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Sustek, Roman

University of Iowa

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Roman Šustek[*]

University of Iowa

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Abstract

In most manufacturing industries output is adjusted in a lumpy way along three margins: shiftwork, weekend work, and closing a plant temporarily down. We incorporate such decisions into a dynamic general equilibrium model and study: (i) if such micro-level nonconvexities magnify business cycles; and (ii) if the aggregate effects of changes in firms' borrowing costs due to monetary policy shocks vary over the cycle. Calibrated to industrial observations, the model implies that aggregate output is in fact 25% less volatile than in an economy without such features, and monetary policy shocks have similar effects on output in recessions as in expansions.

JEL Classification Codes: E22, E23, E32, E52.

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1   Introduction

Over the past three decades or so quantitative dynamic general equilibrium theory has become a dominant tool of analysis in the study of the business cycle. As stressed by Prescott (2005), a crucial element behind the theory's success is its emphasis on consistency with micro observations. At the micro level, economic decisions are often discrete and lumpy. Workers, for example, work either a fixed number of hours per week or do not work at all, plants are either operated or shut down, and investment of production units is characterized by investment spikes followed by long periods of investment inactivity.

Despite such micro-level nonconvexities, the aggregate economy can be convex when individual economic agents are infinitesimal. Incorporating discrete decisions at the micro level into models of the macroeconomy can nevertheless have important aggregate implications. Hansen (1985), for instance, introduces a worker's choice between either working a fixed number of hours or not working into a prototypical business cycle model. The consequence is that the aggregate intertemporal elasticity of labor supply is much higher than in a model in which individual choice sets are convex. As a result the aggregate economy is more volatile. Hansen and Prescott (2005) study the aggregate implications of plant-level decisions to operate or temporarily shut down. They find that in expansionary times the economy's capacity constraint binds and, as in the data, above-trend movements in aggregate output and hours are smoother than fluctuations below trend. In contrast to these two nonconvexities, Thomas (2002), Khan and Thomas (2003), and Khan and Thomas (2008) find that, in general equilibrium, lumpy investment observed at the plant-level has essentially no effect on the cyclical properties of aggregate quantities.

Nonconvexities and lumpiness at the micro level also characterize output adjustments by production plants. In many manufacturing industries plants adjust output by varying capital utilization along three main margins: intermittent production (i.e., temporarily closing down), changing the number of shifts, and adding an extra day to the regular workweek (i.e., running existing shifts on Saturdays).[1] These decisions are discrete and

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[1] The next section provides an overview of the empirical literature on output adjustment in manufacturing.

lead to large changes in production volumes at the plant level.

We incorporate such decisions into an equilibrium business cycle model and investigate two issues.[2] First, we study if such micro-level nonconvexities magnify the volatility of the aggregate economy. In light of the aforementioned results on the effects of micro-level nonconvexities on the macroeconomy it is not clear if the nonconvexities studied here can be ignored in the study of the business cycle. As they affect production directly, they can be of first-order importance. Manufacturing industries, such as the automobile industry, utilizing the margins of output adjustment described above are some of the most volatile in the economy and contribute significantly to fluctuations in real GDP over the business cycle. Is it due to the specific technology used in such industries, which constraints firms to make lumpy output adjustments and thus magnifies the economy-wide effects of a given set of aggregate shocks? Or is it because these industries face larger shocks than other sectors of the economy?

Second, we use the model to investigate the relationship between capacity utilization and the aggregate effects of monetary policy.[3] A large empirical literature documents that monetary policy shocks have larger effects on aggregate output in recessions than in expansions.[4] One hypothesis to explain such asymmetries put forward in this literature is based on cyclical fluctuations in economy-wide capacity utilization – monetary policy easing, the argument goes, should have a larger positive effect on aggregate output in times when most firms operate below capacity than in times when the majority of them are at or near capacity constraints. By explicitly modeling the margins of capacity utilization at

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[2]Previously Kydland and Prescott (1988) and Burnside and Eichenbaum (1996) have studied the cyclical implications of the variation in capital utilization in an equilibrium business cycle model. Hansen and Sargent (1988) introduce straight time and overtime into such a model, while Hornstein (2002) introduces shiftwork. These studies, however, incorporate such margins directly into an aggregate production function operated by a representative plant. Cooley, Hansen and Prescott (1995) and Hansen and Prescott (2005) focus on the aggregate implications of one discrete output adjustment margin at the plant level – a decision to operate or shut down. Hall (2000) constructs a detailed model of output adjustment at the plant level in the automobile industry. In contrast to the above studies, his analysis is carried out in a partial equilibrium.

[3]*Capacity* utilization is a broader concept than *capital* utilization, although the two are closely related; see Corrado and Mattey (1997) and Hornstein (2002) for a discussion. We adopt capital utilization as an operational definition of capacity utilization and use the two terms interchangeably.

[4]See, for instance, Weise (1999), Lo and Piger (2005), Garcia and Schaller (2002), and Peersman and Smets (2002).

the micro level observed in many U.S. industries we provide a model of capacity utilization at the aggregate level consistent with micro observations. As such it offers a valuable laboratory to quantitatively evaluate such an argument.

In our model there is a continuum of plants that differ in terms of their individual productivity. Each plant draws its productivity level from a normal distribution and, given the costs of running a shift during the regular workweek and overtime (which, in line with the practices in manufacturing, takes the form of running the shift on Saturdays), decides whether to operate, how many shifts to run, and whether to schedule overtime work. The mean of the distribution is stochastic, depending on aggregate technology shocks. The cost of running a particular shift during a regular workweek and overtime is determined in general equilibrium by households' preferences for work at different times of the day and week. In addition, the cost depends on costs of financing working capital, which in the short-run are affected by monetary policy shocks, as in Christiano and Eichenbaum (1992) and Christiano, Eichenbaum and Evans (2005).[5]

The model is calibrated to standard long-run features of aggregate data, as well as to cross-sectional observations on capital utilization reported by Mattey and Strongin (1997). Information from U.S. labor market regulations is also used. Given such calibration, we find that the quantitative implications of the model are consistent with the empirical findings of Mayshar and Solon (1993) and Shapiro (1996), discussed in the next section. Specifically, a large fraction of the cyclical movements in employment in our model (about two thirds) is accounted for by shiftwork. In addition, the model implies average workweeks of capital and labor in line with the data. Furthermore, the volatility of capacity utilization at the aggregate level in the model is of the same order of magnitude as in the data, although somewhat smaller.

In terms of our first question we find that output in our economy is about 25% less volatile, for a given set of aggregate technology shocks, than in a model without micro-level

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[5]Empirical evidence documenting the quantitative importance of this transmission channel of monetary policy is provided, for instance, by Barth and Ramey (2002). According to their estimates, it is particularly important in durable-goods manufacturing industries – industries characterized by the use of the output adjustment margins studied here.

production nonconvexities, which we take as a benchmark. Although output adjustments at the plant level are lumpy, in general equilibrium the number of plants making such adjustments varies less in response to technology shocks than would be needed to replicate the volatility of an otherwise identical economy, but without such micro-level nonconvexities. For similar reasons, for our second question, we find that although monetary policy shocks do have larger effects on output in recessions than in expansions, quantitatively the differences are small.

The rest of the paper is organized as follows. Section 2 reviews empirical evidence on output adjustment in U.S. manufacturing industries. Section 3 introduces the model economy with nonconvexities and the benchmark economy. Section 4 describes their calibration. Section 5 discusses the computational method used to solve the model. The method employed preserves nonlinearities in the model economy and thus any potential dependence of the aggregate effects of monetary policy shocks on the state of the economy. Section 6 presents the findings and Section 7 concludes. Details of the computational procedure are provided in an appendix.

2 Output Adjustment in Manufacturing

A number of studies characterize production processes in manufacturing at the plant level.[6] Mattey and Strongin (1997) and Beaulieu and Mattey (1998) classify industries according to the way plants adjust output. They distinguish between 'continuous process industries', which primarily vary production flows per unit of time, and 'variable-workweek industries', which primarily vary the workweek of capital.

Industries in the latter category usually operate assembly-line production, for which it is difficult to change the speed of the line (i.e., to change the production flow per unit of time), but for which the costs of shutting down and re-opening the line are small. Because of these technological constraints, plants in these industries vary output by adjusting the workweek of capital, rather than instantaneous production flows. This is carried out along three main

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[6]To the best of my knowledge, similar studies for other sectors of the economy are not available.

margins: intermittent production, changing the number of shifts, and scheduling Saturday work on existing shifts.[7] Durable-goods industries, such as the automobile, transportation, and machinery industries, are the prime examples belonging in this category.[8] The share of assembly-line production in the economy is quantitatively important. Clark (1996), for example, estimates that it accounts for about 20% of U.S. private sector output.

Various empirical studies document the quantitative importance of these margins of output adjustment, and of shiftwork in particular, at both micro and macro level. At the macro level, Beaulieu and Mattey (1998) and Shapiro (1996) provide estimates of the contribution of the movements in the economy-wide capital workweek to fluctuations in Federal Reserve's measure of capacity utilization.[9] According to Beaulieu and Mattey the growth rate of capital workweek explains about 55% of the variation in capacity utilization, while Shapiro's estimate based on levels is 70%. There is also a large degree of variation in the use of shiftwork, both in the cross-section and over time. Mattey and Strongin (1997) document that 27.3% of plants in variable-workweek industries operate on average one shift, 40.4% operate two shifts, and 32.3% operate three shifts. And Mayshar and Solon (1993) estimate that one half of a decline in employment of manufacturing production workers, and one third of economy-wide employment, occurs due to declines in the use of afternoon and night shifts. Employment per shift within a plant changes only little over the business cycle. These findings are broadly supported by Shapiro (1996).

At the micro level, using panel data from the Survey of Plant Capacity (SPC)[10], Mattey and Strongin (1997) estimate that in variable-workweek industries the workweek of capital explains about 41% of individual plants' variation in capacity utilization. Bresnahan and Ramey (1994), using weekly data for 50 U.S. automobile plants for the period 1972-1983, find that the most frequent output adjustments are made by weekly shutdowns and Saturday work.

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[7]Intermittent production involves closing a plant down for at least a week. Variation in the number of days per week, other than adding a Saturday to a five-day workweek, is highly unusual.

[8]Petroleum or paper industries are examples belonging in continuous process industries.

[9]Their measure of aggregate capital workweek is obtained by aggregating plant-level workweek from Survey of Plant Capacity data described below.

[10]The survey, conducted since 1974, is based on 8,000-9,000 manufacturing establishments. These establishments constitute a subset of the establishments surveyed by the Bureau of Census in its Annual Survey of Manufacturing.

Plants only infrequently change the number of shifts they operate. However, changing the number of shifts is quantitatively the most important margin of output adjustment at quarterly frequency, accounting for about 40% of plant-level output volatility. This is twice as much as weekly shutdowns aimed at inventory adjustment – quantitatively the second most important margin. Together, intermittent production, Saturday work, and shiftwork account for about 80% of output volatility at the plant level. Mattey and Strongin (1997) obtain broadly similar estimates of the contribution of shiftwork to plant-level output movements from SPC data. They find that 32% of the variation in capacity utilization in variable-workweek industries is due to shiftwork. For comparison, the second most important margin in their estimates is weekend work, accounting for only 5% of output variation.

In the following section we incorporate the margins of output adjustment discussed here into an equilibrium business cycle model.

3   The Model Economies

This section first describes the economic environment common to both models, the economy with nonconvexities and the benchmark economy. It then introduces into the common framework the production side of each economy and the labor-leisure choice associated with it.

3.1   The General Economic Environment

The economies are populated by a stand-in, perfectly competitive household, firm, and financial intermediary. In addition, there is a monetary authority that issues fiat money. In the 'benchmark' economy the firm operates a standard aggregate production function. In the 'economy with nonconvexities' it instead operates a continuum of heterogeneous plants.

In both economies business cycles are set off by aggregate productivity shocks, which follow an AR(1) process

$$
(1) \qquad \log(z_{t+1}) = (1 - \rho_z) \log(\bar{z}) + \rho_z \log(z_t) + \xi_{t+1},
$$

where $\rho_z \in (0,1)$, $\bar{z}$ is the nonstochastic steady-state productivity level, and $\xi_t \sim N(0, \sigma_\xi)$.

As in Christiano and Eichenbaum (1992) and Christiano et al. (2005), in both economies the firm finances its wage bill through loans from the financial intermediary. Such a financing requirement comes from the assumption that workers have to be paid before the firm sells its output.[11] In addition, in line with these papers, due to limited participation of the household in the market for fiat money at the time of open market operations, the monetary authority can (at least in the short run) affect the firm's real borrowing costs and thus aggregate output. Because this aspect of our model is borrowed from the two aforementioned papers, in what follows we describe it only briefly and refer the reader to those papers for details. It needs to be stressed that our analysis of the effects of monetary policy shocks on the economy is purely positive. We do not attempt to justify it on normative grounds.

3.1.1 The Household's Problem

The stand-in household enters period t with capital stock k_t and balances of fiat money m_t. After observing the current state of aggregate productivity z_t, but before knowing a (gross) nominal interest rate R_t, it decides how much of its money balances to keep as cash q_t. The remaining part of the balances, $m_t - q_t$, is deposited with the financial intermediary. At the end of the period the household receives gross interest $R_t(m_t - q_t)$. All other decisions of the household are made after observing both z_t and R_t.

The preferences of the stand-in household are characterized by the utility function

$$(2) \qquad E_t \sum_{t=0}^{\infty} \theta^t \left[\log(c_t) - v_t \right],$$

where $\theta \in (0,1)$ is a discount factor, c_t is consumption, and v_t is disutility from work. The expectation operator E_t reflects the information structure described above. In both economies, disutility from work depends on the *amount* of labor supplied to the firm. In the economy with nonconvexities, in addition, it also depends on the *time* of the day and week it is supplied. We therefore describe v_t for each economy separately.

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[11]This timing is intended to capture in a simple way companies' needs to finance working capital observed in actual economies.

The household maximizes the utility function (2) subject to two constraints. First, it must obey a cash-in-advance constraint

(3) $$p_t c_t \leq q_t + e_t,$$

where p_t is the price of goods in terms of money and e_t is nominal labor income received from firms in cash after R_t has been realized. (As firms finance the wage bill through bank loans, e_t is equal to firms' nominal borrowing). Second, it must obey the budget constraint

$$p_t c_t + p_t k_{t+1} + m_{t+1} = q_t + e_t + R_t (m_t - q_t)$$

(4) $$+ p_t (1 + r_t - \delta) k_t + \pi_{It} + \pi_{Ft},$$

where r_t is the real rental rate at which the household rents out capital to the firm, π_{It} is profit of the intermediary, π_{Ft} is profit of the firm, and $\delta \in (0, 1)$ is a depreciation rate.

Ignoring for the moment the household's labor-leisure choice (we can do so as the utility function is additively separable in consumption and labor), the household's problem is to choose plans for c_t, k_{t+1}, q_t, and m_{t+1} in order to maximize (2) subject to (3) and (4).[12] The solution to this problem is characterized by the two constraints and a pair of first-order conditions

(5) $$E_t \left[\frac{1}{p_t c_t} \mid z_t, R_{t-1} \right] = \theta E_t \left[\frac{1}{p_{t+1} c_{t+1}} R_t \mid z_t, R_{t-1} \right],$$

(6) $$p_t E_t \left[\frac{1}{p_{t+1} c_{t+1}} \mid z_t, R_t \right] = \theta E_t \left[\frac{1}{p_{t+2} c_{t+2}} p_{t+1} (1 + r_{t+1} - \delta) \mid z_t, R_t \right].$$

3.1.2   Firms' Borrowing and Monetary Policy

The nominal interest rate is controlled by the central monetary authority. The way changes in the nominal interest rate translate into changes in firms' real borrowing costs is through nonparticipation of households' in the money market at the time of open market operations – injections of fiat money to financial intermediaries. Due to this market segmentation money

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[12]Below we make assumptions on the stochastic process for $(R_t - 1)$ that guarantee that inflation between periods t and $t+1$ is always positive. Constraint (3) thus holds with equality.

injections by the central monetary authority need to be absorbed by firms' borrowing. And, other things being equal, firms are only willing to do so if their real borrowing costs decline. As a result, money injections translate into changes in firms' real borrowing costs, rather than proportional increases in the aggregate price level as in standard cash-in-advance models.[13]

Total loanable funds at the intermediaries' disposal are therefore given by the sum of deposits from households, $m_t - q_t$, and money injections from the monetary authority, X_t. The authority uses X_t to adjust the money stock in the economy such that the money market clears at the interest rate it wants to implement. That is, X_t is chosen so that

$$(7) \qquad\qquad m_t + X_t = e_t + q_t$$

holds for any given R_t (notice that m_t and q_t are predetrmined at the time of the injection). The *net* nominal interest rate is assumed to follow an AR(1) process

$$(8) \qquad \log(R_{t+1} - 1) = (1 - \rho_R) \log\left(\overline{R} - 1\right) + \rho_R \log\left(R_t - 1\right) + \zeta_{t+1},$$

where $\rho_R \in (0, 1)$, $\overline{R}$ is the nonstochastic steady-state gross nominal interest rate, and $\zeta_t \sim N(0, \sigma_\zeta)$.[14]

Intermediation is costless. Perfect competition then ensures that the interest rate charged for loans is equal to the interest rate paid on deposits. At the end of the period, after paying households interest on their deposits, intermediaries are left with net cash position in the amount of $R_t X_t$. This amount is distributed to households in a lump sum way in the form of profits π_{It}.

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[13]See Christiano and Eichenbaum (1992) for details.

[14]By assuming that the nominal interest rate follows an exogenous stochastic process, we are abstracting from any effects of systematic responses of the Federal Reserve to the economy. This makes the nature of our experiments, and their results, easier to interpret.

3.2 Production in the Economy with Nonconvexities

3.2.1 Production Nonconvexities

The firm operates a continuum (of measure one) of production plants. By a 'plant' we mean the smallest production unit in our model at which decisions are made. The plants are indexed by a pair of idiosyncratic productivity shocks (s, ε). These shocks are independently and identically distributed across plants and time. They are drawn, independently of each other, from normal distributions with density functions $f(s; z_t, \sigma_s)$ and $g(\varepsilon; \kappa z_t, \sigma_\epsilon)$. We consider normal distributions a natural choice in the absence of available information on the distribution of plant-level productivity in the U.S. economy. Notice that the mean values of the two distributions are stochastic, depending on the realization of aggregate productivity.

Each period individual plants can adjust output along three margins: closing temporarily down, changing the number of shifts, and scheduling overtime work on existing shifts. More precisely, each plant can remain idle or operate one, two, or three shifts. The shifts can be interpreted as a morning, afternoon, and night shift. Provided a plant operates a shift during a regular workweek, it can also run that shift overtime. In line with practices in U.S. manufacturing discussed in Section 2 overtime work in our model takes the form of Saturday work; i.e., adding an extra day to the regular workweek.

The volume of output generated by shift $j = \{1, 2, 3\}$ during a regular workweek in period t is

$$
(9) \qquad y_{jt}^R(s) = \begin{cases} \left(\frac{5}{7} h_j^R\right) s k_t^\alpha \overline{n}^\beta & \text{if } \eta_{jt} \geq \overline{n} \\ 0 & \text{otherwise.} \end{cases}
$$

Here, $h_j^R \in (0, 1)$ is the length of the shift during the regular workweek, η_{jt} is the number of workers employed on that shift, and $\alpha, \beta \in (0, 1)$ and $\alpha + \beta \in (0, 1)$. As the length of the period is normalized to one, the fraction 5/7 represents the number of days in the regular workweek. If Saturday work is also scheduled on shift j, the additional output of that shift

10

is

$$
(10) \qquad y_{jt}^o(\varepsilon) = \begin{cases} \left(\frac{1}{7}h_j^o\right)\varepsilon k_t^\alpha \overline{n}^\beta & \text{if } \eta_{jt} \geq \overline{n} \\ 0 & \text{otherwise,} \end{cases}
$$

where $h_j^o \in (0,1)$ is the length of the shift on Saturdays, and the fraction $1/7$ represents the extra day that is added to the regular workweek. The shift lengths h_j^R and h_j^o are taken as given.[15] Total output of plant (s, ε) in period t is then

$$
(11) \qquad y_t(s, \varepsilon) = \sum_{i=1}^{3} \left[y_{jt}^R(s) + y_{jt}^o(\varepsilon) \right].
$$

The requirement that the number of workers on each shift must be greater or equal to $\overline{n}$ if the shift is to generate positive output introduces a nonconvexity in the plant's choice set. This nonconvexity makes output adjustment at the plant level lumpy. Without such a minimum-staffing requirement, each plant would operate all three shifts and would adjust output smoothly by varying the number of workers on each shift. Here instead each plant decides how many shifts to run, and whether to use Saturday work. Minimum-staffing requirements are characteristic for assembly-type technology: a minimum number of workers around an assembly line is needed to operate the line and the marginal product of an additional worker beyond the critical number is small (Mattey and Strongin, 1997).

3.2.2 Realization of Plant-level Uncertainty

In each period, after observing z_t and R_t but before observing (s_t, ε_t) of the individual plants, the firm rents capital from the household and allocates it across plants. Once capital is assigned to a plant, it cannot be changed within the period. Since prior to the realization of their productivity shocks the plants are identical, and $\alpha \in (0, 1)$, the firm distributes capital across them equally. After that each plant learns its productivity shock s and decides whether to operate that period. And if it does, how many shifts to run. Once

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[15]The terms $sk_t^\alpha \overline{n}^\beta$ and $\varepsilon k_t^\alpha \overline{n}^\beta$ in the production functions (9) and (10) represent instantaneous production flows. The distinction between production flows and volumes is in the spirit of Lucas (1970) and the subsequent literature on the workweek of capital (e.g., Kydland and Prescott, 1988).

the number of shifts is chosen, it cannot be changed within the period. Each plant then learns its productivity shock ε and decides whether to schedule Saturday work on any of the shifts it operates during the regular workweek. This timing captures in a tractable way the behavior of establishments found in empirical studies summarized in Section 2: Saturday work is used for small changes in production volumes of exiting shifts, while shiftwork is used for medium-term and rather significant output adjustments.

3.2.3   Optimal Plant Utilization

As the marginal product of an additional worker beyond the threshold level $\bar{n}$ is zero, whereas (in equilibrium) the marginal cost is positive, a plant will choose $\eta_{jt} = \bar{n}$ for every shift it operates.

When a plant runs the jth shift during the regular workweek, it incurs a fixed cost

$$(12) \qquad\qquad R_t \left(\frac{5}{7} h_j^R \right) \omega_{jt}^R \bar{n},$$

where ω_{jt}^R is the real hourly wage rate for work on that shift during regular hours. When the plant runs the shift on Saturdays, the cost is

$$(13) \qquad\qquad R_t \left(\frac{1}{7} h_j^o \right) \omega_{jt}^o \bar{n},$$

where ω_{jt}^o is the real hourly wage rate for Saturday work.

After learning ε, and conditional on operating shift j during the regular workweek, a plant schedules Saturday work on that shift only if output produced during these overtime hours is greater or equal to the costs given by (13). Therefore, among the plants operating the jth shift, plants that run the shift on Saturdays are characterized by

$$(14) \qquad\qquad \varepsilon \geq R_t \omega_{jt}^o k_t^{-\alpha} \bar{n}^{1-\beta} \equiv \phi_{jt}$$

and their *conditional* measure is

$$(15) \qquad \widehat{\mu}_{jt}^{o} = \int_{\phi_{jt}}^{\infty} g\left(\varepsilon; \kappa z_t, \sigma_{\varepsilon}\right) d\varepsilon.$$

These plants' output and profits generated from Saturday work on the jth shift are therefore, respectively,

$$(16) \qquad \widehat{y}_{jt}^{o} = \left(\frac{1}{7}h_j^o\right) k_t^{\alpha} \overline{n}^{\beta} \int_{\phi_{jt}}^{\infty} \varepsilon g\left(\varepsilon; \kappa z_t, \sigma_{\varepsilon}\right) d\varepsilon,$$

$$(17) \qquad \widehat{\pi}_{jt}^{o} = \widehat{y}_{jt}^{o} - \widehat{\mu}_{jt}^{o} R_t \left(\frac{1}{7}h_j^o\right) \omega_{jt}^o \overline{n}.$$

After observing s, but before knowing ε, a plant operates the jth shift during the regular workweek only if the shift makes nonnegative expected profit. Plants that operate shift j are therefore characterized by s that satisfies the inequality

$$\left(\frac{5}{7}h_j^R\right) \left[sk_t^{\alpha} \overline{n}^{\beta} - R_t \omega_{jt}^R \overline{n} \right] + \widehat{\pi}_{jt}^{o} \geq 0$$

or (after some manipulation)

$$(18) \qquad s_t \geq k_t^{-\alpha} \left(R_t \omega_{jt}^R \overline{n}^{1-\beta} - \frac{7}{5}(h_j^R)^{-1} (\overline{n})^{-\beta} \widehat{\pi}_{jt}^{o} \right) \equiv \lambda_{jt}.$$

The measure of these plants in the economy is therefore

$$(19) \qquad \mu_{jt}^{R} = \int_{\lambda_{jt}}^{\infty} f(s; z_t, \sigma_s) ds$$

and their combined output and profits from operating the jth shift during the regular workweek are, respectively,

$$(20) \qquad y_{jt}^{R} = \left(\frac{5}{7}h_j^R\right) k_t^{\alpha} \overline{n}^{\beta} \int_{\lambda_{jt}}^{\infty} sf\left(s; z_t, \sigma_s\right) ds$$

$$(21) \qquad \pi_{jt}^{R} = y_{jt}^{R} - R_t \left(\frac{5}{7}h_j^R\right) \omega_{jt}^R \overline{n} \mu_{jt}^{R}.$$

13

As shown below household preferences imply that the first shift, run during the regular workweek, is the least expensive one. Therefore, the measure of plants that are shut down in period t is equal to $\left(1 - \mu_{1t}^R\right)$.

Finally, notice that the measure of plants in the economy with Saturday work on the jth shift is given by

$$(22) \qquad\qquad \mu_{jt}^o = \mu_{jt}^R \widehat{\mu}_{jt}^o,$$

and $\widehat{y}_{jt}^o$ and $\widehat{\pi}_{jt}^o$ contribute to aggregate output and profits, respectively,

$$(23) \qquad\qquad y_{jt}^o \;=\; \mu_{jt}^R \widehat{y}_{jt}^o$$

$$(24) \qquad\qquad \pi_{jt}^o \;=\; \mu_{jt}^R \widehat{\pi}_{jt}^o.$$

3.2.4 Aggregate Output, Employment, and Profits

Aggregate output y_t is given by the sum of output generated across plants by each shift during regular and overtime hours

$$(25) \qquad\qquad y_t = \sum_{j=1}^{3} \left(y_{jt}^R + y_{jt}^o\right) = \widetilde{A}_t k_t^\alpha$$

where

$$(26) \qquad \widetilde{A}_t \equiv \overline{n}^\beta \sum_{j=1}^{3} \left[\left(\frac{5}{7}h_j^R\right)\int_{\lambda_{jt}}^{\infty} s f_t(s)ds + \mu_{jt}^R \left(\frac{1}{7}h_j^o\right)\int_{\phi_{jt}}^{\infty} \varepsilon g_t(\varepsilon)d\varepsilon\right].$$

A worker that works overtime on shift j also works regular hours on that shift (although not every worker who works regular hours also works overtime). In addition, each worker works only on one shift. Aggregate employment n_t is thus obtained as the sum of employment across plants on each shift

$$(27) \qquad\qquad n_t = \overline{n} \sum_{j=1}^{3} \mu_{jt}^R.$$

14

Out of the workers who work on the jth shift, a measure

(28)
$$n_{jt}^{R} = \bar{n}\left(\mu_{jt}^{R} - \mu_{jt}^{o}\right)$$

of them work only regular hours, and a measure

(29)
$$n_{jt}^{o} = \bar{n}\mu_{jt}^{o}$$

of them work overtime, in addition to regular hours. The aggregate wage bill e_t, which is equal to firms' demand for loanable funds, is given by

(30)
$$e_t = \sum_{j=1}^{3}\left[\left(\frac{5}{7}h_j^{R}\right)\omega_{jt}^{R}\left(n_{jt}^{R} + n_{jt}^{o}\right) + \left(\frac{1}{7}h_j^{o}\right)\omega_{jt}^{o}n_{jt}^{o}\right].$$

Through its effect on ϕ_{jt} and λ_{jt} (equations 14 and 18), a fall in R_t (other things being equal) increases the measure of plants that operate any given shift or use overtime. This increases aggregate employment and output. An increase in z_t also increases μ_j^{R} and μ_j^{o}, but in a different way. Other things being equal, a shock to z_t moves the means of the distributions with the density functions f and g, and thus affects the areas under the curves of the density functions between a given λ_{jt}, and ϕ_{jt}, and infinity. Instead, a shock to R_t affects the areas under the curves between $\lambda_{jt}(R_t)$ and $\lambda_{jt}(R_t^*)$ [and between $\phi_{jt}(R_t)$ and $\phi_{jt}(R_t^*)$], where R_t^* is the new interest rate, for f and g characterized by a given z_t. In addition to its effect on μ_j^{R} and μ_j^{o}, an increase in z_t increases the productivity of all plants and thus increases output from any shift that a plant decides to run during regular and overtime hours.

Finally, the firm's profits are obtained by summing its profits from regular and overtime work on the three shifts less rental payments for capital services

(31)
$$\pi_{Ft} = \sum_{j=1}^{3}\left(\pi_{jt}^{R} + \pi_{jt}^{o}\right) - rk_t.$$

At the start of period t the firm chooses k_t in order to maximize (31). Substituting for π_{jt}^{R}

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and π_{jt}^o in (31) from (21) and (24), the first-order condition for this problem is

(32)
$$r_t = \alpha \widetilde{A}_t k_t^{\alpha-1},$$

where $\widetilde{A}_t$ is given by (26).

3.2.5 Aggregate Capacity Utilization

As mentioned in the Introduction, we measure capacity utilization in our model by the workweek of capital – the number of hours per week the average plant is operated

(33)
$$h_{kt} = \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_j^R \right) \mu_{jt}^R + \left(\frac{1}{7} h_j^o \right) \mu_{jt}^o \right].$$

We also define the workweek of labor – the number of hours per week the average household, conditional on being employed, works

(34)
$$h_{lt} = \frac{1}{n_t} \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_j^R \right) n_{jt}^R + \left(\frac{5}{7} h_j^R + \frac{1}{7} h_j^o \right) n_{jt}^o \right].$$

3.2.6 The Household's Optimal Labor Supply

Households (of whom there is a measure one in the economy) face idiosyncratic shocks that determine which households work on which shift (and if they also work overtime) and which do not. A household that is employed on the jth shift receives instantaneous utility $\log(c_{jt}^\tau) + a_j \log(l_j^\tau)$, where

$$l_j^\tau = \begin{cases} 1 - \frac{5}{7} h_j^R & \text{if } \tau = R \\ 1 - \frac{5}{7} h_j^R - \frac{1}{7} h_j^o & \text{if } \tau = o. \end{cases}$$

Here $a_j > 0$ is the relative weight on utility from leisure. A household that does not work gets $\log(c_{0t}) + a_0 \log(l_0)$, where $a_0 > 0$ and $l_0 = 1$. The probability of working only regular hours on the jth shift is n_{jt}^R; the probability of working overtime, in addition to regular hours, is n_{jt}^o; the probability of not working is then $1 - \sum_{j=1}^{3}(n_{jt}^R + n_{jt}^o)$.

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Households can fully insure against this employment risk. An argument similar to that in Hansen (1985) then implies that the stand-in household has instantaneous utility function

$$(35) \qquad \log(c_t) - \sum_{j=1}^{3} \left[b_j^R n_{jt}^R + b_j^o n_{jt}^o \right],$$

where $b_j^R \equiv -a_j \log(1 - h_j^R)$ and $b_j^o \equiv -a_j \log(1 - h_j^R - h_j^o)$. As will be discussed in Section 4, U.S. data on shiftwork and labor market regulations imply that $b_1^R < b_2^R < b_3^R$ and $b_1^o < b_2^o < b_3^o$. The stand-in household thus prefers morning work to afternoon and night work. Notice that the instantaneous utility function is the same as the instantaneous utility function in (2), with v_t defined as

$$(36) \qquad v_t \equiv \sum_{j=1}^{3} \left[b_j^R n_{jt}^R + b_j^o n_{jt}^o \right].$$

The stand-in household chooses $\{n_j^R, n_j^o\}_{j=1}^{3}$ in order to maximize the utility function (2) subject to (3) and (4), where the nominal labor income e_t is given by (30). The optimal labor-leisure choice is characterized by the first-order conditions

$$(37) \qquad \omega_{jt}^R = \frac{7}{5} \left(\frac{b_j^R}{h_j^R} \right) c_t \quad \text{and} \quad \omega_{jt}^o = 7 \left(\frac{b_j^o - b_j^R}{h_j^o} \right) c_t$$

for $j = \{1, 2, 3\}$.

3.2.7 Equilibrium

The equilibrium is characterized by stochastic sequences of c_t, k_{t+1}, q_t, X_t, $\{n_{jt}^R, n_{jt}^o\}_{j=1}^{3}$, p_t, r_t, and $\{\omega_{jt}^R, \omega_{jt}^o\}_{j=1}^{3}$ that satisfy the household's first-order conditions (5), (6), and (37) and the constraints (3) and (4); the firm's optimality conditions (32), (28), and (29); and the money market equilibrium condition (7), where e_t is given by (30).

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3.3 Production and the Optimal Labor Supply in the Benchmark Economy

In the benchmark economy, the firm operates a standard aggregate production function

$$(38) \qquad y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where y_t is aggregate output, n_t is aggregate employment, and $\alpha \in (0, 1)$. After observing z_t and R_t, the firm chooses k_t and n_t in order to maximize profits $\pi_{Ft} = p_t (y_t - R_t \omega_t n_t - r_t k_t)$, where ω_t is the period t real wage rate. The first-order conditions for this problem imply this economy's corresponding conditions to (27) and (32) in the economy with nonconvexities

$$(39) \qquad n_t = (1-\alpha)^{\frac{1}{\alpha}} (z_t)^{\frac{1}{\alpha}} (R_t \omega_t)^{-\frac{1}{\alpha}} k_t,$$

$$(40) \qquad r_t = \alpha A_t k_t^{\alpha-1},$$

where

$$(41) \qquad A_t \equiv z_t n_t^{1-\alpha} = (1-\alpha)^{\frac{1-\alpha}{\alpha}} (z_t)^{\frac{1}{\alpha}} (R_t \omega_t)^{-\frac{1-\alpha}{\alpha}} k_t^{1-\alpha}.$$

Aggregate output is then given as

$$(42) \qquad y_t = A_t k_t^\alpha.$$

Notice that with respect to the economy with nonconvexities, the expression for output differs only in terms of the definition of A_t. The nominal wage bill, and thus the firm's demand for loans, is given by

$$(43) \qquad e_t = (p_t \omega_t) n_t.$$

Households can work either a fixed number of hours, or not work at all. As before, a lottery determines which households work and which do not and households can fully insure

against this risk. The preferences of the stand-in household are thus the same as in Hansen
(1985)

$$\log(c_t) - bn_t, \tag{44}$$

where $b > 0$. The first-order condition characterizing the optimal choice of n_t is

$$\omega_t = bc_t. \tag{45}$$

4 Calibration

Each model economy is calibrated using empirical estimates of steady-state relations among
the model's variables and parameters. We also use information from U.S. labor market
regulations to calibrate the economy with nonconvexities. The calibration targets are sum-
marized in Table 1 and the resulting parameter values in Table 2. Except for the definition
of A, the steady-state relations implied by equations (5), (6), (40), and (42) are the same
in the two models. So is the resource constraint $k_{t+1} - (1 - \delta)k_t = y_t - c_t$. The values
of the parameters obtained from these relations therefore apply to both economies. We
describe their calibration first and then explain separately calibration of parameters that
are economy-specific.

4.1 Common Parameters

We interpret the length of the period as one quarter. The parameter α in the expression
for output (42) equals the steady-state share of capital rental income (capital income less
dividends) in GDP and is set equal to 0.365. This is in line with estimates from the U.S.
National Income and Product Accounts. We use a quarterly depreciation rate equal to
0.026, which (from the resource constraint) is consistent with the postwar average capital
to output ratio of 8.519 and the average share of investment in aggregate output equal to
0.223. Without loss of generality, we choose units so that steady-state output is equal to
one. For the capital to output ratio of 8.519, equation (42) implies a steady-state value of

A equal to 0.438. The discount factor θ is determined by the first-order condition (6) and the rate of return to capital resulting from the pricing function (40). The resulting value of θ is 0.981. The steady-state value of R_t is restricted by (5) and (6). The implied value is large enough to guarantee that inflation is always positive. We choose the other parameters of the stochastic process for R_t separately for each experiment.

4.2 Parameters Specific to the Benchmark Economy

The parameter b in the utility function (44) is specific to the benchmark economy. As in Hansen (1985) we set the steady-state value of n equal to 0.31. The first-order condition (45), once we substitute for ω_t from (39), then restricts b to be 2.516. The autocorrelation coefficient and the standard deviation of the innovation in the stochastic process for $\log(z_t)$ are set equal to 0.9 and 0.0067, respectively. These values come from a linearly detrended Solow residual for the postwar period.

4.3 Parameters Specific to the Economy with Nonconvexities

There are 17 new parameters in the economy with nonconvexities: $\bar{n}$, β, $\{h_j^R, h_j^o\}_{j=1}^3$, $\{b_j^R, b_j^o\}_{j=1}^3$, σ_s, σ_ε, and κ. Moreover, in this economy z_t is not equivalent to the standard Solow residual. We therefore need to parameterize ρ_z and σ_ξ in a different way than we did for the benchmark economy.

As our plant-level production function is similar to one used by Hall (2000), we set β equal to his estimate of 0.58. There is little evidence in the literature that shift lengths differ across shifts and across regular workweek and weekend work. We therefore set $h_j^R = h_j^o = h$ for all j. In addition, we set h equal to 1/3, which implies that plants operate three eight-hour shifts. This is consistent with observations for most manufacturing industries documented by King and Williams (1985). The parameter κ is set equal to 0.38 on the basis that plants using the overtime margin use it 38% of the time (Hall, 2000); i.e., they use 38% of the available Saturdays for weekend work.

For the following discussion, it is convenient to express the wage rates $\{\omega_j^R, \omega_j^o\}_{j=1}^3$ in terms of ω_1^R – the base wage rate – and overtime and shift premia. We define overtime

premia as $\Delta_j^o \equiv (\omega_j^o/\omega_j^R) - 1$, $j = \{1,2,3\}$, and shift premia as $\Delta_2^R \equiv (\omega_2^R/\omega_1^R) - 1$ and $\Delta_3^R \equiv (\omega_3^R/\omega_1^R) - 1$, respectively. The U.S. Fair Labor Standards Act requires that a 50% premium be paid for hours in excess of 40 hours per week. We therefore set Δ_j^o equal to 0.5 for all j.

There are no legal requirements for shift premia. Using data from the Area Wage Survey (AWS), Shapiro (1986) estimates that for the period 1973-75, the average pay differential was 7.8% for work on the second shift and 10.3% for work on the third shift. King and Williams (1985) obtain similar values for 1984 for the manufacturing sector and Bresnahan and Ramey (1994) for the period 1972-83 for a panel of plants in the automobile industry. But Shapiro (1986) argues that because most firms rotate shiftwork among their workforce, a large part of the premium needed to get workers to undertake it is built into base wage rates.[16] Shapiro (1995) takes this practice into account and obtains a premium of about 25%. Kostiuk (1990) finds that labor heterogeneity (such as union membership or firm size) also causes shift premia from AWS to be seriously underestimated. Due to this uncertainty about the true marginal cost of shiftwork to firms, we choose Δ_2^R and Δ_3^R, together with the standard deviations of the idiosyncratic shocks σ_s and σ_ε so that the steady state of the model replicates the observed average use of shiftwork and overtime work across manufacturing establishments.

Mattey and Strongin (1997) provide detailed analysis of the use of the various margins of output adjustment in manufacturing based on plant-level data from SPC. They report that, out of the plants that use technology allowing variation in workweek, conditional on being open, 27.3% of plants operate on average one shift, 40.4% operate two shifts, and 32.3% operate three shifts. In addition, 19% of plants use Saturday work at least on one shift (information on the use of Saturday work across shifts is not available). Mattey and Strongin's estimates also imply that the average plant is shut down for about 0.067 weeks per quarter.

Given the values for overtime premia, we choose Δ_2^R, Δ_3^R, σ_s, and σ_ε such that in steady state: (i) the measure of plants that operate one shift, given by $(\mu_1^R - \mu_2^R)$, is equal

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[16]Labor market regulations do not permit similar practices for overtime premia (Shapiro, 1986).

to $(1 - 0.067) \times 0.273 = 0.255$; (ii) the measure of plants that operate two shifts, given by $(\mu_2^R - \mu_3^R)$, is equal to $(1 - 0.067) \times 0.404 = 0.377$; (iii) the measure of plants that operate three shifts, given by μ_3^R, is equal to $(1 - 0.067) \times 0.323 = 0.301$; and (iv) the measure of plants that use weekend work at least on one shift, given by μ_1^o, is equal to $(1 - 0.067) \times 0.19 = 0.173$. Given these targets we obtain Δ_2^R equal to 0.79, Δ_3^R equal to 1.56, σ_s equal to 0.851, and σ_ε equal to 0.802. Our implied shift premia are thus larger than those estimated by the aforementioned studies. They are, however, similar to those obtained by Hornstein (2002) from a general equilibrium model with shiftwork, but without nonconvexities.

We normalize the base wage rate ω_1^R and the minimum-staffing requirement $\bar{n}$ such that the steady-state average wage rate and aggregate employment, given by

(46)
$$ \omega = \frac{1}{n} \sum_{j=1}^{3} \left[\left(\frac{5}{7} h_j^R \right) \omega_j^R \left(n_j^R + n_j^o \right) + \left(\frac{1}{7} h_j^o \right) \omega_j^o n_j^o \right] $$

and equation (27), respectively, are the same as the steady-state wage rate and employment in the benchmark economy. Using the values for ω_1^R and for shift and overtime premia, we can back out the utility parameters $\{b_j^R, b_j^o\}_{j=1}^{3}$ from the first-order conditions for the labor-leisure choice (37). Their values are provided in Table 2. Finally, the steady-state value of z_t implied by the value of A is 2.35.

Notice that given our calibration, the steady-state values of c, k, y, n, r, and w in the two economies are the same. We have thus intentionally made the two economies observationally equivalent in the long run (in the dimensions along which they can be compared) so that we can focus only on comparing their short-run dynamics.

4.4 Steady-state Workweek of Capital and Labor

Given the observed values for overtime premia and the calibrated values for shift premia and σ_ε, the model implies that in steady state only 0.004 measure of plants use weekend work on the second shift, and $6.602 * 10^{-6}$ measure of them use weekend work on the third shift (as mentioned above there are no available observations on the use of these margins

in the data).

Using μ_j^R, μ_j^o, n, and $\bar{n}$, equations (33) and (34) imply steady-state workweek of capital and labor equal to 0.464 and 0.243, respectively. These values correspond to 77.9 hours per week for capital and 40.7 hours per week for labor.

The model implications are broadly in line with U.S. data. For example, using SPC data for the period 1974-92, Beaulieu and Mattey (1998) estimate that the average workweek of capital in the whole of manufacturing is about 97.0 hours. Based on the same data set, Shapiro (1996) reports estimates of workweek of capital for 2-digit SIC industries. In the transportation equipment industry (a prime example of an industry characterized by the use of shiftwork and Saturday work), the workweek is 73.6 hours. Using Bureau of Labor Statistics' data on Employment and Earnings for the period 1951-90, Shapiro (1996) estimates the workweek of labor for manufacturing production workers to be 40.4 hours. These numbers are close to the steady-state values of capital and labor workweeks implied by our model.

5 Solution Method

For each economy we need to compute aggregate decision rules and pricing functions that generate stochastic sequences of allocations and prices that satisfy the economy's equilibrium conditions. As one of our goals is to investigate if in our model the equilibrium effects of interest rate shocks are different in different states of the economy, in particular for different realizations of z_t, we need to compute the equilibrium using a method that preserves any potential nonlinearities. And as in our experiments we allow z_t to deviate up to three times its standard deviation from steady state, local higher-order approximation methods might not be suitable. We therefore use the projection method, sometimes also known as the weighted residual method (described by Judd, 1992), which produces a global approximation of our model.

The projection method enables us to compute decision rules and pricing functions that, at face value, are linear in R_t, but in which the coefficients that load on to R_t are state

dependent. The quantitative effects of R_t on the economy can thus vary with the state of the economy. As an example, consider an equilibrium decision rule for aggregate output. In the approximate decision rule the terms involving R_t have the form[17]

$$y_t = ... + a_i R_t + a_{i+1} z_t R_t + a_{i+2} z_t R_t^2 + a_{i+3} z_t^2 R_t + a_{i+4} z_t^2 R_t^2 + ... \quad .$$

Notice that we can re-write the right-hand side as $y_t = ... + \widetilde{a}_R R_t + ...$, where $\widetilde{a}_R \equiv a_i + a_{i+1} z_t + a_{i+2} z_t R_t + a_{i+3} z_t^2 + a_{i+4} z_t^2 R_t$ is state-dependent.

5.1 A Dimension Reducing Approach

Before applying the method, we reduce the size of the equilibrium conditions in two respects. First, we reduce the dimension of the state-space. Notice that there are five (continuous) aggregate state variables in our two economies: z_t, R_t, k_t, m_t, and R_{t-1}. By an appropriate normalization of nominal variables we eliminate m_t: we divide p_t, m_t, q_t, and X_t in the equilibrium conditions by m_t; then we define new variables $\widetilde{p}_t \equiv p_t/m_t$, $\widetilde{q}_t \equiv q_t/m_t$, and $\widetilde{x}_{t+1} \equiv m_{t+1}/m_t$.

Second, we reduce the number of equilibrium conditions by substitutions. First, we eliminate $\widetilde{p}_t$ by a substitution from the cash-in-advance constraint, which after the normalization has the form $\widetilde{p}_t c_t = \widetilde{x}_t$. This allows us to write the Euler equations (5) and (6), respectively, as

(47) $$E_t \left[\frac{1}{\widetilde{x}_t} \mid z_t, R_{t-1} \right] = \theta E_t \left\{ \frac{1}{\widetilde{x}_t} R_t E_{t+1} \left[\frac{1}{\widetilde{x}_{t+1}} \mid z_{t+1}, R_t \right] \mid z_t, R_{t-1} \right\}$$

(48) $$\frac{1}{c_t} E_t \left[\frac{1}{\widetilde{x}_{t+1}} \mid z_t, R_t \right] = \theta E_t \left\{ \frac{1}{c_{t+1}} (1 + r_{t+1} - \delta) \right.$$
$$\left. \times E_{t+1} \left[\frac{1}{\widetilde{x}_{t+2}} \mid z_{t+1}, R_{t+1} \right] \mid z_t, R_t \right\}.$$

Further, we re-write the money market equilibrium condition (7), after we have substituted

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[17]In the actual computation described below we use Chebyshev polynomials instead of ordinary polynomials used in this example.

for e_t, as $\widetilde{x}_t = (1 - \widetilde{e}_t)^{-1} \widetilde{q}_t$. Here

$$
\widetilde{e}_t = \begin{cases} b (1 - \alpha)^{\frac{1}{\alpha}} (z_t)^{\frac{1}{\alpha}} (R_t b c_t)^{-\frac{1}{\alpha}} k_t & \text{for the benchmark economy} \\ \\ \overline{n} \sum_{j=1}^{3} \left[b_i^R \mu_{jt}^R + \left(b_i^o + b_i^R \right) \mu_{jt}^o \right] & \text{for the economy with nonconvexities,} \end{cases}
$$

where μ_{jt}^R and μ_{jt}^o are given by (19) and (22), and where the wage rates are eliminated by substitutions from the household's first-order conditions (37). Finally, we eliminate r_{t+1} from the Euler equation (48) by a substitution from the pricing function $r_{t+1} = \alpha A_{t+1} k_{t+1}^{\alpha-1}$. Here A_t is given by equation (41) in the case of the benchmark economy and by equation (26) in the case of the economy with nonconvexities. After these substitutions, we are left with just two Euler equations, (47) and (48), in two unknowns, c_t and $\widetilde{q}_t$.

In order to form expectations about future state of the economy, households use the stochastic processes (1) and (8) to forecast future z_t and R_t, respectively, and the law of motion for capital $k_{t+1} = A_t k_t^\alpha + (1 - \delta) k_t - c_t$ to forecast future capital stock. Here again, A_t is given by equations (41) or (26), depending on which economy we are computing.

The objects we need to compute are approximations to the decision rules $c_t = c(z_t, R_t, k_t)$ and $\widetilde{q}_t = q(z_t, k_t, R_{t-1})$ that satisfy these Euler equations. The approximate decision rules $\hat{c}(z_t, R_t, k_t)$ and $\hat{q}(z_t, k_t, R_{t-1})$ are then used to compute the values of other variables.

5.2 Approximating the Distribution of Plants and Agents' Expectations

Before we can compute $\hat{c}(z_t, R_t, k_t)$ and $\hat{q}(z_t, k_t, R_{t-1})$ we have to approximate the expectation operators in the Euler equations (47) and (48). We do so using Gauss-Hermite quadrature (see Judd, 1998, p. 261). In addition, in the economy with nonconvexities we need to approximate the measures (15) and (19) and the truncated means

$$
\int_{\phi_{jt}}^{\infty} \varepsilon g (\varepsilon; \kappa z_t, \sigma_\varepsilon) \, d\varepsilon \quad \text{and} \quad \int_{\lambda_{jt}}^{\infty} s f (s; z_t, \sigma_s) \, ds,
$$

which appear in expressions for output (16) and (20), respectively. Neither the measures nor the truncated means have closed-form solutions. For the measures we use an approximation

proposed by Bagby (1995). Following Bagby's approach, the measure $\hat{\mu}_{jt}^o$ is approximated as

$$\widehat{\mu}_{jt}^o \simeq \begin{cases} 0.5 - \Phi(\hat{\phi}_{jt}) & \text{if } \hat{\phi}_{jt} < 0 \\ 0.5 + \Phi(\hat{\phi}_{jt}) & \text{if } \hat{\phi}_{jt} \geq 0, \end{cases}$$

where

$$\Phi(\hat{\phi}_{jt}) \equiv 0.5 \left\{ 1 - \frac{1}{30} \left[7 \exp\left(-\frac{\hat{\phi}_{jt}^2}{2} \right) + 16 \exp\left(-\hat{\phi}_{jt}^2 \left(2 - \sqrt{2} \right) \right) \right. \right.$$
$$\left. \left. + \left(7 + \frac{\pi}{4} \hat{\phi}_{jt}^2 \right) \exp\left(-\hat{\phi}_{jt}^2 \right) \right] \right\}^{\frac{1}{2}}$$

and $\hat{\phi}_{jt} \equiv [(\phi_{jt} - \kappa z_t)/\sigma_\varepsilon]$. The truncated mean of ε is then obtained as

$$\int_{\phi_{jt}}^\infty \varepsilon g\left(\varepsilon; \kappa z_t, \sigma_\varepsilon \right) d\varepsilon \simeq \kappa z_t \left[1 - \Phi(\hat{\phi}_{jt}) \right] + \frac{\sigma_\varepsilon}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} \hat{\phi}_{jt}^2 \right).$$

We approximate μ_{jt}^R and the truncated mean of s similarly.

5.3 The Approximate Decision Rules

Using Chebyshev polynomials, the approximate decision rules $\hat{c}\left(z_t, R_t, k_t \right)$ and $\hat{q}\left(z_t, R_{t-1}, k_t \right)$ have the form

$$\widehat{c}\left(z_t, R_t, k_t \right) = \sum_{i=1}^J \sum_{j=1}^J \sum_{k=1}^J a_{ijk} \Psi_i\left(z_t \right) \Psi_j\left(R_t \right) \Psi_k\left(k_t \right)$$

$$\widehat{q}\left(z_t, k_t, R_{t-1} \right) = \sum_{i=1}^J \sum_{j=1}^J \sum_{k=1}^J b_{ijk} \Psi_i\left(z_t \right) \Psi_j\left(R_{t-1} \right) \Psi_k\left(k_t \right)$$

where $\Psi_i\left(z_t \right) \equiv T_{i-1}\left(2\left(\left(z_t - z_m \right) / \left(z_M - z_m \right) \right) - 1 \right)$. Here T_{i-1} is the ith-order Chebyshev polynomial and z_m and z_M are the lower and upper bounds for z_t. $\Psi_j\left(R_t \right)$ and $\Psi_k\left(k_t \right)$ are defined similarly. The lower and upper bounds for the state variables are chosen such that with 99% confidence (verified by simulation) the variables stay within the bounds. The details of the computational procedure, as well as the values of the coefficients of the resulting decision rules, are contained in the Appendix.

6 Findings

6.1 Responses to Aggregate Productivity Shocks

Figure 1 plots responses of some key variables to a 1% positive aggregate productivity shock. We see that in the economy with nonconvexities all quantities respond by less than in the benchmark economy. In particular, while in the benchmark economy output increases on impact by 1.9%, in the economy with nonconvexities the increase is only by 1.45%. That is by 25% less. As a result, in order to generate the same volatility of output as in the benchmark economy, for the same autocorrelation coefficient of z_t, σ_ξ in the economy with nonconvexities needs to be 0.009. We also find that at this level of volatility, the coefficient of variation of the aggregate capital workweek is 0.0145. Based on SPC data Beaulieu and Mattey (1998) report the coefficient of variation to be 0.0293. Our model thus generates variation in the aggregate capital workweek of the same order of magnitude as in the data, but 50% smaller. This suggests that the model does not account for all of the movements in the distribution of plants across the different margins of capital utilization.

Returning to Figure 1, we see that the smaller increase in output in the economy with nonconvexities is largely due to a muted response of employment, which on impact increases by about 50% less than in the benchmark economy. This is for two reasons. First, other things being equal (in particular holding ϕ_{jt} and λ_{jt} constant), the shock to aggregate productivity does not move the mean of the distribution of plants sufficiently enough to increase employment as much as in the benchmark economy. And second, whatever increase is further muted in equilibrium by an increase in the real wage rate. The reason why output increases substantially more than employment (almost twice as much) is that productivity of all plants that are operated increases, which boosts output further, above and beyond its increase due to higher employment.

In terms of output composition, on impact consumption in the economy with nonconvexities increases by almost as much as in the benchmark economy. This is because although output increases by substantially less in the former economy than in the latter on impact, in the economy with nonconvexities it is somewhat more persistent (because employment

is more persistent), as the figure shows. As a result, the lifetime income of the household in the two economies is about the same and therefore the initial increase in consumption in the two economies is similar. Because of this increase in consumption, investment in the economy with nonconvexities thus increases by less on impact than in the benchmark economy.

Finally, we also plot the response of inflation, given as a difference between the log of the price level in period $t - 1$ and the log of the price level in period t which satisfies the cash-in-advance constraint (3), holding with equality. As employment in the economy with nonconvexities increases by less than in the benchmark economy, the real wage bill (and thus also real loans e_t/p_t), increases by less. This is despite the fact that, due to shiftwork, the average real wage rate (not plotted) increases by more in the economy with nonconvexities than in the benchmark economy. And as consumption in the two economies increases by about the same amount, p_t has to fall by more in the economy with nonconvexities than in the benchmark economy in order to clear the goods market.

Figure 2 plots the responses of the measures of plants operating the different shifts. We see that the measure of plants operating shift 2 responds the most (by 0.68%), followed by the measure of plants operating shift 3 (by 0.53%) and shift 1 (by 0.3%). The measure of plants operating shift 1 during overtime responds only little (by 0.15%) and the response of the measure of plants operating shifts 2 and 3 during overtime is minuscule. Notice that, given the steady-state values of these measures and the way employment is calculated (see equation 27), these responses imply that about two thirds of the increase in employment plotted in Figure 1 is due to increase in employment on late shifts (shifts 2 and 3). This is broadly in line with the empirical evidence provided by Mayshar and Solon (1993) and Shapiro (1996) discussed in Section 2.

6.2   Responses to Interest Rate Shocks from Steady State

Figure 3 plots the responses, from steady state, of output and inflation to a 100 basis point decline in the nominal interest rate under the assumption that interest rate shocks are serially uncorrelated (i.e., ρ_R is equal to zero). In the next subsection we carry out similar

experiments, but shock the nominal interest rate not from the economy's steady state, but conditional on the realization of either ξ_t or z_t. (We will also consider serially correlated shocks). We show the responses of output and inflation from steady state in order to set a benchmark with which we can compare the conditional impulse-responses.

The figure shows that both output and inflation in the economy with nonconvexities increases by about 45% less than in the benchmark economy. The reasons behind this result are similar to those discussed in the case of a technology shock – a given decline in real borrowing costs in the economy with nonconvexities does not make a large enough number of plants change their capital utilization as would be necessary to replicate the responses to the same decline in the benchmark economy.

6.3   Responses to Interest Rate Shocks Conditional on Aggregate Productivity Shocks

This subsection investigates if (and by how much) in our model the responses of aggregate output and inflation to nominal interest rate shocks vary with aggregate productivity shocks hitting the economy. Table 4 presents the results for the case of serially uncorrelated shocks, while Table 5 contains the results for the case in which the nominal interest rate shocks are highly autocorrelated (ρ_R equal to 0.97). In both cases the stochastic process for aggregate productivity shocks has the parameter values displayed in Table 2.

In order to facilitate easy comparison of the various responses, we express each response as a ratio with respect to the response from steady state. Each response is labeled by the value of z_t on which we condition the response. In Experiment A we hit the economy with nonconvexities with a negative (i.e., a decline) 100 basis point nominal interest rate shock, conditional on the economy being hit by an aggregate technology shock which moves z_t by one, two, and three σ_ξ either above or below its steady state level $\bar{z}$. In Experiment B we study the interest rate shock responses conditional on a sequence of ξ shocks that move z_t one, two, and three σ_z either above or below its steady-state level $\bar{z}$. Relative to Experiment A, Experiment B thus considers more serious increases/declines in aggregate productivity, and thus in output.

In the middle row of each panel in Table 4 we report deviations of output and inflation from steady state in the impact period plotted in Figure 3 (since the deviations after the impact period are tiny, we only focus on the impact period here). We see that in both experiments the responses of output are larger when the economy is hit by negative aggregate productivity shocks then when it is hit by positive aggregate productivity shocks. Qualitatively, our model supports the hypothesis that monetary policy shocks should have a larger impact on economic activity when aggregate capacity is sparse. Quantitatively though, the differences are small. For example, the response of output is only 10% larger when z_t is $3\sigma_z$ below its steady-state level than when the economy is hit by a negative interest rate shocks from a steady state. And when z_t is $3\sigma_z$ above its steady-state level, the response of output is only 3% smaller than from steady state. The asymmetries in the responses of inflation are similarly small.

Table 5 contains the results for the case of highly serially correlated nominal interest rate shocks. As these shocks are autocorrelated, they have a long-lasting effect, especially on inflation. We therefore show the responses for more than the impact period. They are presented in the same way as in Table 4. We see that in both experiments, as in the case of serially uncorrelated shocks, the asymmetries in the responses of output are small. There are, however, substantial asymmetries in the responses of inflation in the impact period. For example, in Experiment B for z_t being $3\sigma_z$ below its steady-state level, the increase in inflation is about one third as large as when the economy is in steady state. And when z_t is $3\sigma_z$ above its steady-state level, a negative nominal interest rate shock makes inflation increase by almost twice as much as when the economy is in steady state. The aggregate effects of monetary policy shocks, although broadly symmetric across the stages of the business cycle in terms of their effects on output, are highly asymmetric in terms of the responses of inflation. This result is reminiscent of earlier results in the literature (summarized by Taylor and Uhlig, 1990) that while linear approximations work reasonably well for quantities, they might omit important nonlinearities in equilibrium pricing functions.

7 Concluding remarks

This study introduces nonconvex and lumpy margins of capacity utilization at the plant level into an equilibrium business cycle model. As a result it provides a general equilibrium model with cyclical movements in capacity utilization at the aggregate level consistent with observed micro-level behavior of output adjustment in many manufacturing industries. We use the model to answer two questions. First, in light of the debate on the effects of micro-level nonconvexities on the behavior of the aggregate economy we ask if nonconvex margins of capacity utilization, and thus of output adjustment, at the plant level affect the volatility of aggregate output. We find that when our model is calibrated to be consistent with long-run averages of standard aggregates, as well as with cross-sectional distribution of the use of shiftwork and overtime work in U.S. manufacturing, aggregate output is 25% less volatile, for a given set of aggregate technology shocks, than in a model without such features. This is an interesting result. As Hansen (1985) shows, nonconvexities in households' labor supply decisions magnify the responses of the economy to aggregate productivity shocks. We have shown that similar nonconvexities on the production side tend to mitigate such responses. The second question we ask is if cyclical movements in aggregate capacity utilization make monetary injections more effective, in terms of their impact on aggregate output, in recessions than in expansions. We find that although such effects are greater in downturns, when resources are less intensively utilized, than in periods of above-trend growth, quantitatively the differences are small. This suggests that other mechanism than variation in aggregate capacity utilization (for instance borrowing constraints faced by households and firms that bind in recessions) are more likely to be responsible for the asymmetric responses of aggregate output to monetary policy shocks documented in the empirical literature.

As our model accounts for 50% of the cyclical variation in aggregate capital workweek, it clearly omits some important dynamics in the distribution of plants across the margins of capital utilization over the business cycle. This likely makes our estimate of the reduction in aggregate output volatility due to production nonconvexities too large. It also likely underestimates the asymmetries in the aggregate effects of monetary policy shocks. In

the absence of data on plant-level productivity in U.S. manufacturing, we assumed that productivity shocks of individual plants are normally distributed. And we allowed only the mean of the distribution to depend on aggregate productivity shocks, and thus to vary over the business cycle. In addition, we assumed that all plants are identical at the start of each period, before idiosyncratic shocks are realized. As a result of these assumptions, plant-level heterogeneity in our model is exogenous and given by the normal distribution of idiosyncratic productivity shocks with stochastic mean. Although we think this is a natural starting point, it might be desirable to endogenize the distribution of plants in some way. In order to make plant heterogeneity endogenous, in the presence of exogenous idiosyncratic productivity shocks, the plants need to be identified by a time-varying endogenous state variable, in addition to the exogenous productivity level. For example, in the model of lumpy plant-level investment studied by Khan and Thomas (2008) capital plays such a role. It is not clear what variable, other than capital, should do the same in our model. One candidate is the number of workers per shift. There is, however, little evidence that staff numbers within a shift substantially vary over time at the plant level. Inventories or state-dependent pricing are another candidates. We leave such extensions for future research.

Appendix: Details of the Computational Procedure

The solution is obtained in three steps. We start with $J = 2$ and make an initial guess about the coefficients of the approximate decision rules. For $J = 2$ the approximate decision rules have the form

$$(49) \quad c_t = a_{111} + a_{112}\Psi_2(k_t) + a_{121}\Psi_2(R_t) + a_{122}\Psi_2(R_t)\Psi_2(k_t) + a_{211}\Psi_2(z_t)$$
$$+ a_{212}\Psi_2(z_t)\Psi_2(k_t) + a_{221}\Psi_2(z_t)\Psi_2(R_t) + a_{222}\Psi_2(z_t)\Psi_2(R_t)\Psi_2(k_t),$$

$$(50) \quad \hat{q}_t = b_{111} + b_{112}\Psi_2(k_t) + b_{121}\Psi_2(R_{t-1}) + b_{122}\Psi_2(R_{t-1})\Psi_2(k_t) + b_{211}\Psi_2(z_t)$$
$$+ b_{212}\Psi_2(z_t)\Psi_2(k_t) + b_{221}\Psi_2(z_t)\Psi_2(R_{t-1}) + b_{222}\Psi_2(z_t)\Psi_2(R_{t-1})\Psi_2(k_t).$$

The objects we need to compute are the a and b coefficients that satisfy the Euler equations (47) and (48), with the substitutions described in Subsection 5.1, at eight collocation nodes in the state space (for higher J's the number of nodes needs to be adjusted accordingly).

The initial guess is made so that the decision rules are linear and pass through the steady state. In addition, we impose that c_t and $\widetilde{q}_t$ are zero when either z_t or k_t is zero. The coefficients of the initial guess are thus $a_{111} = -0.7772$, $a_{112} = 0.0912$, $a_{211} = 0.7772$, $b_{111} = -0.2190$, $b_{112} = 0.0257$, $b_{211} = 0.2190$, and all other coefficients are set to equal to zero.[18]

We then carry out a couple of initial iterations on the system of 16 equations (the two Euler equations, each evaluated at eight nodes) using the Levenberg-Marquardt algorithm (see Judd, 1998, p. 119) in order to get "near" the solution. The solution is finally obtained using Powell's method (see Judd, 1998, p. 173), which takes the output of the Levenberg-Marquardt algorithm as its input. The solution for $J = 2$ is then used as an initial guess for $J = 3$ and so on.

Chebyshev Approximation Theorem states that as $J \to \infty$, $\widehat{c}(z_t, R_t, k_t) \to c(z_t, R_t, k_t)$ and $\widehat{q}(z_t, k_t, R_{t-1}) \to q(z_t, k_t, R_{t-1})$ uniformly. Furthermore, as $J \to \infty$, the coefficients in the decision rules that load on to the monomials with an increasingly higher order approach zero. In our case, $J = 2$ turns out to be sufficient – the coefficients that load on to the monomials of third order are of the order of magnitude of $1e^{-3}$ or $1e^{-4}$, and the values of the coefficients that load on to the monomials of lower orders change at most in the third decimal place. The approximations to the decision rules that we use in our experiments thus have the form (49) and (50) with the values of the coefficients reported in Table 3.

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[18]We set the steady-state value of z_t equal to one and re-normalize A accordingly.

References

BAGBY, R. J., "Calculating Normal Probabilities," *American Mathematical Monthly* 102 (1995), 46–49.

BARTH, M. J. AND V. A. RAMEY, "The Cost Channel of Monetary Transmission," *NBER Macroeconomics Annual* 16 (2002), 199–256.

BEAULIEU, J. AND J. MATTEY, "The Workweek of Capital and Capacity Utilization in Manufacturing," *Journal of Productivity Analysis* 10 (1998), 199–223.

BRESNAHAN, T. F. AND V. A. RAMEY, "Output Fluctuations at the Plant Level," *Quarterly Journal of Economics* 109 (1994), 593–624.

BURNSIDE, C. AND M. EICHENBAUM, "Factor-Hoarding and the Propagation of Business-Cycle Shocks," *American Economic Review* 86 (1996), 1154–74.

CHRISTIANO, L. J. AND M. EICHENBAUM, "Liquidity Effects and the Monetary Transmission Mechanism," *American Economic Review* 82 (1992), 346–53.

CHRISTIANO, L. J., M. EICHENBAUM AND C. L. EVANS, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113 (2005), 1–45.

CLARK, P. K., "Macroeconomic Implications of Variation in the Workweek of Capital. Comments and Discussion," *Brookings Papers on Economic Activity* 2 (1996), 120–33.

COOLEY, T. F., G. D. HANSEN AND E. C. PRESCOTT, "Equilibrium Business Cycles with Idle Resources and Variable Capacity Utilization," *Economic Theory* 6 (1995), 35–49.

CORRADO, C. AND J. MATTEY, "Capacity Utilization," *Journal of Economic Perspectives* 11 (1997), 151–67.

GARCIA, R. AND H. SCHALLER, "Are the Effects of Monetary Policy Asymmetric?," *Economic Inquiry* 40 (2002), 102–19.

HALL, G., "Nonconvex Costs and Capital Utilization: A Study of Production and Inventories at Automobile Assemmbly Plants," *Journal of Monetary Economics* 45 (2000), 681–716.

HANSEN, G. D., "Indivisible Labour and the Business Cycle," *Journal of Monetary Economics* 16 (1985), 309–27.

HANSEN, G. D. AND E. C. PRESCOTT, "Capacity Constraints, Asymmetries, and the Business Cycle," *Review of Economic Dynamics* 8 (2005), 850–65.

HANSEN, G. D. AND T. J. SARGENT, "Straight Time and Overtime in Equilibrium," *Journal of Monetary Economics* 21 (1988), 281–308.

HORNSTEIN, A., "Towards a Theory of Capacity Utilization: Shiftwork and the Workweek of Capital," *Federal Reserve Bank of Richmond Economic Quarterly* (2002), 65–86.

JUDD, K. L., "Projection Methods for Solving Aggregate Growth Models," *Journal of Economic Theory* 58 (1992), 410–52.

———, *Numerical Methods in Economics* (MIT Press, 1998).

KHAN, A. AND J. K. THOMAS, "Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?," *Journal of Monetary Economics* 50 (2003), 331–60.

———, "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica* 76 (2008), 395–436.

KING, S. L. AND H. B. WILLIAMS, "Shift Work Pay Differentials and Practices in Manufacturing," *Monthly Labor Review* (1985), 26–33.

KOSTIUK, P. F., "Compensating Differentials for Shift Work," *Journal of Political Economy* (1990), 1054–75.

KYDLAND, F. E. AND E. C. PRESCOTT, "The Workweek of Capital and Its Cyclical Implications," *Journal of Monetary Economics* 21 (1988), 343–60.

Lo, M. C. and J. Piger, "Is the Response of Output to Monetary Policy Asymmetric? Evidence from a Regime-Switching Coefficients Model," *Journal of Money, Credit, and Banking* 37 (2005), 865–86.

Lucas, R. E. J., "Capacity, Overtime, and Empirical Production Functions," *American Economic Review* 60, Papers and Proceedings (1970), 23–27.

Mattey, J. and S. Strongin, "Factor Utilization and Margins for Adjusting Output: Evidence from Manufacturing Plants," *Federal Reserve Bank of San Francisco Economic Review* 2 (1997), 3–17.

Mayshar, J. and G. Solon, "Shift Work and the Business Cycle," *American Economic Review* 83 (1993), 224–28.

Peersman, G. and F. Smets, "Are the Effects of Monetary Policy in the Euro Area Greater in Recessions then in Booms?," Working Paper 52, European Central Bank, 2002.

Prescott, E. C., "Nonconvexities in Quantitative General Equilibrium Studies of Business Cycles," in T. J. Kehoe, T. N. Srinivasan and J. Whalley, eds., *Frontiers in Applied General Equilibrium Modeling* (Cambridge University Press, 2005).

Shapiro, M. D., "Capital Utilization and Capital Accumulation: Theory and Evidence," *Journal of Applied Econometrics* 1 (1986), 211–34.

———, "Capital Utilization and the Marginal Premium for Work at Night," Mimeo, 1995.

———, "Macroeconomic Implications of Variation in the Workweek of Capital," *Brookings Papers on Economic Activity* 2 (1996), 79–119.

Taylor, J. B. and H. Uhlig, "Solving Nonlinear Stochastic Growth Models: A Comparison of Alternative Solution Methods," *Journal of Business and Economic Statistics* 8 (1990), 1–17.

Thomas, J. K., "Is Lumpy Investment Relevant for the Business Cycle?," *Journal of Political Economy* 110 (2002), 508–34.

Weise, C. L., "The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach," *Journal of Money, Credit and Banking* 31 (1999), 85–108.

Table 1: Long-run averages of U.S. data used in calibration

Symbol	Value	Description
Both models		
k	8.519	Capital to output ratio
c	0.777	Consumption to output ratio
n	0.310	Fraction of time spent in market activities
Model with nonconvexities		
$\{\Delta_j^o\}_{j=1}^3$	0.500	Overtime premia
$\left(1 - \mu_1^R\right)$	0.067	Fraction of plants that are closed
$\left(\mu_1^R - \mu_2^R\right)$	0.255	Fraction of plants operating one shift
$\left(\mu_2^R - \mu_3^R\right)$	0.377	Fraction of plants operating two shifts
μ_3^R	0.301	Fraction of plants operating three shifts
μ_1^o	0.173	Fraction of plants using weekend work

Table 2: Parameter values

Symbol	Value	Parameter
Both models		
α	0.365	Rental income share of output
δ	0.026	Capital depreciation rate
θ	0.981	Discount factor
Benchmark model		
b	2.516	Parameter for disutility from work
ρ_z	0.9	Persistence in the productivity shock
σ_ξ	0.0067	Standard deviation of innovation
		in the productivity process
Model with nonconvexities		
h	1/3	Shift length
β	0.580	Share of labor in production flow
$\overline{n}$	0.162	Minimum-staffing requirement
κ	0.38	Ratio of the mean of ε to the mean of s
		Parameter for disutility from work on:
b_1^R	1.618	first regular-time shift
b_2^R	2.901	second regular-time shift
b_3^R	4.140	third regular-time shift
b_1^o	2.104	first shift on Saturdays
b_2^o	3.771	second shift on Saturdays
b_3^o	5.381	third shift on Saturdays
σ_s	0.851	Standard deviation of the idiosyncratic shock s
σ_ε	0.802	Standard deviation of the idiosyncratic shock ε
ρ_z	0.9	Persistence in the productivity shock
σ_ξ	0.009	Standard deviation of innovation
		in the productivity process

Table 3: Approximate equilibrium decision rules

| | Coefficients associated with the Chebyshev polynomials for [a] | | | | | | | |
	1	k_t	R_{t+i}	R_{t+i}, k_t	z_t	z_t, k_t	z_t, R_{t+i}	z_t, R_{t+i}, k_t
	Benchmark economy, $\rho_R = 0$							
c_t	0.1453	0.0179	0.0428	0.0125	0.1431	0.0699	0.0078	-0.0479
$\hat{q}_t$	1.3813	-0.0367	0.1417	0.0183	-1.4023	0.0596	-0.2453	0.0003
	Benchmark economy, $\rho_R = 0.96$							
c_t	0.6689	-0.0347	-0.4728	0.0643	-0.0835	0.1375	0.2304	-0.1146
$\hat{q}_t$	-0.4080	-0.0866	1.9084	0.0674	0.0787	0.1090	-1.7086	-0.0481
	Economy with nonconvexities, $\rho_R = 0$							
c_t	-1.5719	0.2282	1.5357	-0.1624	2.0298	-0.1532	-1.6214	0.1373
$\hat{q}_t$	1.0047	-0.0266	-0.4347	0.0810	-0.9075	0.0343	0.4045	-0.0678
	Economy with nonconvexities, $\rho_R = 0.96$							
c_t	-0.3607	0.1522	0.3804	-0.0903	1.1257	-0.0760	-0.7674	0.0642
$\hat{q}_t$	-0.4629	0.0360	1.0968	0.0120	0.4615	-0.0613	-1.0243	0.0335

[a]In the decision rule for c_t the i in the time subscript for R is equal to 0, while in the decision rule for $\hat{q}_t$ it is equal to -1.
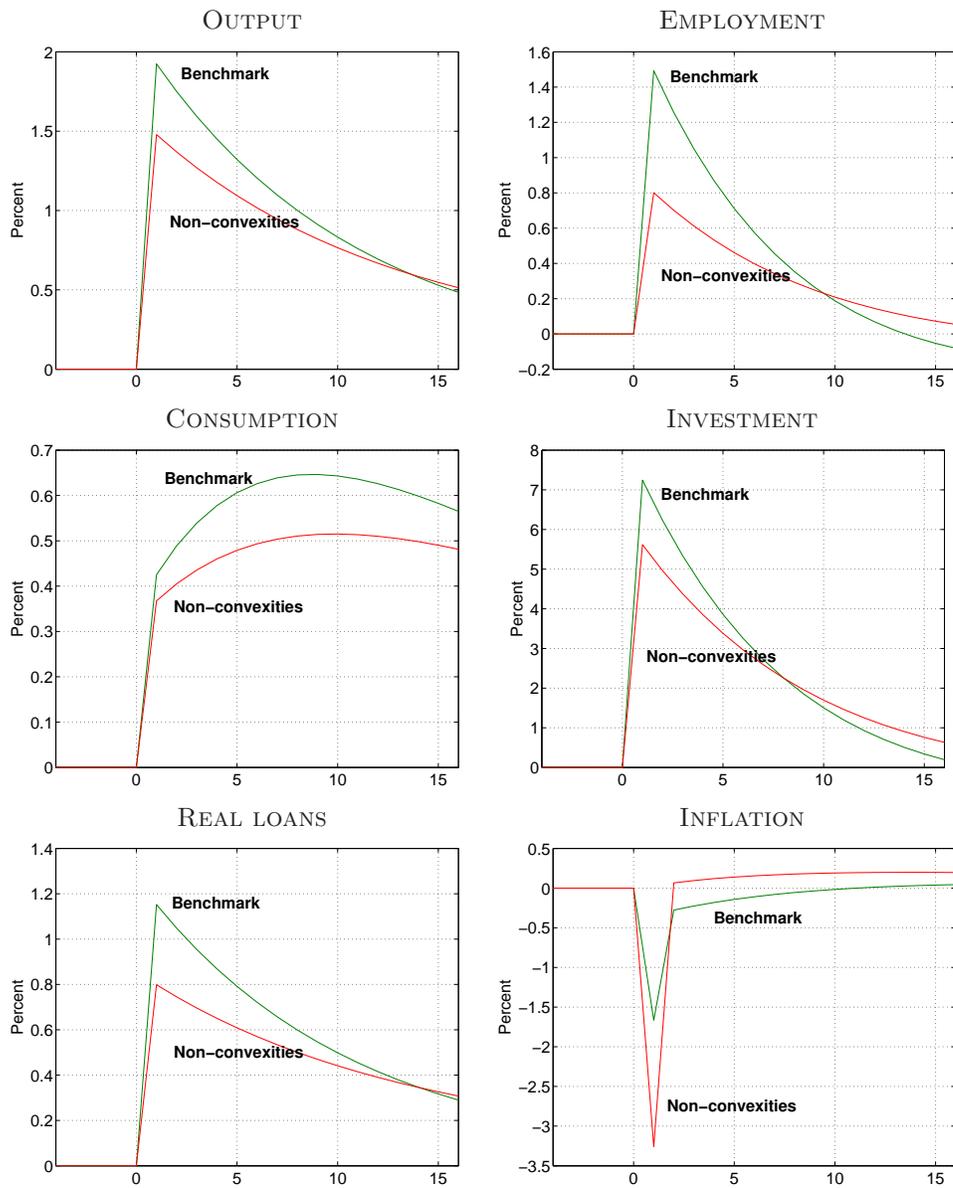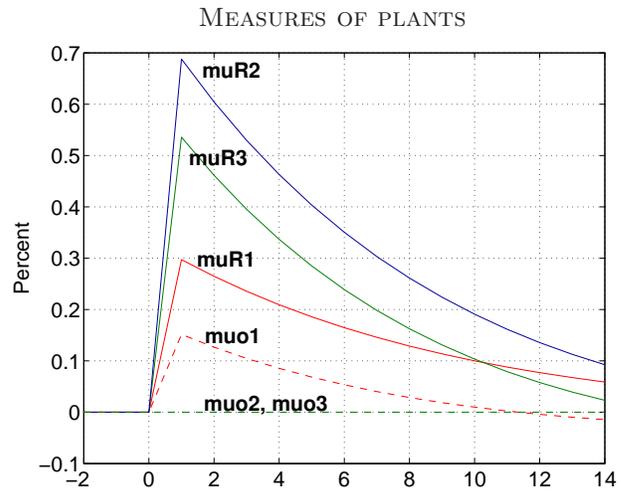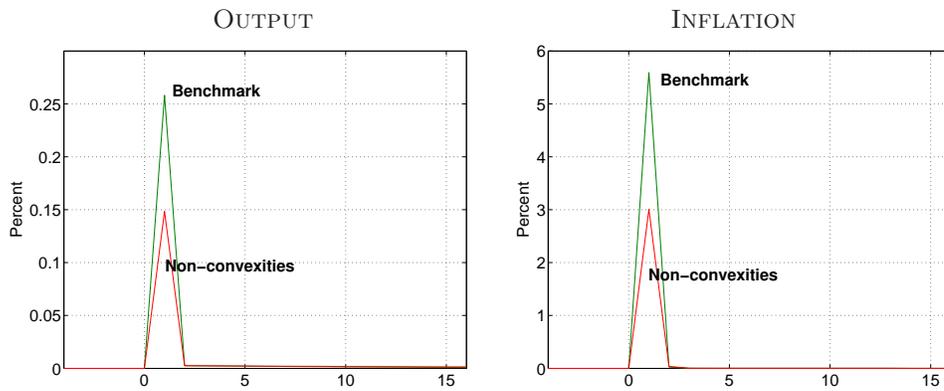
Figure 1: Responses to a 1% increase in aggregate productivity.

MEASURES OF PLANTS

Figure 2: Responses of the measures of plants to a 1% increase in aggregate productivity.

OUTPUT   INFLATION

Figure 3: Responses to a 100 basis point fall in the nominal interest rate (the case of uncorrelated interest rate shocks).

Table 4: Asymmetries in the responses of the economy with nonconvexities to 100 basis point fall in the nominal interest rate (the case of uncorrelated interest rate shocks)

Output		Inflation	
	Experiment A		
$y_t(3\sigma_\xi)/y_t$	0.96	$\pi_t(3\sigma_\xi)/\pi_t$	1.06
$y_t(2\sigma_\xi)/y_t$	0.97	$\pi_t(2\sigma_\xi)/\pi_t$	1.04
$y_t(\sigma_\xi)/y_t$	0.98	$\pi_t(\sigma_\xi)/\pi_t$	1.02
y_t	**0.15**	π_t	**3.00**
$y_t(-\sigma_\xi)/y_t$	1.01	$\pi_t(-\sigma_\xi)/\pi_t$	0.98
$y_t(-2\sigma_\xi)/y_t$	1.03	$\pi_t(-2\sigma_\xi)/\pi_t$	0.97
$y_t(-3\sigma_\xi)/y_t$	1.04	$\pi_t(-3\sigma_\xi)/\pi_t$	0.95
	Experiment B		
$y_t(3\sigma_z)/y_t$	0.97	$\pi_t(3\sigma_z)/\pi_t$	1.04
$y_t(2\sigma_z)/y_t$	0.97	$\pi_t(2\sigma_z)/\pi_t$	1.03
$y_t(\sigma_z)/y_t$	0.98	$\pi_t(\sigma_z)/\pi_t$	1.02
y_t	**0.15**	π_t	**3.00**
$y_t(-\sigma_z)/y_t$	1.02	$\pi_t(-\sigma_z)/\pi_t$	0.98
$y_t(-2\sigma_z)/y_t$	1.06	$\pi_t(-2\sigma_z)/\pi_t$	0.97
$y_t(-3\sigma_z)/y_t$	1.10	$\pi_t(-3\sigma_z)/\pi_t$	0.95

[a]The bold numbers are the actual responses from steady state. All other responses are expressed as a ratio with respect to the response from steady state, shown in Figure 3. Experiment A – responses conditional on the innovation ξ in the stoch. process for aggregate productivity being x standard deviations away from its mean; Experiment B – responses conditional on z being x standard deviations away from its mean.

Table 5: Asymmetries in the responses of the economy with nonconvexities to 100 basis point fall in the nominal interest rate (the case of serially correlated interest rate shocks; $\rho_R = 0.97$)

| | Period after the interest rate shock | | | | | | | |
	1	2	3	4	5	6	7	8
Experiment A								
Output								
$y(3\sigma_\xi)/y_t$	0.97	0.97	0.98	0.98	0.99	0.99	1.00	1.01
$y_t(2\sigma_\xi)/y_t$	0.98	0.98	0.98	0.99	0.99	0.99	1.00	1.01
$y_t(\sigma_\xi)/y_t$	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00
y_t	**0.05**	**0.04**	**0.04**	**0.04**	**0.03**	**0.03**	**0.02**	**0.02**
$y_t(-\sigma_\xi)/y_t$	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00
$y_t(-2\sigma_\xi)/y_t$	1.02	1.02	1.02	1.01	1.01	1.01	1.00	0.99
$y_t(-3\sigma_\xi)/y_t$	1.03	1.03	1.03	1.02	1.02	1.01	1.00	0.99
Inflation								
$\pi_t(3\sigma_\xi)/\pi_t$	1.29	0.97	0.97	0.98	0.98	0.99	0.99	0.99
$\pi_t(2\sigma_\xi)/\pi_t$	1.19	0.98	0.98	0.99	0.99	0.99	0.99	0.99
$\pi_t(\sigma_\xi)/\pi_t$	1.09	0.99	0.99	0.99	0.99	1.00	1.00	1.00
π_t	**0.34**	**-0.66**	**-0.64**	**-0.62**	**-0.59**	**-0.57**	**-0.55**	**-0.53**
$\pi_t(-\sigma_\xi)/\pi_t$	0.91	1.01	1.01	1.01	1.01	1.00	1.00	1.00
$\pi_t(-2\sigma_\xi)/\pi_t$	0.83	1.02	1.02	1.01	1.01	1.01	1.01	1.01
$\pi_t(-3\sigma_\xi)/\pi_t$	0.75	1.03	1.02	1.02	1.02	1.01	1.01	1.01
Experiment B								
Output								
$y(3\sigma_z)/y_t$	1.14	1.14	1.14	1.14	1.14	1.14	1.14	1.14
$y_t(2\sigma_z)/y_t$	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
$y_t(\sigma_z)/y_t$	1.04	1.04	1.04	1.04	1.04	1.04	1.04	1.04
y_t	**0.05**	**0.04**	**0.04**	**0.04**	**0.03**	**0.03**	**0.02**	**0.02**
$y_t(-\sigma_z)/y_t$	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
$y_t(-2\sigma_z)/y_t$	0.93	0.93	0.93	0.93	0.93	0.93	0.93	0.93
$y_t(-3\sigma_z)/y_t$	0.91	0.91	0.91	0.91	0.91	0.91	0.91	0.91
Inflation								
$\pi_t(3\sigma_z)/\pi_t$	1.83	1.02	1.02	1.02	1.02	1.02	1.02	1.02
$\pi_t(2\sigma_z)/\pi_t$	1.54	1.02	1.02	1.02	1.02	1.02	1.02	1.02
$\pi_t(\sigma_z)/\pi_t$	1.26	1.01	1.01	1.01	1.01	1.01	1.01	1.01
π_t	**0.34**	**-0.66**	**-0.64**	**-0.62**	**-0.59**	**-0.57**	**-0.55**	**-0.53**
$\pi_t(-\sigma_z)/\pi_t$	0.76	0.98	0.98	0.98	0.98	0.98	0.98	0.98
$\pi_t(-2\sigma_z)/\pi_t$	0.55	0.95	0.95	0.95	0.95	0.95	0.95	0.95
$\pi_t(-3\sigma_z)/\pi_t$	0.37	0.91	0.91	0.91	0.91	0.91	0.91	0.91

[a]The bold numbers are the actual responses from steady state. All other responses are expressed as a ratio with respect to the response from steady state. Experiment A – responses conditional on the innovation ξ in the stoch. process for aggregate productivity being x standard deviations away from its mean; Experiment B – responses conditional on z being x standard deviations away from its mean.