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The Impacts of Airport Centrality in the EU Network and Inter-Airport Competition on Airport Efficiency

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Abstract

In this paper we study the relationship between airport efficiency and two factors: an airport’s centrality in the EU network, and the intensity of competition from alternative airports in the same catchment area. We apply a two-stage econometric model based on the Simar & Wilson (2007) bootstrap procedure to a balanced sample of 57 European airports. We also design and compute our own measures of airport centrality and competition. The results show that efficiency is positively related to centrality in the European network, as measured by a weighted sum of minimal paths passing through the airport in question. The intensity of competition between airports also has a positive effect on efficiency. Our analysis suggests that air transportation policies should focus on increasing competition within important catchment areas (e.g., by investing in infrastructure facilitating access to alternative airports) and enhancing the connectivity of the EU network (e.g., by subsidizing new point-to-point connections between airports with capacity to spare).

JEL classification: [L930, L590, L110]

Keywords: air transportation, efficiency, network centrality, inter – airports competition.

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1. Introduction

The liberalization of air transport in the European Union (EU) was completed in 1997.\textsuperscript{1} Since then the EU network has expanded as never before, contributing to the economic growth of the sector. Airlines are providing an increasing number of routes, even in remote areas, and moving more and more passengers.\textsuperscript{2} The airports are key factors in this expansion, insofar as they provide benefits to passenger and freight mobility. Airports also spur the development of local economies by acting as gateways to other regions, or connecting nodes of the network.

The benefits produced by airports are proportional to their efficiency. More intense capacity utilization is usually achieved by moving greater numbers of aircrafts and passengers, while this traffic is in turn linked to the airport’s relative importance in the EU network. Moreover, more efficient airports should be able to offer airlines lower fees, yielding (if these savings are transferred to tickets) benefits for consumers and firms.\textsuperscript{3} The latter scenario is more likely in airports facing a higher degree of competition.

Despite the intuitive importance of the airport’s position in the EU network and the intensity of (local) competition among airports, as yet there is no empirical evidence supporting the idea that these factors have an effect on airport efficiency. This paper attempts to bridge the gap by building a two-stage econometric model relating network position, the degree of competition, and airport efficiency. The model is applied to a dataset of 57 European airports, including all those with more than 10 million passengers per year, and 90% of those with 5-10 million passengers per year.

Furthermore, the paper develops some proxies for computing the centrality of airports in the network and the intensity of competition coming from other airports in the same catchment

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\textsuperscript{1} The EU liberalization process was composed of three packages, introduced in 1987, 1990 and 1993 respectively. The “eighth freedom” of cabotage (the right for an airline of one EU Member State to operate a route within another EU Member State) was approved in 1997. This single market was extended to Norway, Iceland and Balkan countries in subsequent years. A strong and very liberal aviation agreement is in force with Switzerland. All the relevant legislation can be found at the website http://ec.europa.eu/transport/air_portal/internal_market/competition_en.htm.

\textsuperscript{2} Recently the EU and the US have signed an Open Skies agreement, which should further improve the sector (see details at http://ec.europa.eu/transport/air_portal/international/pillars/global_partners/us_en.htm).

\textsuperscript{3} The number of passengers has increased from 2003 to 2007 at an average annual rate of 6.15% (CAGR). If we include the beginning of the decade, and hence the consequences of the September 11th 2001 terrorist attack on the air transport industry, this rate is slightly lower (4.36%) (see Malighetti et al., 2008a). However, the growth is still bigger than the average annual increase (2001-2007) of 2% in real GDP for the 15 EU nations (Eurostat). The number of routes has increased by more than 60% since 1997 (European Commission, Regulation No 1008/2008).

\textsuperscript{3} This effect should be reinforced by EC Regulation No. 1008/2008, which came into force on November 1st 2008. This law requires airlines to include all taxes and charges in their published ticket prices, and to itemize tariffs, taxes and airport charges. Hence, airline costumers will soon be able to compare the real prices charged by the various European airports, a change which can only increase their competitive pressure.
area. The importance of an airport within the network and the extent to which an airport is crucial to reach some destinations are linked concepts, both closely related to the idea of centrality. A higher centrality should be associated with a greater number of passengers (due to transfer flights) and therefore more intense utilization of the inputs dedicated to passengers: the terminal area, the number of check-in desks, etc. Moreover, the fact that more flights pass through the airport should have a second-order positive effect on aircraft movements.\(^4\) Hence, our first research hypothesis is that airport centrality has a positive impact on efficiency.

Intense development of the air transportation network has created a significant degree of competition between certain groups of nearby airports. Furthermore, the entrance of new airlines has allowed travellers to choose both origin and destination airports along certain routes. Economic theory suggests that more competition should lead to more efficiency. Consequently, our second research hypothesis is that a higher degree of airport competition has a positive impact on efficiency.

The airports’ technical efficiency is investigated using a two-stage bootstrap procedure, following the methodology of Simar & Wilson (2007). In the first stage, Data Envelopment Analysis (DEA) is employed to obtain consistent but biased estimates of the airports’ efficiency scores (Banker et al., 1984; Charnes et al., 1978). In the second stage, the Simar & Wilson procedure performs a truncated regression on the DEA scores to obtain unbiased estimates of the efficiency scores and identify their determinants. This procedure avoids inference problems concerning the statistical significance of the second-stage covariates.\(^5\)

Our main findings are as follows. First, an airport’s centrality in the EU network has a positive impact on its efficiency, especially if we consider the airport’s capacity for moving passengers. Hence, we have identified an empirical relationship between efficient input utilization and the quality of being a “key” airport for passenger mobility within European countries. We also find a weaker effect on aircraft movements. A possible explanation for this difference is that airlines respond to an increase in passenger traffic at a given airport first by choosing higher capacity airplanes, avoiding the necessity of increasing the number of aircraft.

Second, airport efficiency is greater where the intensity of competition coming from nearby

\(^4\) The effect of airport centrality on aircraft movements also depends on the types of aircrafts employed. For example, higher centrality airports may attract larger aircraft, a change which does not necessarily imply more movement. Nonetheless, the overall effect of centrality should still be positive.

\(^5\) The standard two-stage regression adopted in several papers on airport technical efficiency (e.g. Abbott & Wu, 2002; Barros & Dieke, 2007; Gillen & Lall, 1997) yields inference problems for several reasons. See Section 3 for a detailed explanation.
airports is higher. This result has two important policy implications: (1) the welfare of consumers will be enhanced by policies that increase the indirect competition faced by airports (e.g., investing in roads and railways that reduce the time required to reach alternative nearby airports, or increasing the number of slots available in nearby airports); and (2) the need for airport regulation (e.g., of airport charges) is inversely related to the intensity of indirect competition.  

Third, we find evidence for a scale effect related to managing multiple airports. That is, when two (or more) airports are controlled by the same executive board, we find a positive effect on their efficiency in dealing with passengers. This may be explained by economies of scale or learning effects.

Fourth, we find no evidence that the presence of a dominant airline in an airport has a positive effect on its efficiency. A possible explanation for this result is the growing importance of point-to-point connections within the European network.

Our last result is that European airports are more efficient in dealing with passengers than they are with aircraft movements. When we consider aircraft, only 19% of European airports lie on the efficient frontier (see Section 2), this percentage increases to 33% when we look at passengers. In more detail, we find that the largest (category A) airports tend to be more efficient than the national (category B) ones. These results mainly imply that large European airports are operating at full capacity while the national airports have spare capacity.

Related works. Several authors have already investigated various aspects of airport efficiency. Gillen & Lall (1997) analyze a sample of 21 US airports (all among of the 30 largest US airports), and show that the presence of a hub airline and the number of gates both increase the efficiency of aircraft movements. Other studies on the efficiency of US airports are provided by Oum & Yu (2004), Sarkis (2000), and Sarkis & Talluri (2004). Pels et al. (2001, 2003)

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6 Clearly, a higher index of indirect competition between airports does not guarantee effective competition. The managing bodies of airports may be involved in tacit collusion to fix prices (in this case airport charges). However, under these circumstances antitrust authorities should limit the possibility of collusion.

7 The learning effect inherent in managing more than one airport is mainly linked to the value of dealing with different airlines.

8 According to the European Commission, airports can be divided into four dimensional classes: Great European Airports (those with more than 10 million passengers per year), National Airports (those with fewer than 10 million and more than 5 million passengers per year), Great Regional Airports (with fewer than 5 million but more than 1 million passengers), and Small Regional Airports (with fewer than 1 million passengers).

analyze the efficiency of 34 European airports (from 1995 to 1997) using both parametric and non-parametric techniques, and show that several airports are inefficient. Oum et al. (2008) investigate the impact of different forms of ownership on the efficiency of 109 airports worldwide, and find that those controlled by private firms, autonomous public corporations, or independent authorities are more efficient than public airports.

Other papers analyze airport efficiency within single countries. Parker (1999) investigates the impact of privatization on a sample of 22 British airports, finding that the change has no impact on efficiency. Yoshida (2004) and Yoshida & Fujimoto (2004) focus on Japanese regional airports, which seem to be less efficient than others because they suffer from political pressure. After computing the efficiency scores and Malmquist Indices of Chinese airports, Fung et al. (2008) find that size has a positive effect on efficiency. They also show that airport productivity in China has grown over the period 1995-2004. The Australian airports have been investigated by Hooper & Hensher (1997) and Abbott & Wu (2002), both works showing again that privatization has no impact on efficiency. Fernandes & Pacheco (2002) and Pacheco & Fernandes (2003) provide a benchmark analysis of the performance of Brazilian airports, focusing on domestic airports. Barros & Sampaio (2004) provide benchmarks and determinants of economic efficiency for a sample of 10 Portuguese airports, arguing that Portuguese airports should be privatized. Martin & Roman (2001), Martin Cejas (2002) and Murillo Melchor (1999) have all evaluated the efficiency of Spanish airports. Barros & Dieke (2007, 2008) have published two papers on the efficiency of Italian airports. In the first they analyze 31 airports over a period of 3 years (2001-2003), using balance sheet data to obtain information about certain inputs (labor costs, invested capital, and operational costs). In the second paper they adopt the Simar & Wilson (2007) methodology to perform a second-stage analysis using the same dataset, i.e., they identify the determinants of efficiency. In particular, they show that larger airports and private airports tend to be more efficient than others. Malighetti et al. (2007) also investigate the Italian sector, building a dataset of 34 airports over the period 2005-2006. They collect their physical inputs directly from the airports, rather than working with balance sheet data, and show that efficiency is higher

European airports, and 21 airports from Asian countries. They show that both airport size and capacity constraints (which create costs paid by airlines and passengers) improve airport productivity.

Regional Japanese airports have more capacity than they need (over capacity) because local politicians direct more investment to their region, in order to gain consensus.

Martin & Roman (2001) show that many Spanish airports are inefficient, Martin Cejas (2002) presents evidence that these inefficiencies are related to size, and Murillo Melchor (1999) finds that large airports have decreasing returns to scale.
if the airport is dominated by a single carrier.

This research contributes to the literature by analyzing the efficiency of a large (57 airports) and balanced (all airports with more than 10 millions passengers are included in the sample, and small airports are excluded) dataset of European airports. In both aspects this sample improves on that used by Pels et al. (2001, 2003), the most directly comparable study. Outputs and physical inputs (runways, terminal surface, etc.) for the airports were collected for the year 2006. The sample covers 100% of the largest European airports (of which there are 31) and 90% of those classified by the EU Commission as National airports (26 out of 29).\footnote{The sample does not include the airports of Belfast, Bristol and Nice.}

The paper proceeds as follows. Section 2 describes how we compute the importance of airports in the European network and the index of indirect competition. In Section 3 we present the two-stage Simar & Wilson (2007) econometric model. In Section 4 we describe our dataset and present summary statistics on the European airports. Section 5 presents the DEA results, while the correlations provided by our econometric model are described in Section 6. Concluding comments are found in Section 7.

2. Methodology: airport connectivity index and indirect competition index

This Section describes how to compute several indices of network connectivity and the index of competitive pressure.

2.1 The airport connectivity index

In order to assess an airport’s importance to the European network, we begin by considering measures of connectivity derived from network theory. More precisely, we apply the shortest path length ($SPL$) approach to obtain a proxy for an airport’s centrality in the European network. The $SPL$ between airport $A$ and airport $B$ is defined as the minimum number of steps required to connect $A$ and $B$. For example, if there is a direct link between airport $A$ and airport $B$, then $SPL_{AB} = 1$. If $A$ and $B$ are not directly linked but both are connected to a third airport $C$, then $SPL_{AB} = 2$. To describe a network of $N$ airports, a $N \times N$ adjacency matrix $H$ is used. An element $h_{ij}$ is equal to 1 if and only if there is a direct connection between airport $i$ and airport $j$; otherwise it is set to 0. A standard algorithm is employed to calculate the minimum number of steps
between each pair of airports (Bagler, 2004).

The average SPL of an airport is a proxy for its connectivity, but this measure does not quantify the importance of its role as an intermediate node between airports that are not directly connected. For this purpose, graph theory has developed an indicator known as betweenness centrality (Freeman, 1977; Wasserman & Faust, 1994).

Following the work of Guimerà et al. (2005), we define the “Betweenness” $B_k$ of airport $k$ as the number of minimal paths within the entire network that pass through airport $k$. The higher the value of $B_k$, the more central the airport and the more important its connecting role. $B_k$ is defined formally as follows:

$$B_k = \sum_{i=1,i\neq k}^{N} \sum_{j=1,j\neq k}^{N} \phi_{i,j},$$

where $\phi_{i,j} = 1$ if the shortest connection between airport $i$ and airport $j$ passes through airport $k$, and 0 otherwise.

Calculation of SPLs at the European level shows that many optimal connections have more than one possible path. This is mainly due to the high level of integration associated with the most important European airports, which collectively provide several alternative routes between minor destinations with $SPL \geq 2$. To distinguish those cases where a passenger has no alternative but to pass through airport $k$ to reach a specific destination, we introduce a new measure of centrality called Essential Betweenness ($EB$). This is the number of unavoidable minimal paths passing through an airport, i.e., the number of minimal paths that are unique solutions for their nodes. Figure 1 shows the difference between these two measures of network centrality. In this example, airport $E$ can only be reached by passing through airport $D$. Airports $B$, $C$, and $D$ each have some degree of Betweenness. However, only airport $D$ has a measure of Essential Betweenness.$^{13}$

[insert Figure 1 here]

$^{13}$ When analyzing the connection A-E we find two different Shortest Path Lengths (SPL1 and SPL2), each with 3 steps (A-B-D-E and A-C-D-E). In this case, the A-E connection must be counted when calculating the Betweenness of airports B, C, and D. The A-E connection is counted only for airport D, however, when calculating Essential Betweenness.
The Essential Betweenness for an airport \( k \) is defined as follows:

\[
EB_k = \sum_{i=1, i \neq k}^{N} \sum_{j=1, j \neq k}^{N} \chi_{i,j} \phi_{i,j},
\]

where \( \chi_{i,j} \) equals 1 when all the shortest paths between airport \( i \) and airport \( j \) pass through airport \( k \). The indicator variable \( \chi_{i,j} \) equals 0 when one or more of the shortest paths linking airport \( i \) and airport \( j \) does not pass through airport \( k \). It is clear that \( EB_k \leq B_k \).

The aforementioned measures of connectivity and centrality have some shortcomings. One is that the optimal connection between airport \( i \) and airport \( j \) does not take into account flight frequency. After all, a connection with only one flight per year cannot be compared on equal terms to a daily connection. Second, traditional shortest path approaches neglect routing factors. The routing factor of a path between airport \( i \) and airport \( j \) is defined as the ratio between the sum of in-flight distances and the great circle distance. The higher the routing factor, the longer the detour passengers have to take to complete the connection. Malighetti et al. (2008) show that among the three major airport networks (European, US and Chinese), the average routing factor is 1.07 for 2-step quickest connections and 1.11 for 3-step connections. This result means that on average, optimal 2-step connections allow for a 7% increase in flight distance with respect to the minimum “as the crow flies” distance. An optimal connection with a routing factor of 1.05 cannot be directly compared to an alternative having a routing factor of 1.4.

We thus modify the above definitions of Betweenness and Essential Betweenness to take into account the effects of flight frequency and routing factors. The adjusted Betweenness index, \( \tilde{B}_k \) is defined as

\[
\tilde{B}_k = \sum_{i=1, i \neq k}^{N} \sum_{j=1, j \neq k}^{N} \frac{\phi_{i,j} \cdot f_{i,j}}{r_{i,j}},
\]

where \( f_{i,j} \) is the minimum flight frequency on the optimal connection passing through airport \( k \). For example, given an optimal two-step path \( i-k-j \), \( f_{i,j} \) is the lesser of the \( i-k \) frequency and the \( k-j \) frequency. We use the minimum frequency in order to take into account bottleneck flights which could undermine the connection’s desirability from a passenger perspective. The routing factor of the \( i-j \) connection is \( r_{i,j} \). All else being equal, this measure of Betweenness assigns greater importance to connections with high frequencies and low routing factors.
The Essential Betweenness index is adjusted as follows:

\[
\overline{EB}_k = \sum_{i=1, i \neq k}^{N} \sum_{j=1, j \neq k}^{N} x_{i,j} \cdot \Phi_{i,j} \cdot f_{i,j} \cdot r_{i,j}.
\]

As a proxy for how essential an airport is to reach some destinations, we also compute the ratio \( R_k = \overline{EB}_k / \tilde{B}_k \); the weighted percentage of shortest connections passing through airport \( k \) which have no alternative route of equal length. At \( R_k = 0 \), the airport is never essential to reach other destinations, even if it has a high Betweenness \( \tilde{B}_k \). At \( R_k = 1 \), a passenger must always pass through airport \( k \) when it lies on a shortest path to another destination. In other words, Betweenness merely represents an airport’s potential for indirect connectivity. A high value of \( R_k \), however, indicates that the airport is a vital connector for some portion of the network.

The \( \tilde{B}_k \) and \( \overline{EB}_k \) indices were calculated using the Innovata database\(^{14} \), which publishes information on scheduled flights. For each flight, the following data are available: departure airport, departure time, arrival airport, arrival time, frequency, and operating airline. Our analysis of SPLs includes all 478 European airports with at least one scheduled passenger flight during October 2007.

Figure 2 plots the centrality indices \( \tilde{B}_k \) and \( \overline{EB}_k \) against each other for the 57 European airports in our sample. The main hubs are characterized by high Betweenness levels and low Essential Betweenness levels. Note that Betweenness represents an airport’s potential for indirect connectivity; for instance, Amsterdam Schiphol is a possible intermediate node for more than 33,000 pairs of European airports, but its Essential Betweenness is only about 1.5%. On the other hand, a high level of Essential Betweenness indicates that the airport is vital connector to some nodes of the network (i.e., some regions in Europe). The Scandinavian airports (Stockholm Arlanda, Helsinki Vantaa, Oslo and Copenhagen) act as gateways to local national airports, and thus have the highest values of Essential Betweenness (well over 10%). A similar role is played by Athens in Greece, and by Orly in France. London Stansted is both a local gateway and a connecting node to many European airports, especially those catering to low-cost carriers.

[insert Figure 2 here]

\(^{14}\) Innovata is a provider of Scheduled Reference Services in partnership with IATA. The SRS airline schedules database contains data from over 892 airlines worldwide.
2.2 The airport indirect competition index

Our index of indirect competition for each airport is again based on features of the EU network, and is closely related to the airport’s catchment area. In air transportation, each origin-destination route defines a separate market. Hence, we assume that the airport is subject to some “indirect competition” on a specific route if the same connection is available at other nearby airports. Naturally, the same standard of closeness should be applied at both origin and destination. We consider an alternative airport “close enough” (to generate competition) if its geographic distance from the airport under consideration is no more than 100 km. Thus, for a given connection between two airports, there may be several “close” alternatives to the departure airport (the “origin”) and several “close” alternatives to the arrival airport (the “destination”).

For instance, consider a generic airline’s connection between the Rome Fiumicino and London Heathrow airports. The relevant market is the Rome-London route. Let us now suppose that another (or the same) airline provides a connection between Rome Ciampino and London Stansted. The Rome Ciampino airport lies less than 100 km away from Rome Fiumicino, and London Stansted is less than 100 km from London Heathrow. Both airports are therefore “close” with respect to the route under consideration. Hence, the Rome Fiumicino-London Heathrow connection is subject to indirect competition from the Rome Ciampino-London Stansted connection.

Let $w^k_i$ refer to a connection $i$ available at airport $k$. Let $z^k_j$ denote a connection $j$ at airport $k$ which is also available at an airport “close” to $k$. The indirect competition index of airport $k$ is then defined as $IC_k = \frac{\sum_j z^k_j}{\sum_i w^k_i}$: the ratio of the total number of connections available at both airport $k$ and “close” airports to the total number of connections at airport $k$. In computing the indirect competition index, we replace $w$ and $z$ with the Available Seat Kilometers (ASK) associated with each connection, i.e., the product of offered seats and flight distance. When $IC_k$ is zero, the airport is not subject to indirect competition from other airports. When the ratio is one, all the connections supplied by the airport (and therefore all their ASK) are also offered by “close” airports. In the latter case, the airport is under the highest possible “competitive pressure”.

For each European airport in our sample, we counted the number of “close” airports and identified the connections subject to indirect competition. For each airport, we then sum the ASK of connections subject to indirect competition. Lastly, we calculate $IC_k$, the percentage of total ASK at each airport subject to indirect competition.\textsuperscript{15}

Figure 3 shows the indirect competition indices for all European airports included in the sample. Notice that the major European hubs (e.g. London Heathrow, Paris Charles de Gaulle, Frankfurt and Amsterdam Schiphol) are subject to very low levels of indirect competition, or even a complete absence of indirect competition in the case of Madrid Barajas. The highest levels of indirect competition are observed in airports belonging to the Milan catchment area in Italy (Milan Linate and Bergamo Orio al Serio) and the London area (London Stansted and London Luton).

[insert Figure 3 here]

3. The two-stage Simar & Wilson econometric model

Our main purpose in this section is to estimate the effects of the connectivity and indirect competition indices on airport efficiency. We adopt a two-stage econometric model: in the first stage we obtain efficiency scores for each airport in the sample using Data Envelopment Analysis (DEA), and in the second stage we follow the approach proposed by Simar & Wilson (2007) to estimate the effect of some covariates on the efficiency scores.

The DEA approach, which assumes that all airports have access to the same technology, transforms a vector $\mathbf{x}$ of $L$ inputs into a vector $\mathbf{y}$ of $M$ outputs. The DEA model, through a sequence of linear programming methods, creates a piecewise linear “frontier” encompassing the data.\textsuperscript{16} There are two varieties of DEA analysis to choose from: a Constant Return to Scale (CRS) model (Cooper \textit{et al.}, 1978) and a Variable Return to Scale (VRS) model (Banker \textit{et al.}, 1984). The two models are used to distinguish between Technical Efficiency (TE) and Scale Efficiency

\textsuperscript{15} We acknowledge that this definition presents some drawbacks. For instance, simple distance is always a good proxy for the travelling time, which would be the optimal driver for this analysis. However, we believe that in the absence of information on travelling times, the simple distance may be considered a good approximation of the origin and destination airports’ accessibility.

\textsuperscript{16} Under this approach, the efficiency of an airport is estimated relative to the performance of other airports.
Using Farrell’s (1957) input-oriented measure of technical efficiency, the efficiency score $\delta_i$ of airport $i$ is defined as follows:

$$\delta_i = \min_{\delta} \{ \theta : (\theta \mathbf{x}_i, \mathbf{y}_i) \in P \}.$$  

where $P$ is the technology set, i.e., $P = \{ (\mathbf{x}, \mathbf{y}) : \mathbf{x} \text{ can produce } \mathbf{y} \}$. $\delta_i$ measures the Euclidean distance from the point $(\mathbf{x}_i, \mathbf{y}_i)$ to the boundary $P$, along a direction orthogonal to the output axes and parallel to the input axes ($0 \leq \delta_i \leq 1$, where an airport is efficient if $\delta_i = 1$). The assumption is that some airports are positioned on the frontier of efficiency allowed by available technology; those located on the frontier are “efficient”, while all others are “inefficient”. Since $P$ is unobserved we can use the estimator $\hat{\delta}_i(\mathbf{x}_i, \mathbf{y}_i | P)$ as written. If the CRS model is adopted, it will be expressed in terms of the linear program

$$\hat{\delta}_i = \min \{ \theta > 0 | \mathbf{y}_i \leq \mathbf{y}, \theta \mathbf{x}_i \geq X q, q \in R^n \};$$

if the VRS model is adopted, we instead use the linear program

$$\hat{\delta}_i = \min \{ \theta > 0 | \mathbf{y}_i \leq \mathbf{y}, \theta \mathbf{x}_i \geq X q, s' q = 1, q \in R^n \}; \quad (1)$$

In either case $Y = [ \mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_n ]$ ($n$ being the number of airports in the sample), $\mathbf{y}$ is a vector of

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17 See Charnes et al. (1978), Coelli (1996), and Färe et al. (1994) for a discussion of the DEA model. The choice between CRS and VRS usually depends on the context and purpose of the analysis (managerial benchmarking would use VRS, while a long-run analysis would use CRS), on the length of the time interval covered by the data (VRS is more appropriate for short intervals), and on the relevance of external factors (e.g. regulation, time limits to the hours of operation, weather conditions) which might limit the airport’s ability to operate at the optimal scale. See Barros & Sampaio (2004) and Pels et al. (2003) for a more thorough analysis of the latter point. The size of the available sample may also be relevant to the choice between CRS and VRS. In small samples there are few large units, so the VRS model would tend to find them all efficient for want of meaningful comparison. Our sample is rather homogenous (only the large European airports are considered) and covers a short period, so the VRS model seems more appropriate.

18 We choose an input-oriented DEA model, since we assume that decisions concerning the output levels are beyond the control of airport managements (Gillen & Lall, 1997; Pels et al., 2003).
outputs for airport $i$, $X = [x_1, x_2, \ldots, x_n]$, $x_i$ is a vector of inputs for airport $i$, $q = [q_1, q_2, \ldots, q_n]$ is a vector of intensity variables, and $\mathbf{s}$ a column vector of 1’s. The parameters $(\theta, q)$ are solutions of the problem.\(^{19}\)

As shown by Kneip et al. (1998), $\hat{\delta}_i$ is a consistent estimator of $\delta_i$. It has a low rate of convergence, however, and is biased downward (by construction).\(^{20}\) Hence, a bootstrap procedure appears to offer the only way to approximate the asymptotic distribution of $\hat{\delta}_i$ in a multivariate setting (Simar & Wilson, p. 38). This approach will be useful in the second stage, where we want to estimate the following regression model ($i = 1, 2, \ldots, n$):

$$\delta_i = a + Z_i \beta + \epsilon_i \leq 1,$$

where $a$ is a constant term, $Z_i$ is a row vector of observation-specific environmental variables that may affect the relationship between inputs and outputs, $\beta$ is a column vector of free parameters, and $\epsilon_i$ is a continuous iid random variable. The above relation is estimated using a censored Tobit regression. In the past, it was common to carry the first-stage estimates $\hat{\delta}_i$ over to the second stage as a proxy for the unobserved distances $\delta_i$. Simar & Wilson showed that this approach has inference problems.\(^{21}\) Furthermore, the bias term in the first-stage DEA estimate has a nonzero mean. Fortunately, the bias term may be estimated by bootstrap methods (Simar & Wilson, p. 39). The bootstrap bias estimate is equal to the true bias plus a residual term whose

\(^{19}\)It is important to remember that the environment where we applied the DEA model has to be homogeneous. In Europe the regulations are not uniform across airports, so the costs of several inputs (labor, electricity, fuel, etc.) should vary as well. However, these differences are not so large as to prevent application of the DEA approach to the airport sector, as shown by many past contributions to the literature (e.g. Pels et al., 2003).

\(^{20}\)The rate of convergence is defined by the relation $\hat{\delta}_i = \delta_i + \phi_i (n^{-2/(L+M)})$, where $L$ is the number of inputs and $M$ is the number of outputs. The rate slows as $L + M$ increases, the well-known curse of dimensionality.

\(^{21}\)The $\hat{\delta}_i$ are serially correlated in a complicated and unknown way, since they depend on all observations $(X_i, Y_i)$ in the dataset (through $P$). Consequently, the error term $\epsilon_i$ is also serially correlated. Moreover, $X_i$ and $Y_i$ are correlated with $z_i$ (otherwise the second-stage regression is meaningless) so the $z_i$ are correlated with $\epsilon_i$. This means that maximum likelihood estimates of $\beta$ will be consistent, but will not have the usual parametric rate of $n^{-1/2}$. For $L + M > 3$, correlation among the errors does not disappear quickly enough for standard inference approaches.
variance diminishes as $n \to \infty$. It can therefore be used to construct a corrected estimator of $\delta$.

Maximum likelihood estimation using the bias-corrected distance function estimators yields consistent and efficient results, as shown by Simar & Wilson in Monte Carlo experiments. Hence, we include an estimate of the bias in the second-stage regression, following the procedure described by Simar & Wilson. Specifically, we modify their Algorithm #2 in order to perform an input-oriented DEA estimation. The details of this procedure are given in the Appendix.

3. The data

The data set used in this contribution is composed of information collected directly from a sample of 57 European airports for the year 2006. The sample covers all of the largest European airports (those classified in category A) and 90% of the so-called large national airports (classified in category B). To obtain the needed inputs, for example the number of aircraft parking positions or baggage claim lines, we contacted each airport’s management directly. We investigated 60 airports, but 3 of them (5%) did not provide the necessary information.

Following Gillen & Lall (1997) and Pels et al. (2003), we regard airports as the interface between airlines and passengers. Hence we need to consider two kinds of activity: Air Transport Movements (ATM) and Air Passenger Movements (APM). ATM is both an output (for aircraft movements) and an input (for passenger movements). This means that we can estimate the airport’s efficiency in terms of either ATM (without considering APM) or APM (where ATM is treated as an input).

For each airport we collected information on two output variables: the yearly number of aircraft movements (ATM) and the yearly number of passenger movements (APM). To model the ATM efficiency frontier, we collected the following inputs: the total surface area of the airport (AREA), the total length of its runways (RUNWAYS), and the total number of aircraft parking positions (PARKING). Analysis of the APM frontier involves the following inputs: the yearly number of aircraft movements (ATM), the terminal surface area (TERMINAL), the number of check-in desks (CHECK), the number of the aircraft parking positions (PARKING), and the number of baggage claim lines (CLAIM). Table 1 presents descriptive statistics for each of these output and input variables over the entire sample.

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Their algorithm is designed for output-oriented DEA estimation.
The average number of passengers per airport is about 17 million (considering only European airports with more than 5 million passengers per year). The average number of aircraft movements is about 188,000. The average airport’s infrastructure includes 136,635 square meters of terminal space, 81 aircraft parking positions, 130 check-in desks, and 11 baggage claim lines. It covers a surface area of 906 hectares, including 6,471 meters of runways (most airports in the sample have multiple runways).

4. DEA results

In this section we present the airports’ DEA scores, i.e., their distances from the estimated efficiency frontier. The impact of certain variables on these scores, obtained by applying the Simar & Wilson approach, is described in the next section. We report efficiency scores based on CRS and VRS analysis. The former is obtained by applying the Charnes et al. (1978) model, wherein we assume constant returns to scale between inputs and outputs. VRS efficiency scores are computed by applying the Banker et al. (1984) model, which assumes variable returns to scale and measures only the airports’ technical efficiency (without considering their scale efficiency). We complete the section by computing the direction (increasing or decreasing) of observed returns to scale in individual airports.24

Table 2 shows the DEA efficiency scores of European airports in terms of ATM, reporting distances from both the CRS and VRS frontiers. There are 11 airports on the VRS frontier (i.e., with $TE = 1$): five (16%) category A airports (London Heathrow, Paris Charles De Gaulle, Frankfurt, Dublin, and Manchester), and 6 (23%) category B airports (Larnaca, Milan Linate, Faro, Liverpool, London Luton, and Newcastle). The average distance from the VRS frontier among those airports with $TE < 1$ is 0.27 for category A, and 0.30 for category B. Thus, the average inefficiency is somewhat greater among national airports. These results imply that large

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23 ATM can be considered an intermediate good produced by the airport and consumed in the production of APM.

24 The direction of the returns to scale is obtained by solving the problem shown in expression (1) after substituting the constraint $s'q = 1$ for $s'q \leq 1$. If the new estimate of $\delta$ is less than 1 and the new value of $\theta$ is equal to (less than) the old value of $\theta$, then we have Decreasing (Increasing) Returns to Scale. This result is denoted D(I)RS. CRS indicates Constant Returns to Scale.
European airports are working either at full capacity or close to it, while the national airports have capacity to spare.²⁵

Looking for national differences among the largest countries, we note that in France there is only 1 efficient airport (out of 6), and that the other airports’ average distance from the frontier is 0.43. The same number of efficient airports is reported for Germany (1 out of 9) and Italy (1 out of 7), while no efficient airport belongs to Spain. The average distances of inefficient airports from the frontier in these countries are 0.31, 0.23, and 0.19 respectively. The UK has the greatest number of efficient airports, 5 out of 10, and the average distance from the frontier is equal to 0.27. We conclude that the first country in Europe to implement both liberalization and incentive regulations (based on a price cap) is now reaping the benefits of higher efficiency in airport management.

The very large airports (Heathrow, Paris, Frankfurt, Amsterdam, etc.) exhibit decreasing returns to scale, signaling that they would incur lower average costs by decreasing the scale of operations. Among category A airports, about two-thirds (20) have increasing returns to scale, while 9 have decreasing returns to scale. Only 2 are operating at the optimal scale (Dublin and Manchester). Nearly all the category B airports operate at increasing returns to scale (25 out of 26). Hence, there is evidence that category B airports may benefit from a reduction in average costs if they increase their scale of operations, i.e., the volume of aircraft movements.

The DEA efficiency scores obtained when considering output passengers (APM) are shown in Table 3. There are 19 airports on the frontier, of which 13 (43%) belonging to category A and 6 (22%) to category B. In general, we note that European airports are more efficient with passengers than with aircraft.²⁶ The average distance from the efficient frontier among category A airports (computed for those airports not on the frontier) is 0.23, while that for category B airports is 0.26. Again we see that larger airports are more efficient than national airports, but at the same time both types are somewhat closer to technical efficiency than they appear when considering only aircraft movements.

²⁵ As mentioned before when dealing with the VRS model, some caution is necessary in judging the efficiency scores reported for the very largest airports (i.e., London Heathrow and Paris Charles De Gaulle). Indeed, under the CRS model where these two airports are compared with the entire sample and not just to each other, their efficiency is lower. This is especially true for Paris Charles De Gaulle.
If we compare individual countries, Spain now shows the highest relative efficiency: 7 out of 8 airports lie on the efficient frontier. This is quite a reversal from the ATM scores, where Spain was in the worst position. Italy and the UK each have 3 airports on the frontier (out of 7 and of 10 respectively), while France and Germany have only one efficient airport each. The average distances of inefficient airports from the frontier are as follows: 0.34 for France, 0.30 for Germany, 0.20 for Italy and 0.15 for UK. Hence we can fairly say that Spain is the most efficient country concerning passengers, while France is the least efficient.

Looking at the returns to scale, we again find that most of the largest European airports exhibit decreasing returns. The important exceptions are London Heathrow, Munich (constant returns) and London Gatwick (increasing returns). There are 9 category A airports (out of 31) with decreasing returns to scale, 12 with constant returns to scale and 12 with increasing returns to scale. Considering airport efficiency in terms of passengers thus yields an interesting result: more than one-third of the largest European airports currently enjoy the benefits of operating at the lower bound of the average costs curve. An equal number could reduce their average costs in dealing with passengers by increasing their scale of operations. Almost all of the European category B airports (24 out of 26) have increasing returns to scale; the exceptions are two Spanish airports (Alicante and Tenerife) exhibiting constant returns to scale. Hence, the European national airports can reduce their average costs by increasing the number of passengers. These observations again show that national airports have spare capacity.

These results point out that a similar number of large airports and almost all of the national airports lie far from the efficiency frontier. These installations should first improve the utilization of their capacity by increasing their scale of operations (the number of passengers), in order to take the advantage of lower average costs. We also observe that it is easier to reach the efficiency frontier and an optimal scale by managing passengers rather than aircraft movements.

Figure 4 shows the efficiency scores of each airport, in terms of both aircraft movements and passengers. Only 4 airports lie on both frontiers: Larnaca, Dublin, London Heathrow and Manchester. The majority (35) of the remaining 53 airports are more efficient in dealing with passengers than with aircraft movements. The least efficient European airports at the moment are

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A possible explanation may be a greater degree of input indivisibility when dealing with aircraft movements.
located in the middle of the plot: Marseille, Warsaw and Hanover. The correlation between the two efficiency scores is positive but rather low (0.53), showing that it is rather difficult to achieve efficiency on both counts.

To sum up, this analysis has shown that inefficiency tends to be higher in smaller airports. This is true for both of the outputs considered in this paper (aircraft movements and passengers). This implies that most of the European airports with less than 10 million passengers appear to have spare capacity. In general, airports exhibit a greater ability to manage passengers than aircraft movements. This difference may be due to an exogenous shock (the robust increase in demand for passenger air transportation due to the development of low-cost carriers), to regulatory constraints (e.g., time limits in aircraft movements), or to a lack of competition between airports in attracting carriers.

6. The determinants of efficiency

We now turn to the main goal of this paper, discovering the impact of network centrality and intensity of competition on the efficiency scores of European airports. As mentioned in the Introduction, we set out two main hypotheses: (1) the more an airport is “central” to the European network, the higher its efficiency, since centrality presumably permits a more intense exploitation of the airport’s physical inputs; and (2) the greater the intensity of competition on a given airport, the higher its efficiency, since a more competitive environment incites management to attract airlines by increasing productivity factors.

Assuming cross-sectional heteroskedasticity (see Abbott & Wu, 2002), the DEA efficiency scores are regressed on a number of exogenous variables covering specific characteristics of the airports. As previously described, this regression follows the Simar & Wilson bootstrap methodology. The two dependent variables are $\delta_{ATM}$ and $\delta_{APM}$, the airports’ distances in 2006 from the VRS frontiers regarding aircraft movements and passenger movements respectively. We consider five explanatory variables: BETWEEN (the adjusted Betweenness connectivity index, $\bar{\beta}_k$, showing how well the airport is connected with all other European airports),

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27 The bootstrap analysis was performed using a Matlab program.
ESSEN BETWEEN (the degree to which an airport is necessary to reach other territories within Europe, i.e. \( R_k \)), COMPETITION (the fraction of flights departing from the airport that could be replaced by other flights departing from nearby airports and/or arriving at a nearby destination, i.e. \( IC_k \)), DOMINANCE (a variable reflecting the presence of a dominant carrier, computed as the percentage of total ASK supplied by the first carrier in a given airport), and MULTI_AIRPORT (a dummy variable, equal to 1 if the management of airport \( i \) also controls at least one other airport in the sample). We estimate the following model:

\[
\delta^h_i = \alpha + \beta_B \cdot BETWEEN_i + \beta_{EB} \cdot ESSEN_BETWEEN_i + \beta_C \cdot COMPETITION_i + \\
+ \beta_D \cdot DOMINANCE_i + \beta_{MA} \cdot MULTI_AIRPORT_i + \epsilon_i
\]

where \( \delta^h_i \) is the DEA score of airport \( i \) for output \( h \) (\( h = \{ATM, APM\} \)) obtained after applying the Simar & Wilson procedure. Table 4 presents summary statistics of the explanatory variables. About 35% of the European airports belong to a group, the average index of indirect competition is 35.3% (with a high standard deviation), and the airline with the highest market share in the representative airport is 35%. On average, a given European airport is part of almost 12,000 shortest paths connecting origin/destination pairs (Betweenness). However, the vast majority of these connections can be performed with the same number of steps by passing through other intermediate airports. Only 1.5% of these connections do not have such alternative paths (Essential Betweenness).

Table 5 displays the correlation matrix among the six input variables (CLAIM, CHECK, TERMINAL, PARKING, RUNWAYS and AREA) in its upper rows, and the five Tobit regression covariates in its lower rows. The computed correlation is low for all covariates.
The results of the Simar & Wilson methodology are reported in Table 6. Columns 2-5 of Table 6 provide the estimated coefficients and significance levels, then the lower and upper bounds of the confidence interval obtained after regressing DEA Technical Efficiency (TE) scores on the explanatory variables. This first group refers to efficiency in managing aircraft movements. The second group (columns 6-9) is identical, but refers to efficiency in managing passengers. Significance levels at 1%, 5% and 10% of the confidence intervals are labeled with one, two and three asterisks respectively.

[insert Table 6 here]

The truncated regression seems to fit the data well, with statistically significant \(t\)-statistics for all parameters (if we consider both ATM and APM) except airline dominance. Thus, both of the research hypotheses mentioned above are confirmed in general terms.

While the number of shortest-path interconnections (BETWEEN) is significant under both models (ATM and APM), the quality of being necessary to reach some territories within Europe (ESSEN_BETWEEN) is only significant when we consider aircraft movements (ATM). Furthermore, efficiency is higher for airports with greater values of BETWEEN, and lower for airports with greater values of ESSEN_BETWEEN. The latter result shows that when an airport is essential to certain segments of air traffic, it is more difficult to optimize the use of its capacity. A possible explanation is that the local economies of these European regions have a low degree of attractiveness for the airlines. This, in turn, penalizes the airports that are gateways for these regions.

Efficiency in managing aircraft movements is not significantly influenced by competitive pressure from other airports or the dominance of a single carrier. However, increased competition does make airports more efficient at managing their passenger traffic. The presence of a substitute airport for the same connection probably induces management to reduce fares (or to offer better services) so airlines can attract more customers. Finally, there is evidence that managing more than one airport increases the overall efficiency. Since in our sample it is generally nearby airports that share management, our finding may be due to the deliberate
shifting of passengers toward nearby airports with less congestion; the recent experience of London Heathrow and London Gatwick is one example. Moreover, operators managing multi-airport systems can extract gains from specialization, for example by concentrating domestic flights in one airport and intercontinental flights in another. A further explanation for the scale effect is market power: with more than one airport under its control, the management can reduce the airlines’ buyer power and achieve more efficient asset utilization by optimally allocating flights among airports.

To sum up, our analysis shows that technical efficiency tends to be higher for both outputs considered in this study, among airports that are well connected within the European network. This new result should be taken into account by governments and the European Commission when dealing with airport development programs and policy. It is also of interest to airport managers, who should consider connectivity as one of the driver to improve their efficiency. Another interesting result is that efficiency in dealing with passengers is higher among airports exposed to some degree of competitive pressure (routes offered by other airports within the same catchment areas). Policy makers will find that developing competitive airports is a good way to encourage efficiency in their operations. The efficient utilization of inputs dedicated to aircraft movements (runways, aircraft parking positions, etc.) is less likely, however, when the airport is essential to reaching a particular region within Europe. This fact is probably due to the low degree of attractiveness of these regions for the airlines.

6. Conclusions

This paper has investigated the efficiency of 57 large European airports by applying a two-stage econometric model based on Simar & Wilson’s (2007) bootstrap methodology. The sample covers about 95% of European airports serving more than 5 million passengers per year. The econometric technique used here is superior in many ways to those usually adopted in the literature to estimate airport efficiency. The panel is balanced so that estimation techniques are less influenced by heterogeneity. The paper also proposes some indices to measure the centrality of an airport within the European network and the intensity of competition exerted by airports located in the same catchment area.

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28 During the last decade, British Airways’ response to increased congestion at London Heathrow has been the
We find that an airport’s importance in the European network has a positive impact on its efficiency. This means that the greater an airport’s contribution to the development of the network (and therefore to passenger and freight mobility within the European Union), the more complete its utilization of physical assets. We also identify a positive link between the intensity of competition and efficient utilization of physical inputs. This correlation implies that productivity is higher when the airport cannot operate as a monopolist. When multiple airports compete in the same catchment area, it also means that more passengers are traveling to and from that region, boosting its economic development. Another interesting insight is the positive impact of multi-airport management on airport efficiency. This means that economies of scale and learning effects can be exploited when operating with multiple units in the same region. Finally, we do not find any evidence that the presence of a dominant airline affects airport efficiency. In this we differ from some previous results (e.g. Gilles & Lall, 1997 and Malighetti et al., 2007).

We have shown that efficiency is also related to airport size: airports with more than 10 million passengers are more efficient than medium-sized airports with between 5 and 10 million passengers. Further development in the activities of large airports may lead to an increase in their average costs, since most of these are operating under decreasing returns to scale. In contrast, we find evidence for spare capacity in many medium-sized airports operating under increasing returns to scale.

We can infer some policy recommendations from the above evidence. First, the link between airport competition and efficiency suggests that air transportation policies should focus on increasing the competition within important catchment areas (possibly by investing in infrastructure to reduce the access time to alternative airports). A secondary benefit of competition is that airport charges regulations are less important when more routes are under competition (i.e., when the same origin and destination are available from another airport in the same catchment area).

Second, public policy should provide incentives for airports to develop their role in the European network, for instance by subsidizing routes connecting regional airports to the main European hubs or new point-to-point connections between airports with spare slots. Third, the positive effect of shared management on the efficiency of airport groups suggests that any decision to break up airport groups (e.g. the separation of London Heathrow and London Gatwick now under discussion) should be considered with care. Finally, the absence of a dominant airline transfer of flights to London Gatwick.
effect on efficiency may indicate that competition within an airport is not a factor limiting airport efficiency. All of these suggestions move in the same direction: further development of the EU network, and encouraging competition between airports and airlines belonging to the same catchment area.

Appendix: The Simar & Wilson bootstrap procedure

The procedure consists of 7 steps, and contains two loops.

Step 1. We compute \( \hat{\delta}_i \) using DEA.

Step 2. We apply the maximum likelihood method to obtain the estimates \( \hat{\beta} \) and \( \hat{\sigma}_\varepsilon \) in the truncated regression \( \hat{\delta}_i = a + Z_i \beta + \varepsilon_i \leq 1 \), using the \( m < n \) observations where \( \hat{\delta}_i < 1 \).

Step 3. This loop, performed \( L_1 \) times,\(^{29}\) obtains a set of bootstrap estimates \( \Gamma_i = \{ \hat{\delta}_{ib} \}_{b=1}^L \). It consists of four substeps:

3.1. For each \( i = 1, \ldots, n \), we draw \( \varepsilon_i \) from \( N(0, \hat{\sigma}_\varepsilon) \);

3.2. For each \( i = 1, \ldots, n \), we compute \( \hat{\delta}^*_i = Z_i \beta + \varepsilon_i \);

3.3. We set \( \check{x}_i = \frac{\hat{x}_i}{\hat{\delta}_i}, \check{y}_i = \hat{y}_i \) (for \( i = 1, \ldots, n \));

3.4. For each \( i = 1, \ldots, n \), we compute \( \hat{\delta}^*_i \) using DEA but replacing \( x_i \) with \( \check{x}_i \) and \( y_i \) with \( \check{y}_i \).

Step 4. We compute the bias-corrected estimator \( \hat{\delta}_i = \hat{\delta}_i - \hat{B}(\hat{\delta}_i) \) (\( \hat{B}(\hat{\delta}_i) \) represents the estimated bias) using the bootstrap estimates in \( \Gamma_i \).

Step 5. We use the maximum likelihood method to estimate \( \hat{\delta}_i = a + Z_i \beta + \varepsilon_i \leq 1 \), yielding estimates \( (\hat{\beta}, \hat{\sigma}_\varepsilon) \).

Step 6. This loop, repeated \( L_2 \) times, obtains a set of bootstrap estimates \( \Omega = \{ (\hat{\beta}^*_i, \hat{\sigma}_{\varepsilon}^*_i) \}_{b=1}^{L_2} \).

6.1. For each \( i = 1, \ldots, n \), we draw \( \varepsilon_i \) from the \( N(0, \hat{\sigma}_\varepsilon) \) distribution with left-truncation at \( (1 - Z_i \hat{\beta}) \);

6.2. For each \( i = 1, \ldots, n \), we compute \( \hat{\delta}^{**}_i = Z_i \hat{\beta} + \varepsilon_i \);

\(^{29}\) Simar & Wilson suggest \( L_1 = 100 \). In step 6 below, they suggest \( L_2 = 2000 \).
6.3. We use the maximum likelihood method to estimate the truncated regression of $\delta_i^*$ on $Z_i$, yielding estimates $(\hat{\beta}^*, \hat{\sigma}^*)$.

**Step 7.** We use the bootstrap values in $\Omega$ and the original estimates $(\hat{\beta}, \hat{\sigma})$ to construct estimated confidence intervals for each element of $\beta$ and for $\sigma_\varepsilon$. For instance, looking at $\beta_i^*$, if the distribution of $(\hat{\beta}_i - \beta_i)$ were known, it would be easy to find values $(a_a, b_a)$ such that

$$\Pr[-b_a \leq (\hat{\beta}_i - \beta_i) \leq -a_a] = 1 - \alpha, \quad (2)$$

e.g. with $\alpha = 0.05$. However, since the distribution of $(\hat{\beta}_i - \beta_i)$ is unknown, we can use the $i$th element of each bootstrap value $\hat{\beta}^*$ to find values $(a^*_a, b^*_a)$ such that

$$\Pr[-b_a \leq (\hat{\beta}^*_i - \hat{\beta}_i) \leq -a_a] \approx 1 - \alpha. \quad (3)$$

Substituting $(a^*_a, b^*_a)$ for $(a_a, b_a)$ in (2) leads to an estimated confidence interval $[\hat{\beta}_i + a^*_a, \hat{\beta}_i + b^*_a]$.

**References**

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