A Century of Purchasing Power Parity Confirmed: The Role of Nonlinearity

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Abstract

Taylor (2002) claims that Purchasing Power Parity (PPP) has held over the 20th century based on strong evidence of stationarity for century-long real exchange rates for 20 countries. Lopez *et al.* (2005), however, found much weaker evidence of PPP with alternative lag selection methods. We reevaluate Taylor's claim by implementing a recently developed nonlinear unit root test by Park and Shintani (2005). We find strong evidence of nonlinear mean-reversion in real exchange rates that confirms Taylor's claim. We also find a possible misspecification problem in using the ESTAR model that may not be detected with Taylor-approximation based tests.

Keywords: Purchasing Power Parity, Transition Autoregressive Process, inf-t Unit Root Test

JEL Classification: C22, F31

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1 Introduction

Purchasing power parity (PPP) is a simple theory of real exchange rate determination that continues to serve as a key building block for many open economy macro models. Despite its popularity and extensive studies, however, empirical evidence on PPP still remains elusive. Taylor (2002) constructed over a century-long real exchange rates for 20 countries, and implemented an array of unit root tests. Finding very strong evidence for PPP, he concluded that PPP has held over the 20th century. His claim, however, was upset by Lopez *et al.* (2005) who pointed out that his results were sensitive to the choice of lag selection methods. They reported much weaker evidence of PPP from implementing the same unit root tests for his data with alternative lag selection methods.

This paper takes a different road and reevaluates Taylor's claim by implementing a new nonlinear unit root test proposed by Park and Shintani (2005). Recent theoretical and empirical studies on real exchange rates have shown the importance of nonlinear adjustment of the real exchange rate toward long-run equilibrium value. Dumas (1992) and Sercu *et al.* (1995) show how transaction costs in international arbitrage can induce nonlinear adjustment of the real exchange rates toward PPP. Michael *et al.* (1997) and Obstfeld and Taylor (1997) study nonlinear adjustment process motivated by transaction costs that define a neutral band with profitable commodity arbitrage opportunities at the boundary. It should be noted that a failure to account for such nonlinearity may underlie the difficulties in better understanding real exchange rates dynamics (see, among others, Taylor, 2001).

We also note the low power problem of the conventional linear unit-root tests when the true data generating process is nonlinear mean-reverting process. Pippenger and Goering (1993) find that conventional linear tests perform poorly when the true data generating process is the threshold autoregressive (TAR) model, and are sensitive to the speed of adjustment as well as location of the threshold parameter. Taylor *et al.* (2001) show with Monte Carlo simulations that the Dicky-Fuller test has low power against exponential smooth transition autoregressive (ESTAR) process. This body of work suggests that nonlinear models can provide an explanation for the poor performance of conventional linear unit-root tests on PPP deviations and why the deviations from the PPP appear to be nonstationary or extremely slowly mean-reverting (see, among others, Crucini and

Shintani, 2007).

In this light, we reinvestigate Taylor's (2002) claim by testing the null of unit root for his century long real exchange rate data against nonlinear alternatives. We consider three types of transition autoregressive process: exponential smooth transition autoregression (ESTAR), band logistic smooth transition autoregression (BLSTAR), and band threshold autoregression (BTAR). For this purpose, we implement the inf-t test by Park and Shintani (2005) for Taylor's data extended through 2004. Their test is superior than many previously proposed nonlinear unit root tests in various aspects. The inf-t test does not require stationary threshold variables, while other tests such as the one by Caner and Hansen (2001) does. Unlike the test by Kapetanios *et al.* (2003), the inf-t test does not need any Taylor approximation to deal with the so-called "Davies problem." Their test requires much less stringent assumptions on the parameter space compared with more recently proposed tests that include Kapetanios and Shin (2003), Seo (2006), and Bec *et al.* (2004).

By testing the null of unit root against three types of transition AR models for Taylor's (2002) data, we obtain very strong evidence of PPP. The inf-t test rejects a maximum of 14 out of 16 developed countries with standard lag selection procedures. Our results, thus, confirm Taylor's claim. We also report some evidence against the use of ESTAR models due to a potential misspecification problem that may not be detected when one uses Taylor approximation based tests such as the test by Kapetanios *et al.* (2003).

The remainder of the paper is organized as follows. Section 2 briefly describes Park and Shintani's (2005) inf-t test. In Section 3, we describe the three transition functions we employ in this paper. In Section 4, we provide a brief data description and report some pre-test results. Then, we report our main empirical results. Section 5 concludes.

2 The inf-t Test

Park and Shintani (2005) consider the transition between the following two regimes: the unit root regime,

$$\Delta q_t = u_t \tag{1}$$

and the stationary regime,

$$\Delta q_t = \lambda q_{t-1} + u_t, \tag{2}$$

where $\lambda < 0$ and u_t is the zero mean sequence of possibly serially correlated errors. The transition function $\pi(q_{t-d}|\theta)$ is defined as a weight on the stationary regime. Then, the stochastic process of q_t can be jointly represented by

$$\Delta q_t = \lambda q_{t-1}\pi(q_{t-d}|\theta) + u_t, \tag{3}$$

where q_{t-d} is the potentially *nonstationary* transition variable with delay lag $d \geq 1$.[1] θ is an m-dimensional vector of parameters that can be identified only in the stationary regime and $\pi(\cdot)$ denotes a real-valued transition function on $(m+1)$-dimensional real space. Serial correlation in u_t can be accommodated as usual by adding lagged dependent variables in the right hand side of (3),

$$\Delta q_t = \lambda q_{t-1}\pi(q_{t-d}|\theta) + \sum_{j=1}^{k} \beta_j \Delta q_{t-j} + \varepsilon_t, \tag{4}$$

where ε_t is a martingale difference sequence that generates u_t.[2]

With a broad choice of the transition function $\pi(\cdot)$, the model (4) can represent a wide array of nonlinear partial adjustment AR models. Note that if $\lambda = 0$, the stochastic process of q_t is governed solely by the unit root regime. Therefore, one may test the null of the unit root hypothesis,

$$H_0 : \lambda = 0$$

against the alternative hypothesis

$$H_1 : \lambda < 0,$$

which would imply that q_t obeys a nonlinear mean-reverting process.

The test can be implemented as follows. Let Θ_n denote a random sequence of parameter spaces given for each n as functions of the sample $(q_1, ..., q_n)$. For each $\theta \in \Theta_n$, one obtains the t-statistic

[1] This is one of the very attractive properties of the inf-t test. Caner and Hansen's (2001) test, for example, requires stationary transition variables, which can be a quite stringent requirement in practice.

[2] For detailed explanations about ε_t, see Park and Shintani (2005).

for λ in (4),

$$T_n(\theta) = \frac{\hat{\lambda}_n(\theta)}{s(\hat{\lambda}_n(\theta))}, \tag{5}$$

where $\hat{\lambda}_n(\theta)$ is the least squares estimate and $s(\hat{\lambda}_n(\theta))$ is the corresponding standard error. The inf-t test is then defined as

$$T_n = \inf_{\theta \in \boldsymbol{\Theta}_n} T_n(\theta), \tag{6}$$

which is the infimum of t-ratios in (5) taken over all possible values of $\theta \in \Theta_n$. The limit distribution of inf-t statistic is free from any nuisance parameters and depends only on the transition function and the limit parameter space.

3 The Nonlinear Models of the Real Exchange Rate

Let p_t be the log domestic price level, p_t^* be the log foreign price level, and e_t be the log nominal exchange rate as the unit price of the foreign currency in terms of the home currency. The real exchange rate q_t is defined as $p_t^* + e_t - p_t$. We consider three nonlinear stationary alternatives for the natural logarithm of the real exchange rate (q_t): ESTAR, BLSTAR, and BTAR models described in (7) – (9), respectively.

$$\Delta q_t = \lambda(q_{t-1} - \mu)\left[1 - \exp\left\{-\kappa^2 \left(q_{t-1} - \mu\right)^2\right\}\right] + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t \tag{7}$$

$$\Delta q_t = \lambda\left[\frac{q_{t-1} - \tau_1}{1 + \exp\left\{\kappa\left(q_{t-1} - \tau_1\right)\right\}} + \frac{q_{t-1} - \tau_2}{1 + \exp\left\{-\kappa\left(q_{t-1} - \tau_2\right)\right\}}\right] + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t \tag{8}$$

$$\Delta q_t = \lambda\left[(q_{t-1} - \tau_1)\mathrm{I}\left\{q_{t-1} \leq \tau_1\right\} + (q_{t-1} - \tau_2)\mathrm{I}\left\{q_{t-1} \geq \tau_2\right\}\right] + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t, \tag{9}$$

where μ, τ_1, and τ_2 are either the location or threshold parameters and κ is the scale parameter. All regression equations include an intercept.

These functional forms are considered to properly model the commodity arbitrage view of PPP with fixed transaction cost. When there is a relatively small real exchange rate deviation in either directions, commodity arbitrages may not occur due to prohibitively high transaction costs.

Putting it differently, a real exchange rate may follow a unit root process locally around the long-run equilibrium PPP. Such a property may be well captured by ESTAR models. The BLSTAR and BTAR models can further allow an "inaction" band ($[\tau_1, \tau_2]$) where real exchange rates follow unit root process inside the band. Note also that for a very high value for κ, the smooth transition function collapses to a discrete transition function. For instance, the BLSTAR model becomes the BTAR model in such a case.

For the scale parameter κ, we implement grid search for (6) over the parameter space given

$$[10^{-1}P_n, 10^3 P_n], \tag{10}$$

where $P_n = \left(\sum_{t=1}^{n} q_t^2/n\right)^{-1/2}$ as recommended by van Dijk *et al.* (2002). For the location parameter μ, we choose the interval

$$[\Psi_{n,15}, \Psi_{n,85}], \tag{11}$$

where $\Psi_{n,p}$ denotes the pth percentile of $(q_1, q_2, \cdots, q_n)$ as suggested by Caner and Hansen (2001). For the BLSTAR model, we grid search over the 2-dimensional parameter space of (κ, μ) spanned by (10) and (11).

4 Empirical Results

We consider Taylor's (2002) over a hundred-year long annual real exchange rate data relative to the US dollar. We extend the data through 1998 for Eurozone countries and through 2004 for non-Eurozone countries using the *IFS* CD-ROM. We focus on 16 developed countries by dropping Argentina, Brazil, and Mexico from the original data set.[3] We select the number of lags (k) by the General-to-Specific (GTS) rule for the linear model as recommended by Ng and Perron (2001). For nonlinear models (7) through (9), we employ the Partial Autocorrelation rule (PAR) following Granger and Teräsvirta's (1993) suggestion for the state-dependent autoregressive models. We choose a conventional value for the delay parameter, $d = 1$.

As a pre-test, we implement the conventional linear augmented Dickey-Fuller (ADF) test for

[3] All real exchange rates are CPI-based with the exception of Portugal, which is the deflator-based rate.

the real exchange rates. Results are reported in Table 1. The test rejects the null of unit root for only 9 out of 16 developed countries vis-à-vis the US at the 5% significance level, roughly consistent with the results of Lopez *et al.* (2005).[4]

Table 1

As another pre-test, we implement the ESTAR unit root test by Kapetanios *et al.* (2003), one of the most widely used nonlinear unit root tests. We consider two specifications for the test, one with no serial correlation ($k = 0$) and one that accounts for serial correlation ($k = 1$).[5] Results are reported in Table 2. The test rejects the null of a unit root for 13 and 8 out of 16 countries at the 5% significance level with and without serial correlation, respectively.[6] However, it should be noted that their test requires the Taylor-approximation to avoid "Davies problem." Since the test computes the test statistics without directly estimating the key parameter, error-correction coefficient, it is very difficult to identify potentially serious misspecification problems. In Section 5, we show that there is a misspecification problem by using the ESTAR model for Taylor's data.

Table 2

We conduct the inf-t test for the three nonlinear AR models (7) – (9) and results are presented in Tables 3-5. As mentioned before, one clear advantage of using Park and Shintani's (2005) inf-t test over the Taylor-approximation based test is that it directly estimates all parameters in the model, thus can provide useful information on misspecification problems. Our test results with the ESTAR model clearly demonstrate that this is not a negligible matter. The inf-t test rejects the unit root null for 11 countries at the 5% significance level, roughly consistent with the results in Table 2. For instance, the test rejects the unit root null for Portugal at the 1% significance level.

[4] Lopez *et al.* (2005) reported 8 rejections out of 16 countries at the 5% significance level from the same specification. However, with our extended data through 2004, we were able to reject the unit-root null for one additional country, Australia.

[5] k is set at 1 by the Bayesian Information Criteria (BIC).

[6] As an anonymous referee suggested, we implemented Monte Carlo simulations to obtain critical values for the sample sizes used in the paper. We obtained quantitatively very similar critical values as asymptotic ones so that our statistic inferences are unchanged.

It should be noted, however, that the λ estimate is by far less than -2. Since $k = 0$ for the country, this implies that the real exchange rate is cyclically explosive, inconsistent with stationarity. One can see similar problems for Belgium, Denmark, France, Germany, Italy, Japan, Spain, Sweden, and Switzerland.[7,8] This implies that ESTAR may not be an appropriate model for the data.

Table 3

Next, we implement the inf-t test with the BLSTAR specification and results are reported in Table 4. The test rejects the null of unit root for 14 out of 16 countries favoring the nonlinear stationarity alternative. One interesting finding is that the estimate for κ was often very big, which implies that the data can be successfully approximated by the BTAR model for those countries. Our test with the BTAR specification (Table 5) reveals that this is indeed the case. For example, we find quite similar values for λ and τs as well as the inf-t statistics for Finland using the BTAR and BLSTAR specifications. We find similar observations for many other countries.[9] This is not surprising, because the BLSTAR collapses to the BTAR process as κ increases to infinity. This finding implies that a very simple nonlinear model such as BTAR is good enough to approximate century-long real exchange rate dynamics.[10]

Table 4

Table 5

[7] As an anonymous referee suggested, we implemented some robustness analyses. First, we ran estimations assuming not only λ but also β_is are state-dependent. We obtained quantitatively similar results. Second, we also implemented estimations with demeaned series, again yielding similar results. All results are available from the authors upon request.

[8] When $k > 0$, this may not be a problem, since the sum of autoregressive coefficients can be still less than one in absolute value. This was not the case.

[9] As an anonymous referee suggested, we conducted Monte Carlo simulations to examine precision of λ estimates. Our simulations were carried out with a sample size of 120 and with 5,000 replications for each real exchange rate. For each replication, 620 pseudo observations were generated then the first 500 observations were discarded to minimize the influence of initial values. For most of the cases, we obtained compact 95 confidence bands for λ and the median values were close to the reported point estimates. The simulation results are available from the authors upon request.

[10] We also implemented the inf-t test with the General-to-Specific (Hall 1994) criteria and obtained similar results. The test fails to reject the null of unit root for Canada, Denmark, Japan, and Switzerland at the 5% significance level. At the 10% level, the test rejects the null for one additional country, Switzerland, totalling 13 rejections out of 16 countries.

5 Concluding Remarks

With over hundred-year long real exchange rate data for 20 countries, Taylor report very strong evidence of PPP from an array of linear unit root tests leading him to conclude that PPP has held over the 20th century. Lopez *et al.* (2005), however, question the validity of this conclusion. Implementing the same linear unit root tests with alternative lag selection procedures, they reported much weaker evidence of PPP.

We take a different avenue by investigating nonlinear mean-reversion properties for Taylor's long-horizon data that may be consistent with the commodity arbitrage view of PPP with fixed transaction costs. We test the null of a unit root against three types of stationary transition AR processes with the inf-t test by Park and Shintani (2005), which does not require the Taylor-approximation to avoid the "Davies problem." The test is general enough to include virtually any class of nonlinear AR models.

We apply the inf-t test to Taylor's (2002) data extended through 2004 for non-Eurozone countries and 1998 for Eurozone countries. Our main finding is twofold. First, we obtain very strong evidence of nonlinear mean-reversion as the test rejects the null of a unit root for 14 out of 16 developed countries at the 5% significance level. Our finding seems enough to confirm Taylor's claim that PPP has held over the 20th century.

Second, we find some empirical evidence against the ESTAR specification with unreasonable estimates for the error-correction coefficient (λ) for many countries even when the test statistic lies in the rejection region. It should be noted that the Taylor-approximation based tests such as the one by Kapetanios *et al.* (2003) are not able to detect such a misspecification problem.

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Table 1. Unit Root Test Results: Linear Model

$$\Delta q_t = c + \lambda q_{t-1} + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	Sample	k	ADF	λ
Austria	1870-2004	1	-2.955†	-0.110
Belgium	1880-1998	1	-4.168‡	-0.221
Canada	1870-2004	0	-2.446	-0.087
Denmark	1880-2004	6	-1.334	-0.065
Finland	1881-1998	1	-6.015‡	-0.416
France	1880-1998	2	-2.985†	-0.137
Germany	1880-1998	1	-2.944†	-0.090
Italy	1880-1998	2	-4.286‡	-0.247
Japan	1885-2004	6	-0.474	-0.007
Netherlands	1870-1998	1	-2.791	-0.095
Norway	1870-2004	1	-3.763‡	-0.129
Portugal	1890-1998	5	-2.241	-0.117
Spain	1880-1998	1	-3.244†	-0.125
Sweden	1880-2004	2	-3.192†	-0.155
Switzerland	1880-2004	2	-1.446	-0.038
UK	1870-2004	4	-2.600	-0.144

Notes: i) The number of lags (k) was chosen by the General-to-Specific rule (Hall, 1994) following Ng and Perron (2001). ii) † and ‡ refer to the cases when the unit root null is rejected at the 5% and 1% significance levels, respectively. iii) The asymptotic critical values were obtained from Harris (1992).

Table 2. Unit Root Test Results: Taylor-Approximation Based Exponential Smooth Transition Autoregressive Model by Kapetanios *et al.* (2003)

$$\Delta q_t = \delta q_{t-1}^3 + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	$\text{NLADF}_{k=0}$	$\text{NLADF}_{k=1}$
Australia	-2.801	$-3.542^{\ddagger}$
Belgium	$-7.904^{\ddagger}$	$-10.08^{\ddagger}$
Canada	-1.925	-2.019
Denmark	-2.435	$-3.121^{\dagger}$
Finland	$-5.115^{\ddagger}$	$-8.113^{\ddagger}$
France	$-3.544^{\ddagger}$	$-4.567^{\ddagger}$
Germany	-2.378	$-3.874^{\ddagger}$
Italy	$-3.962^{\ddagger}$	$-4.884^{\ddagger}$
Japan	-1.143	-1.972
Netherlands	-2.246	$-3.129^{\dagger}$
Norway	-2.902	$-4.688^{\ddagger}$
Portugal	$-4.500^{\ddagger}$	$-6.553^{\ddagger}$
Spain	$-3.083^{\dagger}$	$-4.366^{\ddagger}$
Sweden	$-3.728^{\ddagger}$	$-5.125^{\ddagger}$
Switzerland	-1.654	-2.745
UK	$-3.733^{\ddagger}$	$-4.641^{\ddagger}$

Notes: i) NLADF denotes the t-statistic for δ as described in Kapetanios *et al.* (2003). ii) The regression was implemented with the demeaned data. iii) † and ‡ refer to the cases when the unit root null is rejected at the 5% and 1% significance levels, respectively. iv) The asymptotic critical values were obtained from Kapetanios *et al.* (2003). Simulated critical values with actual sample sizes yielded same conclusions.

Table 3. Unit Root Test Results: Exponential Smooth Transition Autoregressive Model

$$\Delta q_t = \lambda(q_{t-1} - \mu)\left[1 - \exp\left\{-\kappa^2 (q_{t-1} - \mu)^2\right\}\right] + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	k	inf-t	λ	κ	μ
Australia	1	-3.648†	-0.317	2.074	-0.190
Belgium	1	-10.01‡	-548.2	0.026	-3.760
Canada	0	-2.500	-0.090	17.82	-0.140
Denmark	0	-3.041	-388.9	0.045	-2.130
Finland	1	-8.157‡	-1.076	1.008	-1.500
France	1	-4.479‡	-683.3	0.058	-1.740
Germany	1	-3.820†	-49.66	0.128	-0.740
Italy	1	-4.950‡	-2470	0.013	-7.380
Japan	1	-2.437	-145.5	0.017	-5.600
Netherlands	1	-3.214	-1.723	0.655	-0.880
Norway	1	-5.517‡	-0.340	1.995	-2.140
Portugal	0	-4.683‡	-28.07	0.169	-5.380
Spain	1	-4.253‡	-1503	0.020	-5.100
Sweden	1	-5.435‡	-1094	0.046	-2.100
Switzerland	1	-2.934	-29.82	0.105	-0.820
UK	0	-3.857‡	-0.392	3.075	0.380

Notes: i) The number of lags (k) was chosen by the Partial Autocorrelation rule following Granger and Teräsvirta (1993). ii) $\exp\{\cdot\}$ is an exponential function. iii) † and ‡ refer to the cases when the unit root null is rejected at the 5% and 1% significance levels, respectively. iv) The asymptotic critical values were obtained from Park and Shintani (2005).

Table 4. Unit Root Test Results: Band Logistic Smooth Transition Autoregressive Model

$$\Delta q_t = \lambda \left[\frac{q_{t-1}-\tau_1}{1+\exp\{\kappa(q_{t-1}-\tau_1)\}} + \frac{q_{t-1}-\tau_2}{1+\exp\{-\kappa(q_{t-1}-\tau_2)\}} \right] + \sum_{i=1}^{k} \beta_i \Delta q_{t-i} + \varepsilon_t$$

Country	k	inf-t	λ	τ_1	τ_2	κ
Australia	1	-3.592†	-0.326	-0.370	-0.039	8.356
Belgium	1	-6.940‡	-0.645	-4.190	-3.559	4.527
Canada	0	-2.464	-0.088	-0.150	-0.089	0.618
Denmark	0	-3.167†	-0.248	-2.290	-1.939	24.70
Finland	1	-7.271‡	-0.720	-1.850	-1.499	4.608
France	1	-4.246‡	-0.734	-1.930	-1.529	6.107
Germany	1	-3.883‡	-0.287	-0.930	-0.539	73.32
Italy	1	-4.278‡	-0.338	-7.670	-7.359	5.215
Japan	1	-3.048	-0.258	-6.380	-4.909	9.020
Netherlands	1	-3.187†	-0.339	-1.130	-0.669	5.352
Norway	1	-5.884‡	-0.410	-2.360	-1.949	20.15
Portugal	0	-5.225‡	-0.839	-5.670	-5.059	12.72
Spain	1	-3.912‡	-0.419	-5.410	-4.879	4.371
Sweden	1	-4.999‡	-0.617	-2.290	-1.959	8.547
Switzerland	1	-3.315†	-0.401	-1.200	-0.449	23.54
UK	0	-3.944‡	-0.371	0.270	0.491	24.89

Notes: i) The number of lags (k) was chosen by the Partial Autocorrelation rule following Granger and Teräsvirta (1993). ii) exp$\{\cdot\}$ is an exponential function. iii) † and ‡ refer to the cases when the unit root null is rejected at the 5% and 1% significance levels, respectively. iv) The asymptotic critical values were obtained from Park and Shintani (2005).

Table 5. Unit Root Test Results: Band Threshold Autoregressive Model

$$\Delta q_t = \lambda\left[(q_{t-1}-\tau_1)\mathrm{I}\left\{q_{t-1}\leq\tau_1\right\}+(q_{t-1}-\tau_2)\mathrm{I}\left\{q_{t-1}\geq\tau_2\right\}\right]+\sum_{i=1}^{k}\beta_i\Delta q_{t-i}+\varepsilon_t$$

Country	k	inf-t	λ	τ_1	τ_2
Australia	1	-3.486†	-0.301	-0.380	-0.039
Belgium	1	-6.674‡	-0.600	-4.190	-3.559
Canada	0	-2.352	-0.120	-0.190	-0.089
Denmark	0	-3.134†	-0.250	-2.300	-1.939
Finland	1	-7.093‡	-0.694	-1.810	-1.499
France	1	-3.849‡	-0.657	-1.930	-1.529
Germany	1	-3.874‡	-0.276	-0.930	-0.549
Italy	1	-4.165‡	-0.316	-7.650	-7.359
Japan	1	-3.007	-0.233	-6.380	-4.909
Netherlands	1	-3.089†	-0.138	-0.980	-0.829
Norway	1	-5.834‡	-0.407	-2.370	-1.949
Portugal	0	-4.955‡	-0.816	-5.680	-5.039
Spain	1	-3.679‡	-0.361	-5.390	-4.879
Sweden	1	-4.905‡	-0.568	-2.310	-1.959
Switzerland	1	-3.268†	-0.386	-1.200	-0.449
UK	0	-3.910‡	-0.343	0.270	0.481

Notes: i) The number of lags (k) was chosen by the Partial Autocorrelation rule following Granger and Teräsvirta (1993). ii) $\mathrm{I}\{\cdot\}$ is an indicator function. iii) † and ‡ refer to the cases when the unit root null is rejected at the 5% and 1% significance levels, respectively. iv) The asymptotic critical values were obtained from Park and Shintani (2005).