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The Evolution of US City Size Distribution from a Long Term Perspective (1900-2000)*

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Abstract

This paper analyses the evolution of city size distribution in the United States throughout the twentieth century. In particular, it tests the validity of two empirical regularities studied in urban economics: Zipf’s law, which postulates that the product between rank and size of a population is constant, and Gibrat’s law or the law of proportionate growth, according to which the growth rate of a variable is independent of its initial size. To achieve this, we use parametric and nonparametric methods. The main contribution of this work is the use of a new database with information on all the cities (understood as incorporated places), thus covering the entire distribution (without size restrictions). Our results enable us to confirm, from a long term perspective, that Gibrat’s law holds (weakly) and that Zipf’s law holds only if the sample is sufficiently restricted at the top, not for a larger sample, because city size distribution follows a lognormal when we consider all cities.

Keywords: Zipf’s law, Gibrat’s law, city size distribution, urban growth.

JEL codes: R00, C14.

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1 Introduction

The United States (US) became an urban nation in the second half of the nineteenth century and the early twentieth century. During this period, the percentage of the population living in cities grew to over 50%, with population growth in the cities strongly concentrated in the emerging industrial belt. As various historians show, (Kim, 2000, 2006; Kim and Margo, 2004), in the second phase of industrialization, from 1850 to 1920, factory production rose in scale, became more mechanized, and spread to numerous industries and to the north-eastern region known as the manufacturing belt, where in turn, the growing urban population was concentrated.

Industrialization and urbanization were strongly correlated in the US, although the direction of causality is disputed (the literature suggests industrialization led urbanization, see Kim and Margo, 2004). In any case, urbanization and industrialization went hand-in-hand. However, in the second half of the twentieth century, this trend seems to be reversed. The largest cities experienced a falling population in relative terms (from 1960 to 1990 the proportion of urban population representing the largest cities with populations of more than 250,000 decreased; see Kim, 2000), there was a substantial transition of employment from the industrial sector to services, and the influence of industrial employment in this period is found to be negative. For example, Glaeser et al. (1995) find that in the period 1960 to 1990, population growth in cities was negatively related to the initial share of employment in manufacturing. Their results suggest that cities followed the fortunes of the industries to which they were exposed initially. This negative effect had been maintained during the 1990s, as Glaeser and Shapiro (2003) observe.

Some authors provide useful historical examples of how the rise or the decline of cities may be joined to their output. Jacobs (1970) provides anecdotal evidence about the role played by emerging industries in city growth. Between the late-nineteenth and mid-twentieth centuries, Rochester, New York, became the new capital of the US film industry and the duplication industry, in place of New York city, and these two industries came to represent an important part of Rochester’s employment. Other typical examples in the literature are the cases of Dalton, Georgia, which became America’s carpet industry capital (Krugman, 1991), and the jewellery industry of Providence, Rhode Island.

More recently, Eeckhout (2004) highlighted the contrast between cities like Detroit and Philadelphia, which have seen a significant drop in population, while at the same time, experiencing a serious decline in their manufacturing industries; and cities in Sil-
icon Valley that have seen higher-than-average population growth rates in the 1990s. He argues that in the last decades, Detroit experienced a decline in population as the manufacturing industry in the area suffered a severe downturn, while at the other extreme, when the high-technology industry was booming, villages, towns, and cities in the San Francisco Bay area experienced higher-than-average population growth. Another example is Glaeser (2005), who carries out an exhaustive survey of the evolution of Boston and finds that, although in 1980 Boston resembled many of the industrial hulks dotting the northeast and Midwest and its future outlook seemed similar to that of cities like Detroit, from 1980 to 2000, Boston was more like San Jose than Detroit. This is because it abandoned manufacturing and specialized in high technology, finance, and education-industries that required skilled workers and that did extremely well over the 1980-2000 period.

Throughout the twentieth century, there have been other economic and social events, which have had an undisputable influence on city size distribution. There have been waves of immigration (although more controlled than in earlier periods); periods of deep economic crisis, such as the Great Depression or the high-oil-prices era of the 1970s, and periods of prosperity, such as the post-war boom (the golden era of American capitalism). Also, the shift of employment from the manufacturing sector to services (by the end of the twentieth century, the percentage of employment in services reached almost three times that of manufacturing; see Kolko, 1999), and specific industry cycles, such as the decline of the Rust Belt and the rise of the Sun Belt, have impacted on city size distribution.

All these factors have affected city size distribution. As shown by Dobkins and Ioannides (2000), new regions and cities have been brought into the US urban system during the nineteenth and twentieth centuries, older regions have grown and declined, and the spatial distribution of economic activity has undergone some remarkable changes. Figure 1 displays two maps, corresponding to the beginning and end of the century, showing the distribution of cities (with populations of more than 10,000) and the changes that took place during the century. Two facts stand out at first glance: (i) there has been a substantial increase in the number of cities (there are more cities), and (ii) the population of the cities has increased (there are more large cities).

Against this background, the aim of this paper is to analyse the evolution of the city size distribution of the United States throughout the twentieth century. In particular, we are interested in testing the validity of two empirical regularities, well-known in urban economics: Zipf’s law, which postulates that the product between rank and size of a population is constant, and Gibrat’s law or the law of proportionate growth,
according to which the growth rate of a variable is independent of its initial size. To achieve this, we use parametric and nonparametric methods. The main contribution of this work is the use of a new database with information on all the cities (understood as incorporated places), thus covering the entire distribution (without size restrictions). To our knowledge, we are the first to document the evolution of the size distribution of all US cities for over a century.

These laws have already been studied for the American case with the most populous cities or with Metropolitan Statistical Areas (MSAs). The difference from earlier studies is that here the entire size distribution is studied, not just the upper tail.

In related literature, both Krugman (1996) and Gabaix (1999) use data from metropolitan areas from the Statistical Abstract of the United States and conclude that for 1991 Pareto’s exponent is exactly equal to 1.005. This implies that Zipf’s law holds for this specific year. For a dynamic analysis, Ioannides and Overman (2003) use data from metropolitan areas from 1900 to 1990 and arrive at the conclusion that Gibrat’s law holds in the urban growth processes and that Zipf’s law is also fulfilled approximately well for a wide range of city sizes. However, their results suggest that local values of Zipf’s exponent can vary considerably with the size of cities. Nevertheless, Black and Henderson (2003) arrive at different conclusions for the same period (because they use different metropolitan areas). Zipf’s law holds only for cities in the upper third of the distribution, while Gibrat’s law would be rejected for any sample size. These results highlight the extreme sensitivity of conclusions to the geographical unit chosen and to sample size.

To close the debate, Eeckhout (2004) demonstrates that the estimated parameter depends on the truncation point, so when he considers all the cities for the period 1990 to 2000, the city size distribution follows a lognormal rather than a Pareto distribution, and the value of Zipf’s parameter is not 1, as earlier works concluded, but is slightly above 1/2, and also, Gibrat’s law holds for the entire sample. The shortcoming of this work is that this is a short term analysis, as only two decades are considered. The aim of the present study is to generalize this analysis for all of the twentieth century and extract long term conclusions. Section 2 presents the database, and sections 3 and 4 concern Zipf’s and Gibrat’s laws respectively. In section 5 we discuss the results and section 6 concludes the paper.
2 The Database

Any study that deals with issues relating to city size distribution faces the problem of what is meant by the term “city”, as there are various ways of defining a city. In this study, we identify cities as what the US Census Bureau denominates as incorporated places. They include governmental units classified under state laws as cities, towns, boroughs, or villages.\(^1\) Alaska, Hawaii, and Puerto Rico have not been considered due to data limitations. Our base, created from the original documents of the annual census published by the US Census Bureau, consists of the available data of all incorporated places without any size restriction, for each decade of the twentieth century. Eeckhout (2004) demonstrates the importance of considering the whole sample. If the underlying distribution is the lognormal distribution, then the estimate of the parameter of the Pareto distribution is increasing in the truncation city size and decreasing in the truncated sample population.

We also use data from Metropolitan Statistical Areas (MSAs) in order to establish comparisons between both geographical units, and between our results and those of other studies.\(^2\) Both units of analysis have advantages. As Glaeser and Shapiro (2003) indicate, MSAs are multi-county units that are meant to capture labour markets. MSAs are attractive because they are more natural economic units. Incorporated places (true cities) are political units that lie within metropolitan areas. Moreover, some factors, such as human capital spillovers, are thought to operate at a very local level.

Two special advantages arise from using data for incorporated places instead of MSAs. First, the US metropolitan areas usually comprise a group of counties that contain a central city with a population of at least 50,000 inhabitants (although this criterion has changed over the course of the twentieth century),\(^3\) meaning that only the largest cities are considered. Figure 2 shows empirical density functions for three representative periods (estimated using adaptive kernels) of the MSAs and our sample of incorporated places without size restrictions. The population is shown in relative terms to the US urban population for the corresponding period.\(^4\) As Eeckhout (2004) shows, the comparison makes it obvious that (i) due to the minimum population threshold the MSAs represent only the largest cities, and (ii) that by considering only the largest cities, the upper tail distribution, most of the cities in the distribution are excluded

\(^{1}\) More details about data sources and definitions are discussed in Appendix A.

\(^{2}\) A third option, intermediate, involves taking the urbanized areas, defined by the US Census Bureau (Garmestani et al., 2008). An urbanized area comprises a central place and the urban fringe, which includes other places.

\(^{3}\) MSAs data sources and definitions are also included in Appendix A.

\(^{4}\) US urban population according to US Census Bureau urban definition.
Second, the sample of incorporated places provides more information about one of the basic characteristics of the distribution of American cities. As Dobkins and Ioannides (2001) point out, the US system is characterized by the entry of new cities. While other countries (such as European countries) have an already consolidated urban structure and new cities are rarely created (urban growth is produced by population increase in existing cities), in the US, urban growth has a double dimension: as well as increases in city size, the number of cities also increases, with potentially different effects on city size distribution. Figure 1, although showing only cities of more than 10,000 inhabitants, illustrates this fact clearly by showing a large increase in the number of cities in the twentieth century. In fact, the number of incorporated places in the sample increased from 10,596 in 1900 to 19,296 in 2000. Table 1 presents the number of cities for each decade, the percentage that the incorporated places in the database represent of the total population of the US, and the descriptive statistics. A glance at the minimum values of each decade enables us to state that absolutely all incorporated places, for which data exist, are included, without size restrictions; even the smallest units, with fewer than 200 inhabitants. Although their urban character is debatable, Eeckhout (2004) suggested considering the whole distribution. In contrast, other authors impose a minimum population threshold. In any case, incorporated places with a population under 2,500 represent only 17.62% of the population of our sample of incorporated places in 1900, and 5.61% in 2000 (8.27% and 3.45% in terms of the total US population in 1900 and 2000, respectively).

The sample reflects the urbanization process that took place throughout the twentieth century. Thus, the population of cities goes from less than half the total population of the US in 1900 (46.99%) to 61.49% in 2000. From the beginning of the century to 1930 there was a rapid increase both in the number of cities and in the percentage of the total population that they represent. This informs us of an urbanization process, which manifests in two ways: on the one hand, already existing cities that are capable of attracting new population (the mean value of inhabitants per city grows over time, as can be seen in Table 1) and on the other hand, growth in the number of cities. After this decade, growth slows and stabilizes at around 64% until the last decades (from 1970 to 2000) when it falls to 61.49%.

As Kim (2000) indicates, data for metropolitan areas provide a different picture of US urban development than that painted above, as the percentage of population in the MSAs grows constantly during the second half of the twentieth century, from 56.55% in 1950 to 82.64% in 2000.
The percentage of the total US population, which our sample of incorporated places represents, can appear low when compared to other studies using MSAs. However, it is similar to that of other works using cities. The population excluded from the sample is what the US Census Bureau calls population not in place. Incorporated places and census designated places (CDPs) do not exhaust the territory of the US. There is quite a bit of territory that is not included in any recognized place. For example, there were more than 74 million people living in a territory that was not in a place in 2000, 26.64% of the total US population in this year. In turn, most of this population not in a place is rural population (61.58% in 2000).

These people living outside incorporated places are excluded from our sample, but they are included in some MSAs, as the MSAs are multi-county units and this population is counted as inhabitants of the counties. MSAs cover huge geographic areas and include a large population living in rural areas, which are not counted as places. This explains why the percentage of total population represented by MSAs is higher than our sample of incorporated places. However, despite the sample of incorporated places covering a lower percentage of the total US population, the population of incorporated places is almost entirely urban, 94.18% in 2000, compared to 88.35% of urban population in the MSAs.

3 Zipf’s law

The aim of this work is to study the temporal evolution of American city size distribution during the twentieth century. For this we will use Pareto’s distribution (1896) as a statistical approximation, also known as power law, originally used to study income distribution. If we use $s$ to denote the relative size of the city and $R$ for its rank, a power law links the relative size of the city and rank as follows:

$$R(s) = As^{-a}, \quad (1)$$

For example, see Kim (2000) and Kim and Margo (2004), where city is defined as an area having a population of greater than 2,500.

Census 2000 data on the population in places and not in places can be found in Table 9 of PHC-3 (US Summary, part 1), available online at: http://www.census.gov/prod/cen2000/index.html

In a long term temporal perspective of stationary equilibrium, it is necessary to use a relative measure of size. The chosen measurement is the relative size, defined as: $s_{it} = \frac{S_{it}}{S_t} = \frac{S_{it}}{\sum_{i=1}^{N_t} S_{it} / N_t}$. The other option most used in the literature is to take the share which represents the size of the city over the total population, $S_{it} / \sum_{i=1}^{N_t} S_{it}$. The results of this section are robust for the three options, size, relative size, and share over the total, as the ratios involve only a change of scale.
where $A$ and $a$ are parameters. This expression is applied to the study of very varied phenomena, such as the distribution of the number of times different words appear in a book, the intensity of earthquakes or the flow of rivers. It has been used extensively in urban economics to study city size distribution (see, for example, Eeckhout (2004) and Ioannides and Overman (2003) for the US case). It has also been used recently to study country size distribution (Rose, 2006; González-Val and Sanso-Navarro, 2009).

Zipf’s law is an empirical regularity, which appears when Pareto’s exponent of the distribution is equal to the unit ($a = 1$). The term was coined after a work by Zipf (1949), which observed that the frequency of the words of any language is clearly defined in statistical terms by constant values. Or, applied to our variable, when ordered from largest to smallest, the relative size of the second city is half that of the first, the relative size of the third is a third of the first, and so on.

### 3.1 Parametric analysis

The expression (1) of Pareto’s distribution is usually estimated in its logarithmic version:

\[
\ln R = K - a \ln s, \tag{2}
\]

where $K$ is a constant.

It is useful to test whether Pareto’s parameter is more or less than 1 and what is the evolution of this coefficient in time. The greater the coefficient, the more homogeneous are the relative city sizes. Also, a growing evolution would mean a process of convergence in city sizes. And the opposite, the smaller the coefficient the less homogeneous are the relative city sizes, and a decreasing evolution would mean a process of divergence in city sizes.

Equation (2) can be represented as a graph. Figure 3 shows the Zipf plots for three periods: 1900, 1950, and 2000. The behaviour of other decades, which is not shown, is similar. Results are shown for incorporated places and for MSAs. Data are fitted by a power law and its exponent is estimated by using the OLS estimator. Moreover, the top 100 data from the incorporated places are also fitted by a power law and its exponent is estimated by using the Hill’s estimator (proposed by Gabaix and Ioannides, 2004). Also shown is the fit by lognormal distribution for the entire range based on the maximum likelihood estimation.

If Zipf’s law were fulfilled, the points would represent a decreasing straight line with a slope equal to minus one. This is the case for the MSAs, for which the power law
provides a very good fit to the real behaviour of the distribution with an estimated Pareto exponent always very close to the unit (the value one is always within the estimation by interval). However, a non-linear and clearly concave behaviour is observed for the incorporated places. In this case, the lognormal distribution provides the better fit for most of the distribution, and the fit improves over time. Although the largest cities’ behaviour is similar to that of the MSAs and the rank-size relationship remains almost linear, so that a power law is also a good description of the behaviour of the upper tail distribution.

Table 2 shows the results of the OLS estimation\(^8\) of Pareto’s exponent. The residues resulting from this regression usually present problems of heteroskedasticity. So, to analyse the significance of the parameters, the corrected standard error proposed by Gabaix and Ioannides (2004) is used: \(GI \text{ s.e.} = \hat{\alpha} \cdot (2/N)^{1/2}\), where \(N\) is the sample size.

The results indicate that when the entire sample is taken, Pareto’s exponent is always less than one. Also, the estimates decrease over time when we consider all incorporated places, which would indicate that for the entire sample (including all the cities for each year) a divergent behaviour was produced. However, if we consider different cross-sections of the sample we can observe different behaviours. Thus, for the 1,000 biggest cities, the exponent grows over time, so that we can state that for the biggest cities, the trend has been convergence: they have become closer in relative size. For the 5,000 biggest cities, the exponent remains stable, and from there the exponents decrease in time for different sample sizes.

We also need to point out that when we consider only the cities in the upper tail distribution the value 1 is always within the estimation by interval, finding evidence in favour of the fulfilment of Zipf’s law in the largest cities. Despite the lognormal distribution gives a better fit for the entire city size distribution (as we see in Figure 3), as noted by Eeckhout (2009), for the largest cities the lognormal tail and the Pareto tail are hard to distinguish.

There are two possible explanations for the decreasing evolution presented by the estimated coefficients when we consider all the incorporated places. First, part of the decrease is purely statistical. As Eeckhout (2004) showed in theory, if the underlying distribution is lognormal the estimated value of Pareto’s exponent depends negatively on the cut-off point, so that, as we increase the sample size and include ever smaller

\(^8\)Gabaix and Ioannides (2004) show that the Hill (Maximum Likelihood) Estimator is more efficient if the underlying stochastic process is really a Pareto distribution. This is not the distribution that the data follow, and so we use the OLS estimator. While the OLS estimate also presents some problems, see Goldstein et al. (2004) and Nishiyama et al. (2008).
cities, the estimated coefficient decreases (but not always; in principle, starting with a small sample and going on to a slightly larger one, as for example from 100 cities to 500, the coefficient can grow).

Secondly, part of this divergence would be explained by the appearance of new cities that enter with very small relative sizes. This second statement implies that (i) cities entering the sample present a relative size lower than the other cities in the sample (on average), and (ii) that greater inequality in the distribution is produced.

Figure 4 shows the empirical density functions of the new entrants (normalized by the average size of the cohort of the entire distribution) in 1910 (the first period of our sample in which new cities appear) and 2000. New entrant cities are those incorporated places that appear in the sample after the first period, 1900. We observe that the estimated density function for the new cities appears to the left of the function for the whole sample, indicating that the new cities enter with smaller relative sizes. However, this difference is greater in the first period, 1910, than in 2000. This is because most cities entering the sample do so in the first decades of the century, in the period 1900 to 1930 (Table 1), so that for 2000, after several decades, their size has become closer to that of all cities in the sample.

We have also run the two-sample Wilcoxon rank-sum test, rejecting, in both periods, the null hypothesis that both samples (new entrants and all cities) are the same. The test also enables us to accept the null hypothesis that the relative size of incorporated places of the whole sample is greater than that of the new cities in both periods. The fact that the sample corresponding to the whole distribution is located to the right and that the difference between both density functions is significant, indicates that, on average, cities enter the sample with a lower relative size.

Regarding the degree of evenness or unevenness of the distribution, Table 3 presents the Gini coefficients for different sample sizes. The Gini coefficients have the advantage of not imposing a specific size distribution (Pareto for rank-size coefficients). It is interesting to note that the coefficients group for the largest cities decreases over time, indicating a convergent behaviour in these subgroups of the upper tail distribution; yet

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9The two-sample Wilcoxon rank-sum test is a nonparametric test for assessing whether two samples of observations come from the same distribution. The null hypothesis is that the two samples are drawn from a single population, and therefore that their probability distributions are equal. Wilcoxon’s test has the advantage of being appropriate for any sample size.

10Wilcoxon rank-sum test results:
Prob{All cities empirical density function in 1910 > New entrants empirical density function in 1910} = 0.654
Prob{All cities empirical density function in 2000 > New entrants empirical density function in 2000} = 0.559.
for the whole sample (including the cities entering the sample in each decade) the coefficient goes from 0.822 in 1900 to 0.851 in 2000. This indicates that the evolution of the whole distribution is divergent: inequality among the relative sizes of the incorporated places has increased.

However, the evolution of the Gini coefficient is not monotonous. There are periods in which the distance between the relative sizes of incorporated places increases (1900-1930, 1940-1950, 1980-2000), and other periods in which the unevenness of distribution is reduced (1930-1940, 1960-1980). While it is beyond the scope of this paper to estimate the specific determinants of these changes in size distribution (unfortunately we only have data on the population of the cities), we can note some possible causes, taking into account the historical context.

Table 1 shows how the beginning of the century (1900-1930) is characterized by a marked increase in the number of cities (which enter the size distribution with lower average relative sizes than the rest of the sample) and in the percentage of the urban population which they represent; to this can now be added a rapid increase in unevenness in city distribution. This is the period that Kim (2000) calls “the era of industrial cities”, in which urban growth went hand-in-hand with industrialization (particularly in the manufacturing belt), ending in 1920, a few years before the Great Depression (1929-1941). During this decade of economic crisis (1930-1940) both the number of cities and the percentage of population they represent remain stable (Table 1). Additionally, the Gini coefficient indicates that unevenness in the distribution is reduced, suggesting that there is some redistribution of the population among the cities. Unevenness also decreased from 1960 to 1980, a period in which the percentage of population in cities peaked at almost 65%, coinciding with the end of the post-war boom and the oil supply shocks of 1973 and 1979. From then, until the last decades of the century (1980-2000), unevenness within the size distribution again increased, although only slightly.

It is notable that if we consider the group of largest cities (the upper tail) the behaviour is the opposite, as the trend, especially during the second half of the century, is clearly convergent; it brought the relative sizes of the largest cities closer together. This convergence coincides in time with a loss of importance for the largest cities. As Kim (2000) points out, in the second half of the twentieth century, urban development in the US was characterized by a decrease in the percentage of the urban population represented by the largest cities, as from 1960 to 1990 the percentage of population in the cities of 250,000 inhabitants or more decreased from 22% to 17.8%.
3.2 Nonparametric analysis

As it has been proven that a power law does not give a good fit for the entire sample of incorporated places, the question is, what distribution best fits the data? For this, we estimate the empirical distribution of the data using an adaptive kernel.

Figure 5 shows the results for three representative decades (the difference from Figure 2 is that now, the population of incorporated places is represented in relative size). It is observed that, starting in 1900, from a very leptokurtic distribution with much density concentrated in the mean value of the distribution, the distribution loses kurtosis and concentration decreases until it reaches a distribution very similar to lognormal, which it maintains until 2000. This evolution can also be seen in the graph on the right of Figure 5, which shows the empirical cumulative density functions estimated for 1900 and for 2000. It can be observed that in the year 2000, probability accumulates much more slowly than in 1900, which indicates a change to a less concentrated distribution. Additionally, as the concentration of the distribution decreases, unevenness increases (the same result obtained in the parametric analysis of the section above); the loss of kurtosis of the centre of the distribution means that the tails gain weight.

4 Gibrat’s law

The above section shows what we consider to be a snapshot of the distribution of American cities during the twentieth century. For different decades we obtained the graphic representation of the distribution and the estimated coefficients of Pareto’s exponent for different sample sizes, which enabled us to conclude if there had been important variations in the distribution, or if concentration had increased or decreased. However, a more rigorous dynamic analysis demands that we work with growth rates. We are particularly interested in seeing if there is fulfilment of Gibrat’s law or the law of proportionate growth, which postulates that the growth of a variable is independent of its initial size; Gibrat (1931) observed that the size distribution (measured by sales or the number of employees) of firms tends to be lognormal, and his explanation was that the growth process of firms could be multiplicative and independent of the size of the firm.

In the field of urban economics, Gibrat’s Law, especially since the 1990s, has given rise to numerous empirical studies contrasting its validity for city size distributions, 11

\[ \text{The results (not shown) of the Wilcoxon rank-sum test indicate that for a 1\% confidence level, the null hypothesis of lognormality would only be rejected in 1920 and 1930.} \]
arriving at a majority consensus, though not absolute, that it holds in the long term. It is useful to test this over the entire twentieth century, from a long term perspective with our sample of all incorporated places.

4.1 Parametric analysis

The parametric approach consists of estimating growth regressions, which relate the growth rate with initial size (the ever popular $\beta$-convergence in economic growth literature). We take two specifications; in one, growth depends on the initial relative size, while in the other the exogenous variable is a mean of the relative size of the two periods:

Specification I:
\[ \frac{s_{t+1}}{s_t} = C + b \cdot \frac{s_t + s_{t+1}}{2}, \]  
(3)

Specification II:
\[ \frac{s_{t+1}}{s_t} = C + b \cdot s_t, \]  
(4)

where $C$ is a constant. Note that the variable is the relative size, so we are checking relative or effective growth, not gross growth. This means that the population of a city may have grown, but if others cities have grown more, the average rises and thus, it has shrunk, in terms of relative size. This can be seen from the decomposition of the ratio $s_{t+1}/s_t$:

\[ \frac{s_{it+1}}{s_{it}} = \frac{S_{it+1}}{S_{it}} = \frac{N_{t+1}}{N_t} \cdot \frac{N_{t+1}}{N_t} \cdot \frac{\sum_{i=1}^{N_{t+1}} S_{it+1/N_{t+1}}}{\sum_{i=1}^{N_t} S_{it/N_t}} \cdot \frac{\sum_{i=1}^{N_{t+1}} S_{it+1}}{\sum_{i=1}^{N_t} S_{it+1}} \cdot \frac{N_{t+1}}{N_t}. \]

This means that relative growth can be produced not only by the increase in population of the city; it also happens if the number of cities rises or the total population of all the cities decreases.

Table 4 shows the results of the OLS estimates, decade by decade, and for a pool of the observations of the whole century. The conclusion is that the parameter $\hat{b}$ is not significant for any period with either of the specifications, which adds evidence in favour of Gibrat’s law and the independence of growth in relationship to relative size. The only exception is the period 1980 to 1990, where the estimated coefficients are significant and positive (although very close to zero), which would indicate that a positive relationship existed between growth and size, with the largest cities gaining the most population. This is the period of least growth in urban population of the entire
twentieth century, about 2.06% (Table 5), and the second lowest decade of growth of the total population in the history of the United States, at 9.8%.  

4.2 Nonparametric analysis

The earlier results confirm that Gibrat’s law holds. However, Quah (1993) points out the problems of regressions towards the mean, which are so usual in studies of economic growth, and proposes using nonparametric methods, specifically, transition matrices. We will use the methodology followed by Ioannides and Overman (2003) and Eeckhout (2004). It consists of taking the following specification:

\[ g_i = m(s_i) + \epsilon_i, \]  

where \( g_i \) is the normalized growth rate\(^\text{13}\) (subtracting the mean and dividing by the standard deviation) and \( s_i \) is the logarithm of relative size, and instead of making suppositions about the functional relationship of \( m \) and supposing a linear relationship, as in equations (3) and (4), \( \hat{m}(s) \) is estimated as a local average around point \( s \) and is smoothed using a kernel, which is a symmetrical, weighted, and continuous function around \( s \).

In order to analyse the entire period 1890 to 2000, all the growth rates are taken between consecutive periods. And the Nadaraya-Watson method is used, exactly as it appears in Härdle (1990), based on the following expression\(^\text{14}\):

\[ \hat{m}(s) = \frac{n^{-1} \sum_{i=1}^{n} K_h(s - s_i) g_i}{n^{-1} \sum_{i=1}^{n} K_h(s - s_i)}, \]  

where \( K_h \) denotes the dependence of the kernel \( K \) (in this case an Epanechnikov) on the bandwidth \( h \) (0.5). Starting from this calculated mean \( \hat{m}(s) \), the variance of the growth}

\(^{12}\)Source: http://www.census.gov/population/censusdata/table-4.pdf

\(^{13}\)Taking normalized growth rates will mean that the choice of the unit of measurement, size, size relative to the average, or share of the total, is indifferent, as it means only a change of scale; the results regarding growth are robust; see Appendix B.

\(^{14}\)The calculation was done with the KERNREG2 Stata module, developed by Nicholas J. Cox, Isaias H. Salgado-Ugarte, Makoto Shimizu and Toru Taniuchi, and available online at: http://ideas.repec.org/c/boc/bocode/s372601.html

This programme is based on the algorithm described by Härdle (1990) in Chapter 5.
rate $g_i$ is also estimated, again applying the Nadaraya-Watson estimator starting from:

$$
\hat{\sigma}^2(s) = \frac{n^{-1} \sum_{i=1}^{n} K_h(s-s_i)(g_i - \hat{m}(s))^2}{n^{-1} \sum_{i=1}^{n} K_h(s-s_i)}.
$$

(7)

The estimator is very sensitive, both in mean and in variance, to atypical values. Thus, the growth rate, both in mean and in variance, of the smallest cities usually has much higher values than for the rest. If we examine the smallest 5% of cities, the differences are even greater.\textsuperscript{15} This is logical; we are considering cities of under 200 inhabitants, where the smallest increase in population is very large in percentage terms. For example, the value which distorts the mean and the variance in 1940 to 1950 is Pine Lake (De Kalb, Georgia), which goes from 2 inhabitants in 1940 to 566 in 1950. However, we need not consider such extreme changes; any city with fewer than 50 inhabitants that sees some population growth, increases a great deal in percentage terms. Thus, we decided to eliminate this 5% of the smallest distribution observations, as they are characterized by very high dispersion in mean and in variance, and they distort the results. This is not a great loss in terms of representativeness of the sample, as the size of the last city excluded is under 180 inhabitants.

Gibrat’s law implies that growth is independent of size in mean and in variance. As growth rates are normalized, if Gibrat’s law were strictly fulfilled and growth were independent of size, the estimated kernel would be a straight line on the zero value. Values different to zero involve deviations from the mean. And variance would also be a straight line, supposing that variance does not depend on the size of the variable analysed.

Figure 6, shows the estimated kernels of growth and the variance of growth for all the twentieth century (a pool of 162,403 observations). Bootstrapped 95% confidence bands, calculated using 500 random samples with replacement, are also displayed. It is noticeable that the estimation of growth is nearly a straight line around zero, meaning that on average, we can accept that during the whole period, growth was independent of size and Gibrat’s law holds. Regarding variance, even if the smallest 5% of observations are eliminated, the smallest cities present greater variance than the rest of the sample. However, it should be noted that starting from the zero value (in a logarithmic scale, this corresponds to a city relative size equal to 1, i.e., cities of a size that is equal to the contemporaneous mean), variance stabilizes, becoming much more homogenous.

\textsuperscript{15}The specific values are available from the author on request.
indicating that the variance of growth is independent of size for cities with a population equal to or greater than the mean (a little over 3,000 inhabitants at the beginning of the century and almost 9,000 at the end).

Our results, obtained with our sample of all incorporated places without any size restriction, are similar to those obtained by Ioannides and Overman (2003), with their database of MSAs.\textsuperscript{16} The main difference stems from the estimation of variance, much higher in our sample of incorporated places for the smallest cities. The estimated kernels show that while average growth appears to be independent of size, variance in growth seems to depend negatively on size: the smallest cities present a variance that is clearly higher than the rest. This points to Gibrat’s Law holding weakly (growth is proportionate on average, but not in variance). This possibility has already been considered theoretically, both by Gabaix (1999), who examines the case in which cities grow randomly with expected growth rates and standard deviations that depend on their sizes, and by Córdoba (2008), who introduces a parsimonious generalization of Gibrat’s law that allows size to affect the variance of the growth process but not its mean.

5 Discussion

There is debate concerning the laws governing the size distribution. The debate has converged recently (explaining for example that a Pareto upper tail of the distribution can be reconciled with a lognormal distribution for the entire sample; see Eeckhout, 2009), and our results clearly contribute to making that point. In addition, given the novelty of the early data, our work also establishes the robustness of the findings.

Specifically, our database of all incorporated places enables us to confirm, from a long term perspective that:

(1) Gibrat’s law holds (weakly). Growth is proportionate on average but not in variance. Although the variance of growth is independent of size for cities with a population equal to or greater than the contemporary mean, the smallest cities present a variance clearly higher than the rest.

(2) Proportionate growth implies a lognormal distribution (Gibrat, 1931; Kalecki, 1945; Eeckhout, 2004). City size distribution follows a lognormal when we consider all cities without any size restriction.

(3) Zipf’s law holds only if the sample is sufficiently restricted at the top, not for a

\textsuperscript{16}See Ioannides and Overman (2003), Figure 2.
larger sample. For the largest cities the lognormal tail and the Pareto tail are hard to
distinguish.

(4) The incorporation of new cities to the sample, together with other social and
economic factors, throughout the twentieth century, leads to rising unevenness in city
size distribution. These new cities appear with a smaller relative size (on average) than
the rest of the cities in the sample.

Underlying these empirical regularities are the changes that city size distribution
has undergone during the twentieth century. For example, in terms of actual cities
(incorporated places), what does it mean that the rank-size rule does not hold when
we consider the whole size distribution, but Gibrat’s law does hold? The size distribu-
tion being lognormal is a statistical consequence of the proportionate growth process.
In turn, growth being proportionate imposes a high degree of persistence in city size
distribution, although this does not mean that distribution remains static. In section 3
we have obtained the conclusion that, in the twentieth century, the level of unevenness
in the distribution increased, something that we associated with the appearance of new
cities with smaller relative sizes (on average) than the rest of the sample, but which
undoubtedly relates to city growth rates. How does this result relate to proportionate
growth?

Proportionate growth does not mean that all cities grow the same way. The evolution
over time of city growth rates (and of the total population) depends on the historical
and social context. Table 5, shows both the mean growth rates for the whole period ($g_p$),
calculated from gross growth rates, defined as $g_{it} = \frac{S_{it} - S_{i(t-1)}}{S_{i(t-1)}}$, where $S_{it}$ is the population
of the city $i$ in the year $t$, and the annual mean growth rates ($g_a$), which are calculated
from the mean growth rates for the whole period applying that $(1 + g_a)^{10} = (1 + g_p)$.
It can be observed that indeed, the first decades of the century saw strong growth rates
for city sizes, as well as a marked increase, both in the number of cities (which entered
the size distribution with average relative sizes below the rest of the sample) and in
unevenness within the distribution. However, this period of growth came to an end in
1920-1930. Between 1940 and 1980, the high growth rates seem to recover, and then fall
in the last two decades. The two periods of lowest growth, 1930-1940 and 1980-1990,
coincide with the two periods of lowest growth of the total population in US history,
7.3% and 9.8%, respectively, and are very close to two profound economic crises (the
Great Depression and the second oil supply shock in 1979).

If we disaggregate these growth rates we find interesting differences according to
period. As pointed out by Gabaix and Ioannides (2004), the casual impression of the
authors is that in some decades, large cities grow faster than small cities, but in other
decades, small cities grow faster. This would suggest that Gibrat’s law for means holds only as a long-run average (with periods in which urban growth may be convergent or divergent). Figure 7, shows growth rates by distribution quartiles, which corroborate this assertion. Despite Gibrat’s law holding over the long term when considering all the twentieth century, we can find differentiated behaviour in each decade. When distinguishing the growth rates of groups, we see how periods in which the cities with most growth are the largest incorporated places (1910-1930, 1940-1970, 1980-2000) are interspersed with others in which the very small communities of the distribution take the lead (1900-1910, 1930-1940, 1970-1980). It is notable that in periods of high economic growth, the largest cities are the ones that gain most in population, while in periods of crisis the smallest cities are the ones that grow most.\textsuperscript{17} In contrast, the medium-sized incorporated places, the cities in the two middle quartiles of the distribution (Q2 and Q3), present a much more stable evolution, with growth rates very close to each other and to the total average for the period.

6 Conclusion

In this paper we have analysed the evolution of US city size distribution using data for all US cities (understood as incorporated places) for over a century. Our results enable us to confirm, from a long term perspective, that Gibrat’s law holds (weakly; growth is proportionate on average but not in variance, as the smallest cities present a clearly higher variance). Additionally, Zipf’s law holds only if the sample is sufficiently restricted at the top, not for a larger sample, because city size distribution follows a lognormal when we consider all cities with no size restriction.

Underlying these empirical regularities are the changes which city size distribution has undergone during the twentieth century. Behind the long term trend represented by Gibrat’s law, we find that periods in which the cities with most growth are the largest incorporated places alternate with others in which the very small communities of the distribution take the lead. In addition, the unevenness of the distribution has increased over the century, especially the first decades in which a large number of new cities appear with a smaller relative size (on average) than the rest. In contrast, the last decades are characterized by stability in the number of cities and the percentage of the US total population they represent, indicating a shift to a stable city size distribution and a more consolidated urban landscape.

\textsuperscript{17}The role of small cities has received little attention in the literature. One exception is Partridge et al. (2008).
Appendix A: Data definitions and sources

Cities, 1900-2000

In the same way as Eeckhout (2004), we identify cities as what the US Census Bureau denominates as places. This generic denomination, since the 2000 census, includes all incorporated and unincorporated places.

The US Census Bureau uses the generic term incorporated place to refer to a type of governmental unit incorporated under state law as a city, a town (except the New England states, New York, and Wisconsin), a borough (except in Alaska and New York city), or a village and having legally prescribed limits, powers, and functions. On the other hand, there are unincorporated places (which were renamed Census Designated Places, CDPs, in 1980), which designate a statistical entity, defined for each decennial census, according to Census Bureau guidelines, comprising a densely settled concentration of population that is not within an incorporated place, but is locally identified by a name. Evidently, the geographical boundaries of unincorporated places may change if settlements move, so that the same unincorporated place may have different boundaries in a different census. They are the statistical counterpart of the incorporated places. The difference between them, in most cases, is merely political and/or administrative. Thus, for example, due to a state law of Hawaii, there are no incorporated places; they are all unincorporated.

The unincorporated places began to be accounted for from 1950. The US Census Bureau established size restrictions for their inclusion (except in 2000, when they were all counted). Although the overall criterion is usually that they have over a thousand inhabitants, there are differences in each decade. However, these settlements did exist earlier, so their inclusion presents a problem of inconsistency in the sample. As a result, we decided to exclude unincorporated places from the sample, in order to carry out a long term analysis of the twentieth century with a homogenous sample. Also, their elimination is not quantitatively important; in fact there were 1,430 unincorporated places in 1950, representing 2.36% of the total population of the US, which, by 2000, would increase to 5,366 places and 11.27%.

Our base, created from the original documents of the annual census published by the US Census Bureau, consists of the available data of all incorporated places without any size restriction, for each decade of the twentieth century. While the data of only the last two decades are computerized (US Bureau of the Census, County and City Data Book, Washington DC), the data corresponding to other decades is available in
the original documents (US Bureau of the Census, Census of Population, Washington DC). We have created our database from these.

Source: http://www.census.gov/prod/www/abs/decennial/

**MSAs, 1900, 1950 and 2000**

The definition of a metropolitan area is from the Office of Management and Budget (OMB), based on data provided by the US Census Bureau. Standard definitions of metropolitan areas were first issued in 1949 by the then Bureau of the Budget (predecessor of OMB), under the designation “standard metropolitan area” (SMA). The term was changed to “standard metropolitan statistical area” (SMSA) in 1959, and to “metropolitan statistical area” (MSA) in 1983. The term “metropolitan area” (MA) was adopted in 1990 and referred collectively to metropolitan statistical areas (MSAs), consolidated metropolitan statistical areas (CMSAs), and primary metropolitan statistical areas (PMSAs). Finally, the term “core based statistical area” (CBSA) became effective in 2000 and refers collectively to metropolitan and micropolitan statistical areas.

Without entering into each definition (these can be consulted at http://factfinder.census.gov - American FactFinder Help), what interests us is the basic criterion used to define a MSA, as CMSAs and PMSAs are still MSAs, which fulfil certain conditions. Thus, according to the OMB definition, qualification as an MSA requires the presence of a city with 50,000 or more inhabitants, or the presence of an urbanized area and a total population of at least 100,000 (75,000 in New England) – an urbanized area, according to the Census Bureau, consists of a central place(s) and adjacent territory with a general population density of at least 1,000 people per square mile of land area that together have a minimum residential population of at least 50,000 people. However, this criterion has changed over the course of the twentieth century. Thus, the original criterion of 1950 only required a city of 50,000 inhabitants.

For the years 1900 and 1950 we use the data of Bogue’s Standard Metropolitan Areas (1953). He took the definitions of SMAs for 1950 and reconstructed the population of these areas for the period 1900 to 1940. The problem is that applying the 1950 definitions to 1900 means that some of these SMAs are much smaller than the minimum threshold of 50,000 inhabitants. For this reason, for 1900, we exclude all SMAs that do not reach this minimum threshold, reducing the sample size of 162 SMAs in 1950 to 112 in 1900.

For the year 2000, we take the data of the Metropolitan Statistical Areas corresponding to the 2000 census of the US Census Bureau, available at:
Appendix B: Normalized growth rates and the different measurements of city size

When growth rates are normalized, subtracting the mean and dividing by the standard deviation, the choice of measurement of size (size, relative size, or share of the total) makes no difference, as it means only a change of scale.

If we take size \( S_{it} \):

\[
\frac{g_i - \bar{X}_g}{\sigma_g} = \frac{\left( \frac{S_{it+1} - S_{it}}{S_{it}} \right) - \frac{1}{N} \sum_i \left( \frac{S_{it+1} - S_{it}}{S_{it}} \right)}{\left[ \frac{1}{N} \sum_i \left( \frac{S_{it+1} - S_{it}}{S_{it}} \right) \right]^2} = \frac{\left( \frac{S_{it+1} - S_{it}}{S_{it}} \right) - \frac{1}{N} \sum_i \left( \frac{S_{it+1} - S_{it}}{S_{it}} \right)}{\left[ \frac{1}{N} \sum_i \left( \frac{S_{it+1} - S_{it}}{S_{it}} \right) \right]^2}^{1/2}.
\]

We arrive at the same expression taking the relative size, \( s_{it} = \frac{S_{it}}{N_t} \):

\[
\frac{g_{relative\_size} - \bar{X}_{g_{relative\_size}}}{\sigma_{g_{relative\_size}}} = \frac{\left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right) - \frac{1}{N} \sum_i \left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right)}{\left[ \frac{1}{N} \sum_i \left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right) \right]^2} = \frac{\left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right) - \frac{1}{N} \sum_i \left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right)}{\left[ \frac{1}{N} \sum_i \left( \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it+1}}{\frac{1}{N_t} \sum_{i=1}^{N_t} s_{it}} - 1 \right) \right]^2}^{1/2}.
\]
And also taking the share of the total, $S_{it}/\sum_{i=1}^{N_t} S_{it}$:

$$\frac{g_{Share_t} - \bar{X}_{g_{Share}}}{\sigma_{g_{Share}}} = \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{S_{i(t+1)}}{\sum_{i}^{N_t} S_{it}} - \frac{1}{N} \sum_{i}^{N} \left( \frac{S_{i(t+1)}}{\sum_{i}^{N_t} S_{it}} - 1 \right) \right) \right]^{1/2}$$

$$= \left[ \frac{1}{N_t} \sum_{i=1}^{N_t} \left( \frac{S_{i(t+1)}}{\sum_{i}^{N_t} S_{it}} - \frac{1}{N} \sum_{i}^{N} \left( \frac{S_{i(t+1)}}{\sum_{i}^{N_t} S_{it}} \right) \right)^2 \right]^{1/2}$$

**References**


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Note: (GI s.e.) Gabaix-Ioannides (2004) corrected standard error. All coefficients are significantly different from zero at the 0.05 level.
Table 3: Incorporated places (relative size) Gini’s coefficients

<table>
<thead>
<tr>
<th>Year</th>
<th>Top 100</th>
<th>Top 500</th>
<th>Top 5000</th>
<th>All</th>
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</thead>
<tbody>
<tr>
<td>1900</td>
<td>0.598</td>
<td>0.669</td>
<td>0.768</td>
<td>0.822</td>
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<tr>
<td>1910</td>
<td>0.583</td>
<td>0.668</td>
<td>0.767</td>
<td>0.833</td>
</tr>
<tr>
<td>1920</td>
<td>0.575</td>
<td>0.663</td>
<td>0.771</td>
<td>0.844</td>
</tr>
<tr>
<td>1930</td>
<td>0.576</td>
<td>0.656</td>
<td>0.776</td>
<td>0.859</td>
</tr>
<tr>
<td>1940</td>
<td>0.578</td>
<td>0.648</td>
<td>0.760</td>
<td>0.855</td>
</tr>
<tr>
<td>1950</td>
<td>0.567</td>
<td>0.637</td>
<td>0.754</td>
<td>0.858</td>
</tr>
<tr>
<td>1960</td>
<td>0.527</td>
<td>0.589</td>
<td>0.717</td>
<td>0.855</td>
</tr>
<tr>
<td>1970</td>
<td>0.509</td>
<td>0.568</td>
<td>0.708</td>
<td>0.854</td>
</tr>
<tr>
<td>1980</td>
<td>0.488</td>
<td>0.544</td>
<td>0.685</td>
<td>0.844</td>
</tr>
<tr>
<td>1990</td>
<td>0.474</td>
<td>0.527</td>
<td>0.683</td>
<td>0.850</td>
</tr>
<tr>
<td>2000</td>
<td>0.473</td>
<td>0.516</td>
<td>0.674</td>
<td>0.851</td>
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</table>
Table 4: Estimated coefficients of parametric growth regressions

<table>
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<tr>
<th>Initial year</th>
<th>Final year</th>
<th>N</th>
<th>( b ) (s.e.)</th>
<th>( b ) (s.e.)</th>
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</thead>
<tbody>
<tr>
<td>1890</td>
<td>1900</td>
<td>7,531</td>
<td>8.28E-04</td>
<td>7.17E-04</td>
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<tr>
<td>1900</td>
<td>1910</td>
<td>10,502</td>
<td>4.43E-04</td>
<td>4.51E-04</td>
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<tr>
<td>1910</td>
<td>1920</td>
<td>13,578</td>
<td>4.83E-04</td>
<td>3.50E-04</td>
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<tr>
<td>1920</td>
<td>1930</td>
<td>15,310</td>
<td>3.14E-04</td>
<td>3.75E-04</td>
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<tr>
<td>1930</td>
<td>1940</td>
<td>16,211</td>
<td>-1.03E-04</td>
<td>2.42E-04</td>
</tr>
<tr>
<td>1940</td>
<td>1950</td>
<td>16,420</td>
<td>1.73E-04</td>
<td>1.13E-03</td>
</tr>
<tr>
<td>1950</td>
<td>1960</td>
<td>17,075</td>
<td>6.26E-04</td>
<td>6.66E-04</td>
</tr>
<tr>
<td>1960</td>
<td>1970</td>
<td>17,832</td>
<td>2.17E-04</td>
<td>7.29E-04</td>
</tr>
<tr>
<td>1980</td>
<td>1990</td>
<td>18,991</td>
<td>1.07E-03*</td>
<td>3.38E-04</td>
</tr>
<tr>
<td>Pool</td>
<td>Pool</td>
<td>170,950</td>
<td>3.43E-04</td>
<td>1.91E-04</td>
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</table>

Note: * Significant coefficients for a confidence level of 95%
<table>
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<th>Period</th>
<th>N</th>
<th>Period mean</th>
<th>Annual mean</th>
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<td>7,531</td>
<td>31.52%</td>
<td>2.78%</td>
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<tr>
<td>1900-1910</td>
<td>10,502</td>
<td>30.53%</td>
<td>2.70%</td>
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<tr>
<td>1910-1920</td>
<td>13,578</td>
<td>19.08%</td>
<td>1.76%</td>
</tr>
<tr>
<td>1920-1930</td>
<td>15,310</td>
<td>14.99%</td>
<td>1.41%</td>
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<tr>
<td>1930-1940</td>
<td>16,211</td>
<td>10.47%</td>
<td>1.00%</td>
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<tr>
<td>1940-1950</td>
<td>16,420</td>
<td>16.25%</td>
<td>1.52%</td>
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<tr>
<td>1950-1960</td>
<td>17,075</td>
<td>20.77%</td>
<td>1.91%</td>
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<tr>
<td>1960-1970</td>
<td>17,832</td>
<td>17.29%</td>
<td>1.61%</td>
</tr>
<tr>
<td>1970-1980</td>
<td>18,321</td>
<td>19.13%</td>
<td>1.77%</td>
</tr>
<tr>
<td>1980-1990</td>
<td>18,991</td>
<td>2.06%</td>
<td>0.20%</td>
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<tr>
<td>1990-2000</td>
<td>19,179</td>
<td>12.44%</td>
<td>1.18%</td>
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</tbody>
</table>
Figure 1: US Cities by Population Size.
Figure 2: Adaptive Kernels of the Share of US Urban Population (ln scale) by Incorporated Places or MSAs.
Figure 3: Rank-Size Plots (ln scale).
Figure 4: Empirical Density Functions of the New Entrants.
Figure 5: Empirical Density and Cumulative Density Functions (ln scale).
Figure 6: Kernel Estimates (bandwidth 0.5), (US, 1900-2000), 162,403 observations.
Figure 7: Decennial Growth Rates by Quartiles.