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Abstract

We study students placement in Egyptian colleges under the current demand/supply placement mechanism implemented in Egypt (ϵ-mechanism). We show that the ϵ-mechanism is not Pareto efficient nor strategy proof and, moreover, it cannot be improved to accommodate Pareto efficiency nor strategy proofness. The final conclusion is that it is better, from an efficiency point of view, to adopt a matching algorithm, like the Gale-Shapley mechanism, in students placement.

JEL classification: C78, D71.

Keywords: Student placement, Gale-Shapley mechanism, ϵ-mechanism, Egypt.

1 Introduction

The problem of how to place high school students to colleges/universities has always taken the consideration of the public in Egypt. Despite the vitality of the subject there is little academic attention on the topic of how does student placement take place in Egypt[1]? The problem of matching students with schools/universities is a widely discussed topic in the education literature which started originally in Milton Friedman (1955, 1962). The question of how to design matching mechanisms satisfying certain criteria, like Pareto optimality and strategy proofness, forms the basis for a growing academic literature. Some prominent examples in the matching literature are the house allocation problem (Ergin 2000) and school choice model (Abdulkadiroglu and Sonmez 2003). We can have one-to-one matching like in the Marriage problem formulated by Gale and Shapley (1962) where each male is matched to one and only one female. The model was latter generalized to many-to-one matching like the college admission problem where a one i.e college is matched to many i.e students. There are three closing paradigms of studying matching students with schools/universities. The first literature is on the *College admission* litera-

1. To the extent of our limited knowledge there are no papers on student placement in Egypt directly related to the Gale-Shapley algorithm.

ture introduced by Gale and Shapley (1962) where schools are considered to have preferences over prospect students (see Roth and Sotomayor (1990) for an excellent survey). The second paradigm is *School choice* literature (Abdulkadiroglu and Sonmez 2003) which has a very similar mathematical structure to college admission, but differs in schools preferences where schools are assumed to have priorities and not preferences over students. These priorities are determined by local laws which differs from one country to another. The third literature is the *Student placement* model due to Balinski and Sonmez (1999) which deals with matching between students and universities, not schools. A key difference between student placement model and school choice or college admission models is that students are assumed to have score profiles which play a central role in the admission process, an aspect that was not important in the school choice or college admission problems. In this paper we are seeking how high school students are assigned their places in universities under the current matching system (based on demand and supply) implemented in Egypt, by adopting a simple version of the student placement model which includes *Pareto efficiency, strategy proofness and fairness*. The paper is organized as follows: Section 2 introduces Balinski and Sonmez (1999) model formally and shows its main results. Section 3 describes the mechanism implemented in Egypt and shows the drawbacks associated with it. Section 4 discusses the possibility of improving the current mechanism. Section 5 concludes the paper and summarizes its main findings.

2 Student Placement

2.1 The Model

In this section we present Balinski and Sonmez (1999) model in a formal way since it provides the cornerstone of our investigation of the process of matching high school students to colleges (universities). First we start with basic definitions and then move to the main concepts used. Students and Colleges are represented by two vectors $S = \{s_1, \ldots, s_n\}$ and $C = \{c_1, \ldots, c_m\}$ respectively. Capacity of college c_i is q_{c_i} where $q = \{q_{c_1}, \ldots, q_{c_m}\}$ is the capacity of all colleges. Student preferences is $P_S = \{P_{s_1}, \ldots, P_{s_n}\}$ of student s_i. Two new components, that are absent from school choice model, are *skill categories* and *student test scores*. The set of skill categories is $T = \{t_1, \ldots, t_k\}$ and the list of student test scores $f = (f^{s_1}, \ldots, f^{s_n})$, and the score of each student at each category is $f^{s_i} = (f^{s_i}_{t_1}, \ldots, f^{s_i}_{t_k})$. Finally, the function $t\colon C \to T$ represents what each college requires from the set of skill categories.

A weak preference relation of students over the set of colleges including the no college option (c_0) is denoted by R_S while strong preference relation is denoted by P_S[2]. A matching problem is defined by the vector (P_S, f, q). An allocation of college slots to students in a way that each student is allocated in only one college slot is a *matching*. Formally, a matching is a function $\mu: S \rightarrow C \cup \{c_0\}$ where the inverse function $\left|\mu^{-1}\right| \leqslant q_{c \in C}$ is used to overcome the problem of student surplus i.e. the number of students exceeds the number of college slots. Three more concepts are needed to be mentioned before stating the main results:

a) A matching μ *Pareto dominates* any other matching σ if there exists no student who prefers σ to μ and there exists at least one student who prefers μ to σ. Formally, $\sigma(s_i) R_{s_i} \mu(s_i)\ \forall\ s_i \in S$ and $\exists s_j \in S$ such that $\mu(s_i) P_{s_i} \sigma(s_i)$.

b) A matching μ is *fair* if it assigns students with higher scores their higher preferences or choices. Formally, if $\bar{c}\ P_s\, c$ implies that $f^{\bar{s}}_{t(\bar{c})} > f^{s}_{t(\bar{c})}$, where $\bar{c}$, $c \in C$ and $\bar{s}$, $s \in S$.

c) A matching μ is *strategy proof* if it provides students with incentives to reveal their *true* preferences ordering or to play an *incentive compatible* strategy.

2.2 Gale-Shapley mechanism

In order to evaluate the student placement mechanism followed in Egypt (ϵ-mechanism), we need a benchmark mechanism for economic efficiency. The benchmark in our paper will be the *Gale-Shapley mechanism*. The Gale-Shapley mechanism (or Gale-Shapley deferred acceptance algorithm) was primary introduced by Gale and Shapley (1962) in the context of two sided matching between males and females in a marriage problem. Then it became extensively used in college admission model where schools are matched to students, like in Roth (1985) and Roth and Sotomayor (1990). In Abdulkadiroglu and Sonmez (2003), the Gale-Shapley algorithm is used in school choice model which differs slightly from college admission model[3]. Below, we describe the Gale-Shapley mechanism given schools quotas and students preferences, next we give some important results that explain why the Gale-Shapley mechanism is so popular. The student assignment under Gale-Shapley student optimal stable mechanism works as follows:

2. The matching literature use these notations rather than the conventional $\succsim$ and $\succ$.

3. College admission model assumes that schools have *preferences* over students while school choice model assumes that schools have *priorities*.

Step 1: Each student proposes to his first choice. Then each school tentatively assigns its seats to its proposers one at time in their priority order until the quota is reached. Any remaining proposers are rejected.

Generalizing to step k,

Step k: Each student who was rejected in the previous step proposes to his next choice if one remains. Each school considers the set consisting of the students it has been holding and its new proposers, and tentatively assigns its seats to these students one at a time in priority order. Any students in the set remaining after all seats are filled are rejected.

The algorithm terminates when no student is rejected, and each student is assigned his final tentative assignment. The main results are summarized in the following propositions, based on (a), (b) and (c) in section 2.1.

Proposition 1. (Balinski and Sonmez 1999) *Gale-Shapley student optimal mechanism Pareto dominates any other fair mechanism.*

A placement mechanism (a rule by which a matching mechanism is selected) is said to be Pareto efficient if it selects a Pareto efficient matching. Thus, the Gale-Shapley placement mechanism is Pareto efficient. The next result is about strategy proofness.

Proposition 2. (Dubins and Freedman 1981, Roth 1982). *The Gale Shapley student optimal mechanism is strategy-proof.*

Analogously, A placement mechanism is said to be strategy proof if it selects a strategy proof matching. Thus, the Gale-Shapley placement mechanism is strategy proof. Therefore the Gale-Shapley mechanism satisfies the two desired properties, *inter alia,* that we require. The reason for choosing the Pareto optimality criterion is for welfare purpose i.e. to maximize students' welfare with no extra costs. The choice of strategy proofness criterion is to avoid matching students with colleges that are *less* preferred by them, as shown later, which is going to affect their job market performance later in a negative way.

3 The ϵ-Mechanism

This section aims to describe the student placement mechanism implemented by the ministry of higher education in Egypt, which we call the ϵ-mechanism[4]. Actually, the system implemented in Egypt is not a 'mechanism' with the full sense of word, rather it is a matching system based on demand of students and supply of quotas. There are fundamental differences between the Gale-Shapley and the ϵ-mechanism as shown in table 1.

Time line	Gale-Shapley	ϵ-mechanism
1	Exams	Exams
2	Student scores	Student scores
3	MAS announcement	Student preferences
4	Student preferences	MAS announcement
5	Tentative placement	Certain placement

Table 1.

The matching process of the ϵ-mechanism works as follows:

a) Students submit their preferences without knowing the minimum admission scores (MAS) for each college at each category.

b) Colleges announce their minimum admission scores based on demand of students and supply of quota.

c) Each student who's score is equal or above his preferred college minimum admission score guarantees a place there.

As noticed, the mechanism implemented in Egypt mainly depends on supply and demand analysis. Chade, Lewis and Smith (2009) shows that in a simple college admission model the classical result of demand and supply analysis is still applicable. Namely if minimum admission scores are treated as prices then *when a college rises its admission standard, then the enrollment level falls* (theorem 2 in Chade, Lewis and Smith (2009)). Note that in step (a) students do not have *priori* certain information about minimum admission scores. A direct result of that uncertainty is that students submit their preferences based on *rational expectations* about prospect MAS. These expectations are based on many variables like number of students and quotas, last year minimum admission scores, exams difficulty, and so on. Student's rational expectations plays a central role in shaping students' strategies when submitting their preferences and hence affects the final matching result. The next part gives a description of the ϵ-mechanism.

4. The letter ϵ is a shortcut for 'Egypt'.

3.1 How do Things Work

In Egypt there are only two skill categories: Arts and Science and there are two colleges categories c_A and c_S. Students enrolled in arts can *only* proceed to c_A while students enrolled in science can proceed to c_A and c_S i.e. $t(c_A) = (t_A, t_S)$ and $t(c_S) = t_S$. As shown in table 1, students submit their preferences before knowing the minimum admission scores, here rises an important question: On what bases do students submit their preferences? The answer is not as simple as it looks as a big proportion of students do not reveal their *true* preferences for the following two reasons. Firstly, the law prohibits students (after official college starting) from transferring from a college with a *lower* minimum admission score to any college with a *higher* minimum admission score even if their original scores were acceptable for the transferred college. Later on we will see the result of that law on students decisions and how it leads to a *non* strategy proof student placement. Secondly, students do not know colleges MAS which force them to submit their preferences based on rational expectations and not stating their preferences truthfully. Now we investigate students' strategies of preferences submission in more details. There are two variables that interfere in students decisions: First, the possibility of future transfer[5]; second, the choice of whether to reveal or not to reveal true preferences rankings. Here are the three possible cases:

a) *To reveal his true preferences with no future transfer possibility*: In this case, he will rank his preferences according to his free will without sophisticated planning.

b) *To reveal his true preferences with future transfer possibility*: The problem that arises here is that he does not know whether he will be accepted or not in his 1st, 2nd, ..., nth option since the minimum admission scores are not announced. So if his preferred colleges minimum admission score were *below* his score, he will be risking his transfer possibility in the future.

c) *Not to reveal his true preferences*: He will submit his preferences according to *rational expectations* about colleges ranking. Expectations about colleges ranking are almost common knowledge since it rarely differs substantially from one year to another from an ordinal perspective and not from a cardinal one.

5. The transfer possibility plays an important role in students' decisions as many of them prefer to keep a room for changing their current college in case they do not like their current choices.

It is now clear that the law mentioned above will provide an incentive for many students to choose option (c) as students may regard options (a) and (b) to be too risky relative to option (c).

3.2 Main Results

Our main results are related to Pareto efficiency and strategy proofness of the ϵ-mechanism. If it does not satisfy these criteria or at least one of them, we may regard as an *inferior* mechanism comparing to the Gale-Shapley mechanism. The following assumption is essential in proving our main results.

Assumption 1. *There exists a nonempty set of students $\sigma \subset S$ who are considering future transfer possibility such that $\sigma \geqslant \delta$.*

Assumption 1 is to ensure, without loss of generality, that category (c) students is not empty, moreover it is large enough relative to S. Hence we can deduce the following result.

Proposition 3. *The ϵ-mechanism is not strategy-proof.*

Proof. The proof is straightforward and follows from assumption 1 $\square$

The drawback of the ϵ-mechanism of being non strategy-proof leads to deeper effects than it looks. It actually leads to the following effect.

The Transfer effect. If the set σ is large enough i.e. up to a certain critical value $\sigma \geqslant \delta$, then there will be a large proportion of preferences submission biased towards colleges with *expected highest* minimum admission scores[6]. A direct result is the increase of demand for these colleges raising minimum admission scores without a true intention from students. Hence creating an externality effect on students of categories (a) and (b) above. Note that the transfer effect vanishes in case σ was *not* large enough i.e. $\sigma < \delta$; in that case the continuum assumption[7] will apply.

6. Like colleges of Medicine, Pharmacy, Engineering, and so on.

Now we move to another question, is the ϵ-mechanism Pareto efficient? The answer is No. The question is important from the social welfare point of view, especially when we have already a Pareto efficient mechanism (Gale-Shapley) in comparison. To show this simple fact, the following assumption is stated

Assumption 2. *The ϵ-mechanism is fair.*

Assumption 2 is justified by the structure of the ϵ-mechanism that allows students with higher scores to get their top choices *ex ante* MAS announcement.

Proposition 4. *The ϵ-mechanism is not Pareto efficient.*

Proof. We proceed by providing a counterexample to Pareto efficiency under the ϵ-mechanism. Consider a market consisting of only two students in the following example where student 1 is drawn from category (c) and student 2 is drawn from either (a) or (b). Let $S = \{s_1, s_2\}$, $C = \{c_1, c_2\}$, $q = (q_{c_1}, q_{c_2}) = (1, 1)$, $P_S = (P_{s_1}, P_{s_2})$, $f = (f^{s_1}, f^{s_2})$ be the (students, colleges, quotas, students preferences and student scores) profiles, respectively. Let the skill categories be limited only to one $T = \{t_1\}$ where c_1, c_2 require skill category t_1 i.e. $t(c_1, c_2) = t_1$. Student 1 and 2's preferences are as follows respectively:

s_1:	*Rational expectations-based preferences* (submitted)	$\rightarrow$	$c_1 \, P_{s_1} \, c_2 \, P_{s_1} \, c_0$
	Truth-based preferences (not submitted)	$\rightarrow$	$c_2 \, P_{s_1} \, c_1 \, P_{s_1} \, c_0$
s_2:	*Truth-based preferences* (submitted)	$\rightarrow$	$c_1 \, P_{s_2} \, c_2 \, P_{s_2} \, c_0$

Now consider three possible cases according to students' average score and based on assumption 2.

7. The continuum assumption consider the case when individual agents have a very tiny effect to affect the whole market (c.f. Aumann 1966)

Case 1: $f^{s_1} > f^{s_2}$ leads to the following placement $c_1 \longleftrightarrow s_1$ and $c_2 \longleftrightarrow s_2$

Case 2: $f^{s_1} = f^{s_2}$ leads to tie[8]

Case 3: $f^{s_1} < f^{s_2}$ leads to the following placement $c_2 \longleftrightarrow s_1$ and $c_1 \longleftrightarrow s_2$

It is clear that case 1 is not Pareto efficient since as each student is matched with his *second* best choice despite that each student's best choice is attainable according to the fairness criterion. □

Indeed the analysis can be further complicated by examining bigger number of students with different combinations from categories (a), (b) and (c) and more counterexamples will be found. Fortunately, the previous simple example was enough to prove the point.

4 Are Improvements possible ?

After reaching the basic results in the previous section, here are few more questions: Can we improve the ϵ-mechanism? If yes, how? And if not, can we replace it with Gale-Shapley mechanism? These questions are crucial in the context of improving the current educational system and we attempt to provide answers in this section. As a start our findings, until now, are summarized in the following table

	Strategy Proofness	Pareto Efficiency	Fairness
Gale-Shapley	Yes	Yes	Yes
ϵ-mechanism	No	No	Yes

Table 2.

8. In case of ties, which are rare, additional criteria are used for selection like age or geographical location. For example the Turkish Placement office uses student's age as an additional criteria (see Balinski and Sonmez (1999))

Our goal is to seek for means to improve the ϵ-mechanism strategy-proofness or Pareto efficiency or both. The Time line difference between Gale-Shapley mechanism and ϵ-mechanism, as shown in table 1, is the reversal order of steps (2) and (3). Note that in the ϵ-mechanism students submit their preferences before the MAS announcement which creates an environment of uncertainty while the opposite happens in the Gale-Shapley mechanism. Unfortunately, we cannot solve this problem by reversing steps (2) and (3) in the ϵ-mechanism since MAS are determined based on students' demand expressed by their preferences. Indeed, this is not the case in the Gale-Shapley mechanism as it is based on a matching algorithm and not on demand and supply (see section 2.2). A clear issue is that our solution have to concentrate on the *elimination* of rational expectations-based preferences and thus eliminating category (c) of students responsible for non strategy proofness and Pareto inefficiency. In other words, we need to make a modification that forces student to play an *incentive compatible* strategy when submitting their preferences. One solution is simply by canceling the transfer law mentioned above and hence permitting students who met MAS to transfer from *lower* colleges to *higher* ones. This will create both a positive and a negative effect. On one hand it will reduce both the transfer effect and category (c) students, but on the other hand it will raise a deeper problem that rises from the fact that we have different types of degrees qualifying for college admission like Thanweya amma, IGCSE and American Diploma, and others.

4.1 A problem raised by the transfer effect

Consider n pools of degrees qualifying for college admission: P_1, P_2,.., P_n where each pool has one type of degree students e.g. IGCSE. If the transfer law mentioned above is removed we will have the following problem. If student s_1 is from P_1 and transfers from a college with higher admission score ($\square_{C_iH}$) to a lower one ($\square_{C_jL}$), then there will be an empty place which will be filled by s_n from P_n which makes it impossible to transfer back to a higher college.

$$s_1 \colon \square_{C_iH}, \rightarrow \square_{C_jL}$$

$$s_1 \colon \square_{C_jL} \nrightarrow \square_{C_iH}$$

The problem will even exist if there is only one pool of students because student placement under the ϵ-mechanism is certain and not tentative like the Gale-Shapley mechanism. Therefore canceling the transfer law is not a feasible solution and we deduce the following corollary.

Corollary 5. *The ϵ-mechanism cannot be improved to compensate for Pareto inefficiency and non strategy proofness.*

The only solution is to replace the ϵ-mechanism with a Gale-Shapley mechanism which is already shown to be strategy-proof and Pareto efficient.

5 Conclusion

In this paper we have investigated the structure of the mechanism (ϵ-mechanism) used in Egypt to match high school students with colleges/universities. The noticed point is that the ϵ-mechanism is based on demand and supply and not on a matching algorithm like *serial dictatorship* mechanism or *Gale-Shapley* mechanism. The main finding is that the ϵ-mechanism is not *Pareto efficient* nor *strategy proof*. This is a direct result of requiring students to submit their preferences before knowing colleges' MAS which creates a general environment of uncertainty and forces students to make *un*informed decisions. Another problem is the transfer law that prohibits students (after official college starting) from transferring from a college with a *lower* minimum admission score to any college with a *higher* minimum admission score even if their original scores were acceptable for the transferred college which leads to the *transfer effect* mentioned above. The uncertainty in the ϵ-mechanism combined with the transfer effect leades to the following:

1. Rational expectations-based preferences dominates Truth-based preferences.
2. Students' preferences tends to be biased towards colleges with highest MAS.

Point 1 simply restates that the ϵ-mechainsm is not strategy proof and point 2 states that the continous rise in MAS of some colleges in Egypt like Medical and Engeineering colleges may not result from *real* demand presure, but rather because a large proportion of students write these colleges as their first preferences without truthfully prefering them. Hence it is better to adopt a matching algorithm, like the Gale-Shapley mechanism, is implemented widely in matching students with public schools in Boston area (Abdulkadiroglu *et* al 2005), and also in NewYork city (Abdulkadiroglu, Pathak and Roth 2008) and have shown to be quite successful there. However, implementing the Gale-Shapley mechanism will require a carefull study of its degree of applicability in Egypt.

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