115:              ENDDO
116:              CALL LSRCH(A,M,VI,NSTEP,F,I)
117:              IF(FI.LT.F(I)) THEN
118:                  F(I)=FI
119:                  DO IN=1,M
120:                      BST(IN)=A(IN)
121:                  ENDDO
122:      C F(I) CONTAINS THE LOCAL BEST VALUE OF FUNCTION FOR ITH INDIVIDUAL
123:      C XX(I,J) IS THE M-TUPLE VALUE OF X ASSOCIATED WITH LOCAL BEST F(I)
124:                  DO J=1,M
125:                      XX(I,J)=A(J)
126:                  ENDDO
127:              ENDIF
128:          ENDDO
129:      C NOW LET EVERY INDIVIDUAL RANDOMLY COSULT NN(<<N) COLLEAGUES AND
130:      C FIND THE BEST AMONG THEM
131:      DO I=1,N
132:      C -----
133:      IF(ITOP.GE.3) THEN
134:      C RANDOM TOPOLOGY *****

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135: C      CHOOSE NN COLLEAGUES RANDOMLY AND FIND THE BEST AMONG THEM
136:         BEST=1.0D30
137:         DO II=1,NN
138:             CALL RANDOM(RAND)
139:             NF=INT(RAND*N)+1
140:             IF(BEST.GT.F(NF)) THEN
141:                 BEST=F(NF)
142:                 NFBEST=NF
143:             ENDIF
144:         ENDDO
145:     ENDIF
146: C-----
147:     IF(ITOP.EQ.2) THEN
148: C      RING + RANDOM TOPOLOGY *****
149: C      REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
150:         BEST=1.0D30
151:         CALL NEIGHBOR(I,N,I1,I3)
152:         DO II=1,NN
153:             IF(II.EQ.1) NF=I1
154:             IF(II.EQ.2) NF=I
155:             IF(II.EQ.3) NF=I3
156:             IF(II.GT.3) THEN
157:                 CALL RANDOM(RAND)
158:                 NF=INT(RAND*N)+1
159:             ENDIF
160:             IF(BEST.GT.F(NF)) THEN
161:                 BEST=F(NF)
162:                 NFBEST=NF
163:             ENDIF
164:         ENDDO
165:     ENDIF
166: C-----
167:     IF(ITOP.LE.1) THEN
168: C      RING TOPOLOGY *****
169: C      REQUIRES THAT THE SUBROUTINE NEIGHBOR IS TURNED ALIVE
170:         BEST=1.0D30
171:         CALL NEIGHBOR(I,N,I1,I3)
172:         DO II=1,3
173:             IF(II.NE.I) THEN
174:                 IF(II.EQ.1) NF=I1
175:                 IF(II.EQ.3) NF=I3
176:                 IF(BEST.GT.F(NF)) THEN
177:                     BEST=F(NF)
178:                     NFBEST=NF
179:                 ENDIF
180:             ENDIF
181:         ENDDO
182:     ENDIF
183: C-----
184: C      IN THE LIGHT OF HIS OWN AND HIS BEST COLLEAGUES EXPERIENCE, THE
185: C      INDIVIDUAL I WILL MODIFY HIS MOVE AS PER THE FOLLOWING CRITERION
186: C      FIRST, ADJUSTMENT BASED ON ONES OWN EXPERIENCE
187: C      AND OWN BEST EXPERIENCE IN THE PAST (XX(I))
188:         DO J=1,M
189:             CALL RANDOM(RAND)
190:             V1(J)=A1*RAND*(XX(I,J)-X(I,J))
191: C      THEN BASED ON THE OTHER COLLEAGUES BEST EXPERIENCE WITH WEIGHT W
192: C      HERE W IS CALLED AN INERTIA WEIGHT 0.01< W < 0.7
193: C      A2 IS THE CONSTANT NEAR BUT LESS THAN UNITY
194:             CALL RANDOM(RAND)
195:             V2(J)=V(I,J)
196:             IF(F(NFBEST).LT.F(I)) THEN
197:                 V2(J)=A2*W*RAND*(XX(NFBEST,J)-X(I,J))
198:             ENDIF
199: C      THEN SOME RANDOMNESS AND A CONSTANT A3 CLOSE TO BUT LESS THAN UNITY
200:             CALL RANDOM(RAND)
201:             RND1=RAND

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202:          CALL RANDOM(RAND)
203:          V3(J)=A3*RAND*W*RND1
204: C          V3(J)=A3*RAND*W
205: C          THEN ON PAST VELOCITY WITH INERTIA WEIGHT W
206:          V4(J)=W*V(I,J)
207: C          FINALLY A SUM OF THEM
208:          V(I,J)= V1(J)+V2(J)+V3(J)+V4(J)
209:          ENDDO
210: ENDDO
211: C          CHANGE X
212: DO I=1,N
213: DO J=1,M
214: RANDB=0.D00
215: C          -----
216: IF(NSIGMA.EQ.1) THEN
217:   CALL RANDOM(RAND) ! FOR CHAOTIC PERTURBATION
218:   IF(DABS(RAND-.5D00).LT.SIGMA) RANDB=RAND-0.5D00
219: C   SIGMA CONDITIONED RANDB INTRODUCES CHAOTIC ELEMENT IN TO LOCATION
220: C   IN SOME CASES THIS PERTURBATION HAS WORKED VERY EFFECTIVELY WITH
221: C   PARAMETER (N=100,NN=15,MX=100,NSTEP=9,ITRN=100000,NSIGMA=1,ITOP=2)
222: ENDF
223: C          -----
224: X(I,J)=X(I,J)+V(I,J)*(1.D00+RANDB)
225: ENDDO
226: ENDDO
227: DO I=1,N
228:   IF(F(I).LT.FMIN) THEN
229:     FMIN=F(I)
230:     II=I
231:     DO J=1,M
232:       BST(J)=XX(II,J)
233:     ENDDO
234:   ENDF
235: ENDDO
236: IF(LCOUNT.EQ.NPRN) THEN
237:   LCOUNT=0
238:   WRITE(*,*) 'OPTIMAL SOLUTION UPTO THIS (FUNCTION CALLS=',NFCALL,')'
239:   WRITE(*,*) 'X = ',(BST(J),J=1,M), ' MIN F = ',FMIN
240: C   WRITE(*,*) 'NO. OF FUNCTION CALLS = ',NFCALL
241: ENDF
242: LCOUNT=LCOUNT+1
243: 100 CONTINUE
244: WRITE(*,*) 'COMPUTATION OVER:',TIT(KF)
245:
246: END
247: C          -----
248: SUBROUTINE LSRCH(A,M,VI,NSTEP,FI)
249: IMPLICIT DOUBLE PRECISION (A-H,O-Z)
250: COMMON /KFF/KF,NFCALL
251: COMMON /RNDM/IU,IV
252: INTEGER IU,IV
253: DIMENSION A(*),B(100),VI(*)
254: AMN=1.0D30
255: DO J=1,NSTEP
256:   DO JJ=1,M
257:     B(JJ)=A(JJ)+(J-(NSTEP/2)-1)*VI(JJ)
258:   ENDDO
259: CALL FUNC(B,M,FI)
260: IF(FI.LT.AMN) THEN
261:   AMN=FI
262:   DO JJ=1,M
263:     A(JJ)=B(JJ)
264:   ENDDO
265: ENDF
266: ENDDO
267: FI=AMN
268: RETURN

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269:      END
270: C -----
271: C THIS SUBROUTINE IS NEEDED IF THE NEIGHBOURHOOD HAS RING TOPOLOGY
272: C EITHER PURE OR HYBRIDIZED
273:      SUBROUTINE NEIGHBOR(I,N,J,K)
274:      IF (I-1.GE.1 .AND. I.LT.N) THEN
275:        J=I-1
276:        K=I+1
277:      ELSE
278:      IF (I-1.LT.1) THEN
279:        J=N-I+1
280:        K=I+1
281:      ENDIF
282:      IF (I.EQ.N) THEN
283:        J=I-1
284:        K=1
285:      ENDIF
286:      ENDIF
287:      RETURN
288:      END
289: C -----
290:      SUBROUTINE RANDOM(RAND1)
291:      DOUBLE PRECISION RAND1
292:      COMMON /RNDM/IU,IV
293:      INTEGER IU,IV
294:      RAND=REAL(RAND1)
295:      IV=IU*65539
296:      IF (IV.LT.0) THEN
297:        IV=IV+2147483647+1
298:      ENDIF
299:      RAND=IV
300:      IU=IV
301:      RAND=RAND*0.4656613E-09
302:      RAND1=DBLE(RAND)
303:      RETURN
304:      END
305: C -----
306:      SUBROUTINE FUNC(X,M,F)
307: C TEST FUNCTIONS FOR GLOBAL OPTIMIZATION PROGRAM
308:      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
309:      COMMON /RNDM/IU,IV
310:      COMMON /KFF/KF,NFCALL
311:      INTEGER IU,IV
312:      DIMENSION X(*)
313:
314:      PI=4.D+00*DATAN(1.D+00)
315:      NFCALL=NFCALL+1
316: C -----
317:      IF (KF.EQ.1) THEN
318: C FUNCTION #1 MIN AT -0.18467 APPROX AT (-8.4666, -9.9988) APPROX
319:      F=0.D00
320:      FP=0.D00
321:      DO I=1,M
322:        IF (DABS(X(I)).GT.10.D00) FP=FP+DEXP(DABS(X(I)))
323:      ENDDO
324:      IF (FP.NE.0.D00) THEN
325:        F=FP
326:      RETURN
327:      ELSE
328:      F=DABS(DCOS(DSQRT(DABS(X(1)**2+X(2)))))**0.5 +0.01*X(1)+.01*X(2)
329:      RETURN
330:      ENDIF
331:      ENDIF
332: C -----
333:      IF (KF.EQ.2) THEN
334: C FUNCTION #2 MIN = -0.199409 APPROX AT (-9.94112, -9.99952) APPROX
335:      F=0.D00

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336:      FP=0.D00
337:      DO I=1,M
338:      IF (DABS(X(I)) .GT. 10.D00) FP=FP+DEXP(DABS(X(I)))
339:      ENDDO
340:      IF (FP .NE. 0.D00) THEN
341:      F=FP
342:      RETURN
343:      ELSE
344:      F=DABS(DSIN(DSQRT(DABS(X(1)**2+X(2)))))**0.5 +0.01*X(1)+.01*X(2)
345:      RETURN
346:      ENDIF
347:      ENDIF
348: C -----
349:      IF (KF.EQ.3) THEN
350: C      FUNCTION #3 MIN = -1.01983 APPROX AT (-1.98682, -10.00000) APPROX
351:      F=0.D00
352:      FP=0.D00
353:      DO I=1,M
354:      IF (DABS(X(I)) .GT. 10.D00) FP=FP+DEXP(DABS(X(I)))
355:      ENDDO
356:      IF (FP .NE. 0.D00) THEN
357:      F=FP
358:      RETURN
359:      ELSE
360:      F1=DSIN((DCOS(X(1))+DCOS(X(2)))**2)**2
361:      F2=DCOS((DSIN(X(1))+DSIN(X(2)))**2)**2
362:      F=(F1+F2+X(1))**2 ! IS MULTIMODAL
363:      F=F+ 0.01*X(1)+0.1*X(2) ! MAKES UNIMODAL
364:      RETURN
365:      ENDIF
366:      ENDIF
367: C -----
368:      IF (KF.EQ.4) THEN
369: C      FUNCTION #4 MIN = -2.28395 APPROX AT (2.88631, 1.82326) APPROX
370:      F=0.D00
371:      FP=0.D00
372:      DO I=1,M
373:      IF (DABS(X(I)) .GT. 10.D00) FP=FP+DEXP(DABS(X(I)))
374:      ENDDO
375:      IF (FP .NE. 0.D00) THEN
376:      F=FP
377:      RETURN
378:      ELSE
379:      F1=DSIN((DCOS(X(1))+DCOS(X(2)))**2)**2
380:      F2=DCOS((DSIN(X(1))+DSIN(X(2)))**2)**2
381:      F3=-DLOG((F1-F2+X(1))**2)
382:      F=F3+0.1D00*(X(1)-1.D00)**2+0.1D00*(X(2)-1.D00)**2
383:      RETURN
384:      ENDIF
385:      ENDIF
386: C -----
387:      IF (KF.EQ.5) THEN
388: C      QUINTIC FUNCTION: GLOBAL MINIMA, EXTREMELY DIFFICULT TO OPTIMIZE
389: C      MIN VALUE = 0 AT PERMUTATION OF (2, 2, ..., 2, -1, -1, ....., -1)
390:
391: C      CHECK FOR OPTIMUM SOLUTION : SET X(I) = -1 OR 2 SUCH AS X=(-1,-1)
392: C      X=(2,2) OR X=(2,-1) OR X=(-1,2) OR TURN ALIVE THE FOLLOWING BY
393: C      REMOVING C FROM THE FIRST COLUMN -----
394: C      X(1)=? ! SET IT TO -1 OR 2
395: C      X(2)=? ! SET IT TO -1 OR 2
396: C      OR FOR M => 1 TURN THE FOLLOWING ALIVE BY REMOVING C IN THE FIRST
397: C      COLUMN OF THE LINE -----
398: C      DO I=1,M ! TURN IT ALIVE
399: C      CALL RANDOM(RAND) ! TURN IT ALIVE
400: C      IF (RAND.LT.0.5D00) THEN ! TURN IT ALIVE
401: C      X(I)=-1.D00 ! TURN IT ALIVE
402: C      ELSE ! TURN IT ALIVE

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403: C      X(I)=2 ! TURN IT ALIVE
404: C      ENDIF ! TURN IT ALIVE
405: C      ENDDO ! TURN IT ALIVE
406: C      TEST OVER -----
407:
408:      FP=0.D00
409:      F=0.D00
410:      DO I=1,M
411:      IF (DABS(X(I)).GT.10.D00) FP=FP+DEXP(DABS(X(I)))
412: ENDDO
413:      IF (FP.NE.0.D00) THEN
414:      F=FP
415:      RETURN
416:      ELSE
417:      CALL QUINTIC(M,F,X)
418:      RETURN
419:      ENDIF
420:      ENDIF
421: C      -----
422:      IF (KF.EQ.6) THEN
423: C      NEEDLE-EYE FUNCTION M=>1;
424: C      MIN = 1 IF ALL ABS(X) ARE SMALLER THAN THE EYE
425: C      SMALLER THE VALUE OF ZZ, MORE DIFFICULT TO ENTER THE EYE
426: C      LARGER THE VALUE OF M, MORE DIFFICULT TO FIND THE OPTIMUM
427:      F=0.D00
428:      EYE=0.0001D00
429:      FP=0.D00
430:      DO I=1,M
431:      IF (DABS(X(I)).GT.EYE) THEN
432:      FP=1.D00
433:      F=F+100.D00+DABS(X(I))
434:      ELSE
435:      F=F+1.D00
436:      ENDIF
437:      ENDDO
438:      IF (FP.EQ.0.D00) F=F/M
439:      RETURN
440:      ENDIF
441: C      -----
442:      IF (KF.EQ.7) THEN
443: C      ZERO SUM FUNCTION
444: C      MIN = 0 AT SUM(X(I))=0
445:      F=0.D00
446:      FP=0.D00
447:      DO I=1,M
448:      IF (DABS(X(I)).GT.10.D00) FP=FP+DEXP(DABS(X(I)))
449: ENDDO
450:      IF (FP.NE.0.D00) THEN
451:      F=FP
452:      RETURN
453:      ELSE
454:      SUM=0.D00
455:      DO I=1,M
456:      SUM=SUM+X(I)
457: ENDDO
458:      IF (SUM.NE.0.D00) F=1.D00+(10000*DABS(SUM))**0.5
459:      RETURN
460:      ENDIF
461:      ENDIF
462: C      -----
463:      IF (KF.EQ.8) THEN
464: C      CORANA FUNCTION
465: C      MIN = 0 AT (0, 0, 0, 0) APPROX
466:      F=0.D00
467:      FP=0.D00
468: C      -1000 TO 1000 M=4
469:      DO I=1,4

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470:         IF (DABS (X (I)) .GT. 1000.D00) FP=FP+X (I) **2
471:         ENDDO
472:         IF (FP .GT. 0.D00) THEN
473:           F=FP
474:         ELSE
475:         DO J=1, 4
476:           IF (J .EQ. 1) DJ=1.D00
477:           IF (J .EQ. 2) DJ=1000.D00
478:           IF (J .EQ. 3) DJ=10.D00
479:           IF (J .EQ. 4) DJ=100.D00
480:           ISGNXJ=1
481:           IF (X (J) .LT. 0.D00) ISGNXJ=-1
482:           ZJ=(DABS (X (J) / 0.2D00) + 0.49999) * ISGNXJ * 0.2D00
483:           ISGNZJ=1
484:           IF (ZJ .LT. 0.D00) ISGNZJ=-1
485:           IF (DABS (X (J) - ZJ) .LT. 0.05D00) THEN
486:             F=F+0.15D00 * (ZJ - 0.05D00 * ISGNZJ) **2 * DJ
487:           ELSE
488:             F=F+DJ * X (J) **2
489:           ENDIF
490:         ENDDO
491:         ENDIF
492:         RETURN
493:       ENDIF
494: C -----
495:       IF (KF .EQ. 9) THEN
496: C     MODIFIED RCOS FUNCTION MIN=-0.179891 AT (-3.196989, 12.52626) APPRX
497:       F=0.D00
498:       FP=0.D00
499:       IF (X (1) .LT. -5.D00 .OR. X (1) .GT. 10.D00) FP=FP+DEXP (DABS (X (1)))
500:       IF (X (2) .LT. -0.D00 .OR. X (2) .GT. 15.D00) FP=FP+DEXP (DABS (X (2)))
501:       IF (FP .NE. 0.D00) THEN
502:         F=FP
503:       RETURN
504:       ELSE
505:       CA=1.D00
506:       CB=5.1 / (4 * PI **2)
507:       CC=5.D00 / PI
508:       CD=6.D00
509:       CE=10.D00
510:       CF=1.0 / (8 * PI)
511:       F1=CA * (X (2) - CB * X (1) **2 + CC * X (1) - CD) **2
512:       F2=CE * (1.D00 - CF) * DCOS (X (1)) * DCOS (X (2))
513:       F3=DLOG (X (1) **2 + X (2) **2 + 1.D00)
514:       F=-1.0 / (F1 + F2 + F3 + CE)
515:       RETURN
516:       ENDIF
517:     ENDIF
518: C -----
519:       IF (KF .EQ. 10) THEN
520: C     FREUDENSTEIN ROTH FUNCTION
521: C     MIN = 0 AT (5, 4)
522:       F=0.D00
523:       FP=0.D00
524:       DO I=1, M
525:         IF (DABS (X (I)) .GT. 10.D00) FP=FP+DEXP (DABS (X (I)))
526:       ENDDO
527:       IF (FP .NE. 0.D00) THEN
528:         F=FP
529:       RETURN
530:       ELSE
531:       F1=(-13.D00 + X (1) + ((5.D00 - X (2)) * X (2) - 2) * X (2)) **2
532:       F2=(-29.D00 + X (1) + ((X (2) + 1.D00) * X (2) - 14.D00) * X (2)) **2
533:       F=F1 + F2
534:       RETURN
535:       ENDIF
536:     ENDIF

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537: C -----
538: IF (KF.EQ.11) THEN
539: C ANNS XOR FUNCTION (PARSOPOULOS, KE, PLAGIANAKOS, VP, MAGOULAS, GD
540: C AND VRAHATIS, MN "STRETCHING TECHNIQUE FOR OBTAINING GLOBAL
541: C MINIMIZERS THROUGH PARTICLE SWARM OPTIMIZATION")
542: C MIN = 0.959789 AT X = (0.99999, 0.99993, -0.89414, 0.99994,
543: C 0.55932, 0.99994, 0.99994, -0.99963, -0.08272).
544: C F=0.D00
545: C FP=0.D00
546: C DO I=1,M
547: C IF (DABS(X(I)).GT.1.D00) FP=FP+DEXP(10.D00+DABS(X(I)))
548: C ENDDO
549: C IF (FP.NE.0.D00) THEN
550: C F=FP
551: C RETURN
552: C ELSE
553: C F11=X(7)/(1.D00+DEXP(-X(1)-X(2)-X(5)))
554: C F12=X(8)/(1.D00+DEXP(-X(3)-X(4)-X(6)))
555: C F1=(1.D00+DEXP(-F11-F12-X(9)))**(-2)
556: C F21=X(7)/(1.D00+DEXP(-X(5)))
557: C F22=X(8)/(1.D00+DEXP(-X(6)))
558: C F2=(1.D00+DEXP(-F21-F22-X(9)))**(-2)
559: C F31=X(7)/(1.D00+DEXP(-X(1)-X(5)))
560: C F32=X(8)/(1.D00+DEXP(-X(3)-X(6)))
561: C F3=(1.D00-(1.D00+DEXP(-F31-F32-X(9)))**(-1))**2
562: C F41=X(7)/(1.D00+DEXP(-X(2)-X(5)))
563: C F42=X(8)/(1.D00+DEXP(-X(4)-X(6)))
564: C F4=(1.D00-(1.D00+DEXP(-F41-F42-X(9)))**(-1))**2
565: C F=F1+F2+F3+F4
566: C RETURN
567: C ENDIF
568: C ENDIF
569: C -----
570: IF (KF.EQ.12) THEN
571: C PERM FUNCTION #1 MIN = 0 AT (1, 2, 3, 4)
572: C BETA => 0. CHANGE IF NEEDED. SMALLER BETA RAISES DIFFICULTY
573: C FOR BETA=0, EVERY PERMUTED SOLUTION IS A GLOBAL MINIMUM
574: C BETA=50.D00
575: C F=0.D00
576: C FP=0.D00
577: C DO I=1,M
578: C IF (DABS(X(I)).GT.M) FP=FP+X(I)**2
579: C ENDDO
580: C IF (FP.NE.0.D00) THEN
581: C F=FP
582: C RETURN
583: C ELSE
584: C DO K=1,M
585: C SUM=0.D00
586: C DO I=1,M
587: C SUM=SUM+(I**K+BETA)*((X(I)/I)**K-1.D00)
588: C ENDDO
589: C F=F+SUM**2
590: C ENDDO
591: C RETURN
592: C ENDIF
593: C ENDIF
594: C -----
595: IF (KF.EQ.13) THEN
596: C PERM FUNCTION #2 MIN = 0 AT (1/1, 1/2, 1/3, 1/4, ..., 1/M)
597: C BETA => 0. CHANGE IF NEEDED. SMALLER BETA RAISES DIFFICULTY
598: C FOR BETA=0, EVERY PERMUTED SOLUTION IS A GLOBAL MINIMUM
599: C BETA=10.D00
600: C F=0.D00
601: C FP=0.D00
602: C DO I=1,M
603: C TO CHECK MIN=0, TURN THE FOLLOWING STATEMENT [(X(I)=1.D00/I) ALIVE

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604: C      X(I)=1.D00/I      ! TURN IT ALIVE
605:      IF (DABS (X (I)) .GT.1.D00) FP=FP+(1.D0+DABS (X (I)))**2
606:      ENDDO
607:      IF (FP.NE.0.D00) THEN
608:      F=FP
609:      RETURN
610:      ELSE
611:      DO K=1,M
612:      SUM=0.D00
613:      DO I=1,M
614:      SUM=SUM+(I+BETA)*(X(I)**K-(1.D00/I)**K)
615:      ENDDO
616:      F=F+SUM**2
617:      ENDDO
618:      RETURN
619:      ENDIF
620:      ENDIF
621: C      -----
622:      IF (KF.EQ.14) THEN
623: C      POWER SUM FUNCTION; MIN = 0 AT PERM(1,2,2,3) FOR B=(8,18,44,114)
624: C      0 =< X <=4
625:      F=0.D00
626:      FP=0.D00
627:      DO I=1,M
628: C      TURN THE FOLLOWING STATEMENTS ALIVE TO CHECK SOLUTION FOR ANY
629: C      PERMUTATION OF X=(1,2,2,3), FOR EXAMPLE
630: C      IF (I.EQ.1) X(I)=3.D00 ! TURN IT ALIVE
631: C      IF (I.EQ.2) X(I)=1.D00 ! TURN IT ALIVE
632: C      IF (I.EQ.3) X(I)=2.D00 ! TURN IT ALIVE
633: C      IF (I.EQ.4) X(I)=2.D00 ! TURN IT ALIVE
634: C      ANY PERMUTATION OF (1,2,2,3) WILL GIVE MIN = ZERO
635:      IF (X(I) .LT.0.D00 .OR. X(I) .GT.4.D00) FP=FP+(10.D0+DABS (X (I)))**2
636:      ENDDO
637:      IF (FP.NE.0.D00) THEN
638:      F=FP
639:      RETURN
640:      ELSE
641:      DO K=1,M
642:      SUM=0.D00
643:      DO I=1,M
644:      SUM=SUM+X(I)**K
645:      ENDDO
646:      IF (K.EQ.1) B=8.D00
647:      IF (K.EQ.2) B=18.D00
648:      IF (K.EQ.3) B=44.D00
649:      IF (K.EQ.4) B=114.D00
650:      F=F+(SUM-B)**2
651:      ENDDO
652:      RETURN
653:      ENDIF
654:      ENDIF
655: C      -----
656:      IF (KF.EQ.15) THEN
657: C      GOLDSTEIN PRICE FUNCTION
658: C      MIN VALUE = 3 AT (0, -1)
659:      F=0.D00
660:      FP=0.D00
661:      DO I=1,M
662:      IF (DABS (X (I)) .GT.10.D00) FP=FP+DEXP (DABS (X (I)))
663:      ENDDO
664:      IF (FP.NE.0.D00) THEN
665:      F=FP
666:      RETURN
667:      ELSE
668:      F11=(X(1)+X(2)+1.D00)**2
669:      F12=(19.D00-14*X(1)+ 3*X(1)**2-14*X(2)+ 6*X(1)*X(2)+ 3*X(2)**2)
670:      F1=1.00+F11*F12

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671:      F21=(2*X(1)-3*X(2))**2
672:      F22=(18.D00-32*X(1)+12*X(1)**2+48*X(2)-36*X(1)*X(2)+27*X(2)**2)
673:      F2=30.D00+F21*F22
674:      F= (F1*F2)
675:      RETURN
676:      ENDIF
677:      ENDIF
678: C -----
679:      IF (KF.EQ.16) THEN
680: C      BUKIN'S 6TH FUNCTION  MIN = 0 FOR (-10, 1)
681:      FP=0.D00
682: C      -15. LE. X(1) .LE. -5 AND -3 .LE. X(2) .LE. 3
683:      IF (X(1) .LT. -15.D00 .OR. X(1) .GT. -5.D00) FP=FP+X(1)**2
684:      IF (DABS(X(2)) .GT. 3.D00) FP=FP+X(2)**2
685:      IF (FP .GT. 0.D00) THEN
686:      F=FP
687:      ELSE
688:      F=100*DSQRT(DABS(X(2)-0.01D00*X(1)**2)) + 0.01*DABS(X(1)+10.D0)
689:      ENDIF
690:      RETURN
691:      ENDIF
692: C -----
693:      IF (KF.EQ.17) THEN
694: C      NEW N#8 FUNCTION (MULTIPLE GLOBAL MINIMA)
695: C      MIN VALUE = -1 AT (AROUND .7 AROUND, 0.785 APPROX)
696:      F=0.D00
697:      FP=0.D00
698:      DO I=1,M
699:      IF (X(I) .LT. 0.5D00 .OR. X(I) .GT. 1.D00) FP=FP+DEXP(2.D00+DABS(X(I)))
700:      ENDDO
701:      IF (FP .NE. 0.D00) THEN
702:      F=FP
703:      RETURN
704:      ELSE
705:      F=-DEXP(-DABS(DLOG(.001D00+DABS((DSIN(X(1)+X(2))+DSIN(X(1)-X(2)))+
706:      & (DCOS(X(1)+X(2))*DCOS(X(1)-X(2))+.001)**2))+
707:      & .01D00*(X(2)-X(1))**2)))
708:      ENDIF
709:      RETURN
710:      ENDIF
711: C -----
712:      IF (KF.EQ.18) THEN
713: C      DEFLECTED CORRUGATED SPRING FUNCTION
714: C      MIN VALUE = -1 AT (5, 5, ..., 5) FOR ANY K AND ALPHA=5; M VARIABLE
715:      CALL DCS(M,F,X)
716:      RETURN
717:      ENDIF
718: C -----
719:      IF (KF.EQ.19) THEN
720: C      FACTORIAL FUNCTION, MIN =0 AT X=(1,2,3,...,M)
721:      CALL FACTOR1(M,F,X)
722:      RETURN
723:      ENDIF
724: C -----
725:      IF (KF.EQ.20) THEN
726: C      DECANOMIAL FUNCTION, MIN =0 AT X=(2, -3)
727:      FP=0.D00
728:      IF (X(1) .LT. -4.D00 .OR. X(1) .GT. 4.D00) FP=FP+(100+DABS(X(1)))**2
729:      IF (X(2) .LT. -4.D00 .OR. X(2) .GT. 4.D00) FP=FP+(100+DABS(X(2)))**2
730:      IF (FP .GT. 0.D00) THEN
731:      F=FP
732:      ELSE
733:      CALL DECANOM(M,F,X)
734:      ENDIF
735:      RETURN
736:      ENDIF
737: C -----

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738:      IF (KF.EQ.21) THEN
739: C      JUDGE'S FUNCTION  $F(0.864, 1.23) = 16.0817$ ; M=2
740:      CALL JUDGE (M, X, F)
741:      RETURN
742:      ENDIF
743: C      -----
744:      IF (KF.EQ.22) THEN
745: C      DODECAL FUNCTION
746:      CALL DODECAL (M, F, X)
747:      RETURN
748:      ENDIF
749: C      -----
750:      IF (KF.EQ.23) THEN
751: C      WHEN  $X(1)*X(2)=X(1)*X(2)$  ? M=2
752:      CALL SEQP (M, F, X)
753:      RETURN
754:      ENDIF
755: C      -----
756:      IF (KF.EQ.24) THEN
757: C      WHEN ARITHMETIC MEAN = GEOMETRIC MEAN ?
758: C      M =>1
759:      CALL AMGM (M, F, X)
760:      RETURN
761:      ENDIF
762: C      -----
763:      IF (KF.EQ.25) THEN
764: C      M =>2
765:      CALL FUNCT2 (M, F, X)
766:      RETURN
767:      ENDIF
768: C      -----
769:      IF (KF.EQ.26) THEN
770: C      M =>2
771:      CALL FUNCT3 (M, F, X)
772:      RETURN
773:      ENDIF
774: C      -----
775:      IF (KF.EQ.27) THEN
776: C      M =>2
777:      CALL FUNCT4 (M, F, X)
778:      RETURN
779:      ENDIF
780: C      -----
781:      IF (KF.EQ.28) THEN
782: C      M =>2
783:      CALL FUNCT6 (M, F, X)
784:      RETURN
785:      ENDIF
786: C      -----
787:      IF (KF.EQ.29) THEN
788: C      M =>2
789:      CALL FUNCT7 (M, F, X)
790:      RETURN
791:      ENDIF
792: C      -----
793:      IF (KF.EQ.30) THEN
794: C      M =>2
795:      CALL FUNCT12 (M, F, X)
796:      RETURN
797:      ENDIF
798: C      -----
799:      IF (KF.EQ.31) THEN
800: C      M =>2
801:      CALL FUNCT13 (M, F, X)
802:      RETURN
803:      ENDIF
804: C      -----

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872: C      FACTORIAL FUNCTION; MIN (1, 2, 3, . . . ., M) = 0
873: C      FACT = FACTORIAL(M) = 1 X 2 X 3 X 4 X . . . . X M
874: C      FIND X(I), I=1,2,...,M SUCH THAT THEIR PRODUCT IS EQUAL TO FACT.
875: C      LARGER THE VALUE OF M (=>8) OR SO, HARDER IS THE PROBLEM
876: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
877: C      DIMENSION X(*)
878: C      F=0.D00
879: C      FACT=1.D00
880: C      P=1.D00
881: C      DO I=1,M
882: C      FACT=FACT*I
883: C      P=P*X(I)
884: C      F=F+DABS(P-FACT)**2
885: C      ENDDO
886: C      RETURN
887: C      END
888: C      -----
889: C      SUBROUTINE DECANOM(M,F,X)
890: C      DECANOMIAL FUNCTION; MIN (2, -3) = 0
891: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
892: C      DIMENSION X(*)
893: C      X(1)=2 ! TO CHECK TURN IT ALIVE - REMOVE C FROM IST COLUMN
894: C      X(2)=-3 ! TO CHECK TURN IT ALIVE - REMOVE C FROM IST COLUMN
895: C      F1= DABS(X(1)**10-20*X(1)**9+180*X(1)**8-960*X(1)**7+
896: C      & 3360*X(1)**6-8064*X(1)**5+13340*X(1)**4-15360*X(1)**3+
897: C      & 11520*X(1)**2-5120*X(1)+2624.D00)
898: C      F2= DABS(X(2)**4+12*X(2)**3+54*X(2)**2+108*X(2)+81.D00)
899: C      F=0.001D00*(F1+F2)**2
900: C      RETURN
901: C      END
902: C      -----
903: C      SUBROUTINE JUDGE(M,X,F)
904: C      PARAMETER (N=20)
905: C      THIS SUBROUTINE IS FROM THE EXAMPLE IN JUDGE ET AL., THE THEORY
906: C      AND PRACTICE OF ECONOMETRICS, 2ND ED., PP. 956-7. THERE ARE TWO
907: C      OPTIMA: F(0.86479,1.2357)=16.0817307 (WHICH IS THE GLOBAL MINIMUM)
908: C      AND F(2.35,-0.319)=20.9805 (WHICH IS LOCAL). ADAPTED FROM BILL
909: C      GOFFE'S SIMMAN (SIMULATED ANNEALING) PROGRAM
910: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
911: C      DIMENSION Y(N), X2(N), X3(N), X(*)
912: C      DATA (Y(I),I=1,N)/4.284,4.149,3.877,0.533,2.211,2.389,2.145,
913: C      & 3.231,1.998,1.379,2.106,1.428,1.011,2.179,2.858,1.388,1.651,
914: C      & 1.593,1.046,2.152/
915: C      DATA (X2(I),I=1,N)/.286,.973,.384,.276,.973,.543,.957,.948,.543,
916: C      & .797,.936,.889,.006,.828,.399,.617,.939,.784,.072,.889/
917: C      DATA (X3(I),I=1,N)/.645,.585,.310,.058,.455,.779,.259,.202,.028,
918: C      & .099,.142,.296,.175,.180,.842,.039,.103,.620,.158,.704/
919: C
920: C      F=0.D00
921: C      DO I=1,N
922: C      F=F+(X(1) + X(2)*X2(I) + (X(2)**2)*X3(I) - Y(I))**2
923: C      ENDDO
924: C      RETURN
925: C      END
926: C      -----
927: C      SUBROUTINE DODECAL(M,F,X)
928: C      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
929: C      DIMENSION X(*)
930: C      DODECAL POLYNOMIAL MIN F(1,2,3)=0
931: C      CHECK TURN THESE VALUES ALIVE
932: C      X(1)=1 !TURN ALIVE PY REMOVING C FROM THE FIRST COLUMN
933: C      X(2)=2 !TURN ALIVE PY REMOVING C FROM THE FIRST COLUMN
934: C      X(3)=3 !TURN ALIVE PY REMOVING C FROM THE FIRST COLUMN
935: C      F=0.D00
936: C      F1=2*X(1)**3+5*X(1)*X(2)+4*X(3)-2*X(1)**2*X(3)-18.D00
937: C      F2=X(1)+X(2)**3+X(1)*X(2)**2+X(1)*X(3)**2-22.D00
938: C      F3=8*X(1)**2+2*X(2)*X(3)+2*X(2)**2+3*X(2)**3-52.D00

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939:      F=(F1*F3*F2**2+F1*F2*F3**2+F2**2+(X(1)+X(2)-X(3))**2)**2
940:      RETURN
941:      END
942: C -----
943:      SUBROUTINE SEQP(M,F,X)
944:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
945:      DIMENSION X(*)
946: C      FOR WHAT VALUES X(1)+X(2)=X(1)*X(2) ? ANSWER: FOR (0,0) AND (2,2)
947: C      CHECK THE ANSWER FMIN = 0 FOR X=(0, 0) OR X=(2, 2)
948: C      X(1)=2 !SET X(1) TO 0 OR 2
949: C      X(2)=2 !SET X(2) TO X(1) -----
950: C      X(1)=DABS(X(1)) ! ONLY NON-NEGATIVE VALUES
951: C      X(2)=DABS(X(2)) ! ONLY NON-NEGATIVE VALUES
952:      F1=X(1)+X(2)
953:      F2=X(1)*X(2)
954: C      SET EITHER OF THE TWO ALIVE BY REMOVING C FROM THE FIRST COLUMN
955:      F=(F1-F2)**2 ! TURN ALIVE THIS XOR
956: C      F=DABS(F1-F2) ! TURN ALIVE THIS - BUT NOT BOTH -----
957:      RETURN
958:      END
959: C -----
960:      SUBROUTINE AMGM(M,F,X)
961:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
962:      DIMENSION X(*)
963: C      FOR WHAT VALUES ARITHMETIC MEAN = GEOMETRIC MEAN ? THE ANSWER IS:
964: C      IF X(1)=X(2)=...=X(M) AND ALL X ARE NON-NEGATIVE
965: C      TAKE ONLY THE ABSOLUTE VALUES OF X
966:      SUM=0.D00
967:      DO I=1,M
968:      X(I)=DABS(X(I))
969:      ENDDO
970: C      SET SUM = SOME POSITIVE NUMBER. THIS MAKES THE FUNCTION UNIMODAL
971:      SUM= 100.D00 ! TURNED ALIVE FOR UNIQUE MINIMUM AND SET SUM TO
972: C      SOME POSITIVE NUMBER. HERE IT IS 100; IT COULD BE ANYTHING ELSE.
973:      F1=0.D00
974:      F2=1.D00
975:      DO I=1,M
976:      F1=F1+X(I)
977:      F2=F2*X(I)
978:      ENDDO
979:      XSUM=F1
980:      F1=F1/M ! SUM DIVIDED BY M = ARITHMETIC MEAN
981:      F2=F2**(1.D00/M) ! MTH ROOT OF THE PRODUCT = GEOMETRIC MEAN
982:      F=(F1-F2)**2
983:      IF (SUM.GT.0.D00) F=F+(SUM-XSUM)**2
984:      RETURN
985:      END
986: C -----
987:      SUBROUTINE FUNCT2(M,F,X)
988: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
989: C      IN FOGEL, L.J., ANGELIN, P.J. AND BACK, T. (ED) PROCEEDINGS OF THE
990: C      FIFTH ANNUAL CONFERENCE ON EVOLUTIONARY PROGRAMMING, PP. 451-460,
991: C      MIT PRESS, CAMBRIDGE, MASS.
992: C      MIN F (0, 0, ..., 0) = 0
993:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
994:      DIMENSION X(*)
995:      F=0.D00
996:      F1=1.D00
997:      FP=0.D00
998:      DO I=1,M
999:      IF (DABS(X(I)).GT.10.D00) FP=FP+(100.D00+DABS(X(I)))**2
1000:      ENDDO
1001:      IF (FP.NE.0.D00) THEN
1002:      F=FP
1003:      RETURN
1004:      ELSE
1005:      DO I=1,M

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1006:      F=F+DABS(X(I))
1007:      F1=F1*DABS(X(I))
1008:      ENDDO
1009:      F=F+F1
1010:      RETURN
1011:      ENDIF
1012:      END
1013: C
1014:      SUBROUTINE FUNCT3(M,F,X)
1015: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1016: C      MIN F (0, 0, ..., 0) = 0
1017:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1018:      DIMENSION X(*)
1019:      F=0.D00
1020:      F1=0.D00
1021:      FP=0.D00
1022:      DO I=1,M
1023:      IF (DABS(X(I)) .GT.100.D00) FP=FP+(100.D00+DABS(X(I)))**2
1024:      ENDDO
1025:      IF (FP.NE.0.D00) THEN
1026:      F=FP
1027:      RETURN
1028:      ELSE
1029:      DO I=1,M
1030:      F1=0.D00
1031:      DO J=1,I
1032:      F1=F1+X(J)**2
1033:      ENDDO
1034:      F=F+F1
1035:      ENDDO
1036:      RETURN
1037:      ENDIF
1038:      END
1039: C
1040:      SUBROUTINE FUNCT4(M,F,X)
1041: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1042: C      MIN F (0, 0, ..., 0) = 0
1043:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1044:      DIMENSION X(*)
1045:      F=0.D00
1046:      FP=0.D00
1047:      DO I=1,M
1048:      IF (X(I) .LT.0.D00 .OR. X(I) .GE.M) FP=FP+(100.D00+DABS(X(I)))**2
1049:      ENDDO
1050:      IF (FP.NE.0.D00) THEN
1051:      F=FP
1052:      RETURN
1053:      ELSE
1054: C      FIND MAX(X(I))=MAX(ABS(X(I))) NOTE: HERE X(I) CAN BE ONLY POSITIVE
1055:      XMAX=X(1)
1056:      DO I=1,M
1057:      IF (XMAX.LT.X(I)) XMAX=X(I)
1058:      ENDDO
1059:      F=XMAX
1060:      RETURN
1061:      ENDIF
1062:      END
1063: C
1064:      SUBROUTINE FUNCT6(M,F,X)
1065: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1066: C      MIN F (-.5, -.5, ..., -.5) = 0
1067:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1068:      DIMENSION X(*)
1069:      F=0.D00
1070:      FP=0.D00
1071:      DO I=1,M
1072:      IF (DABS(X(I)) .GT.100.D00) FP=FP+(100.D00+DABS(X(I)))**2

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1073:      ENDDO
1074:      IF (FP.NE.0.D00) THEN
1075:      F=FP
1076:      RETURN
1077:      ELSE
1078:      DO I=1,M
1079:      F=F+(X(I)+0.5D00)**2
1080:      ENDDO
1081:      RETURN
1082:      ENDIF
1083:      END
1084: C -----
1085:      SUBROUTINE FUNCT7(M,F,X)
1086: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1087: C      MIN F(0, 0, ..., 0) = 0
1088:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1089:      COMMON /RNDM/IU,IV
1090:      INTEGER IU,IV
1091:      DIMENSION X(*)
1092:      F=0.D00
1093:      FP=0.D00
1094:      DO I=1,M
1095:      IF (DABS(X(I)).GT.1.28D00) FP=FP+(100.D00+DABS(X(I)))**2
1096:      ENDDO
1097:      IF (FP.NE.0.D00) THEN
1098:      F=FP
1099:      RETURN
1100:      ELSE
1101:      F=0.D00
1102:      DO I=1,M
1103:      CALL RANDOM(RAND)
1104:      F=F+(I*X(I)**4)
1105:      ENDDO
1106:      CALL RANDOM(RAND)
1107:      F=F+RAND
1108:      RETURN
1109:      ENDIF
1110:      END
1111: C -----
1112:      SUBROUTINE FUNCT12(M,F,X)
1113: C      REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1114:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1115:      DIMENSION X(100),Y(100)
1116:      DATA A,B,C /10.D00,100.D00,4.D00/
1117:      PI=4.d00*DATAN(1.D00)
1118:      F=0.D00
1119:      FP=0.D00
1120:      DO I=1,M
1121: C      MIN F (-1, -1, -1, ..., -1) = 0
1122: C      X(I)=-1.D00 ! TO CHECK, TURN IT ALIVE
1123:      IF (DABS(X(I)).GT.50.D00) FP=FP+(DABS(X(I)))**2
1124:      ENDDO
1125:      IF (FP.NE.0.D00) THEN
1126:      F=FP
1127:      RETURN
1128:      ELSE
1129:      F1=0.D00
1130:      DO I=1,M
1131:      XX=DABS(X(I))
1132:      U=0.D00
1133:      IF (XX.GT.A) U=B*(XX-A)**C
1134:      F1=F1+U
1135:      ENDDO
1136:      F2=0.D00
1137:      DO I=1,M-1
1138:      Y(I)=1.D00+.25D00*(X(I)+1.D00)
1139:      F2=F2+ (Y(I)-1.D00)**2 * (1.D00+10.d00*(DSIN(PI*X(I+1))**2))

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1140:      ENDDO
1141:      Y(M)=1.D00+.25D00*(X(M)+1.D00)
1142:      F3=(Y(M)-1.D00)**2
1143:      Y(1)=1.D00+.25D00*(X(1)+1.D00)
1144:      F4=10.d00*(DSIN(PI*Y(1)))**2
1145:      F=(PI/M)*(F4+F2+F3)+F1
1146:      RETURN
1147:    ENDIF
1148:  END
1149: C -----
1150:  SUBROUTINE FUNCT13(M,F,X)
1151: C REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1152:  IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1153:  DIMENSION X(100)
1154:  DATA A,B,C /5.D00,100.D00,4.D00/
1155:  PI=4*DATAN(1.D00)
1156:  F=0.D00
1157:  FP=0.D00
1158:  DO I=1,M
1159: C MIN F (1, 1, 1, ..., 4.7544 APPROX) = -1.15044 APPROX
1160: C X(I)=1.D00 ! TO CHECK, TURN IT ALIVE
1161: C X(M)=-4.7544 ! TO CHECK, TURN IT ALIVE
1162:  IF(DABS(X(I)).GT.50.D00) FP=FP+(DABS(X(I)))**2
1163:  ENDDO
1164:  IF(FP.NE.0.D00) THEN
1165:  F=FP
1166:  RETURN
1167:  ELSE
1168:  F1=0.D00
1169:  DO I=1,M
1170:  XX=DABS(X(I))
1171:  U=0.D00
1172:  IF(XX.GT.A) U=B*(XX-A)**C
1173:  F1=F1+U
1174:  ENDDO
1175:  F2=0.D00
1176:  DO I=1,M-1
1177:  F2=F2+(X(I)-1.D00)**2*(1.D00+(DSIN(3*PI*X(I+1)))**2))
1178:  ENDDO
1179:  F3=(X(M)-1.D00)*(1.D00+(DSIN(2*PI*X(M)))**2)
1180:  F4=(DSIN(3*PI*X(1)))**2
1181:  F=0.1*(F4+F2+F3)+F1
1182:  RETURN
1183:  ENDIF
1184:  END
1185: C -----
1186:  SUBROUTINE FUNCT14(M,F,X)
1187: C REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1188: C MIN F (-31.98, 31.98) = 0.998
1189:  PARAMETER (N=25,NN=2)
1190:  IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1191:  DIMENSION X(2), A(NN,N)
1192:  DATA (A(1,J),J=1,N) /-32.D00,-16.D00,0.D00,16.D00,32.D00,-32.D00,
1193: & -16.D00,0.D00,16.D00,32.D00,-32.D00,-16.D00,0.D00,16.D00,32.D00,
1194: & -32.D0,-16.D0,0.D0,16.D0,32.D0,-32.D0,-16.D0,0.D0,16.D0,32.D0/
1195:  DATA (A(2,J),J=1,N) /-32.D00,-32.D00,-32.D00,-32.D00,-32.D00,
1196: & -16.D00,-16.D00,-16.D00,-16.D00,-16.D00,0.D00,0.D00,0.D00,0.D00,
1197: & 0.D00,16.D00,16.D00,16.D00,16.D00,16.D00,16.D00,32.D00,32.D00,
1198: & 32.D00,32.D00,32.D00/
1199: C FOR TEST TURN VALUES OF X(1) AND X(2) BELOW ALIVE
1200: C X(1)=-31.98 ! TURN ALIVE
1201: C X(2)=-31.98 ! TURN ALIVE
1202:  F=0.D00
1203:  FP=0.D00
1204:  DO I=1,M
1205:  IF(DABS(X(I)).GT.100.D00) FP=FP+(DABS(X(I)))**2
1206:  ENDDO

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1207:         IF (FP.NE.0.D00) THEN
1208:           F=FP
1209:           RETURN
1210:         ELSE
1211:           F1=0.D00
1212:           DO J=1,N
1213:             F2=0.D00
1214:             DO I=1,2
1215:               F2=F2+(X(I)-A(I,J))**6
1216:             ENDDO
1217:             F2=1.D00/(J+F2)
1218:             F1=F1+F2
1219:           ENDDO
1220:           F=1.D00/(0.002D00+F1)
1221:           RETURN
1222:         ENDIF
1223:       END
1224: C -----
1225:       SUBROUTINE FUNCT15(M,F,X)
1226: C       REF: YAO, X. AND LIU, Y. (1996): FAST EVOLUTIONARY PROGRAMMING
1227: C       MIN F(.19, .19, .12, .14) = 0.3075
1228:       PARAMETER (N=11)
1229:       IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1230:       DIMENSION X(*), A(N),B(N)
1231:       DATA (A(I),I=1,N) /.1957D00,.1947D00,.1735D00,.16D00,.0844D00,
1232: & .0627D00,.0456D00,.0342D00,.0323D00,.0235D00,.0246D00/
1233:       DATA (B(I),I=1,N) /0.25D00,0.5D00,1.D00,2.D00,4.D00,6.D00,8.D00,
1234: & 10.D00,12.D00,14.D00,16.D00/
1235:       DO I=1,N
1236:         B(I)=1.D00/B(I)
1237:       ENDDO
1238:       F=0.D00
1239:       FP=0.D00
1240:       DO I=1,M
1241:         IF (DABS(X(I)).GT.5.D00) FP=FP+DEXP(DABS(X(I)))
1242:       ENDDO
1243:         IF (FP.NE.0.D00) THEN
1244:           F=FP
1245:           RETURN
1246:         ELSE
1247:           DO I=1,N
1248:             F1=X(1)*(B(I)**2+B(I)*X(2))
1249:             F2=B(I)**2+B(I)*X(3)+X(4)
1250:             F=F+(A(I)-F1/F2)**2
1251:           ENDDO
1252:           F=F*1000
1253:           RETURN
1254:         ENDIF
1255:       END
1256: C -----
1257:       SUBROUTINE LINPROG1(M,F,X)
1258: C       LINEAR PROGRAMMING : MINIMIZATION PROBLEM
1259: C       IN THIS PROBLEM : M = NO. OF DECISION VARIABLES = 2
1260: C       MIN F (2.390, 2.033) = -19.7253 APPROX
1261: C       MIN F = OVER J=1, M : DO SUM(A(1,J)*X(J)) SUBJECT TO CONSTRAINTS
1262: C       OVER J=1, M : DO SUM(A(I,J)*X(J)) <= C(I) ; I=2
1263: C       . . . . .
1264: C       OVER J=1, M : DO SUM(A(I,J)*X(J)) <= C(I) ; I=N
1265: C       ALL X(I) => 0
1266:       PARAMETER (N=3) ! N IS THE NO. OF CONSTRAINTS + 1
1267:       IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1268:       DIMENSION X(*),A(20,10),C(20),FF(20)
1269:       DATA (A(1,J),J=1,2),C(1)/4.D0,5.D0,0.0D0/!COEFF OF OBJ FUNCTION
1270:       DATA (A(2,J),J=1,2),C(2)/10.D0,3.D0,30D0/!COEFF OF 1ST CONSTRAINT
1271:       DATA (A(3,J),J=1,2),C(3)/6.D0,20.D0,55.D0/!COEFF OF 2ND CONSTRAINT
1272: C -----
1273: C       USING ONLY NON-NEGATIVE VALUES OF X(I)

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1274:      DO I=1,M
1275:      X(I)=DABS(X(I))
1276:      ENDDO
1277: C      EVALUATION OF OBJ FUNCTION AND CONSTRAINTS
1278:      DO I=1,N
1279:      FF(I)=0.D00
1280:      DO J=1,M
1281:      FF(I)=FF(I)+A(I,J)*X(J)
1282:      ENDDO
1283:      ENDDO
1284:      F=-FF(1) ! CHANGE OF SIGN FOR MINIMIZATION
1285: C      CHECK FOR SATISFYING OR VIOLATING THE CONSTRAINTS
1286:      DO I=2,N
1287:      FF(I)=FF(I)-C(I) ! SLACK
1288: C      PENALTY FOR CROSSING LIMITS
1289:      IF(FF(I).GT.0) F=F+(10+FF(I))**2
1290:      ENDDO
1291:      RETURN
1292:      END
1293: C      -----
1294:      SUBROUTINE LINPROG2(M,F,X)
1295: C      LINEAR PROGRAMMING : MINIMIZATION PROBLEM
1296: C      IN THIS PROBLEM : M = NO. OF DECISION VARIABLES = 3
1297: C      MIN F (250, 625, 0) = -3250
1298: C      MIN F = OVER J=1, M : DO SUM(A(1,J)*X(J)) SUBJECT TO CONSTRAINTS
1299: C      OVER J=1, M : DO SUM(A(I,J)*X(J)) <= C(I) ; I=2
1300: C      . . .
1301: C      OVER J=1, M : DO SUM(A(I,J)*X(J)) <= C(I) ; I=N
1302: C      ALL X(I) => 0
1303:      PARAMETER (N=4) ! N IS THE NO. OF CONSTRAINTS + 1
1304:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1305:      DIMENSION X(*),A(20,10),C(20),FF(20)
1306:      DATA (A(1,J),J=1,3),C(1)/30.D0,40.D0,20.D0,0.0D0/! COEFF OF OBJ FUNCTION
1307:      DATA (A(2,J),J=1,3),C(2)/10.D0,12.D0,7.D0,10000.0D0/!COEFF OF 1ST CONSTRAINT
1308:      DATA (A(3,J),J=1,3),C(3)/7.D0,10.D0,8.D0,8000.D0/! COEFF OF 2ND CONSTRAINT
1309:      DATA (A(4,J),J=1,3),C(4)/1.D0,1.D0,1.D0,1000.D0/! COEFF OF 3RD CONSTRAINT
1310: C      -----
1311: C      USING ONLY NON-NEGATIVE VALUES OF X(I)
1312:      DO I=1,M
1313:      X(I)=DABS(X(I))
1314:      ENDDO
1315: C      EVALUATION OF OBJ FUNCTION AND CONSTRAINTS
1316:      DO I=1,N
1317:      FF(I)=0.D00
1318:      DO J=1,M
1319:      FF(I)=FF(I)+A(I,J)*X(J)
1320:      ENDDO
1321:      ENDDO
1322:      F=-FF(1) ! CHANGE OF SIGN FOR MINIMIZATION
1323: C      CHECK FOR SATISFYING OR VIOLATING THE CONSTRAINTS
1324:      DO I=2,N
1325:      FF(I)=FF(I)-C(I) ! SLACK
1326: C      PENALTY FOR CROSSING LIMITS
1327:      IF(FF(I).GT.0.D00) F=F+(100.D00+FF(I))**2
1328:      ENDDO
1329:      RETURN
1330:      END
1331: C      -----
1332:      SUBROUTINE TRIGON(M,F,X)
1333:      IMPLICIT DOUBLE PRECISION (A-H, O-Z)
1334:      DIMENSION X(*)
1335: C      F MIN (0, 0, 0, ..., 0) OR (PI, 0, 0, ..., 0) = 0
1336:      PI=4*DATAN(1.D00)
1337:      F=0.D00
1338:      DO I=1,M
1339:      IF(x(i).lt.0.d00 .or. X(I).GT.PI) THEN
1340:      F=F+(DABS(X(I))+10.D00)**2

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1341:         ENDIF
1342:         ENDDO
1343:         DO I=2,M
1344:           F=F+(dcos(I+0.D00)*DSIN(X(I)-X(I-1)))**2 +
1345:           & (i-1.D00)*(1.D0-DCOS(X(I)))**2
1346:         ENDDO
1347:         RETURN
1348:         END
1349: C =====
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