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Multi-Tasking vs. Screening: A Model of Academic Tenure

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Abstract

The paper develops a model of academic tenure based on multi-tasking and screening. A professor has two tasks, researching and teaching. We assume that researching performance is easy to measure but teaching performance is immeasurable. Then Holmstrom and Milgrom’s (1991) classical multi-task principal-agent model implies that the only way for the university to “incentivize” teaching activity is decreasing the incentive power to researching activity. This justifies the low-powered contract to tenured professors. However, with low-powered contract, the university will face serious informational problem in the process of enrollment, either transferring rents to the candidates with low ability if the wage level is high, or suffering from the potential occupational vacancy if the wage level is low. To this dilemma, the up-or-out contract is a possible solution.

Key Words: Multi-tasking, Screening, Academic Tenure, Up-or-Out Contract
JEL Classifications: D86, J41, J44, M55

1 Introduction

Compared to other organizations, modern universities are characteristic of its tenure system, under which the tenured professors usually earn performance-independent wage, and can not be fired unless they have serious moral problems. It is quite strange given that economics emphasizes the fundamental role of incentive for efficiency. The way that the tenure position is granted is also varied. Usually, a well-established senior scholar is offered a tenure position directly while a junior with uncertain academic prospect has to experience a probationary period, by the end of which he will obtain tenure position if he has met a predetermined academic criterion and will be fired otherwise.

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Academic tenure system since its advent has always been controversy and under heated debate. Advocates consider it to be extremely necessary for protecting academic freedom and original innovations, as articulated in "1915 Declaration of Principles on Academic Freedom and Academic Tenure" by American Association of University Professors (AAUP). However, opponents contend that tenure system, depriving the universities of the right to fire their employees, has spawned plenty of "lazy professors" with low academic productivity or some eccentric professors who only care about own academic interests while overlooking the important practical needs. Because fiscal patronage is the important financial sources for the universities (even the private ones), tenure system should be abolished to save public resource waste.

Despite the strong oppositions, one stunning phenomenon is that many universities whether public or private have adopted the tenure system "voluntarily". Since the private universities, in principle, have the right to reject it, simple revealed preference argument show that the universities are the very beneficiary from this system. Any institution, once established, will evolve in its inherent logic which may be quite different from it its historical origin. So, besides protecting academic freedom, are there other rationales for tenure system?

Freeman (1977) offers a risk-sharing explanation. He emphasizes the potential tension between the facts that researching activities are in essence highly risky on the one hand and that the researchers are usually risk-averse on the other hand. In his opinion, the combination of tenure system and minimum wage policy is a risk-sharing mechanism encouraging risk-averse researcher to do the risky but socially beneficial research projects. Furthermore, McPherson and Whinston (1999) argue that scientific progress necessitates knowledge specialization with which, however, the researchers risk being stranded. For example, a researcher highly specialized in one frontier field might not be rightly understood and evaluated by other colleagues, especially in short term. This implies that, without enough occupational protection, risk-averse researcher will have insufficient incentive to specialize, which in turn retards scientific progress. Plausible as an explanation to tenure system, the risk-sharing argument begs the question why one rarely observes all life employment (as counterpart of tenure system in universities) in industries where technological progress is also characteristic of risk and specialization. Maybe, law bar is a special case.

Siow (1998) argues that tenure system may help improve the social efficiency of a researcher allocating his time on teaching and research in life cycle. Teaching productivity is assume to be irrelevant to age while research productivity has an inverse-U-shaped relationship in age, first increasing, then peaking and finally decreasing. Under this assumption, a professor should spend more time on teaching as his age increases. However, insofar as only academic publications can be observed by other universities as potential competing employers, the professors will exert excessive effort on research in order to have a better outside option as a credible threat to quit in the bargaining process with his current employer for a better compensation. Siow suggests that tenure system helps alleviate the inefficiency. The reason is as follows: Only when a professor has well accomplished the tasks, will the current employer have incentive to match
the outside bid increasing the compensation for the professor. In our opinion, this explanation faces a problem of credibility. The fact that the professor has well accomplished the tasks in the past does not imply it is still the case in the future.

Carmichael (1988) understands the university as an internal labor market whose efficiency may be plagued by asymmetric information. Given an exogenous budget constraint, the university always wants to select the most competent employees. Thanks to knowledge specialization, the university has to delegate the enrollment tasks to the incumbent professors who have more expertise for estimating the academic prospect of the candidates. However, without occupational protection, the incumbents will be inclined to enroll those less competent than themselves to minimize the potential threat to their position. In this vein, Carmichael argues that the main function of academic tenure is supply the incumbent professor with "right" incentive to enroll new employees by offering full occupational protection to them. Quite plausible as it seems, this story is still not enough to justify academic tenure. Indeed, if the main motivation of introducing tenure system is to correct incumbents’ enrollment incentive, why will the universities not peg the new employees’ performance to the incumbents’ compensation?

The above literature only analyze the not-fire-employee feature of tenure system, but omit its up-or-out feature. However, in practice an assistant professor often faces an up-or-out contract under which he will be fired if he can not fulfill the predetermined academic criteria by the end of the probational period. Kahn and Huberman (1988) justifies the "up-or-out" contract when the employer-employee relationship is characteristic of double moral hazard. Consider a simple two-period model. In the first period, the employee determines whether to make a costly investment to accumulate specific human capital. If he does invest, he is more likely to have a high productivity in the second period in the sense of first-order stochastic dominance. Here double moral hazard means that only the employee knows whether he has made the investment period while only the employer observes the employee’s productivity in the second period (say, due to team work). To be meaningful, the investment is socially desirable. However, since in the second period the realization of the employee’s productivity can be either high or low, the employer will always have incentive to claim low productivity and pay low wage if the employee can not be fired for low productivity. With rational expectation, the employee will have no incentive to invest in the first period. This is an inefficient outcome. However, if the employee can be fired in the second period, the employer will be able to make a credible commitment that any employee, if not fired, should be paid high wage. Clearly, the employees with realized high productivity will be retained. In this scenario, the employees are also willing to make investment in the first period. Although Kahn and Huberman emphasized human capital specificity, Waldman (1990) shows that their insight are quite general, still holding as long as the incumbent employer can observe the employee’s productivity earlier than potential employers, i.e., when there is a time lag for informational diffusion. Since human capital in higher education is highly general, Waldman’s (1990)
model actually provides an important rationale of tenure system with up-or-out contract. However, since the key to the double moral hazard story is that the university can fire the employees with low productivity, one may wonder why the universities will totally give up the option to fire the tenured professors, which may be very costly when there is productivity fluctuation of the professors doing research in the future.

Although the existing literature has shed light on tenure system from different perspectives, there are still some open problems. As suggested above, either they neglect to discuss the up-or-out dimension of tenure system, or they fail to explain why the universities will voluntarily give up the right to fire its employees in an once-for-all way. Having these in mind, this paper tries to propose a novel rationale to academic tenure system, highlighting that it might be important for the universities to solve the interwoven problems of multi-task incentive and ability screening in the process of enrollment. More concretely, we put two questions: Firstly, why would the universities like to adopt a tenure system voluntarily? and secondly, conditional on that the universities will have tenure system, what is the best way for them to offer tenure position to candidates whose ability is private information?

Our answer to the first problem is a direct application of Holmstrom and Milgrom’s (1991) classic analysis on multi-task incentive. The university is a locale for knowledge production and dissemination. Consequentially, professors have two main tasks, teaching and research, both of which should be incentivized. Given there are many academic journals publicly available and ranking differently, designing a performance-dependent incentive contract for researching activity seems quite simple. Say, the university can just count how many articles one teacher has published, while putting feasible weights on journal rankings and impact factors.

By contrast, it is very difficult to estimate a professor’s teaching performance. One difficulty arises from the inherent double moral hazard problem in the process of knowledge dissemination in which "teaching" performance depends not only on the teaching effort at the teachers’ side but also on the absorbing effort at the students’ side (Dewatripont and Tirole, 2005). Even when a bad teaching performance is observed, it is not clear which party, either the teacher or the students, is to blame. Another difficulty comes from the fact that teachers in universities are often self-estimated. This implies that student score is not a good proxy for teaching performance. If ever used, it must lead to score "inflation". ¹ Finally, we should emphasize that, the teaching service obtained as a package by the students in universities is, in essence, like a public good supplied by several individual teachers. Therefore, it is not easy to attribute good or bad performance to one specific teacher. This is especially true if we take into consideration the long time for full revelation of the reaching performance.

To simplify, we assumes that research performance is measurable while teaching performance is not. Since any implementable contract should be made on

¹However, in practice, enrollment rate is often used estimating the teaching performance of the teachers in primary or high schools, for these teachers usually can not control the enrollment rate in concours.
some observable and verifiable signals, this arouses truly serious problems for universities to incentive teaching and research at the same time. Typically, a high-powered incentive contract on observable academic performance must be at the expense of teaching incentive, which might not accord to the interest of the universities. Thus, according to Holmstrom and Milgrom (1991), the only way to "incentivize" teaching activity is to decrease the incentive power to researching activity. In our opinion, this explains why professors often have fixed wages largely independent on their after-tenure academic performance. Furthermore, the tenured professors should not be fired even when their after-tenure academic performance is unsatisfactory. Otherwise, the fixed-wage compensation together with conditional firing is actually a high-powered incentive mechanism rather than a low-powered one. Or in Lazear’s (2000) terminology, it is a discontinuous incentive mechanism. As it is clear now, giving up the option to fire tenured professors can be understood as a credible commitment from the university to "incentivize" teaching activities whose performance is hard to observe and estimate.

Based on our justification of tenure system, the paper is focused on how the university should offer tenure contract when the productivity or ability of the candidates are their private information. More concretely, should the university directly offer a tenure contract to the candidates with a fixed wage, or should it offer an up-or-out contract with a probational period and predetermined academic criterion?

From the viewpoint of multi-tasking incentive, the university should offer tenure position directly to professors such that they will have "right" incentive for teaching activity. However, this kind of direct tenure contract confronts serious adverse selection. Given candidates with high ability have better outside option and occupational positions in university are scarce, direct tenure contract leads to a dilemma for the university between rent extraction and efficiency. By offering a direct tenure contract with high wage satisfying the high-ability candidates' participation constraint, the university will transfer information rents to the low-ability candidates. By offering a direct tenure contract with low wage just satisfying the low-ability candidates' participation constraint, the university will suffer from occupational vacancy.

The main point of this paper is that, when multi-tasking incentive is mingled with ability screening in the process of enrollment, an up-or-out tenure contract is a potential way out for the university. Only if a candidate meets the predetermined academic criterion in probational period, can he be granted tenured position. If there is a monotonicity relationship between ability and academic performance, this contract will have the effect of screening the candidates. Meanwhile, the tenured professors have "right" incentive for teaching activity since they get fixed-wage compensation and can not fired in case of bad academic performance.

Section II is the basic model, in which research performance can be perfectly measured. Since a candidate's ability is either high or low, we consider three scenarios of employment: the first is direct tenure contract with low wage, the second is direct contract with high wage, the third is up-or-out contract with
a probational period and a predetermined academic criteria for promotion. By comparison between these three scenarios, we obtain the best the employment contract. We also do comparative static analysis. When academic performance can be perfectly measured, the optimal up-or-out contract may lead to first-best allocation if the ability difference between candidates is small. Although, in this case, the optimal up-or-out contract always dominates the low-wage direct tenure contract, it results in distortion and will be dominated by high-wage direct tenure contract when the ability difference is very large and the candidate is most likely to have high ability. Finally, as we shall show in the extended model in section III, the up-or-out contract will be dominated by a low-wage direct tenure contract when the academic performance can not be perfectly measured and the candidate is most likely to have low ability. Section IV concludes.

2 Basic Model

2.1 Model Setting

Consider a representative university. For expositional convenience, one employee of the university is simply called a "teacher" when it is unclear whether he has obtained tenure position. A potential employee is called a candidate. A teacher who has obtained tenure position is called a tenured professor. A teacher who has not obtained tenure position is called an assistant professor.

Every teacher has two tasks, i.e., research (task 1) and teaching (task 2). Both the university and the candidates are long-lived, risk neutral. They aim to maximize respective profits. Time is discrete with discount factor $\delta < 1$. Following Siow (1998), the vacant position in the university are scare. Without loss of generality, there is only one vacant position left for candidates. Each period enters only one candidate, having either high ability ($\bar{\theta}$) or low-ability ($\underline{\theta}$), with probability $v$ and $1 - v$ respectively. The candidate knows exactly his type, but the university only knows $v$. Any time, if one candidate is not employed or has been fired by the university, he exits forever, thereafter working in a competitive market with net payment $\theta - c$, where $\theta$ and $c > 1$ can be understood as the corresponding output and cost.

Consider the contract for a candidate $\theta$. If he exerts effort $x_i$ on task $i$ ($= 1, 2$), his output in task $i$ is

$$q_i = x_i \hat{\theta}.$$ (1)

This assumption has two implications. On the one hand, for a specific teacher, the marginal contribution of teaching and researching effort is the same; on the other hand, with same effort, a teacher with higher ability will have a larger contribution. However, as argued before, only academic performance $q_1 = x_1 \hat{\theta}$ can be observed and used for contract design. As a convention, when one variable has upper bar (or lower bar), it is related to type $\bar{\theta}$ (or $\underline{\theta}$).
It is important how to characterize the multi-task cost. Teaching and researching may be complementary. For example, the fruits of research can be used as teaching materials, hence decreasing the preparation cost of teaching. On the contrary, as an interaction between teachers and students, teaching activity may help teachers to find excellent research assistants, and hence decreasing the researching cost. However, teaching and research can also be substitutes. Besides direct crowding-out effect of time, frequent transformation between different tasks may incur some extra cost. No matter teaching or research, it may need continuous attention or meditation. Stopping on half way often entails duplication cost. To synthesize both possibilities, we assume that, if a teacher exerts effort $x_i$ on task $i$, then his multi-task cost is

$$C(x_1, x_2) = x_1^2 + \gamma x_1 x_2 + x_2^2 \quad (2)$$

where $\gamma \in [-2, 2]$ characterizes the relationship between teaching and researching. They are complementary if $\gamma < 0$ but substitutes if $\gamma > 0$.

In this paper, our focus is not on the overall effort of a teacher. Instead, by assuming that a teacher’s overall time (effort) is fixed, we want to see how a teacher will allocate his fixed time across teaching and researching activities under different employment contract. Without loss of generality, we normalize the overall time to 1:

$$x_1 + x_2 = 1 \quad (3)$$

Substituting (3) into (2), the multi-task cost becomes:

$$C(x) = 1 - (2 - \gamma)x + (2 - \gamma)x^2, \quad (4)$$

where we define $x \equiv x_1$ for national convenience.

From (4), $C'(x) \geq 0$ when $x \leq 1/2$. That is, $C(x)$ is a parabola curve opening upwards. Define

$$C_m = \frac{1}{2} + \frac{\gamma}{4} = \min_x C(x) \quad (5)$$

Clearly, with inelastic overall time assumption, we simplify the multi-task problem to a single-task one. But is it meaningful to make this assumption? We want to say that this assumption, in essence, is consistent with Holmstrom and Milgrom (1991) in which the marginal effort cost of the agent is assumed to be negative when the effort is below a threshold, and then increases when effort increases. The reason that they make this assumption is to guarantee that the agent has incentive to exert some effort under a fixed-wage contract. So is true for our paper.

Before going to market outcome, we first consider the first-best allocation. Obviously, for each candidate, whether high- or low-ability, once employed by the university, his contribution to social welfare, net opportunity cost, is

$$W(\theta) = c - C(x). \quad (6)$$
Noting $C(x) < 1$, the technical assumption $c > 1$ implies that there should be no positional vacancy in the university since the social benefit from the candidate working in university is higher than in other competitive lines. Further noting that $C(x)$ is minimized when $x = 1/2$, a candidate, once employed by the university, should spend equal time on teaching and researching activities.

**Proposition 1** Under first-best outcome, there should be positional vacancy and the fixed time should be equally allocated to teaching and researching.

### 2.2 Market Equilibrium

We want to discuss what is the best way for the university to offer tenure contract. As told before, there are three possibilities, the low-wage direct tenure, the high-wage direct tenure contract and the up-or-out tenure contract.

The timing is as follows: The university first offers one kind of tenure contract which a candidate can accept or reject it. In case of acceptance, the teacher chooses how to allocate the fixed time across teaching and researching activities. Then the contract is implemented. If the scare position is still vacant (maybe, it is because the candidate reject the offer or because the university has just fired an unqualified assistant professor), then the game starts again in the next period; otherwise, the game is over since all the position in the university have been filled with tenured professors.

#### 2.2.1 Direct Tenure Contract

Since a candidate may be low- or high-ability. There are two options for direct tenure contract. If the university offers the direct tenure contract with high wage $\bar{w}$ such that

$$
\bar{w} = \bar{\theta} - c + C_m,
$$

the high-ability candidate will accept it since (7) is just his participation constraint. Of course, $\bar{\theta}$ will also be happy to accept this offer, with current rent $\bar{\theta} - \bar{\theta}$ in each period. As both both types will accept this contract, the vacant position is filled since the first period by tenured professors who, in turn, will choose the time allocation plan that minimizes multi-task cost. After discounting, the total benefit of the university from this contract is

$$
K^T = \frac{v\bar{\theta} + (1 - v)\hat{\theta} - \bar{w}}{1 - \delta} = \frac{c - C_m}{1 - \delta} - \frac{(1 - v)(\hat{\theta} - \bar{\theta})\bar{w}}{1 - \delta}.
$$

If the university offers the direct tenure contract with low wage $w$ such that

$$
w = \hat{\theta} - c + C_m,
$$

then only $\hat{\theta}$ will accept this offer and then chooses time allocation plan that minimizes multi-task cost. Therefore, if the candidate is $\hat{\theta}$ (with probability $1 - v$), the vacant position will be filled afterwards. However, if the candidate
is $\overline{\theta}$ (the probability is $v$), he will reject the offer and the position keeps vacant, the game going to the next period. After discounting, the total benefit of the university from this contract is

$$K^{II} = (1-v)\frac{\overline{\theta} - w}{1-\delta} + \delta v K^{II} = \frac{(1-v)(c - C_m)}{(1-\delta v)(1-\delta)}$$  (10)

### 2.2.2 Up-or-Out Contract

With direct tenure contract, the university faces such a dilemma, either suffering from possible positional vacancy when the wage is low or transferring informational rents to low-ability candidate when the wage is high. An up-or-out tenure contract may be a way out. We denote an up-or-out contract to be a triple $\{w^n, w^t, R\}$, where $w^n$ is the fixed wage for assistant professor in probational period, $w^t$ is the fixed wage for tenured professor all the life, and $R$ is the academic criterion for promotion. Sometimes, without confusion, we also call the up-or-contract $R$. Depending on the type difference, we consider the up-or-out contract in the following cases.

- $\overline{\theta} \geq 2\overline{\theta}$

In this case, the type difference is very big. Even when $\overline{\theta}$ chooses his cost-minimizing time allocation $\overline{\pi} = \frac{1}{2}$ and $\overline{\theta}$ spends all his time on researching activity, $\overline{\theta}$’s academic performance ($\overline{\theta}$) is still less than $\overline{\theta}$’s ($\overline{\theta}/2$). With these observations, the optimal up-or-out tenure contract can be easily characterized by the following conditions:

$$R^* = \overline{\theta}/2 \quad (\overline{\theta} \text{ cannot imitate } \overline{\theta})$$  (11)

$$w^n - C_m + \frac{\delta (w^t - C_m)}{1-\delta} \geq \frac{\overline{\theta} - c}{1-\delta} \quad (\overline{\theta} \text{ s ex ante participation constraint})$$  (12)

$$w^t - C_m \geq \overline{\theta} - c \quad (\overline{\theta} \text{ s interior participation constraint})$$  (13)

$$w^n = \overline{\theta} - c + C_m \quad (\overline{\theta} \text{ s participation constraint})$$  (14)

Under this up-or-out tenure contract, the university’s net benefit is

$$K^{III} = v \left[ \overline{\theta} - w^n + \frac{\delta (\overline{\theta} - w^t)}{1-\delta} \right] + (1-v) \left[ \overline{\theta} - w^n + \delta K^{III} \right] = \frac{c - C_m}{1-\delta}$$  (15)

Note that $K^j$ ($j = I, II, III$) are all continuous in $v$. Note further that $\partial K^j / \partial v > 0$, $\partial K^{II} / \partial v < 0$, $\partial K^{III} / \partial v = 0$, $K^{III} = K^I$ at $v = 0$, and $K^{III} = K^{II}$ at $v = 1$. We get the following proposition.
Proposition 2 If $\bar{\theta} > \frac{2\bar{\theta}}{ar{\beta}}$, then the up-or-out contract always dominates the direct tenure contracts, and leads to first-best outcome.

This proposition shows that, if the candidate’s type difference is very big, then the up-or-out tenure contract characterized by (11) through (14) is the best tenure contract for the university. It is quite intuitive. By (12) (with equality), the rent of high-ability candidate is zero; by (13), the rent of low-ability candidate is also suppressed to zero. As a result, the up-or-out contract not only extracts all the information rent of the low-ability candidate (compared to high-wage direct tenure contract), but also avoids positional vacancy (compared to low-wage direct tenure contract). Clearly, this up-or-out contract results in effort distortion as well.

- $\bar{\theta} < \frac{2\bar{\theta}}{ar{\beta}}$

Now the type difference is not so large as before. If the university still sets academic criterion $R^* = \frac{\bar{\theta}}{2}$, $\bar{\theta}$ can meet this criterion if he distorts his researching effort upwards to $\frac{R^*}{\bar{\theta}} < 1$. If this does happen, there will be no screening. On the contrary, if the university wants full screening, it can simply set an academic criterion $R = \frac{\bar{\theta}}{2}$ which $\bar{\theta}$ will never succeed to fulfill. However, when academic criterion is $R = \frac{\bar{\theta}}{2}$, $\bar{\theta}$’s optimal effort in probational period should be $\frac{\bar{\theta}}{2}$, larger than $\frac{1}{2}$, the cost-minimizing (also the socially optimal) effort. We want to ask whether the university can find an academic criterion $\hat{R} \in (\bar{\theta}/2, \bar{\theta})$ that also fulfills perfect screening with less distortion than $R = \frac{\bar{\theta}}{2}$.

Suppose that the university has set a criterion $\hat{R} \in (\bar{\theta}/2, \bar{\theta})$, under which promotion entails the research effort $\bar{x} = \hat{R}/\bar{\theta}$ for $\bar{\theta}$ and $\hat{x} = \hat{R}/\hat{\theta}$ for $\hat{\theta}$ in the probational period. Since $\hat{x} > \bar{x} > 1/2$, neither $\bar{\theta}$ nor $\hat{\theta}$ would like to choose higher research effort for promotion. Therefore, if some $\hat{R} \in (\bar{\theta}/2, \bar{\theta})$ can lead to perfect screening, it must be the case that it is better for $\bar{\theta}$ to choose $1/2$ than $\hat{x}$. Based on above analysis, a separating equilibrium $\hat{R} \in (\bar{\theta}/2, \bar{\theta})$ (if any) can be characterized by the following conditions:

\begin{align*}
    w^n - C(\hat{x}) + \frac{\delta(w^n - C_m)}{1-\delta} &= \frac{\bar{\theta} - c}{1-\delta} \quad (\bar{\theta}’s \text{ ex ante participation constraint}) \quad (16) \\
    w^t - C_m &\geq \bar{\theta} - c \quad (\bar{\theta}’s \text{ interior participation constraint}) \quad (17) \\
    w^n - C(\hat{x}) + \frac{\delta(w^n - C_m)}{1-\delta} &< w^n - C_m + \frac{\delta(\bar{\theta} - c)}{1-\delta} \quad (\hat{\theta} \text{ does not mimick } \bar{\theta}) \quad (18) \\
    w^n &= \frac{\bar{\theta}}{2} - c + C_m \quad (\bar{\theta}’s \text{ participation constraint}) \quad (19) \\
    \hat{R} &\in (\bar{\theta}/2, \bar{\theta}) \text{ or } \hat{x} \in (1/2, \bar{\theta}/\bar{\beta}) \quad (\hat{x} = 1/2 \text{ cannot be equilibrium}) \quad (20)
\end{align*}
For above conditions, only (20) needs further explanation. It means that if the university sets the academic criterion $R^*$, then $\bar{\theta}$ will have incentive to mimic. This condition is essential to welfare analysis and comparative statics later. Indeed, if this condition is violated, the university’s benefit under up-or-out contract is still $K^{IV}$, hence always dominating the direct tenure contracts.

Using (16), (18) is equivalent to

$$w^n > \bar{\theta} + \frac{\delta(\bar{\theta} - \theta)}{1 - \delta} + C_m - c + C(\bar{\pi}) - C(\bar{\pi}), \quad (21)$$

which in turn implies that the key to equilibrium $R$ is: the right side of (19) should be larger than the right side of (21). Using the fact that $\bar{\pi} = \bar{\theta} = R$, this requires

$$D(\bar{\pi}) = C\left(\frac{\bar{\theta}}{\bar{\pi}}\right) - C\left(\bar{\pi}\right) > \frac{\bar{\theta} - \theta}{1 - \delta} \quad (22)$$

Substituting the concrete form of $C(x)$ into (22) and taking equality, (22) becomes

$$(2 - \gamma)\bar{\pi} \left(1 + \frac{\bar{\theta}}{\bar{\pi}}\right) - 1 = \frac{\theta}{1 - \delta}, \quad (23)$$

Note that $D'(.) > 0$ because research effort is now larger than $1/2$ in probational period. Depending on parameters, there may arise two kinds of equilibria.

**Case I:** $D(\bar{\pi}) > \frac{\bar{\theta} - \theta}{1 - \delta}$ or $(2 - \gamma)(1 - \delta) > \frac{\bar{\pi}}{\bar{\pi}}$

In this case, there exists a unique $\hat{x} \in (1/2, \bar{\theta}/2)$ such (23) is satisfied. The separating equilibrium characterized by (18) through (20) exists: $\hat{\pi}$ is given by (23), then $R = \bar{\pi} = \bar{\theta}/2$, and $\hat{x} = R/\bar{\theta}$. $\bar{\theta}$ chooses research effort $\bar{\pi}$ in probational period and then gets tenure position. Thereafter, he always chooses $x = 1/2$. $\bar{\theta}$ chooses $x = 1/2$ in probational period but then gets fired, going to competitive industry. With a little algebraic calculation, under this up-or-out tenure contract, the university gets expected profit

$$K^{IV} = v(\bar{\theta} - w^n) + (1 - v)(\bar{\theta} - w^n) + v \frac{\delta(\bar{\theta} - w^n)}{1 - \delta} + (1 - v)\delta K^{IV} \quad (24)$$

$$= \frac{c - C_m}{1 - \delta} - v \frac{C(\bar{\pi}) - C_m}{1 - \delta + v\delta}$$

However, there are more intuitive way to get(24). Note that $\bar{\theta}$ does not distort his effort and neither type gets positive rent. Suppose for the moment that $\bar{\theta}$ also does not distort his effort. Then the university’s profit will be $\frac{c - C_m}{1 - \delta}$. Now what remains is to subtract $\bar{\theta}$’s distortion loss in probational period. Noting that the distortion occurs in the next period only if the current assistant
professor is \( \theta \), the expected distortion loss is

\[
L = v \left[ C(\theta) - C_m \right] + (1 - v)\delta L = \frac{v \left[ C(\theta) - C_m \right]}{1 - \delta + v\delta}.
\]  

(25)

**Case II:**

\( D(\frac{\pi}{2}) < \frac{\pi - \theta}{2\delta} \) or \( (2 - \gamma)(1 - \delta) < \frac{\pi^2}{2\delta} \)

In this case, the equilibrium \( \bar{R} \in (\frac{\theta}{2}, \theta) \) characterized by (18) through (20) does not exist. However, the university can simply set the academic criterion \( \bar{R} = \theta \) to implement perfect screening equilibrium. In this equilibrium, \( \theta \) chooses research effort \( \bar{\pi} = \frac{\theta}{2\delta} \) in probational period, then gets tenure position and chooses \( \bar{\pi} = 1/2 \) forever. \( \theta \) chooses \( \bar{x} = 1/2 \) in probational period but then get fired going to competitive industry. Like (24), in this equilibrium, the university

gets expected profit

\[
K^V = \frac{c - C_m}{1 - \delta} - \frac{v \left[ C(\bar{\pi}) - C_m \right]}{1 - \delta + v\delta}.
\]  

(26)

Note that \( K^V = K^{IV} > K^I \) at \( v = 0 \), \( K^V < K^{IV} < K^I \) at \( v = 1 \), \( \partial K^I / \partial v > 0 \), \( \partial K^{IV} / \partial v < 0 \) and \( \partial K^V / \partial v < 0 \). Then there must be a unique \( \bar{v} \) such that \( K^I \leq K^{IV} \) if \( v \leq \bar{v} \). There also exists a unique \( \bar{v} \) such that \( K^I \leq K^V \) if \( v \leq \bar{v} \). Furthermore, because \( \bar{\pi} > \frac{\pi}{2} > 1/2 \) and hence \( C(\bar{\pi}) > C(\bar{\pi}) > C_m \), then \( K^V < K^{IV} \), \( \bar{\pi} > \bar{\pi} \). Figure (??) illustrates the university’s payoff when \( \bar{\pi} < \theta < 2\theta \).

Figure 1: Comparison between up-or-out contract and direct tenure contracts when academic performance can be perfectly measured.
Proposition 3 When $\bar{\theta} \leq 2\bar{\theta}$, the optimal up-or-out contract still dominates low-wage direct tenure contract. However, if $v$ is very large ($v > \bar{v}$ in equilibrium $\bar{R} = (\bar{\theta}/2, \bar{\theta})$; $v > \bar{v}$ in equilibrium $\bar{R} = \bar{\theta}$), then the optimal up-or-out contract will be dominated by high-wage direct tenure contract. None of the tenure contracts can realize first-best outcome.

Everyday experience tells us that it is difficult to distinguish two things with tiny difference. Or to tell the difference between them is very costly. Similarly, when the type difference of the candidate is very small, the up-or-out contract will inevitably result in efficiency loss even it can still implement perfect screening given academic performance is perfectly observed. When the candidate is unlikely to be low ability type, the social value of screening is very small, and hence the up-or-out contract will be dominated by the high-wage direct tenure contract. The reason that the up-or-out contract still dominates low-wage direct contract is that it does not lead to distortion but can suppress informational rents when the candidate is low-ability type.

2.3 Comparative Statics

Now we consider the effects of parameters $\delta$ and $\gamma$ on the optimal contract choice. According to model setting, $\gamma$ characterizes the multi-task cost. When $\gamma$ is larger, teaching and research activities are more likely to be substitutes, or the complementarity between them decreases. Furthermore, when $\delta$ is bigger, the future benefit is more important, which in some sense represents a shorter probational period.

Note that the equilibrium $\bar{v}$ is determined by $K^I = K^V$, or

$$
\frac{v(2 - \gamma)(\bar{\theta} - \frac{1}{2})^2}{1 - \delta + v\delta} = \frac{(1 - v)(\bar{\theta} - \bar{\theta})}{1 - \delta}.
$$

(27)

From this equation, the implicit function theorem immediately leads to the following proposition.

Proposition 4 $\partial \bar{v}/\partial \delta > 0, \partial \bar{v}/\partial \gamma > 0$

The results are very intuitive. In the equilibrium $\bar{R} = \bar{\theta}, \bar{\pi} = \bar{\theta}/\bar{\theta}, \bar{\theta}$’s research effort in probational period, depends neither on $\delta$ nor on $\gamma$. So, ceteris paribus, a bigger $\delta$ implies that the university can screen the candidate in a shorter time, which necessarily increases the attractiveness of up-or-out contract. Furthermore, referring to (25) and (26), a larger $\gamma$ means that $C(\bar{\pi}) - C_m = (2 - \gamma)(\bar{\pi} - \frac{1}{2})^2$ is smaller. That is, compared to the high-wage direct tenure contract, $\bar{\theta}$’s effort distortion in probational period is smaller, which necessarily increases the attractiveness of up-or-out contract.

Relatively, the comparative statics of $\bar{v}$ is a little more complicated. Like $\bar{v}$, $\bar{v}$ is determined by $K^I = K^IV$, or
\[ \frac{v(2 - \gamma)(\bar{\tau} - \frac{1}{2})^2}{1 - \delta + v\delta} = \frac{(1 - v)(\bar{\theta} - \theta)}{1 - \delta}. \] (28)

However, now \( \bar{\tau} \) is endogenously determined by (23). As a result, we must combine (28) with (23) to see how the parameters affect \( \bar{\nu} \).

**Proposition 5** \( \frac{\partial \bar{\nu}}{\partial \delta} < 0, \frac{\partial \bar{\nu}}{\partial \gamma} < 0. \)

**Proof.** Together with (28) and (23), we define an implicit function

\[ \hat{F} = \frac{v(\bar{\tau} - \frac{1}{2})^2}{(1 - \delta + v\delta)\bar{\tau}(\bar{\tau} \left(1 + \frac{\bar{\tau}}{\theta}\right) - 1)} - \frac{(1 - v)(\bar{\theta} - \theta)}{\theta} = 0, \] (29)

from which we immediately have \( \frac{\partial \hat{F}}{\partial \tau} > 0, \frac{\partial \hat{F}}{\partial \nu} > 0, \frac{\partial \hat{F}}{\partial \gamma} = 0, \) and \( \frac{\partial \hat{F}}{\partial \tau} \propto \frac{\bar{\tau} - \frac{1}{2}}{\theta} > 0, \) where the last inequality uses the separating condition \( \bar{\tau} > \frac{1}{2} \). From (23), we have \( \frac{\partial \hat{F}}{\partial \nu} > 0 \) and \( \frac{\partial \hat{F}}{\partial \gamma} > 0. \) Thus, we have \( \frac{\partial \hat{F}}{\partial \delta} = \frac{\partial \hat{F}}{\partial \gamma} + \frac{\partial \hat{F}}{\partial \tau} \frac{\partial \tau}{\partial \delta} > 0, \frac{\partial \hat{F}}{\partial \gamma} = \frac{\partial \hat{F}}{\partial \tau} \frac{\partial \tau}{\partial \gamma} > 0. \) Based on above analysis, the implicit function theorem immediately implies that \( \frac{\partial \bar{\tau}}{\partial \tau} > 0 \) and \( \frac{\partial \bar{\nu}}{\partial \delta} < 0. \]

Surprisingly, the results of proposition 5 are completely opposite to those in proposition 4. Now the up-or-out contract becomes less attractive when the probationary period is shorter, or teaching and research activities are less complementary. What is the reason for this sharp contrast?

Compared to \( \bar{\nu} \), the equilibrium \( \hat{\nu} \) has two remarkable characteristics. First, as we have mentioned, parameters \( \delta \) and \( \gamma \) not only have direct effect on \( \bar{\nu} \) in (29), but also have indirect effect on \( \bar{\nu} \) through \( \bar{\tau} \) which is endogenously determined by (23), hence directly dependent on parameters \( \delta \) and \( \gamma \). Second, \( \gamma \) and \( \delta \) should not be too large to satisfy \( (2 - \gamma)(1 - \delta) > \bar{\theta}^2/\bar{\theta} \), the condition for this case to be true.

With these observations and referring to (18), we know that when \( \delta \) becomes larger, ceteris paribus, \( \bar{\theta} \) will have stronger incentive to mimick \( \hat{\theta} \). Put in another word, for perfect screening, now the the university should correspondingly increase the academic criterion, but this exacerbates \( \bar{\theta} \)'s effort distortion in probational period. Since \( \bar{\theta} \) has no rents, this distortion will be burdened by the university in the end. This implies that up-or-out contract becomes less attractive. By contrast, in equilibrium \( \bar{\nu} \), \( \bar{\theta} \) can not mimick \( \hat{\theta} \), so a slight change of \( \delta \) does not increase effort distortion.

Similarly, when \( \gamma \) increases, \( \bar{\theta} \) will have more incentive to mimick \( \hat{\theta} \) since \( C(\bar{\tau}) - C_m = (2 - \gamma)(\bar{\tau} - \frac{1}{2})^2 \) decreases. Then perfect screening also requires the university to increase academic criterion, which in turn exacerbates \( \bar{\theta} \)'s effort distortion in probational period, and hence decreases the attractiveness of the up-or-out tenure contract.
3 Extension

In above analysis, we discuss how the university should offer tenure contract under the assumption that academic performance can be perfectly measured. Two results are worthwhile mentioning again. First, the up-or-out contract always dominates the low-wage direct tenure contract; second, the up-or-out contract, once employed, always realizes perfect screening. In this extension, we want to see whether these results will be changed if we introduce measurement error for academic performance.

To this aim, now we assume that the observed academic performance of teacher $\theta$ is

$$q(x, \theta) = x\theta + \varepsilon,$$  \hspace{1cm} (30)

which depends not only on his ability $\theta$ and research effort $x$, but also on random shock $\varepsilon$. To simplify, we assume that $\varepsilon$ is iid in each period, following a normal $H(.)$ with mean 0 and not correlated with $\theta$.

Except for introducing $\varepsilon$, other model setting is the same as before. Since both the university and candidates are risk neutral, introducing this random shock does not change the direct tenure contracts.

Now denote the up-or-out contract in question to be $\{R^\#, w^m, w^t\}$. If both $\bar{\theta}$ and $\bar{\theta}$ accept this contract, and choose $\bar{x}$ and $x$ in probational period, then $\bar{\theta}$ gets tenure position with probability

$$\bar{P}(\bar{x}, R^\#) = \Pr(\varepsilon > R^\# - \bar{\theta}) = 1 - H(R^\# - \bar{x}\bar{\theta}),$$ \hspace{1cm} (31)

and $\bar{\theta}$ with probability

$$P'(x, R^\#) = \Pr(\varepsilon > R^\# - x\theta) = 1 - H(R^\# - x\theta)$$ \hspace{1cm} (32)

Because a cumulative distribution function is always nondecreasing, then for any $x$ and $R^\#$, there must be

$$\bar{P}(x, R^\#) \geq P'(x, R^\#),$$ \hspace{1cm} (33)

that is, $\bar{P}(x, R^\#)$ is a first-order stochastic dominance to $P'(x, R^\#)$.

If $\bar{\theta}$ accepts contract $\{R^\#, w^m, w^t\}$, he will choose his effort $\bar{x}$ to maximize his expected benefit:

$$w^m - C(\bar{x}) + \bar{P}(\bar{x}, R^\#)\frac{\delta(w^t - C_m)}{1 - \delta} + (1 - \bar{P}(\bar{x}, R^\#))\frac{\delta(\bar{\theta} - c)}{1 - \delta}$$ \hspace{1cm} (34)

where the first two terms represent $\bar{\theta}$'s benefits in probational period, the third term represents his discounted present benefits since after obtaining tenure position, and the fourth term represents the present value of his benefit in outside competitive market if he fails to get tenure position. The first-order condition for the program is

$$\frac{\partial C(\bar{x})}{\partial \bar{x}} = \frac{\partial \bar{P}(\bar{x}, R^\#)}{\partial \bar{x}}\frac{\delta(w^t - C_m) - (\bar{\theta} - c)}{1 - \delta}$$ \hspace{1cm} (35)
Similarly, if $\theta$ accepts contract $\{R^#, w^n, w^t\}$, he will choose $x$ to maximize his expected profits

$$w^n - C(x) + P(x, R^#) \frac{\delta (w^t - C_m)}{1 - \delta} + (1 - P(x, R^#)) \frac{\delta(\theta - c)}{1 - \delta}, \quad (36)$$

and the corresponding first order condition is

$$\frac{\partial C(x)}{\partial x} = \frac{\partial P(x, R^#) \delta([w^t - C_m] - (\theta - c))}{1 - \delta} \quad (37)$$

Denote the results of the two programs above to be $x^#$ and $\bar{x}^#$, and $P^*$ and $\bar{P}^*$. Note that, under the normal distribution assumption, $P^*$ and $\bar{P}^*$ are strictly positive. Now we characterize the conditions for the contract $\{R^#, w^n, w^t\}$.

$\theta$'s ex ante participation constraint is

$$w^n - C(\bar{x}^#) + P^# \frac{\delta(w^t - C_m - \bar{\theta} + c)}{1 - \delta} \geq \bar{\theta} - c. \quad (38)$$

$\theta$'s ex ante participation constraint is

$$w^n - C(\bar{x}^#) + P^# \frac{\delta(w^t - C_m - \bar{\theta} + c)}{1 - \delta} \geq \bar{\theta} - c. \quad (39)$$

$\bar{\theta}$'s interior participation constraint is

$$w^t - C_m \geq \bar{\theta} - c. \quad (40)$$

Clearly, if possible, $\bar{\theta}$ will necessarily accept the tenure offer.

**Lemma 1** $\bar{x}^# > 1/2$. That is, $\bar{\theta}$ will distort upwards the research effort in probational period.

**Proof.** Let us see the first-order condition (37). By assumption of the normal distribution, $\frac{\partial P(x, R^#)}{\partial x} > 0$ for any $R^#$ and $\bar{x}$. However, from $\bar{\theta}$'s interior participation constraint (40), we have $w^t - C_m > \bar{\theta} - c$. So, the right side of (37) must be positive. Recall that $C'(x) \geq 0$ if $x \geq \frac{1}{2}$. There must be $\bar{x}^# > 1/2$. □

**Proposition 6** Assume $\varepsilon$ follows a normal distribution with mean zero and variance $\sigma^2 < \infty$. If $\nu$ is very small, then the low-wage direct tenure contract dominates up-or-out contract.

This proposition is a direct corollary of the above lemma. As it is clear, given that the measurement error follows a normal distribution with mean zero and finite variance, $\bar{\theta}$ facing the up-or-out tenure contract will distort upwards his research effort in probational period for a larger promotion probability. For the tenure wage $w^t$ satisfying $\bar{\theta}$'s interior participation constraint (40) is very attractive. However, when $\nu$ is very small, i.e., when the candidate is most likely to be $\bar{\theta}$, this distortion effect from the up-or-out contract will dominate.
its benefit from avoiding vocational vacancy. By contrast, in this case, the low-wage direct tenure contract results in no distortion of although the university may indeed suffer from vocational vacancy when the candidate proves to be \( \theta \). But as it is assumed, this expected loss is very small when \( v \) is very small.

**Definition 1** The up-or-out contract \( \{ R^\#, w^n, w^t \} \) implements effective screening in the sense of probability if it leads to \( \bar{P}^# \geq \underline{P}^# \).

Here we say it is an efficient screening in the sense of probability because once employed \( \bar{\theta} \) will have a positive probability to get the tenure position. This is a very important difference with the perfect screening case, which, of course, can be seen as a special case with zero measurement error.

**Proposition 7** Assume \( \varepsilon \) follows a normal distribution with mean zero and variance \( \sigma^2 \). If \( \sigma^2 \to \infty \), the up-or-out contract \( \{ R^\#, w^n, w^t \} \) characterized above implements effective screening in the sense of probability. Furthermore, it realizes the first-best outcome unless there is a binding minimum-wage restriction.

**Proof.** Consider an arbitrary contract \( \{ R^\#, w^n, w^t \} \). \( \theta \)'s promotion probability is \( P(x, R^\#) = \int_{R^\#-x}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{z^2}{2\sigma^2}} dz \). So, \( \frac{\partial P(x, R^\#)}{\partial x} \to 0 \) when \( \sigma^2 \to \infty \). Then the first order conditions (35) and (37) immediately imply that both types will choose socially optimal effort allocation, i.e., \( x^\# \to 1/2 \) and \( x^\# \to 1/2 \), which, by the first-order stochastic dominance, in turn, imply \( \bar{P}^# > \underline{P}^# \), the effective screening in the sense of probability. Then, combining two participation constraints (38) and (39) taking equality, we get

\[
(\bar{P}^# - \underline{P}^#) \frac{\delta(w^t - C_m - \bar{\theta} + c)}{1 - \delta} = (\bar{\theta} - \underline{\theta}) \left( 1 + \frac{\bar{P}^# - \underline{P}^#}{1 - \delta} \right)
\]

(41)

where all terms are independent on \( w^n \). Note that the right side of (41) is positive, and \( \bar{P}^# - \underline{P}^# > 0 \), then there must exist a unique \( w^t \) such that (41) holds. Note further that \( \bar{\theta} \)'s interior participation constraint is strictly satisfied, i.e., \( w^t - C_m > \bar{\theta} - c \). Finally, substituting the \( w^t \) determined in (41) back to (38) or (39), we can get the unique corresponding \( w^n \).

At first glance, this proposition seems astonishing since it implies that extreme measurement error will improve efficiency. However, this is reasonable. If measurement error of academic performance tends to infinity, the benefit of the candidate by distorting research effort in probation period tends to zero since it will not increase his probability of being promotion to tenure position. So no effort distortion arises. Insofar as \( \bar{\theta} \)'s interior participation constraint is strictly satisfied, the university actually "rewards" the tenured professors. Because \( \bar{\theta} \) has less valuable outside option, his actual reward, if promoted to tenure position, is larger. However, with same effort \( x \), \( \bar{P}(x, R^\#) < \underline{P}(x, R^\#) \), so \( \bar{\theta} \) is also less likely to be promoted. Adjusted by opportunity cost and promotion probability, the ex ante expected reward for both types is the same. As no term in (41) depends on \( w^n \), the university can then choose a low \( w^n \)
to "punish" the candidate in the probational period taking away all the rents if there is no minimum wage restriction. In this sense, minimum wage policy may decrease the attractiveness of up-or-out contract to the university. Put in another word, given the university offers an up-or-out contract, the minimum wage policy probably restricts the freedom of its rent-extraction, and hence protecting the teachers. This implication is consistent to Freeman’s (1977) analysis on minimum wage policy as a insurance mechanism to professors under tenure system.

4 Concluding Remarks

The university can be seen as an economic organization that produces and sells knowledge. Consequentially, the professors produce knowledge by researching activity while instructing knowledge by teaching activity. In long term, research and teaching activities are equally important for the university. Therefore, both need to be incentivized. However, designing a multi-task incentive contract has essential difficulty when one task, teaching activity in this paper, is immeasurable. In this case, by Holmstrom and Milgrom (1991), the only way to incentive teaching activity is to decrease the incentive power to the measurable researching activity. As such, we justify the low-powered contract for tenure professors from the multi-task perspective.

However, multi-tasking is not the only problem facing the university. It also want to select able employees while extracting the informational rents. For this aim, the low-powered contract performs badly. Typically, when the candidates have their abilities as private information, the university faces a dilemma by using a direct tenure contract, either transferring informational rents to the candidate with low ability, or suffering from vocational vacancy. This paper shows that an up-or-out tenure contract may be a solution to this dilemma.

Finally, we think that the tenure system is a very complicated institution. Perhaps no single insight is enough for its justification. In this sense, our explanation is complementary to the existing literature on academic tenure system.

References


