The Theorem of Proportionality in Mainstream Capital Theory: An Assessment of its Conceptual Foundations

Bitros, George C.

Athens University of Economics and Business, Department of Economics

29 July 2009

(This paper has evolved from Discussion Paper No. 192 under the title "The Hypothesis of Proportionality in Capital Theory: An Assessment of the Literature." The present covers only the theoretical literature and it will be supplemented by another paper covering the empirical literature.)

The Theorem of Proportionality in Mainstream Capital Theory: An Assessment of its Conceptual Foundations

By

George C. Bitros*

Abstract

It is ascertained that the theorem of proportionality, which maintains that replacement investment is a constant proportion of the outstanding capital stock, has several fundamental shortcomings. It derives from a model founded on assumptions that are highly restrictive and unlikely to hold in reality. It is alien to the thinking of researchers in industrial organization and other neighboring fields to economics that treat the durability of capital goods as a choice variable. It ignores several thorny conceptual and methodological issues and, perhaps most important, it may have restrained seriously the progress towards developing models based on more realistic approaches of production. However, despite its shortcomings, the theorem continues to dominate mainstream capital theory, most probably because of: a) its simplicity, and b) the lack of a model that might yield a better theorem in terms of standard criteria, like explanatory and predictive power, simplicity, fruitfulness, etc. For this reason attention is drawn to recent research which shows that a model centered on the heterogeneous structure of capital and the useful lives of its components is both feasible and exceedingly rich in theoretical and empirical implications.

JEL Classification: E220

Keywords: Capital longevity, replacement, depreciation, scrappage, maintenance, utilization, obsolescence.

Correspondence: Professor George C. Bitros, Emeritus
Athens University of Economics and Business
76 Patission Street, Athens 104 34, Greece
Tel: ++30 210 8203740 Fax: ++30 210 8203301,
E-mail: bitros@aueb.gr

1. Introduction

Once durable goods are put in place, in the overwhelming majority of cases their earning capability starts to decline. This happens for many reasons. One is the intensity with which they are used, because frequently it is responsible for their wear and tear or physical deterioration. Another is that all durable goods are designed and built for normal usage under certain conditions of maintenance; so if owners cut corners with regard to manufacturers recommendations for proper maintenance, the quantity and at times the quality of their services decline. Lastly a third reason is that with the passage of time older durables become economically inferior because there appear newer ones that are able to produce the same amount of services with less resources, since they embody the most recent advances in science and technology. However, in as much as the owners of durables have significant control over these and other influences, to transform the non-stationary replacement problem that Preinreich (1940) had posed, Terborgh (1949) introduced initially two simplifications. These were that the operating costs of durables in place increase and that the operating costs of newer vintages decline at constant rates per unit of time. As a result, he did managed to derive the optimal useful life of durables in the steady state of a perpetual stream of reinvestments, but at the cost of quashing the effects of utilization, maintenance and technological obsolescence on the processes of replacement and scrapping.

The field remained in the above state until Smith (1961) revisited it in a truly remarkable contribution to the theory of capital-using enterprise. As Terborgh did over a decade earlier, he continued to approximate the operating cost and salvage value functions involved in the perpetual replacement problem with linear forms. But his modeling of the process by which market and engineering factors combine to reduce the efficiency of capital services was ingenious. In particular, he hypothesized that these factors work through two channels. The one of them is the useful live and the other is the multitude of non-age related forces that are responsible for the normal wear and tear of durable goods. Thus, to capture their impact on capital services, he postulated that the firm solved the following problem:[1]

$$\begin{aligned} &min \quad C = (m + bT)x + (\hat{\delta} + aT + q/T + rq)K \quad \text{(i)} \\ &S.T. \qquad\qquad O = f(x, K) \qquad\qquad\qquad \text{(ii)} \end{aligned} \qquad (1)$$

where the various symbols have the following meanings: $C =$ total cost; O output; $x =$ variable input like the amount of energy consumed; $K =$ quantity of durable goods used in production; $T =$ average useful life of the stock of durable goods; $m =$ unit cost of variable input; $q =$ purchase unit cost of the stock of durable goods; $b =$ age related rate of deterioration in the usage of the variable input; $a =$ age related rate of deterioration in the services from the incumbent durables due to embodied technological change in newer vintages; $r =$ a constant rate of interest, and $\hat{\delta} =$ a constant non-age related proportional rate of deterioration in capital services. Now from (1) it is clear that the notion of proportionality was adopted as a hypothesis for the first time by Smith (1961, p. 166) and it was motivated by his concern to allow for the impact on capital services of the numerous non-age related factors.

By contrast to the above, in a very influential paper that appeared three years later, Jorgenson (1963) stipulated in different but equivalent terms that the firm solved the problem:

$$\begin{array}{lll} Min & C = mx + (\hat{\delta} + rq)K = mx + q(r + \delta)K & \text{(i)} \\ S.T. & O = f(x, K) & \text{(ii)} \end{array} \qquad (2)$$

Clearly this conceptualization constituted a major break from all past endeavors in this area, which centered primarily on the role of useful life in the services of the stock of durables.[2] Therefore, the justifications that warranted this far-reaching departure from the received theory were of particular importance. In this regard, here is how Jorgenson (1963) supported his assertion that the rate of deterioration of capital services is a constant proportion δ of the stock of durables involved:

> "The justification for this assumption is that the appropriate model for replacement is not the distribution of replacements of a single investment over time but rather the infinite stream of replacements generated by a single investment; in the language of probability theory, replacement is a recurrent event. It is a fundamental result of renewal theory that replacements for such an infinite stream approach a constant proportion of capital stock for (almost) any distribution of replacements for a single investment and for any initial age distribution of capital stock. This is true for both constant and growing capital stocks…" (p. 251).

Thus, in view of its grounding in renewal theory and the forcefulness with which Jorgenson (1965) returned with further details to defend its validity, this theorem[3] started to take hold in economic theory as well as in econometric studies and policy applications.

However, soon after it was launched, several researchers began to raise doubts about its underpinnings. Some of them emanated from theoretical considerations. Some other derived from empirical studies; and still some other sprung from the nature of the theories and practices adopted in neighboring scientific fields. But all shared a common feature. Namely, they refuted the theoretical and empirical foundations on which its validity rested. Consequently one would have expected that after a while it would have been abandoned or at least reconsidered.[4] Instead what transpired was that the theorem came to dominate mainstream capital theory and its applications in computing capital stocks at various levels of aggregation through the perpetual inventory method. So the questions that come naturally to mind are: Why have all arguments against this theorem failed to attract significant following among researchers and practitioners? What inferences might we draw in this regard from the program of research in macroeconomics, including the theories of economic growth, business cycles and investment? Where else outside the confines of mainstream economics might we look for insights regarding the processes of depreciation and replacement of durable goods? Are there alternative grounds on which to judge the validity of the theorem of proportionality? My objective is to assess the relevant literature in the expectation that it may shed some light on these questions and perhaps help settle the controversy that surrounds this theorem, if not in general, at least in those fields of economics where procedural consistency and precision in the measurement of capital is of utmost importance.

The paper is organized as follows. Section 2 presents the views that have been expressed in favor and against the theorem of proportionality by researchers working in the core areas of mainstream economics. In particular, while early on the focus is on the theories of replacement, subsequently the attention turns to macroeconomics, including economic growth and business cycles, and industrial organization. Then, Section 3 highlights the insights that may be derived mainly from the Austrian theory of capital, which, by stressing the notions of roundaboutness and other essential characteristics of durable goods, is closely related to the issues under consideration. Section 4 assesses the theorem of proportionality from a methodological point of view and sketches the rudiments of model that may yield a better theorem, and, lastly, Section 5 closes with a synopsis of main findings and conclusions.

2. Views from within the mainstream theory of capital

In retrospect the most robust and persuasive element in Jorgenson's (1963; 1965) arguments was the claim that the theorem could be derived as a general proposition from renewal theory. Accordingly the researchers who adhered to the earlier tradition, which stressed the importance of *longevity* or *durability* or *useful life* of capital goods, confronted two tasks. The one was to challenge the validity of the theorem from a theoretical standpoint, whereas the other was to subject it to more discriminating empirical tests. From the research efforts in these directions there developed a large body of literature, which is partly theoretical and partly empirical. On the other hand, since the process by which durable goods depreciate has significant implications for, and hence it is of interest to, all scientific fields that are concerned with capital as a factor of production, the frontier of research was expanded into such neighboring specializations as operations research, operations management, finance, capital budgeting, accounting, etc. As a result the relevant literature has grown so vast that defies the capability, and most likely the purpose, of surveying it in a single pass. For this reason the presentation below adopts two delimitations. In particular, it restricts attention to the literature only in the core areas of economics, with only sporadic references to advances in related fields, and does so by focusing on the theoretical part of the literature, leaving the assessment of the empirical literature for a companion paper.

2.1 Standing in the theory of replacement

For several years Jorgenson's (1963; 1965) claim that the theorem of proportionality constituted a fundamental result of renewal theory went unchallenged. In particular, while the empirical evidence that was reported did cast doubts about its applicabulity, a proof that refuted it by recourse to theory was missing. This situation lasted until Feldstein and Rothschild (1972/1974) in a widely cited paper argued that:

> "Except for numerical accidents of no economic interest, … a constant replacement ratio will emerge only if either: (i) each piece of equipment is subject to output decay at the same constant exponential rate or (ii) the entire capital stock, and therefore both net and gross investment, grow at a constant exponential rate" (p. 397).

Based on the theorems from which these arguments derived, the balance of professional opinion was expected to tip on the side of the conclusion that the theorem of proportionality lacked theoretical foundations. But in the same year Jorgenson (1974) provided a step-by-step counterproof by showing that, irrespective of whether the analysis concerns a single or multiple investments and constant or changing capital stock, the sequence of replacement ratios converges to a constant for almost any mortality distribution of investment(s). So apparently the issue remained undecided because, even though both sides obtained their results from renewal theory, they arrived at diametrically opposite conclusions.

Responsible to some extent for the uncertainty that ensued was the failure of the protagonists to delineate clearly the time frame over which their results applied. To see why, assume that the question of interest is what happens to the replacement ratio in the short run or, alternatively, in the course of the business cycle. As indicated above, the theorem of proportionality was obtained from renewal theory as the limit to which a sequence of replacement ratios converges after a lengthy process. Therefore, by construction, it was meant to apply in the long run. But Jorgenson (1965; 1974) asserted that it provided also a good approximation in the short run and this provoked a particularly forceful rebuttal from Feldstein (1972/1974) on three planes. In the first one, he drew on several theoretical arguments to establish the proposition that:

> "Even if a long-run tendency towards proportional replacement followed from a generalized renewal theorem as Jorgenson (1965; 1974) has suggested, this would provide no basis for making the same assumption in the short run."(1972, pp. 2-3).

This stressed the possibility that the replacement ratio in the short run might differ from that in the long run and implied that its behavior ought to be investigated separately. In the second plane, he estimated a replacement investment equation using the same data as in Feldstein and Foot (1971) and found that the above proposition was confirmed with comfortable levels of confidence.[5] Finally, in the third plane, he made the following critical remarks:

> "Although Jorgenson and Stephenson (1967; 1969) claim to have tested the proportionality replacement hypothesis, their calculations actually test a quite different proposition. In estimates of two-digit investment behavior with gross investment as dependent variable, they included the lagged capital stock among the regressors and interpreted its coefficient as an estimate of the replacement rate, $\hat{\delta}$. They then tested and confirmed that $\hat{\delta}$ was different from zero but not different from the average annual rate used by the Office of Business Economics to construct the capital stock series. Neither of these tests refers to the stability and constancy of the annual replacement ratio. They show only that on average the amount of replacement is related to the capital stock, a very much weaker proposition than the proportionality replacement hypothesis used in investment studies."(1972, p. 3, ft 2).

These raised two issues of wider and enduring methodological importance. The first had to do with the proper testing of the theorem of proportionality in the short run. In this regard he suggested that tests based on the average replacement ratio usually obtained in investment studies ought to be supplemented with tests of the stability and constancy of the annual replacement ratios. As for the second issue, this concerned the nature of the average replacement ratio itself. Could it be conceived as an estimate of the replacement ratio in the long run? If yes, was it constant or variable? If not, how might the theorem of proportionality be tested in the long run? But all were left open for later consideration.

Moreover, the case was that the assumptions that Jorgenson (1974) adopted were extremely restrictive because they ignored the crucial role of utilization, maintenance and embodied technological change. For if the owners of an investment vary deliberately the respective policies in response to changing market and technological conditions, most likely the coefficients of output efficiency from the one vintage to the next will vary, and thus they may not follow any given distribution. Therefore, a way to bypass the controversy was to look at the implications of these processes for replacement investment. To this end it suffices to mention that by the mid-1970s, i.e. when the above debate broke open, there existed already a large volume of theoretical and empirical literature establishing that utilization, maintenance and repair costs, and obsolescence influence significantly the deterioration of capital services, and hence replacement. Just to cite a few examples, Smith (1957) had ascertained this linkage in the case of trucks; Thompson (1968) and Kamien and Schwartz (1971) had highlighted respectively the relationship of maintenance to the sale date of a machine under conditions of stochastic failure and deterioration; Taubman and Wilkinson (1970) had shown how utilization affects gross investment via replacement, whereas the exhaustive survey by Winston (1974) regarding capital utilization and idleness left no doubt that output efficiency varies with utilization, and Malcomson (1975) had demonstrated how serious is the omission of obsolescence from the analysis of replacement investment. Hence, all indications were that the distribution of output coefficients would follow any particular distribution only accidentally.

Consistent with this view were also the findings by most other replacement theorists. For a few examples, consider first the results obtained by Nickell (1975). In the concluding remarks to the section where he investigates the implications of a constant scrapping age to the ratio of replacement investment to capital stock he writes:

> "…It is perhaps worth mentioning that the above analysis indicates that the conditions under which the replacement/capital ratio is constant are very restrictive. It therefore seems very unlikely that it would be constant in reality" (p. 63).

Next, take the widely acclaimed study by Rust (1987), which focused on the relationship of maintenance of bus-engines to the timing of their replacement. By setting up a stochastic dynamic programming model of bus-engine replacement and testing it with monthly data from 104 buses over a 10 year period, he found that mileage and maintenance and repair expenditures explained most of the variance in the decisions of bus-engine replacement. Finally, it is worth noting that in Bitros and Flytzanis (2002; 2005; 2009) and Bitros, Hritonenko and Yatsenko (2007) we traced the influences that reinvestment opportunities exercise on the decisions to replace or scrap in the presence of embodied technological change and active utilization and maintenance policies.

Additionally the theorem of proportionality came under attack from two other camps. The first consisted of theorists who worked in the area of two-sector growth models, whereas the second comprised econometricians and other theorists who labored in such research frontiers as aggregation, growth accounting and total factor productivity. In particular, while investigating the conditions for collapsing a static multisectoral economy into a two-sector model, Zarembka (1975) found that it was impossible to aggregate heterogeneous types of capital that deteriorate at different constant exponential rates. Here is how he concluded:

> "In a steady-state model it is reasonable to assume that depreciation is some constant fraction of the stock of a particular capital good and that the rate does not vary substantially according to the goods produced (with some exceptions)… But if the depreciation rate varies substantially among capital goods, then the reduction of equation (10) to (11) in the capital goods sectors does not obtain (and similarly for the consumer goods sectors). Therefore, in comparing steady-state equilibria, it is not possible to aggregate capital goods with different depreciation rates (and thus one reason why capital in structures and equipment needs to be disaggregated" (p. 113).[6]

Apparently these findings contradicted sharply all previous theoretical constructs and national income accounting measures, which relied on the presumption that a sufficiently good approximation to the economy or sector-wide capital stocks could be obtained by adding the depreciated magnitudes of the underlying investment expenditures through perpetual inventory methods. To be sure, the contribution by Brown and Chiang (1976) one year later created the impression that the above impossibility theorem had been bypassed. But it had not because of two reasons. First, because the possibility theorems by these authors rested on conditions that were extremely unlikely to hold, and second because the work by Miller (1982; 1990) in the 1980s left little doubt about the inconsistencies for which the theorem of proportionality was responsible in econometric applications.

2.2 Standing in macroeconomics

In the early years after Keynes (1936) launched his far-reaching ideas, mainstream macroeconomic theorists paid little or no attention to the problems of capital as a factor of production. The root cause of this neglect was Keynes's conviction that, if he solved the problem of insufficient aggregate demand for achieving full employment, then automatically we would be in the world of classical economics, where Say's Law and supply-side constraints would determine the path of the economy. In his words:

> "If our central controls succeed in establishing an aggregate volume of output corresponding to full employment as nearly as is practicable, the classical theory comes into its own again from this point onward"(p. 378).

Apparently, assuming that investment in the short run increased the stock of capital insignificantly, he relegated all issues regarding its role as a factor of production to economic growth and business cycle specialists.

Responding to this conceptualization, Tinbergen (1942), Domar (1946), Harrod (1948) and Solow (1956) took the lead by presenting models in which investment drove the growth rate of the economy and determined the path of employment. But these models could not shed light on the processes of retirements, depreciation and replacement of capital because they were *net* in the sense that they defined saving, investment and national output as *net* saving, *net* investment and *net* national output. As a result, soon it became clear that research efforts ought to be directed towards modeling gross saving, investment, and national output, which meant that it was imperative to integrate into growth and business cycles models the useful life of durable goods. Working in this direction, researchers came through with several seminal contributions, which highlighted the linkages of depreciation and replacement to aggregate demand under various circumstances. In particular, treating the useful life of capital as a parameter, Eisner (1952) and Domar (1953) showed that in a growing economy with stationary prices depreciation allowances would exceed replacement requirements, thus leading eventually to deficient aggregate demand and unemployment. Eisner (1956) extended these results to the case where the useful life of capital varied with technological progress. Johansen (1959) traced the implications that emerged if ample substitution between labor and capital were allowed in the context of the vintage capital model; and last, but not least, Massell (1962) and Solow (1962) incorporated in variations of these models embodied technological change. Thus, in the early 1960s mainstream theorists were well on their way to formulating a model of economic growth, which, by placing the emphasis on the replacement of old by new capital that embodies the most recent advances in science and technology, might achieve the coupling of the short run Keynesian with the long run classical analysis. But then suddenly interest in the vintage capital model and the useful life of capital as an economic variable eclipsed.

A glimpse into what happened in the next three decades may be obtained from the following assessment that Solow (1997) made in the late 1990s:

> "One major weakness in the core of macroeconomics is the lack of real coupling between the short-run picture and the long-run picture. Since the long run and the short run merge into one another, one feels that they cannot be completely independent"(p. 231).

This revealed lack of progress but not the factors that were responsible. Yet viewed in conjunction with the possibility that the root cause might be related to capital as a factor of production, Solow's assessment implied that the theorem of proportionality, which had rendered retirements, depreciation and replacement invariant with respect to the useful lives of capital goods, might not be innocuous after all. That this is a reasonable conjecture is corroborated by certain trends that prevailed in the areas of economic growth, business cycles and investment,[7] which are summarized immediately below in the same order.

2.2.1 The theorem of proportionality in the field of economic growth

Growth theorists knew from Solow et al. (1966) and Sheshinski (1967) that in the vintage capital model the convergence to the balanced growth path was monotonic and that the investment solution paths were markedly smooth. Also they knew that lumps and bumps characterize investment activities at the firm level. So, from then already it was clear that progress in the coupling of the short run picture with long run picture would require building a model that would allow the investment solution paths to be nonmonotonic. Thus researchers were expected to turn their attention in this direction. But, for reasons that might not be unrelated with the dominance of the theorem of proportionality and the results mentioned above, most shied away. One who did not was Brems (1968). By grafting into the vintage capital model a mechanism that optimized the timing of replacement of the capital goods in each vintage, he was able to show how the effects from a change in the interest rate and the rate of technological progress would work their way to a new balanced growth path through changes in gross saving, gross investment and gross output, while maintaining full employment. The thread that linked the short run picture with the long run picture was the relationship between the useful life of capital on the one hand and the rates of interest and technological progress on the other. However, despite its promising advances, Brems' model did not attract the attention it deserved and all related research came to a standstill.

This phase lasted for almost 25 years and ended with the contribution by Benhabib and Rustichini (1991). Not unexpectedly what these researchers found was that the theorem of proportionality restrained significantly the dynamic properties of growth models in which it was embedded. In their words:

> "The assumption of exponential depreciation suffers by virtue of its own simplicity (that is, by dramatically reducing the possible dynamics that an optimal growth model can describe)" (p. 324).

So, by moving away from it, they were able to show that, for some non-exponential depreciation rules, like for example the "one-hoss shay", the optimal growth model with vintage capital gave rise to periodic solutions, thus opening a whole range of possibilities for obtaining more realistic representations of lumpy investment activities. But, unlike Brems (1968), their framework of analysis treated depreciation as an engineering process and in this respect it served only as a point of departure in the quest for a more general model, i.e. one in which the useful life of capital would be determined endogenously. Actually Benhabib and Rustichini (1993) did consider this possibility, but they did not characterize explicitly the dynamics of investment.[8]

In the next phase, researchers sought to enhance the capability of the vintage capital model[9] to yield non-monotonic solutions by introducing the useful life of capital, or scrapping time, as an endogenous variable. Boucekkine, Germain and Licandro (1997) and Boucekkine, Germain, Licandro, and Magnus (1998) spearheaded this program of research in Europe by focusing on the existence of replacement echoes, i.e. the ability of investment to reproduce its own past behavior when it is dominated by replacement activity.[10] They did so in two well-known vintage models with linear and non-linear utility functions and found that: a) the optimal paths of consumption, investment and production showed periodicity beginning at a well-specified date; b) under linear utility the average age of capital remained constant; c) under non-linear utility the average age of capital was variable, and d) under non-linear utility the results were consistent with the observed dynamics of investment. From these results it became clear that a growth model that would provide for vintage capital in conjunction with some non-linear utility and non-exponential depreciation rule stood a good chance to shed ample light on how a sequence of short run equilibria converge seamlessly to the long-run balanced growth path of the economy, where of course Say's law and classical supply-side ideas apply.

Working in this direction Boucekkine et al. (2005) presented a state-of-the-art analysis of the simple AK vintage capital model with concave utility in which, instead of the traditional assumption of exponential depreciation, they postulated that machines have a finite lifetime, i.e. the one-hoss shay depreciation assumption. As could be expected from Boucekkine et al. (1997; 1998), this small departure from exponential depreciation modified dramatically the off-balanced growth paths in this class of models. In particular, they found that:

> "The introduction of vintage capital into an otherwise standard AK-type optimal growth model leads to three main conclusions. First, persistent oscillations in investment can occur with concave utility when we allow for some non-smooth depreciation scheme. Second, since investment involves creation and destruction as separate activities, those oscillations are the result of replacement echoes. Third, there is a trade-off between rapid expansion and hence rapid net investment and longer lasting fluctuations" (p. 63).

In turn, these powerful results confirmed the explanatory advantages of vintage capital models with endogenously determined useful life of capital and set the standard for this line of research in the current phase.

Equally promising with the above are also the results from very recent research efforts in the front of the two-sector vintage capital model. For two cases in point, consider the studies by Boucekkine et al. (2008) and Bitros (2008c). The models proposed in them have a common feature. This is that both consider the lifetime of capital goods as an endogenous variable. But at the same time they are characterized by certain fundamental differences, which emanate from their structure and mechanisms that drive the process of depreciation. For example, with regard to the structure of the models, the crucial difference is that, whereas in the latter study the model comprises two sectors, i.e. one that builds producer's durables and another that employs them in the production of final goods, in the former study the model consists of two final goods sectors, employing durable goods that are built by the corresponding representative firms in house. However, despite this and other differences, both studies establish, among other significant propositions, that the rate of depreciation increases when investment-specific technical progress accelerates due to a shorter optimal lifetime of capital.[11] This finding contrasts sharply with the theorem of proportionality, which stands on the assumption that all depreciation comes from output decay, and is consistent with the evidence, according to which the rate of deprecation rose in the 1990s because of the spectacular progress in the information processing technology.

2.2.2 The theorem of proportionality in the field of business cycles

Efforts by researchers to construct a dynamic general equilibrium model have followed two directions. On the one hand, there are those who are working to extend the neoclassical model of capital accumulation so as to admit cycles. In this group belong, among many others, the leading growth theorists whose contributions were summarized above. On the other hand, there are those who strive to formulate business cycle models in which the variables of interest converge to their long-term values through a seamless sequence of short runs positions. The latter may be distinguished further into four subgroups. The first of them is composed of theorists who attribute economic fluctuations to the existence of various sources of adjustment costs and the fact that capital takes time to build and install. The second subgroup comprises the theorists who trace economic fluctuations to exogenous and unpredictable shocks emanating from technological change, uncertainty, government policies, etc. The third is made up of theorists who take issue with mainstream capital theory by stressing the endogenous nature of retirements, depreciation, replacement and maintenance and repair expenditures; and finally the last subgroup consists of theorists who focus on the role of money and the functioning of financial markets. The theorem of proportionality has significant implications in all these contexts. However, it relates primarily to capital as a factor of production. For this reason, the focus below will be restricted to this particular segment of the relevant literature.

Unlike growth theorists, a sizable group of researchers in the area of business cycles started to suspect that the theorem of proportionality constrained unduly the explanatory power of their models well before the appearance of the paper by Benhabib and Rustichini (1991). Actually their efforts to relax it begun early in the 1980s and followed three directions. Those who worked in the first direction aimed at modeling depreciation as an endogenous variable. They did so as follows. Initially, drawing on the notion of *depreciation-in-use*, Epstein and Denny (1980), Merrick (1984) and Hercowitz (1986) introduced depreciation as a function of the rate of utilization in a model of intertemporal entrepreneurial choice. Then, by generalizing the model to the case where the marginal efficiency of capital shifts, due to shocks from technological change, Greenwood, Hercowitz and Huffman (1988) obtained strong theoretical and empirical evidence according to which:

> "A variable capacity utilization rate may be important for the understanding of business cycles. It provides a channel through which investment shocks via their impact on capacity utilization can affect labor productivity and hence equilibrium employment. Such a mechanism may allow for a smaller burden to be placed on intertemporal substitution in generating observed patterns of aggregate fluctuations" (p. 415).

Subsequently, Burnside and Eichenbaum (1996) analyzed the model under the conceptualization that the endogeneity of depreciation derives from the theory of *factor hoarding* and their results confirmed the conclusions reached by Greenwood et al. (1988) in the above passage; and, lastly, Choi and Kollintzas (1985), Collard and Kollintzas (1998), McGrattan and Schmitz (1999), Licandro and Puch (2000), Dueker and Fischer (2003), and others, analyzed the implications when depreciation depends on utilization and maintenance and improvement expenditures. Thus, on account of this literature and the convincing evidence offered more recently by Chatterjee (2005) regarding the role of utilization in the speed and the nature of convergence to the long-run growth path, more and more researchers in the areas of business cycles and dynamic stochastic general equilibrium chose to model depreciation explicitly.

Consistent with this trend have been also the findings of studies in the second direction, which address the aggregate implications of investment decisions at the firm level. Caballero, Engel, and Haltiwanger (1995) modeled depreciation-in-use as a fixed proportion of the outstanding capital stock and at the same time they introduced retirements explicitly into the equation that traces the time evolution of capital. This implied that they abandoned the theorem of proportionality because, if retirements are modeled endogenously as a function of the useful life of capital goods, replacement cannot be invariant with respect to time and the property of duality between depreciation and replacement does not hold any more. Thus, they were able to conclude that ignoring retirements:

> " can yield potentially large measurement errors in the evolution of the capital stock at the plant level, because the average service life distributions are applied to all plants in the same industry" (p. 12).

A few years later, Cooley, Greenwood and Yorukoglu (1997), Gylfanson and Zoega (2002) confirmed that, if technological change is modeled explicitly, the convergence paths to the steady state should be markedly different from the standard neoclassical model, which is based on the theorem of proportionality; and more recently Bitros (2008b) highlighted the potential gains in explanatory power by modeling explicitly the uncertainty that surround the pace of technological change.

Support in favor of modeling depreciation as an economic process has come also from studies in the third direction, which emphasize the Schumpeterian process of creative destruction. One standard example in this regard is the model analyzed by Caballero and Hammour (1994; 1996) in which cyclical variations in demand influence the creation of productive units that embody the latest technology and the retirement of older ones that are obsolete. Among other important results they find that, while during expansions creating new productive units exerts an "insulating" effect in the sense that it prolongs the useful lives of the units already in place, in recessions the same process acts in a "cleansing" fashion, because it precipitates the removal of unprofitable units from production. Another example is the model presented by Boucekkine and Martinez (2003), which shows how adoption costs influence the lifetime of existing productive units and the timing of replacement of the oldest ones. Still a third example is the model by Dosi, Fagiolo and Roventini (2006) in which the scrapping of an incumbent productive unit depends on the degree of its obsolescence and the market price of new capital goods. From this particular literature it turns out that, since mostly the process of creative destruction drives scrapping, models that gloss over retirements by adopting the theorem of proportionality miss an important source of fluctuations.

Aside from the above, it is worth noting that Cummins and Violante (2002), on the one hand, and Ambler and Paquet (1994) and Dueker, Fischer and Dittmar (2002), on the other, have introduced two different approaches to modeling depreciation. More specifically, assuming that depreciation-in-use is related to the age of real assets and economic depreciation to their loss of earning power due to technological obsolescence, the former estimate capital stock series using the perpetual inventory method in conjunction with depreciation rates that vary with time. To be sure this is a computational technique with little foundation in theory. But it does highlight the problem and its far-reaching implications for the study of fluctuations. As for the latter, here is how Dueker, Fischer and Dittmar (2002) rationalize the treatment of the depreciation rate as a stochastic variable:

> "Endogenous depreciation equates margins at less than full capital utilization or introduces a role for large, counter cyclical expenditures on maintenance and repair. In this way, endogenous depreciation serves to amplify and augments the persistence of the effects of technology shocks on output. But full endogenous depreciation does not allow for random changes in the depreciation rate as independent source of economic fluctuations. Alternatively, the depreciation rate can be stochastic in DSGE models, putting depreciation shocks on a par with technology shocks as fundamental driving forces behind macroeconomic fluctuations" (pp. 1-2).

This concludes the survey of the literature that recognizes the limitations of the theorem of proportionality in the field of business cycles and attempts to deal with them by treating depreciation either as an economic process or as a parameter that shifts randomly over time.

2.2.3 The theorem of proportionality in the field of investment

Aggregate investment is composed of three components. These include spending by business firms for structures and equipment, spending by the state for public capital, and spending by households for consumer durables. All three are very important in the determination of the level and the evolution of national income. But historically the development of investment theory has been associated with the analysis of the first component, because it constitutes one of the main forces that drive the process of economic growth. For this reason, even though more recently the contribution of public infrastructure to the productivity of private sector has attracted some attention, the focus below will be restricted to the spending for business fix investment.

The last time someone reviewed the literature in this area was Chirinko (1993). Not including the references, his survey extended over 30 printed pages and covered all crucial issues in a detailed and balanced way. Yet the problem that concerns us here was not among them. This was due most likely to the view expressed earlier that modeling of depreciation as a fixed proportion of the outstanding capital stock dominated the thinking of researchers and practitioners up to that time. But looking forwards he made the following assessment:

> "An important characteristic of the capital accumulation decisions that has not been considered here is that investment is partly or fully irreversible. An emerging literature examines the investment dynamics that arise from irreversible investment . Including endogenous depreciation (which will attenuate the effect of irreversibility)…within this analytic framework should prove particularly informative" (p. 1905).

In retrospect it turns out that his recommendation proved quite insightful because in the following years economic growth and business cycle theorists made great strides in their efforts to endogenize depreciation. However, the same was not the case with the study of investment, because utilization, maintenance and repair expenditures, retirements, shifts in the age structure of capital due to technical change, and other depreciation-related variables, continued to be ignored. At least this is the view that emerges from the various evaluations of their performance.

2.3 Standing in the theory of industrial organization

While the debate about the theorem of proportionality raged among capital and investment theorists, another group of researchers working independently investigated the factors that determine the durability of durable goods. To be sure this literature was not concerned with the question when is it optimal to discard or replace a durable. But since a more durable good would last longer than an identical good of lesser durability, the two goods could not be expected to deteriorate at the same exponential rate, as the theorem of proportionality would predict. For if, *ceteris paribus*, the two durables deteriorated at the same exponential rate, the demand for the more durable and presumably more costly good would cease to exist and only the less durable would be offered. Hence, this literature had a crucial implication for the issue under consideration.

This is that the amount of durability built into producer's durables is not a technological datum but an attribute determined by market forces. More specifically, in the 1960s the model presented by Kleiman and Ophir (1966) established that under perfect competition a rise in the interest rate reduces durability, increases the number of units produced by the manufacturing firms, but may either increase or decrease the total number of units produced by the industry. Then in the 1970s Swan (1971; 1977), Coase (1972), Barro (1972), Schmalensee (1974), Kamien and Schwartz (1974), Parks (1974; 1979) and others, investigated the relationship of market structure to durability and maintenance. From their studies emerged several results. One was that, irrespective of whether a monopoly produces durables of higher or lower durability than the firms under perfect competition, the structure of the market does influence the amount of durability produced. Another was that the nature of ownership affects durability. In particular, if manufacturers chose to sell rather than rent their durables, the durability they build into them would be different. Last, but not least, was the result that the ability to change the useful lives of durables through maintenance influences the choice of durability at the time of their production. Therefore, this literature left no doubt about the endogenous nature of durability and hence the rate of deterioration of producers durables.

Moreover, other research efforts parallel to the above reinforced this view even further. Bulow (1986), Rust (1986) and Waldman (1993) added significantly to the results that had been achieved earlier by Swan (1971) in the front of planned obsolescence. Contrary to the views held by neoclassical theorists, who insist on the like-for-like perpetual inventory replacement of capital goods, Mann (1992) showed that when used durables are relatively good substitutes for new ones, a durable goods monopolist is better off encouraging maintenance and reducing depreciation. Kinokuni (1999) traced the effects of repair market structure on the choice of durability; and a sizable group of researchers focused on the intricacies introduced in the analysis of durability by moral hazard and adverse selection in the selling or renting of durables by their manufacturers.

2.4 Summary of findings

The theorem of proportionality suffers from several major limitations. Its derivation from renewal theory requires adopting the heroic assumption that all depreciation derives from output decay, and hence that it is invariant with respect to utilization, maintenance, technological change, and the reinvestment opportunities that market conditions afford to enterprises. Even then it may apply only in the long run, so that the behavior of depreciation in the short run is left unexplained. In models with multiple sectors of production and consumption it renders aggregation impossible and thus prohibits the comparison of steady-state equilibria. When applied in the context of the perpetual inventory method it leads to measurements of the capital input from which it is impossible to identify the parameters of the production function, etc. As a result, since the early 1990s an increasing number of macroeconomic theorists have been moving away from it by joining the ranks of researchers in industrial organization and other neighboring fields, like operations research and operations management, who treat depreciation as an endogenous variable. So far this trend has been particularly strong in the fields of economic growth and business cycles, where the theorem of proportionality has proved exceedingly restrictive. But soon it may spread to other areas with strong microeconomic foundations, because postulating depreciation as a fixed proportion of the outstanding capital stock in the presence of rapid technological change limits drastically the explanatory and predictive properties of models.

3. Views from outside the mainstream theory of capital

The researchers who contributed to the voluminous literature that was surveyed above differ significantly in the way they model depreciation. Those in the dominant majority consider it to be a proportion of the outstanding capital stock. Some others are content to postulate that depreciation follows the one-hoss shay pattern by assuming that capital goods have a fixed useful life. A narrow minority perceives depreciation as a decision variable related explicitly either to the average age of capital or to such endogenous variables as utilization, maintenance and technological change; and lastly there are even a few who ignore depreciation by assuming that capital goods have infinite useful lives. But all share a common view. Namely, that the great assortment of producer durables employed in the economy can be aggregated consistently into a homogeneous mass of "capital-in-general". In turn this led to the so-called Cambridge controversy, which had to do mainly with the definition and appropriate units of measurement of aggregate capital. Consequently, to complete the survey, it is interesting to look into the issues that were debated and do so with an eye towards their implications for the theorem of proportionality.

In his celebrated contribution to the theory of economic growth, Solow (1956) assumed that: a) the production function of the economy took the general form: $O = f(L, K)$, where $O =$ output, $K =$ capital, and $L =$ labor; b) physically capital lasted forever, and c) all technological change was of the "Hicks neutral type". By implication, capital did not depreciate and the only issue was how to define and measure its quantity. Solow (1956, p. 101) thought that, if K were measured in the same physical units as O, say the labor hours it takes to produce one unit of output, the fundamental issues that Robinson (1953) had raised would be settled for good. But even after the introduction by Samuelson (1962) of the surrogate production function, where the various types of capital goods defined distinct productive activities and depreciated at their own fixed proportional rates, the issues that Robinson (1959) had reiterated would not go away. According to Samuelson (1966), who summed up the debate, this was due to the recognition that the issues were insurmountable for two reasons: First, because there exists no unique way to define and measure capital; and, secondly, because the relationship of the interest rate to the capital intensity entails points of reswitching in the sense that a decline in the interest rate may lead to the choice not of more but of less capital intensive techniques.

Both these issues hold serious implications for the theorem of proportionality. To see why, consider first the possibility of reswitching by assuming a simple economy with two sectors, each populated by a single representative firm. Also let each firm produce output by means of a neoclassical production function like the one displayed above. Apparently, in the neighborhood of reswitching, a change in the interest rate may render the firm in one of the sectors unprofitable, thus leading to its shut down. This would be an instance of interest rate induced scrapping that would vitiate the validity of the theorem of proportionality. But according to Robinson (1975), in reality reswitching of techniques is extremely unlikely, and hence the weight of the preceding argument should be discounted.

Actually doing so would be most warranted for the additional reason that the oddity of re-switching is closely related to the first issue, which concerns the definition and the appropriate units of measuring capital.

Turning to it, Garrison's (2006) assessment of the relevant literature offers a superb synopsis. According to his account, there have been three compromise approaches to the definition and measurement of capital. These are the neoclassical, the Austrian, and the Cantabrigian, i.e. favored by proponents of the views of the University of Cambridge, England, economic theorists who started the controversy. Researchers in the neoclassical tradition define capital in terms of its output producing capacity and measure its quantity in *monetary units* of constant purchasing power. Also, frequently they adjust the acquisition prices of producer durables for improvements in their *quality*, but ignore invariably all of their other attributes, including the *time* over which such goods remain physically and economically viable in the productive process. This lack of attention to the *durability* or *roundaboutness* of capital constitutes the main difference that distinguishes them from the researchers who adhere to the Austrian approach. Because, while the latter do concur that money is the only conceivable means to agglomerate the great assortment of durables employed in production, they have come to know from Böhm-Bawerk (1889) and the other titans of the Austrian theory of capital that at least the durability of capital goods is too important to be ignored. Finally, with regard to the Cantabrigian approach, the researchers who adopt it insist on the definition and measurement of capital in physical units, which implies a framework of analysis totally alien to the process of depreciation, because physical wear and tear is the least reason for which such goods are scrapped or replaced.

From the preceding it follows that, if the Cambridge controversy on capital has any significant implications for the theorem of proportionality, these should stem from the differences between the neoclassical and the Austrian approaches. More specifically, they should stem from the emphasis that Austrian theorists place on the durability or roundaboutness of producer durables, which requires that capital be measured in some composite unit of standard purchasing power and durability, like, say, so many 2000 dollars for so many years. Under this convention Garrison (2006) has argued that:

> "The neoclassical school allows for a market determination of the interest rate (the loanable-funds theory) but does not allow for changes in the interest rate to have any significant effect on the intertemporal structure of capital…. The Austrian theory allows for a market determination of the interest rate and allows for changes in the interest rate to govern the intertemporal allocation of resources within the economy's capital structure"(p. 205).

Therefore, the difference in the units of measurement of capital between the neoclassical and the Austrian approaches translates into the finding that the latter allows for an economic theory of depreciation and replacement, whereas the former does not, and hence it ascertains that the theorem of proportionality has not been as innocuous as thought. Because by standing as a wedge between the neoclassical and Austrian approaches it delayed progress in the direction that Solow (1962) recommended in the following passage:

> "All machines, regardless of types, are one-hoss shays of fixed life L. Other assumptions are possible but it will be seen later that, for the kind of application I have in mind the assumption I have made is particularly easy to handle. It would be a genuine generalization of this model to extend it to cover different life-times for different machine-types, to permit the lifetime to be one of the unknowns of the problem"(p. 207).

In support of this conjecture consider Brems (1968). His model drew heavily on the structure of the model suggested by Solow (1962) and at the same time allowed for the lifetime of capital to be one of the unknowns of the problem. However, in view of the influence that the theorem of proportionality exercised at the time, its achievements were ignored.[12]

In conclusion, what is surprising is not that the theorem of proportionality contrasts sharply with the Austrian theory of capital. This contradiction was known from very early, because Hayek's (1939) theory of the business cycles, which was based on the relationship of the interest rate to the time structure of capital, never eclipsed. But to ascertain that at about the same time that Jorgenson (1963) launched the theorem of proportionality Solow (1962) recommended a generalization that would have put the neoclassical and Austrian theories of capital on a path to convergence is truly revealing. This piece of evidence more than any other demonstrates how a hypothesis gone awry may retard the progress of theory.

4. Views from a methodological perspective

The last task to be undertaken here seeks to expand the preceding assessment in three directions. In particular, the first of them is to speculate on the possible reasons for which mainstream economic theorists, as well as national and international agencies that publish data on capital stocks, have shunned all evidence against the theorem of proportionality. The second is to identify the rudiments of a model in which the time structure of capital takes center stage; and, finally, the third direction is to draw attention to an attempt by Bitros (2008a; 2008c) to formulate a simple two-sector model where the useful life of capital is determined endogenously in the presence of embodied technological change that evolves at a constant exogenous rate.

4.1 Implications from the method of economics

Economic theorists do not hold the same views regarding the proper approach to articulating scientific propositions and employing them to explain the structure of observed economic phenomena or predict their evolution. Machlup (1955) classified various researchers from this perspective into three categories, i.e. the extreme apriorists, the ultraempiricists and the logical positivists. Following Mises (1959), extreme apriorists believe that: a) the premises and axioms of economic theory are absolutely true; b) the theorems and propositions deduced from them by the laws of logic are absolutely true; c) there is no need to verify empirically either the axioms or the theorems; and d) the theorems could not be tested even if it were desirable to do so. In the realm of capital theory very close to this position are authors of neo-Austrian persuasions like Lachmann (1947; 1956) and Kirzner (1966; 1976), who object to the neoclassical notion of capital as a mass of *homogeneous* capital goods and instead emphasize their *heterogeneity* on the basis of their physical differences, the diversity of purposes to which they are assigned by individuals, and other attributes. As a result they find the idea of arriving at a single aggregate to represent the size of the stock of capital goods at the firm, the sector, or the economy level absolutely unacceptable, and hence for them the theorem of proportionality is vacuous.

The ultraempiricists hold the same view, but for different reasons. In particular, the researchers in this group reject all economic theories that are based on assumptions, postulates, premises or axioms that cannot be verified independently by reference to experience. From hypotheses that are not grounded in facts, they argue, only unfounded conclusions may follow, no matter how consistently the latter may be deduced from the former by applying the laws of logic. Instead, they suggest that a proper approach to research must start always with observations collected by statistical investigations, questionnaires to consumers and entrepreneurs, the examination of family budgets and the like. Thus, for them, the theorem of proportionality is devoid of empirical content, because Jorgenson (1974) derived it, for example, on the assumption that all depreciation takes the form of output decay, which cannot be verified independently in the presence of embodied technological change.[13]

By implication of the above, the great majority of economic theorists who side with the theorem of proportionality must fall in the third group, i.e. that of logical positivists. Consequently it is in their approach to economics that we must search for clues for its dominance. To this end, consider the following passage from Schumpeter (1954):

> “Economic theory… cannot indeed, any more than can theoretical physics, do without simplifying schemata or models that are intended to portray certain aspects of reality and take some things for granted in order to establish others according to certain

> rules of procedure. So far as our argument is concerned, the things (propositions) that we take for granted may be called indiscriminately either hypotheses or axioms or postulates or assumptions or even principles, and the things (propositions) that we think we have established by admissible procedure are called theorems" (p. 15).

From this it follows that positive economists proceed by constructing "models". But the passage does not give any hints as to how they select better over good "models". The case is that on this issue they have split into two main groups. The first group consists of those who maintain that the appropriate criterion by which to gauge the success of "models" is their ability to *predict* the phenomena to which they pertain, without regard to the empirical validity either of the "models" themselves or the "hypotheses or axioms or postulates or assumptions or even principles" on which they rest.[14] As for the second group, this comprises all those who place the emphasis on the ability of "models" to *explain* the phenomena under consideration, which in turn requires that both the "models" and the premises on which they stand must be empirically valid.[15] To see the rationale for their view, assume that one aspires to explain why and how a particular economic phenomenon may happen. Then one would need to construct a model linking causes to effects. Denote the model by T, its givens by C and the effects to which it leads by R. What T tells us is that, under conditions C the perceived process that drives the phenomenon should result in R or in symbols: $C \rightarrow R$. But suppose that R does not obtain. This would imply that C or T or both would not be true. Even in one case where the predicted R would not obtain, C or T or both would have to be rejected, since they would have been falsified, and then at least they would have to be revised. Consequently, if we are looking for *explanations* and not just *predictions*, we need to make sure that C and T are true or close to truth. For reasons of classification call the economists in the first group "instrumentalists" and those in the second group "structuralists." Accordingly, while all positive economists build "simplifying schemata or models that are intended to portray certain aspects of reality," instrumentalists select better over good "models" on the basis of their predictive power, whereas structuralists do the same by stressing their *explanatory* power. The question then that comes to mind is this. Has the method of positive economics something to do with the apparent invincibility of the theorem of proportionality or not?

It has for at least three reasons. Starting from the most innocuous, suppose that what Jorgenson (1963; 1965; 1974) wished to explain was *net* investment. Since the latter cannot be observed directly, the only way to factor it out from *gross* investment is to estimate depreciation. To do so it sufficed for him to invoke the theorem of proportionality, not as an apparatus to explain the sources and the processes of depreciation, but as a means to predict its magnitude. Thus, even though the model of depreciation that he adopted was based on several highly unrealistic assumptions, it remained consistent with the instrumentalist approach to economic analysis and this explains why the theorem of proportionality that derived from it proved so robust in the face of all criticisms from the structuralist standpoint. On the contrary, if he were interested to *explain* depreciation, certainly he would have modeled explicitly its determinants.

The second reason stems from Hulten and Wykoff (1981), Hulten, Robertson, and Wykoff (1989), and others, who claim that using a single number to characterize the process of depreciation helps achieve "a major degree of simplification". These researchers are right in that *simplicity* is a significant criterion for choosing among alternative hypotheses. But according to the following passage from Friedman (1953) the same is true with the criterion of *fruitfulness*:

> "The choice among alternative hypotheses equally consistent with the available evidence must to some extent be arbitrary, though there is general agreement that relevant considerations are suggested by the criteria of "simplicity" and "fruitfulness," themselves notions that defy completely objective specification"(p. 10).

Hence, even if a constant depreciation rate were equally consistent with an endogenously determined one, choosing it for its simplicity may entail substantial losses in explanatory and predictive power, as well as serious narrowing of the prospects for further fruitful research. Yet despite all evidence that a constant depreciation rate is indeed inferior in these respects, instrumentalist positive economists have opted exclusively for simplicity.

Lastly, and most importantly, the third reason is that changing over to a new approach would render obsolete much of the investment that has gone into the publication of capital stock series by national and international organizations. Certainly, if these data were produced in the private sector under competitive conditions, one would hope that at some point capital stock series based on a more fruitful approach would start to emerge and perhaps also supply might create its own demand. Yet under the present government driven system of producing and distributing such data, the rate of obsolescence of perpetual inventory based capital stock series is bound to be slow, if not nil. So this may be the hardest impediment to confront, if the incumbent theorem is to give way to one that would provide for an endogenously determined rate of depreciation.

From the preceding it emerges that, if we had to single out just one reason for the continued dominance of the theorem of proportionality in mainstream capital theory, this would be the lack of a model that might yield a better theorem in terms of standard selection criteria like explanatory and predictive power, simplicity, fruitfulness, parsimony of assumptions, etc.

Hence the issue is how to construct a model that might yield such a theorem. This is addressed briefly in the two sub-sections that follow.

4.2 Dethroning of non-reliable assumptions in the dominant model

Above it was established that, if a model is built to explain a particular phenomenon, the model itself as well as the premises on which it is erected must be amenable to empirical verification or better falsification. Thus, since the objective below is to present a model to explain depreciation, the first priority is to review and if necessary modify the main assumptions in the dominant depreciation model.

Among other less restrictive conditions, Jorgenson (1974) derived the theorem of proportionality by assuming that: a) producer durables can be aggregated at any desired level into a quantity of "capital-in-general"; b) all depreciation is due to output decay, and c) as initial investment depreciates, it gives rise to a stream of replacement investments, each of which generates a new stream of subsequent replacements, and this process repeats itself indefinitely. Now regarding the first assumption, in previous sections we saw that the possibility of obtaining a measure of "capital-in-general" is a highly contested issue for three reasons: First, because more than any other input producer durables differ in many respects, including measurement units, rates of physical depreciation, maintenance requirements, etc. Second, because, as Garrison (2006) has argued, using money values alone to capture the heterogeneity of producers durables won't do the trick, since money values do not allow for the time dimension, i.e. the durability or longevity or useful life of capital goods; and thirdly, because even if the problem of measurement units is resolved by adopting a composite scale like money and time, from Zarembka (1976) and Brown and Chang (1976) it follows that aggregation would require that all producer durables depreciate at the same proportional rate.[16] Therefore, to achieve progress what we need is a general equilibrium model of depreciation that will allow for the heterogeneity of producer durables by focusing simultaneously on the money and time structure of the capital stock.

The second key assumption has to do with the sources that are responsible for the depreciation of a piece of equipment. In principle, depreciation springs from three sources. The first, called *output decay,* emanates from the possibility that as a machine ages it may produce less output due to wear and tear. The second, called *input decay*, signifies that an older machine may absorb more inputs of materials, labor, maintenance, etc. per unit of output. Finally, the third source of depreciation is technological change, because the appearance of newer and more productive machines gradually renders older machines obsolete, thus leading to a gradual decline in their prices. Perhaps, at the time the theorem of proportionality was proved, output decay was the dominant source of depreciation, so this assumption was justified. But now the circumstances have changed and the dominant source of depreciation is technological obsolescence. Besides, as Bitros and Flytzanis (2009) show, under active policies of maintenance and utilization, depreciation-in-use, may vanish. Consequently, the general equilibrium model sought should place the emphasis of depreciation in the presence of embodied technological progress.

Next, let us turn to the third assumption that provides for the process of reinvestment to repeat itself indefinitely. For this to be even remotely likely, re-investment opportunities at the same terms and conditions as those that prevailed initially must be available for all future times. But if we know anything with certainty from the theories of business cycles and economic growth, this is that re-investment opportunities vary with market conditions. For example, re-investment opportunities may be steadily available in sectors producing necessities with low demand elasticities and slow technological progress, whereas in sectors producing luxuries with high demand elasticities and rapid technological change, reinvestment opportunities may vary widely. Hence, the general equilibrium model pursued should provide for an endogenous approach to the horizon of re-investment opportunities, because this may facilitate the generalization suggested by Solow (1962).

From a methodological standpoint successful research in empirical sciences quite often involves reviewing an established model and dethroning its non-reliable assumptions. In the case of the model from which the theorem of proportionality has been derived there is not one but at least three such assumptions. Therefore, research efforts to develop models of depreciation in which these assumptions are relaxed hold good prospects for success.

4.3 Towards a model of endogenous depreciation, scrapping and replacement[17]

In Bitros (2008a; 2008c) I proposed and analyzed a general equilibrium model of an economy with the following structure. The economy consists of two sectors. The representative firms *X* and *Y* that operate in them are characterized by three fundamental differences. The first of them is that, whereas firm *X* supplies electricity, which is a necessity with relatively inelastic demand that lasts forever, firm *Y* supplies tennis rackets, which is a luxury with highly elastic demand that may vanish at any time due to shifts in tastes. The second difference springs from the implication that, because of the inherent difference in the nature of their products, the two firms are bound to view their re-investment opportunities differently. Firm *X* would plan for the indefinite future by adopting a capital policy of perpetual replacements, whereas firm *Y* would adopt a scrapping policy, which would give it an option to decide at the end of the useful life of its current investment whether to exit or reinvest, depending on the demand for tennis rackets at that time.[18] Finally, the third difference is that technical progress increases the productivity of more recent vintages of the durables in each sector at different constant and exogenous rates. Otherwise firms *X* and *Y* are similar. In particular, they face downward sloping demand curves, implying that they behave as monopolists. They deter other firms from entering into their markets to take advantage of the higher productivity of newer durables by applying a pricing rule that transfers all benefits from technological change to final consumers; and last, but not list, while the durables they build internally are fixed in the sense that they cannot be moved from the one sector to the other, workers move freely in the economy.

Due to the structural and behavioral differences of firms *X* and *Y*, the model that emerges leads to different useful lives for their durables. To be sure, drawing on Bitros and Flytzanis (2005), this finding would be expected even if the firms differed only with respect to their capital policies, i.e. replacement vs. scrapping. But in the richer modeling environment of these papers the differences in the useful lives arise also because the two firms operate in markets with different elasticities of demand and different rates of embodied technical change. Thus, as soon as the attention turns from microeconomics to macroeconomics, the analysis confronts the question of how to aggregate the two durables, since: a) they are not substitutes and hence their physical quantities cannot be translated into an index of homogeneous units; b) older vintages differ from newer vintages because the latter embody the more recent advances in technology, and c) depending on the elasticities of demand for electricity and tennis rackets, the rates of embodied technological change, and other market parameters, the durables of firm *X* may last longer than those of firm *Y*. To tackle it, the analysis of the model starts from the realization that at the sectoral level the quantities of the two durables are expressed in uniform monetary values of constant prices. This implies that, if they did not differ in any other respect and their relative prices remained constant, we could invoke Hicks (1946, p. 33) theorem to obtain an index of "capital-in-general". But the two capital goods differ in durability and this approximation would be open to serious objections from both the theoretical and the empirical standpoints. For this reason the model is endowed with a Haavelmo (1960, pp. 95-102) type mechanism, which, by expressing the two capital goods in units of standard durability, permits their aggregation in a more efficient manner.[19]

Finally, the proposed model is put through several tests to assess its performance in comparison to that of the dominant model. The results from these tests have shown that, by focusing on the structure of capital and the useful lives of its components, the model gains significantly in explanatory and predictive power. It deals with the aggregation problem in a more efficient way;

and it realizes these gains without introducing excessive complexity, mathematical or computational. In short, its fruitfulness is far superior to that of the dominant model.

5. Overall summary of findings and conclusions

Shortly after the theorem of proportionality was launched, a few researchers at the beginning and many more later on started to question its theoretical foundations by arguing that: a) the assumptions of the model from which the theorem derived were highly restrictive and unlikely to hold in reality; b) it constrained seriously the efforts by business cycle and economic growth theorists to characterize properly the dynamics of investment and thus come up with models in which the short run Keynesian analysis merged seamlessly with the classical long run analysis; c) it failed to account for the findings by industrial organization theorists which showed that how durable the producer durables are built is decided at the time of their production on the basis of economic criteria, whereas how long they last is determined also by such deliberate economic processes as the intensity of utilization and maintenance; and d) its implication that firms cannot affect the manner in which their durables decay is completely alien to the modes of thinking in neighboring fields to economics, like operations research, operations management, capital budgeting, and accounting. However, even though all above criticisms were well reasoned, they failed to attract significant attention by the community of interested researchers and the theorem has continued to carry the day in mainstream capital theory.

In retrospect, and in the light of the assessment that was carried out above, it appears that responsible for this failure has been one reason. Namely, that the researchers who objected to the theorem did not realize that the only way to bypass it was to obtain a better theorem in terms of certain criteria like explanatory and predictive power, simplicity, fruitfulness, etc. By taking this route in more recent years a group of researchers in the areas of business cycles and economic growth have demonstrated that switching to an endogenous theory of depreciation, scrapping and replacement is technically feasible and can be very rewarding in terms of implications. Yet a model aimed at the generalization first suggested by Solow (1962), and exploited a few years later to some extent by Brems (1968), is missing.

Bitros (2008c) has attempted to contribute in this direction by presenting a two-sector vintage capital model with emphasis on the heterogeneous structure of capital and the useful lives of its components. From the results it turns out that, modeling of depreciation, scrappage and replacement as endogenous variables in the presence of technological change is technically feasible and that it yields rich implications for theory and policy. For example, whereas on the one hand the model brings into the forefront the channel through which the rates of interest and embodied technological change determine the structure of capital by influencing the decisions to replace and scrap capital, on the other, the structure of capital is linked directly to the structure of demand for final output, thus opening the way for the study of how changes in the structure of final output may influence the structure of capital over the business cycle and in the long-run. But the fruitfulness of the model does not end with the above. Because, as Bitros (2008a) showed more recently, the same model could be employed to assess the biases involved in the computing of perpetual inventory capital stock series, which may bias further the results from models in which such series are employed for the purposes of econometric estimation.

In conclusion, simplicity is not any more a good reason for the continued dominance of the theorem of proportionality in mainstream capital theory. A justification for this is that its employment thwarts the advances that can be achieved in economic theory and econometric applications by returning to a general equilibrium model centered on the time structure of capital and the useful lives of its components. Indicative of how technically feasible and substantively fruitful such advances might turn out to be are the achievements in recent years in the fronts of economic growth and business cycles, where the adoption of the theorem of proportionality has retreated. Moreover, the returns in terms of precision and robustness of models with endogenous depreciation, scrappage and replacement can be expected to be even higher in growth accounting, productivity studies, and various other applications, where presently researchers employ estimates of capital stocks based on the perpetual inventory method.[20] On these grounds, the sooner the theorem of proportionality is abandoned or at least revised, the better for economic theory and policy.

6 Appendix: Contemporary methodological guidelines for research in the empirical sciences

Notwithstanding important disagreements among philosophers of science, what is accepted today as appropriate methodological approach to science can be laid down briefly in the following four principles:

Principle I A scientific theory (physics, biology, economics, sociology, medicine but NOT mathematics, logic, philosophy and other non-empirical disciplines) must be empirically testable. It must be verifiable said the logical positivists in the 1930's, falsifiable as Popper (1935) insisted then and later. The two are not equivalent: there is an asymmetry between verification and falsification, but that need not bother us here. The important thing is that scientific theories must be empirically testable. We can call this principle, the principle of empirical accountability. No empirical accountability, no science. Instead of science you have metaphysics.

Principle II Some metaphysics is instrumentally useful. It can serve heuristically. One may engage in a 'metaphysical' research programme from which certain empirical hypotheses can be deduced. We may call this principle, the principle of scientific speculation or hypothesis construction. One can use experience or imagination or metaphysical ideas as background; Certainly experience, which serves as background knowledge; But not induction.

Principle III There is no induction. What we call induction is unwarranted generalization from a finite number of observations. Whenever you believe you are using inductive thinking, you are really engaged in an activity described in Principle II above. There are no neutral observations. They are always theory-laden (or theory-impregnated). They contain theory. So you can't use a number of supposedly neutral observations to form a universal theory.

Principle IV What this boils down to is that usually theories (hypotheses) in empirical sciences are to be compared, say T_1 (the old one) and T_2 (the newly proposed one) and we judge their merits and demerits using various criteria. If we opt for T_2 and decide to discard T_1, it will be because the newer one has greater explanatory and/or predictive (or 'postdictive') power.

7. Endnotes

* Professor D. Dimitrakos, of the National and Kapodistrian University of Athens, took me to task with his highly perceptive remarks regarding my arguments in Section 4 and the Appendix. Professor A. Karayiannis, of the University of Piraeus, read the paper with the critical eye of a specialist in the history of economic theories and save me from several pitfalls of omission and wrong attribution; and Professor R. Garrison injected me with ample doses of encouragement. To all of them, as well as to the many colleagues who have read various versions of this paper over the years and helped me improve it stylistically and substantively, I express my sincere appreciation. However, all responsibility for the views expressed in the paper rests solely with me.

1 Note that the symbols used below may not correspond to those employed in the original sources.

2 For the sake of historical accuracy it should be noted that at about the same period other leading contributors to the neoclassical theory of capital adopted various ad hoc approaches to modeling depreciation. For example, Solow (1956) ignored depreciation altogether, whereas Samuelson (1962) introduced proportionality on the grounds that:

> "To keep the alpha good homogeneous independently of age, one has to assume a force of mortality independent of age (or an exponential life table). This means that physical depreciation is always directly proportional to the physical stock of alpha, K_a: Depreciation equals $_a$ times K_a where the average length of life of alpha is the reciprocal of the $_a$ factor."(p. 197)

3 Notice the switch from the term "hypothesis" in Smith (1961, p. 166) to the term "theorem" in Jorgenson (1963, p. 251). This reflects the major difference mentioned above, because the latter obtained what was hypothesized by the former author as a general result from renewal theory.

4 In the period during which Jorgenson (1963, 1974) launched and defended the theorem of proportionality, there appeared several related surveys. Dean (1962) reviewed the literature on replacement theory. McCall (1965) assessed the literature on the maintenance of stochastically failing equipment and shortly thereafter Jorgenson, McCall, and Radner (1967) summarized the literature on optimal replacement policy for both deteriorating and failing equipment. Lastly, shortly thereafter, Pierskalla and Voelker (1976) reviewed the literature regarding maintenance models of deteriorating systems, etc. Having gone through these surveys, I failed to find even a single reference to the theorem of proportionality. Hence, given that a) all known replacement models at the time aimed at the determination of the optimal useful life of equipment, and b) the primary focus was on the reliability, maintainability, and reparability of equipment and systems thereof, I surmise that this theorem is alien to such fields as operations research, operations management, capital budgeting and accounting.

5 As it will be argued shortly this proposition received further confirmation from the research in the area of Real Business Cycle (RBC) analysis, which has shown that the replacement ratio cannot possibly remain constant, because output induced changes in investment over the business cycle shift the time structure of the capital stock.

6 Just for the clarity of the term depreciation it should be noted that under certain conditions depreciation is dual to replacement investment, which constitutes a component of gross investment equal to retirements plus the loss of efficiency in the surviving part of the capital stock.

7 According to Hicks (1965, p. 4) there is no compelling methodological or other reason for studying separately economic growth from business cycles. But in mainstream literature it has been common to think of business cycles models as distinct from models of economic growth and to view business cycles as deviations from some smooth, usually deterministic, trend that stands as a proxy for growth. Consequently, this arrangement is adopted only for convenience of presentation.

8 From now on the terms retirements, scrapping or scrappage will be used interchangeably.

9 Parallel to these efforts, some growth theorists drew on Epstein and Denny (1980), Auernheimer (1986), Bischoff and Kokkenlenberg (1987), Rubos and Auernheimer (1997) and others to investigate the implications of depreciation, utilization and maintenance within homogeneous capital neoclassical-type growth models. But, as the paper by Licandro, Puch and Ruiz-Tammarit (2001) indicates, apart from finding that the speed of convergence to the steady state was substantially reduced relative to the standard neoclassical growth model with adjustment costs, the results did not shed much light on the issue of coupling under investigation. For this reason the review here is re-

stricted to the achievements in the context of vintage capital models.

10 During this period a comparable research effort was underway in the United States. For example, Cooley, Greenwood and Yorukoglu (1997) analyzed the decision to replace old technologies with new ones in a vintage model of growth with endogenous depreciation and found that the transitional dynamics differed markedly from the standard neoclassical growth model.

11 It should be noted that this result is not specific to the vintage capital model because more recently Gylfanson and Zoega (2002) derived it in a Solow model and a model of endogenous growth with learning-by-doing in which the social planner could decide on the optimal durability of capital and its depreciation, including obsolescence.

12 Ignored were also the results obtained by Ioyha (1971) in the context of the one-sector growth model, where the useful life of capital was ingeniously linked to maintenance. But at least in this case one might invoke the reason that this model was expressed in *net* saving, *net* investment, and *net* capital stock, i.e. a modeling approach that had gone out of fashion.

13 To ascertain why it is impossible to discriminate between output decay and technological change see Hall (1968).

14 Samuelson (1947) and Friedman (1953) introduced this approach into economics following the epistemologist Duhem (1908), who recommended using theories as instruments and without concern if they are true or if their assumptions are realistic. According to the latter, what is important is whether the predictions derived from theories match appearances (phainomena), thus implying that models are useful not as causal explanations, but 'as if' ways of highlighting what appears before us.

15 For a brief but more detailed account of the principles that guide contemporary research in the empirical sciences, see the Appendix.

16 In retrospect, what this literature implies is that in deriving the theorem of proportionality Jorgenson (1974) may have proved what he has assumed, thus falling into the trap of circular logic. But this is beside the point discussed in this juncture.

17 For a very general model of capital in which explicit emphasis is also placed on net or expansionary investment as well as the whole range of operating and capital policies see Bitros and Flytzanis (2002).

18 At this point the choice of capital policy may appear to be imposed on the two firms exogenously. But it is not, because in an earlier stage the two firms solve the problem posed by Bitros and Flytzanis (2005), where the horizon of re-investment is determined endogenously. In other words, here it is assumed that, if the firms solved this problem in the light of the differences in the nature of and the demand for their products, firm *X* would apply replacement and firm *Y* scrapping.

19 In the context of the vintage capital model Bhattacharyya (1965) has proposed a different mechanism to standardize the equipment of various vintages for differences in durability. But in comparison to that of Haavelmo it is much harder to apply because it is based on two parameters whose value is difficult to estimate.

20 To appreciate the differences that they may result in these areas from the new age of capital centered research approach, compare the study by Jorgenson (1966) with those, for example, of McHugh and Lane (1987) and more recently Whelan (2005).

8. References

1. Ambler, S. and Paquet, A., (1994), “Stochastic Depreciation and the Business Cycle,” International Economic Review, 35, 101-116.
2. Auernheimer, L., (1986), “Variable Depreciation and Some of its implications,” Canadian Journal of Economics, 19, 99-113.
3. Barro, R. J., (1972) “Monopoly and Contrived Depreciation,” Journal of Political Economy, 80, 598-602.
4. Benhabib, J. and Rustichini, A., (1991), “Vintage Capital, Investment, and Growth,” Journal of Economic Theory, 55, 323-339.
5. ---------------, (1993), “A vintage capital model of investment and growth: Theory and evidence,” in R. Becker et al. (Eds.), General Equilibrium, Growth and Trade. II The legacy of Lionel W. Mackenzie, Academic Press.
6. Bhattacharyya, S. K. (1965), “Capital Longevity and Economic Growth,” Review of Economics Studies, 32, pp. 39-46.
7. Bischoff, C. W. and Kokkenlenberg, E. C., (1987), “Capital Utilization and Depreciation in Use,” Applied Economics, 19, 995-1007.
8. Bitros, G. C., (2008a) “Aggregation of Producer Durables with Exogenous Technological Change and Endogenous Useful Lives,” Athens University of Economics and Business, Department of Economics, Discussion Paper No. 193.
9. ---------------, (2008b), “The optimal lifetime of assets under uncertainty in the rate of embodied technical change,” Metroeconomica, 59, 173-188.
10. ---------------, (2008c), “Why the Structure of Capital and the Useful Lives of its Components Matter: A Test Based on a Model of Austrian Descent,” Review of Austrian Economics, DOI 10.1007/s11138-008-0049-1.
11. ---------------, and Flytzanis, E., (2009), “Utilization and Maintenance in a Model with Scrapping,” European Journal of Operational Research, 194, pp. 551-573.
12. ----------------------, and Flytzanis, E., (2002), “Towards a General Theory of Real Capital,” in Bitros, G. C. and Katsoulakos, Y., (Eds.), Essays In Economic Theory, Growth and Labor Markets: A festschrift in Honor of E. Drandakis, Cambridge UK: Edward Elgar.
13. ------------------------, (2005), “On the Optimal Lifetime of Assets,” Athens University of Economic and Business, Department of Economics, Discussion paper No. 170.
14. ----------------, Hritonenko, N., and Yatsenko, Y., (2007), “Investment, Replacement and Scrapping in a Vintage Capital Model with Embodied Technological Change,” Athens University of Economic and Business, Department of Economics, Discussion paper No. 186.
15. Brems, H., (1968), Quantitative Economic Theory: A Synthetic Approach, New York: John Wiley & Sons Inc.
16. Brown, M. and Chang, W. W. (1976) “Capital Aggregation in a General Equilibrium Model of Production,” Econometrica, 44, 1179-1200.

17. Böhm-Bawerk, E. (1889) Positive Theory of Capital, 2nd volume in the 3-volume work Capital and Interest, translated by G. D. Hunke and H. F. Sennholtz, South Holland, Ill.: Libertarian Press, 1959. (The three volumes were originally published in German in 1984, 1989, and 1909).
18. Boucekkine, R., Germain and Licandro, O., (1997), "Replacement Echoes in the Vintage Capital Growth Model," Journal of Economic Theory, 74, 333-348.
19. ----------------, and Magnus, A., (1998), "Creative Destruction, Investment Volatility, and the Average Age of Capital," Journal of Economic Growth, 3, 361-384.
20. ----------------, and Martinez, B., (2003), "Replacement, adoption and economic dynamics: lessons from a canonical creative destruction model," Structural Change and Economic Dynamics, 14, 339-359.
21. Bulow, J. (1986) "An Economic Theory of Planned Obsolescence," Quarterly Journal of Economics, 101, 1986.
22. Caballero, R., Engel, M. R. A., Haltiwanger, J. C., Woodford, M., and Hall, R. E., (1995), "Plant-Level Adjustment and Aggregate Investment Dynamics," Brookings Papers on Economic Activity, 1995, 1-54.
23. Caballero, R. and Hammour, M., (1994), "The Cleansing Effect of Recessions," American Economic Review, 84, 1350-1368.
24. ----------------, (1996), "On the Timing and Efficiency of Creative Destruction," Quarterly Journal of Economics, 111, 805-852.
25. Chatterjee, S. (2005) "Capital Utilization, Economic Growth and Convergence," Journal of Economic Dynamics and Control, 29, 2093-2124.
26. Chirinko, R. S., (1993), Business Fixed Investment Spending: Modeling Strategies, Empirical Results, and Policy Implications, Journal of Economic Literature, 31, 1875-1911.
27. Choi, J. B. and Kollintzas, T. (1985), " A linear Rational Expectations Model of Aggregate Investment with Endogenous Capital Utilization and Maintenance," University of Pittsburgh, unpublished mimeo.
28. Coase, R., (1972),"Durability and Monopoly," Journal of Law and Economics, 15, 143-149.
29. Collard, F. and Kollintzas, T., (1998), ""δ" along the business cycle," Athens University of Economics and Business, Department of Economics, Discussion Paper No. 93.
30. Cooley T. F., Greenwood, J. and Yorukoglu, M. (1997) "The Replacement Problem," Journal of Monetary Economics, 40, 457-499.
31. Cummings, J. G. and Violante, G. L., (2002), "Investment-Specific Technical Change in the United States (1947-2000): Measurement and Macroeconomic Consequences," Review of Economic Dynamics, 5, 243-284.
32. Dean, B. V., (1962), "Replacement Theory," in R. L. Ackoff (ed.) Progress In Operations Research, New York: J. Wiley & Sons, 330-362.
33. Domar, E. D., (1946), "Capital Expansion, Rate of Growth, and Employment," Econometrica, 14, 137-147.
34. -----------------, (1953),"Depreciation, Replacement, and Growth," Economic Journal, 63, 1-32.

35. Dosi, G., Fagiolo, G., and Roventini, A., (2006), "An Evolutionary Model of Endogenous Business Cycles," Computational Economics, 27, 3-34.
36. Dueker, M. J. and Fischer, A. M., (2003), "Fixing Swiss Potholes: The Importance of Improvements," Federal Reserve Bank of St. Louis, Working Paper 2001-025B.
37. ----------------, and Dittmar, R., (2002), "Stochastic Capital Depreciation and the Comovement of Hours and Productivity," CEPR Discussion Paper No. 3192.
38. Duhem, P., (1908), Sozein Ta Phainomena, Essai sur la Notion de Theories Physique de Platon a Galilee, Paris: Herman. English translation Edmond Dollard and Chaninah Marschler, To Save the Phainomena, Chicago: University of Chicago Press, 1969.
39. Eisner, R., (1952), "Depreciation Allowances, Replacement Requirements and Growth," American Economic Review, 42, 820-831.
40. ------------, (1956), "Technological Change, Obsolescence and Aggregate Demand," American Economic Review, 46, 92-105.
41. Epstein, L. and Denny, M., (1980), "Endogenous Capital Utilization in a Short-Run Production Model," Journal of Econometrics, 12, 189-207.
42. Feldstein, M. S., (1972/1974), "Tax Incentives, Stabilization Policy, and The Proportional Replacement Hypothesis: Some Negative Conclusions," Southern Economic Journal, 40, 544-552. This paper appeared under the same title in 1972 as Discussion Paper Number 249, Harvard Institute of Economic Research.
43. --------------- and M. Rothschild, (1972/1974), "Towards an Economic Theory of Replacement Investment", Econometrica, Vol. 42, 393-423. This paper appeared under the same title in 1972 as Discussion Paper Number 249, Harvard Institute of Economic Research.
44. Friedman, M., (1953), "The Methodology of Positive Economics," in Essays in Positive Economics, Chicago: University of Chicago Press, pp. 3-43.
45. Garrison, R. W., (2006), "Reflections on Reswitching and Roundaboutness," in Roger Koppl (ed.) Money and Markets: Essays in Honor of Leland B. Yeager, New York: Rutledge.
46. Greenwood, J., Herkowitz, Z., and Huffman, G. W., (1988), "Investment, Capacity Utilization, and the Real Business Cycle," American Economic Review, 78, 402-417.
47. Gylfanson, T. and Zoega, G., (2002), "Obsolescence," Centre for Economic Policy Research, Discussion Paper 2833.
48. Haavelmo, T. (1960) A Study in the Theory of Investment, Chicago: The University of Chicago Press.
49. Hall, R. E., (1968), "Technical Change and Capital from the Point of View of the Dual," Review of Economic Studies, 35, pp. 35-46.
50. Harrod, R. F., (1948), Towards a Dynamic Economics, New York: St Martin's Press Inc.
51. Hayek, F. A., (1939), Profits, Interest and Investment, London: Routledge & Kegan Paul, Ltd.
52. Herkowitz, Z., (1986), "The real interest rate and aggregate supply," Journal of Monetary Economics, 18, 121-145.
53. Hicks, J., (1965), Capital and Growth, Oxford: Oxford University Press.

54. -----------------, (1946), Value and Capital, Oxford: Oxford University Press, 2nd edition.
55. Hulten, C. R. and Wykoff, F., (1981), "The Estimation of Economic depreciation Using Vintage Asset Prices," Journal of Econometrics, 15, 367-396.
56. -----------------, Robertson, J.W. and Wykoff, F. (1989) "Energy, Obsolescence, and the Productivity Slowdown," in D. W. Jorgenson and R. Landau (eds.) Technology and Capital Formation, Cambridge MA: MIT Press, 225-258.
57. Ioyha, M. A., (1971), "Capital Durability and the Golden Rule of Capital Accumulation: A Note," Southern Economic Journal, 38, 93-96.
58. Johansen, L., (1959), "Substitution versus Fixed Production Coefficients in the Theory of Economic Growth: A Synthesis," Econometrica, 27, 157-176.
59. Jorgenson, D. W. (1963) "Capital Theory and Investment Behavior," American Economic Review, 52, 247-259.
60. -----------------, (1965), "Anticipations and Investment Behavior," in The Brookings Quarterly Econometric Model of the United States Chicago: Rand McNally and Co, 35-92.
61. -----------------, (1966), "The Embodiment Hypothesis," Journal of Political Economy, 74, 1-17.
62. -----------------, (1974), "The Economic Theory of Replacement and Depreciation," in W. Sellekaerts (ed.) Econometrics and Economic Theory: Essays in Honour of Jan Tinbergen, New York: International Arts and Sciences Press. From Feldstein (1974) it follows that he was aware of this paper as forthcoming from 1972.
63. -----------------, McCall, J. J., and Radner, R., (1967), Optimal Replacement Policy, Chicago: Rand McNally & Company.
64. ----------------- and Stephenson, J., (1967), "Investment Behavior in U. S. Manufacturing, 1947-1970," Econometrica, 35, 169-200.
65. ----------------- and Stephenson, J., (1969), "Issues in the Development of the Neoclassical Theory of Investment," Review of Economics and Statistics, LI, 346-353.
66. Kamien, M. I., and Schwartz, N. L. (1971) "Optimal Maintenance and Sale Age for a Machine Subject to Failure", Management Science, 17, 495-504.
67. -----------------, (1974), "Product Durability under Monopoly and Competition," Econometrica, 42, 289-301.
68. Keynes, J. M., (1936), The General Theory of Employment, Interest, and Money, New York: Harcourt, Brace, and Company.
69. Kinokuni, H., (1999) "Repair Market Structure, Product Durability, and Monopoly," Australian Economic Papers, 38, 343-353.
70. Kirzner, I. M., (1966), An Essay on Capital, New York: Augustus M. Kelley.
71. ----------------- (1976), "The Theory of Capital", in E. G. Dolan (ed.), The Foundations of Modern Austrian Economics, Kansas City: Sheed and Ward.
72. Kleiman, E. and Ophir, T. (1966) "The Durability of Durable Goods," Review of Economic Studies, 33, 165-178.

73. Lachmann, L. M., (1956/1978) Capital and its Structure, Kansas City: Sheed, Andrews, and McMeel.
74. -----------------, (1947), "Complementarity and Substitution," Economica, 14, 108-119.
75. Licandro, O. and Puch, L. A. (2000) "Capital Utilization, Maintenance Costs and Business Cycle," Annales d' Economie et de Statistique, 58, 144-164.
76. -----------------, and Ruiz-Tamarit, J. R., (2001), "Optimal Growth under Endogenous Depreciation, Capital Utilization and Maintenance Costs," Investigaciones Economicas, XXV, 543-559.
77. Machlup, F., (1955), 'the Problem of Verification in Economics," Southern Economic Journal, 22, pp. 1-21. Also in G. C. Bitros (ed.), Selected Economic Writings of Fritz Machlup, New York: New York University Press, 1976, pp.57-78.
78. Malcomson, J. M. (1975) "Replacement and the Rental Value of Capital Equipment Subject to Obsolescence," Journal of Economic Theory, 10, 24-41.
79. Mann, D. P. (1992) "Durable Goods Monopoly and Maintenance," International Journal of Industrial Organization, 10, 65-79.
80. Massell, B. F., (1962), "Investment, Innovation, and Growth," Econometrica, 30, 239-252.
81. McCall, J. J., (1965), "Maintenance Policies for Stochastically Failing Equipment: A Survey," Management Science, 11, 493-524.
82. McGrattan, E. R. and Schmitz Jr. J. A., (1999) "Maintenance and Repair: Too Big to Ignore" Federal Reserve Bank of Minneapolis Quarterly Review, 2-13.
83. McHugh, R. and Lane, J. (1987) "The age of capital, the Age of Utilized Capital and the Embodiment Hypothesis," Review of Economics and Statistics, 69, 362-367.
84. Merrick, J. J., (1984), "The anticipated real interest rate, capital utilization and the cyclical pattern of real wages," Journal of Monetary Economics, 13, 17-30.
85. Miller, E. M., (1982), "Capital Aggregation in the Presence of Obsolescence-Inducing Technical Change," Review of Income and Wealth, 29, 284-296.
86. -----------------, (1990), "Can a Perpetual Inventory Capital Stock be used for Production Function Parameter Estimation?" Review of Income and Wealth, 36, 67-82.
87. Mises, L. V., (1959), Human Action, New Haven, Con: Yale University Press.
88. Nickell, S, (1975) "A Closer Look at Replacement Investment," Journal of Economic Theory, 10, 54-88.
89. Parks, R. W., (1974), "The Demand and Supply of Durable Goods and Durability," American Economic Review, 64, 37-55.
90. -----------------, (1979), "Durability, Maintenance and the Price of Used Assets," Economic Enquiry, 17, 197-217.
91. Pierskalla, W. P., and Voelker, J. A., (1976), "A Survey of Maintenance Models: The Control and Surveillance of Deteriorating Systems," Naval Research Logistics Quarterly, 23, 353-388.
92. Popper, K. R., (1935), Logik der Forschung, Vienna: Springer Verlag.Translated into English as The Logic of Scientific Discovery, London: Hutchinson, 1959.

93. Preinreich, G. A. D., (1940), "The Economic Life of Industrial Equipment", Econometrica, Vol. 8, 12-44.
94. Robinson, J., (1953), "The Production Function and the Theory of Capital," Review of Economic Studies, 21, 81-106.
95. -----------------, (1959), "Some Problems of Definition and Measurement of Capital," Oxford Economic Papers, 11, 157-166.
96. -----------------, (1975), "The Unimportance of Reswitching," Quarterly Journal of Economics, 89, 32-39.
97. Rubos, B. and Auernheimer, L., (1997), "Endogenous Capital Utilization in a Neoclassical Growth" Model, ITAM, DP 9704, Mexico.
98. Rust, J. (1987) "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," Econometrica, 55, 999-1033
99. -----------------, (1986), "When is it Optimal to Kill off the Market for Used Durables Goods," Econometrica, 54, 65-86.
100. Samuelson, P. A., (1947), Foundations of Economic Analysis, Cambridge Mass.: Harvard University Press.
101. -----------------, (1962), "Parable and Realism in Capital Theory: The Surrogate Production Function, Review of Economic Studies, 29, 193-206.
102. -----------------, (1966), "A Summing Up," Quarterly Journal of Economics, 80, 568-583.
103. Schmalensee, R. (1974) "Market Structure, Durability, and Maintenance Effort," Review of Economic Studies, 41, 277-287.
104. Schumpeter, J. A., (1954), History of Economic Analysis, New York: Oxford University Press.
105. Sheshinski, E., "Balanced growth and stability in the Johansen vintage model," Review of Economic Studies, 34, 239-248.
106. Smith, V. L., (1957), "Economic Equipment Policies: An Evaluation." Management Science, 4, 20-37.
107. ----------------- (1961) Investment and Production: A Study in the Theory of the Capital-Using Enterprise, Cambridge, Mass: Harvard University Press.
108. Solow, R. M., (1956), "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, 70, 65-94.
109. -----------------, (1962), "Substitution and Fixed Proportions in the Theory of Capital," Review of Economic Studies, 29, 207-218.
110. -----------------, (1997), "Is There a Core of Usable Macroeconomics We Should All Believe In?" American Economic Review, 87, 230-32.
111. -----------------, Tobin, J., Von Weizsacker, C. and Yaari, M., (1966), "Neoclassical growth with fixed factor proportions," Review of Economic Studies, 33, 79-115.
112. Swan, P. L., (1971), "Optimum Durability, Second-Hand Markets, and Planned Obsolescence," Journal of Political Economy, 80, 575-585.
113. -----------------, (1977), "Product Durability under Monopoly and Competition: Comment," Econometrica, 45, 229-236.
114. Taubman, P. and Wilkinson, M. (1970) "User Cost, Output, and Unexpected Price Changes," in E. D. Phelps et al (Eds) Microeconomic Foundations of Employment and Inflation Theory, New York: W. W. Norton and Company Inc., 411-420.
115. Terborgh, G., (1949), Dynamic Equipment Policy, New York: McGraw-Hill.

116. Thompson, G. L. (1968) “Optimal Maintenance Policy and Sale Date of a Machine,” Management Science, 14, 543-550.
117. Tinbergen, J., (1942), “Zur Theorie der langfristigen Wirtschaftsentwicklung,” Wirtschatliches Archiv, 55, 511-549.
118. Waldman, M. (1993) “A New Perspective on Planned Obsolescence,” Quarterly Journal of Economics, 108, 273-283.
119. Whelan, K., (2005), “Embodiment, Productivity, and the Age Distribution of Capital,” http://mpra.ub.uni-muenchen.de/5912/.
120. Winston, G. C. (1974) “The Theory of Capital Utilization and Idleness,” Journal of Economic Literature, 12, 1301-1320.
121. Zarembka, P. (1975) “Capital Heterogeneity, Aggregation, and the Two-Sector Model,” Quarterly Journal of Economics, 89, 103-114.