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Abstract

We analyze the limit behavior of sequences of oligopolistic equilibria in which firms follow objectives consistent with their shareholders' interests. We show that the efficiency of the limit allocation depends on how firms' shares are distributed across consumers, and provide a characterization of the class of ownership structures that lead to Walrasian equilibrium allocations in the limit.

1 Introduction

Perfectly competitive (or price taking) behavior is believed to arise – and is generally justified in the literature – when the number of economic agents

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that interact in the market is large, and each agent is small relative to the whole economy. There are, however, examples that show how monopoly profits and inefficient allocations can persist in equilibrium, even with an arbitrarily large number of small, competing agents. In an environment without uncertainty (or with uncertainty but a complete set of contingent securities) this happens if, as the economy grows larger, the sequence of its (oligopolistic) equilibria approaches a *critical* equilibrium point of the limit economy (Roberts 1980). The results of this paper uncover yet another possible source of inefficiency in large economies: the firms' ownership structures. If firms pursue their shareholders' interests, the way shares are allocated across consumers plays an important role in achieving efficiency in the limit.

For a firm that has market power, the choice of a production plan affects shareholders' real wealth in two ways: through the profits it generates (we will call this the *income effect*), and through the change in market prices it induces (we call this the *price effect*). It is well-known that these can be opposite effects (see, for example, Dierker and Grodal (1999), Bejan (2008)) and thus the production plan that maximizes firm's profit, under some price normalization,[1] may not maximize the welfare of firm's shareholders.

One would expect a firm's production choice to be consistent with its shareholders' interests but, typically, no production plan will be unanimously supported by all shareholders. We say that a production plan chosen by a firm is compatible with its shareholders' interests (given the production plans chosen by the other firms) if no other production plan makes all shareholders better off (provided that the other firms do not change their plans). Such a production plan is therefore efficient (or Pareto undominated) from the point of view of the firm's shareholders and will be called S-efficient (with S standing for "shareholders"). We are interested in the strategic interaction of a large number of firms whose objective is compatible with their share-

[1]Profit maximization is not well-defined in this context unless it is specifically linked to a particular price normalization. For a discussion of this well-known issue the reader is referred to Gabszewicz and Vial (1972) or Dierker and Grodal (1999).

holders' interests, in the sense of selecting S-efficient production plans. The Cournot-Nash equilibria of such game played by the firms must then have the property that every firm's equilibrium production plan is S-efficient given the production plans of the others. We call such equilibrium a Cournot S-equilibrium. Although we assume, for simplicity, that the interests of *all* shareholders govern the decisions of a firm, our results also hold under the weaker assumption that a firm's objective is shaped by the interests of a smaller "control group" such as the Board of Directors.

We study the limit behavior of Cournot S-equilibrium production plans of a sequence of private ownership economies and show that, depending on the ownership structure, the equilibria may or may *not* approach a Walrasian equilibrium of the limit economy. A sufficient condition for convergence to competitive equilibrium is to have a uniform lower bound on the number of shares owned by all (controlling) shareholders in any firm. The result is fairly intuitive. For sufficiently large economies (i.e., large number of competing firms), the price effect of each firm's action on its shareholders' welfare becomes almost negligible. However, if the ownership of a given firm is dispersed among a large number of shareholders, so that each of them holds only a tiny fraction of the firm, the income effect of that firm's choices on their wealth must be negligible as well. Thus the price effect, albeit becoming negligible itself, may still dominate the income effect. As a result, shareholders may disapprove the maximization of profits in arbitrarily large economies. Our results suggest that, while perfect portfolio diversification might be optimal from an investor's point of view (as suggested by CAPM-style models) it may not lead to efficiency economy-wide when firms pursue their shareholders' interests.

One of the major difficulties in studying the limit behavior of a sequence of Cournot S-equilibria is defining a notion of "closeness" on the space of private ownership production economies. For the case of pure-exchange, representing an economy as a distribution on the space of agents' characteristics

(Hildenbrand 1970, Hildenbrand 1975) enables the use of weak convergence of measures to define a topology on the space of economies. For a production economy, the space of characteristics must be enlarged to include firms' production sets *and* ownership structure. However, as opposed to preferences, endowments or production sets, an ownership structure is intrinsically related to a space of consumers and a space of firms, and it is not obvious how such ownership structure can be included in a space of characteristics that is agent-independent. Even when restricting attention to economies with a finite number of types of consumers and firms, the separation of ownership from the actual *names* of consumers and firms is difficult, unless one is willing to make very restrictive symmetry assumptions on the ownership structure. A familiar example of such symmetry requirement is that every consumer of type i owns equal shares in all the firms of type j (Gabszewicz and Vial 1972, Mas-Colell 1980). As some of our results show, focusing on such specific symmetry of the ownership structure, one is bound to miss important insights that are revealed only in more heterogeneous economies.

We construct here a general framework that embeds any private ownership production economy and allows for a natural topological structure, which generalizes other topologies defined in the literature, over more restrictive spaces of economies. We also show that a continuum production economy is a good approximation for a large finite economy, since it can be written as the limit of finite economies.

Hart (1979) proves, in related work, that if a firm maximizes profits (under a specific price normalization), then each shareholder's gain from switching to his most preferred production plan diminishes as the economy grows larger. Thus, Hart's result implies that profit maximization by oligopolistic firms is *approximately* in the best interest of each firm's shareholder if the economy is large enough. By contrast, we show that if a firm follows an objective that is consistent with its shareholders' interests, then it will choose a plan that is *close* to the (Walrasian) profit maximizing plan, if the economy is large enough and the ownership is not too diffuse. Hart's results do not imply ours, but can rather be seen as a converse to this paper. The standard oligopolistic equilibria and Cournot S-efficient equilibria may not be Pareto ranked. Indeed, a change in one firm's production plan to a plan that improves the welfare of its shareholders may result in a decreased utility for the shareholders of *other* firms, via the price effects. Hence, although each firm can improve the welfare of its shareholders by a unilateral deviation from the (normalized) profit maximization objective, a Cournot S-equilibrium plan that Pareto dominates the standard Cournot-Nash equilibrium plan may not exist.

The paper is organized as follows. We start, in section 2, by giving an example that illustrates the main points of the paper. In section 3 we set up a general framework for describing private ownership production economies and show how the standard Arrow-Debreu economies and their replicas can be embedded in this framework. The Cournot S-equilibrium concept and some of its properties are described in section 4. Section 5 defines a topology on the space of private ownership economies and provides conditions on the ownership structure such that convergence to Walrasian equilibrium obtains. Section 6 concludes.

2 An illustrative example

Let $\mathcal{E}_1$ be an Arrow-Debreu economy with two goods, two consumers and one firm. Consumers have identical preferences over consumption of the two goods, represented by the utility $u : \mathbb{R}^2_+ \to \mathbb{R}$, $u(c_1, c_2) = \ln c_1 + \ln c_2$. The endowment of goods of consumer 1, respectively 2, are $e^1 = (4, 4)$, respectively $e^2 = (4, 2)$. The first consumer is the sole owner of the firm, whose production set is $Y := \{(-\alpha, \alpha) \mid \alpha \in [0, 1]\}$. We will refer to the economy $\mathcal{E}_1$ as the *prototype economy*.

It is assumed throughout that consumers are price takers in all markets while every firm behaves strategically, internalizing the effect of its choices of production plans on the market prices. Unlike the standard Cournot-Walras model (Gabszewicz and Vial 1972), firms do not maximize profits, but rather choose production plans that are non-dominated from the point of view of their shareholders, taking as given the choices of other firms, but internalizing the effect of its own choice on the equilibrium market prices. We call such equilibria Cournot S-efficient equilibria (or simply Cournot S-equilibria). The term hints to the fact that such a production plan is efficient from the point of view of the shareholders who, while price takers as consumers, are aware of the market power of the firms they own. Cournot S-efficient equilibria are typically different from the standard Cournot-Walras equilibria due to price effects on shareholders' wealth.

Without loss of generality, we normalize prices to lie in the unit simplex.[2] Given a choice of a production plan $(-\alpha, \alpha)$, in the resulting competitive exchange equilibrium, (normalized) prices are $\left(\frac{6+\alpha}{14}, \frac{8-\alpha}{14}\right)$, and first consumer's utility is $v(\alpha) = 2\ln(28 + \alpha(1-\alpha)) - \ln(6+\alpha) - \ln(8-\alpha)$. As the sole owner of the firm, consumer 1 would want the firm to choose a production plan $(-\alpha^*, \alpha^*)$ with $\alpha^* \in [0, 1]$ that maximizes his utility (which is strictly concave in α). The first order conditions show that α^* is the unique solution of the equation $\alpha^3 - 3\alpha^2 - 67\alpha + 20 = 0$ that belongs to $[0, 1]$, hence $\alpha^* \approx 0.3$.

Hence, the unique equilibrium of the prototype economy in which the firm acts strategically in the market but follows an objective that is consistent with its owner's interests corresponds to a production vector $(-\alpha^*, \alpha^*) \approx (-0.3, 0.3)$ and the equilibrium prices $\left(\frac{6+\alpha^*}{14}, \frac{8-\alpha^*}{14}\right) \approx (0.45, 0.55)$.

To study the limit behavior of Cournot S-equilibria we construct sequences of replica economies, in the spirit of Debreu and Scarf (1963). An n-fold replica of $\mathcal{E}_1$, denoted $\mathcal{E}_n$, is an economy with $2n$ consumers and n firms. All firms, indexed by $j \in \{1, ..., n\}$, have the same production set as

[2]Our results are independent of this normalization since the objective of each firm is formulated in terms of shareholders' indirect utilities, which only depend on *relative* equilibrium prices and thus are immune to the normalization chosen.

the firm of the prototype economy. Consumers are indexed by (i, k) with $i \in \{1, 2\}$ and $k \in \{1, .., n\}$. We will refer to i as the *type* and to k as the *name* of the consumer (i, k). Every consumer of type i has the same preferences and endowment of goods as consumer i of the prototype economy. Similarly, a *continuum replica*, (also called the *limit* replica) $\mathcal{E}_\infty$, is an economy with a continuum of identical firms and consumers of each type.

Due to log-utilities, the exchange equilibrium prices following a choice of production plans by the firms in any of these replicas do not depend on the way shares are distributed across consumers. For the n-fold replica economy, given a production plan $y = ((-\alpha_j, \alpha_j))_{j=1}^n$, let

$$\kappa(y) := \frac{1}{n} \sum_{j=1}^{n} \alpha_j. \tag{2.1}$$

Simple computations reveal that the unique exchange equilibrium price vector following the choice of production plan y is $\left(\frac{6+\kappa(y)}{14}, \frac{8-\kappa(y)}{14}\right)$. The same formula is valid in the continuum replica economy $\mathcal{E}_\infty$, with a proper reinterpretation of κ, i.e.,

$$\kappa(y) := \int_{[0,1]} \alpha(j) d\lambda(j), \tag{2.2}$$

where λ is the Lebesgue measure on $[0, 1]$. For a continuum replica, a feasible production plan is a Lebesque measurable function $y : [0, 1] \to Y$.

Cournot S-equilibrium production plans of the n-fold replica economy depend on the ownership structure, which is the deciding factor in whether Cournot S-equilibria of large economies become close to Walrasian equilibria of the limit economy $\mathcal{E}_\infty$. There are various ways to replicate the ownership of firms' shares. We will outline here two different types of ownership structures, and show that they bear very different implications on the issue whether Cournot S-equilibrium allocations approach the competitive allocations of the limit economy.

1. **Concentrated ownership replication.** In this replication, every

consumer of type 1 is the sole owner of the firm with the same name (i.e., consumer $(1, j)$ is the sole owner of firm j), and all consumers of type 2 have no firm ownership. We denote (finite and continuum) replicas bearing this ownership structure by $\mathcal{E}_n^c$ and $\mathcal{E}_\infty^c$.

Following a choice $y = ((-\alpha_j, \alpha_j))_{j=1}^n \in \mathbb{R}^{Ln}$ of production plans by the firms, the wealth and utility of consumer $(1, j)$ in such replica are given by:

$$w(\kappa(y), \alpha_j) = \frac{1}{7}\left(28 + \alpha_j(1 - \kappa(y))\right), \tag{2.3}$$

$$V(\kappa(y), \alpha_j) = 2\ln\left(28 + \alpha_j(1 - \kappa(y))\right) - \ln[(6 + \kappa(y))(8 - \kappa(y))]. \tag{2.4}$$

Thus, in accordance with its owner's preferences, each firm chooses $\alpha \in [0, 1]$ to maximize $V(\kappa, \alpha)$. Since the problem has a unique solution, Cournot S-equilibria of the economy $\mathcal{E}_n^c$ must be symmetric, i.e., $\alpha_j = \kappa$ for $j = 1, .., n$. The first order condition implies that κ satisfies

$$\frac{2\kappa - 2}{\kappa^2 - \kappa - 28} = \frac{1}{n}\left(\frac{1}{\kappa - 8} + \frac{1}{\kappa + 6} - \frac{2\kappa}{\kappa^2 - \kappa - 28}\right) \tag{2.5}$$

According to the implicit function theorem, the solution of (2.5), denoted $\kappa(1/n)$, is a continuous function of $1/n$, hence when $n \to \infty$, $\kappa(1/n)$ converges to the solution of $(2\kappa - 2)/(\kappa^2 - \kappa - 28) = 0$, which is $\kappa^* = 1$. This corresponds to every firm choosing the competitive production plan. Thus, the sequence of Cournot S-efficient equilibria of $\mathcal{E}_n^c$ converges to the Walrasian equilibrium of $\mathcal{E}_\infty^c$.

2. **Diffuse ownership replication.** In this replication, every firm is equally owned by *all* consumers of type 1, while consumers of type 2 still have no ownership. The finite n-fold replica will be denoted by $\mathcal{E}_n^d$, while the continuum replica will be denoted by $\mathcal{E}_\infty^d$.

   Since each firm in $\mathcal{E}_n^d$ is owned by n identical consumers, at a Cournot

S-equilibrium, firms maximize the utility of their representative owner. The wealth and utility of consumer $(1, j)$ in $\mathcal{E}_n^d$ depend only on the average production, $\kappa(y)$, and have the same expressions as (2.3),(2.4) with α_j replaced by $\kappa(y)$. Thus owners of each firm are identical in terms of preferences and wealth, and therefore finding S-efficient allocations amounts to maximizing the utility of the representative consumer, which reduces to the case analyzed for the prototype economy. Hence a production plan y_n is a Cournot S-equilibrium plan if and only if it satisfies $\kappa(y_n) = \alpha^* \approx 0.3$. In particular, the production plan y_n^* in which all firms choose $(-\alpha^*, \alpha^*)$ is a Cournot S-equilibrium. Hence, the monopolistic choice persists in arbitrarily large economies.

Note that every consumer of type 1 has the same total ownership of shares in the two examples. In the concentrated ownership example, each consumer remains the sole owner of a firm, irrespective of the size of the economy. Thus, as the economy grows larger, that firm's production choice essentially affects its owner's budget constraint and thus the income effect of a firm's production choice persists in arbitrarily large economies. On the other hand, price effects vanish and will be dominated by the income effect, and thus every type 1 consumer would want his firm to choose a production plan close to the profit maximizing plan at the limit competitive price. By contrast, in the diffuse ownership example, a type 1 consumer's ownership in any firm diminishes, as the economy grows larger. Thus, the income effect of a firm's choice vanishes and shareholders tend to be indifferent among that firm's feasible production plans. This is the mechanism through which a monopolistic equilibrium can persist in arbitrarily large economies.

It should also be noted that what drives the results is firms' behavior: i.e., their choice of production plans in accordance with their shareholders' interests. Whether a firm's shareholders' interests are aligned (as in this example) or not (as in the main theorem or the example of Section 5) is inconsequential.

3 Finite-type production economies

Let $\mathcal{I} = \{1, ..., I\}$ be the set of consumers' types and $\mathcal{J} = \{1, ..., J\}$ be the set of firms' types. For every $j \in \mathcal{J}$ let $Y_j \subseteq \mathbb{R}^L$ be the production set of a type-j firm. Y_j is assumed to satisfy the following standard conditions: (a) Y_j is closed, convex and contains the origin and (b) $Y_j \cap \mathbb{R}^L_+ = \{0\}$ (i.e., Y_j excludes "free lunches"). For every $i \in \mathcal{I}$, let $(\mathbb{R}^L_+, u^i, e^i)$ be the characteristics of a type-i consumer, where $\mathbb{R}^L_+$ is the consumption set, $u^i : \mathbb{R}^L_+ \to \mathbb{R}$ a utility representation of his preferences, and $e^i \in \mathbb{R}^L_{++}$ the endowment of goods. It is assumed that the utility functions u^i are continuous, monotonic and strictly quasi-concave.

The *space of firms* is $(\Omega_F, \mathcal{G})$, where $\Omega_F = \mathcal{J} \times [0,1]$ and $\mathcal{G}$ is a finite or countably generated σ-algebra on Ω_F such that $2^{\mathcal{J}} \times [0,1] \subset \mathcal{G}$; thus the projection $\jmath$ of Ω_F on $\mathcal{J}$, defined as $\jmath((j,a)) = j$, for all $(j,a) \in \Omega_F$, is measurable. A *firm* is an atom of the σ-algebra $\mathcal{G}$.[3] For every $t \in \Omega_F$, the unique atom that contains t is denoted by $\mathcal{G}(t)$, and is called *firm* $\mathcal{G}(t)$, or simply *firm* t, when no confusion can arise.[4] Since $2^{\mathcal{J}} \times [0,1] \subseteq \mathcal{G}$, every atom's projection on $\mathcal{J}$ must be a singleton, and therefore any firm $\mathcal{G}(t)$ can be written as a pair (j, A) for some $j \in \mathcal{J}$ and $A \subset [0,1]$. We will refer to $j = \jmath(t)$ as the *type* of firm t.

The *consumers'* side of the economy is represented by the probability space $(\Omega_C, \mathcal{F}, \mu_C)$, where $\Omega_C = \mathcal{I} \times [0,1]$, $\mathcal{F}$ is a σ-algebra on Ω_C and μ_C is a probability measure on $\mathcal{F}$. We assume that $2^{\mathcal{I}} \times [0,1] \subset \mathcal{F}$, hence the projection function $\imath$ of Ω_C on $\mathcal{I}$ is measurable. We assume that either $(\Omega_C, \mathcal{F})$ is a Polish space,[5] or that $\mathcal{F}$ is finite. A *consumer* is an atom of the σ-algebra $\mathcal{F}$. For every $s \in \Omega_C$, the unique atom that contains s will be

[3]A non-empty set B is called an atom of the σ-algebra $\mathcal{G}$ if and only if $B \in \mathcal{G}$ and for all $C \in \mathcal{G}$, either $B \subseteq C$ or $B \cap C = \emptyset$ (Dudley 2002, p.87).

[4] Since $\mathcal{G}$ is countably generated, the atoms of $\mathcal{G}$ form a partition of Ω_F, and the atom $\mathcal{G}(t)$ equals the intersection of all sets in $\mathcal{G}$ containing t (see Appendix A for details).

[5]The space $(\Omega_C, \mathcal{F})$ is Polish if $\mathcal{F}$ is the Borel σ-algebra generated by a topology on Ω_C induced by a complete and separable metric.

denoted by $\mathcal{F}(s)$ and referred to as consumer $\mathcal{F}(s)$, or simply as consumer s by an abuse of notation.[6] The *type* of consumer $\mathcal{F}(s)$ is $\imath(s) \in \mathcal{I}$. The relative size of type-i consumers to the size of the economy is $\mu_C(\{i\} \times [0,1])$.

The *ownership structure* of the economy is described by a measure kernel $\theta : \Omega_C \times \mathcal{G} \to \mathbb{R}_+$. Thus, for all $s \in \Omega_C$, $\theta(s, \cdot)$ is a finite measure on $\mathcal{G}$ (interpreted as consumer s's allocation of shares across firms) and, for every $B \in \mathcal{G}$, the map $\theta(\cdot, B)$ is $\mathcal{F}$-measurable. For every $s \in \Omega_C$, $\theta(s, \Omega_F)$ represents the total "number" of shares (in various firms) owned by consumer s. We assume that $\theta(\cdot, \Omega_F)$ is bounded. Note that this definition allows consumers of the same type to have different endowments of shares. Therefore, consumers of the same type are identical only in terms of their preferences and endowments of goods.

Let $\mu_C \otimes \theta$ be the measure on $\mathcal{F} \otimes \mathcal{G}$ (Kallenberg 2002, p.21) defined by

$$(\mu_C \otimes \theta)(B) := \int_{\Omega_C} \int_{\Omega_F} \mathbf{1}_B(s,t)\theta(s,dt)\mu_C(ds), \quad B \in \mathcal{F} \otimes \mathcal{G}, \tag{3.1}$$

where $\mathbf{1}_B$ denotes the indicator function of set B.[7] Since $\theta(\cdot, \Omega_F)$ is bounded, $\mu_C \otimes \theta$ is a finite measure. The *composition* $\mu_C\theta$ of μ_C and the kernel θ (Kallenberg 2002, p.22) defines a measure $\mu_F := \mu_C\theta$ on the the space of firms, given by

$$\mu_F(T) := \int_{\Omega_C} \theta(s,T)\mu_C(ds), \quad T \in \mathcal{G}. \tag{3.2}$$

Notice that $\mu_F(\cdot) = (\mu_C \otimes \theta)(\Omega_C \times \cdot)$ and thus μ_F is also a finite measure. Hence, there exists a probability kernel[8] $\gamma : \Omega_F \times \mathcal{F} \to [0,1]$, such that: (i) for every $t \in \Omega_F$, $\gamma(t, \cdot)$ is a probability measure on $\mathcal{F}$, (ii) for every

[6]The Polish space assumption imposed on $(\Omega_C, \mathcal{F})$ implies that $\mathcal{F}$ is countably generated, and hence the results of Appendix A apply. See also footnote 4.

[7]The indicator function $\mathbf{1}_B$ is defined as $\mathbf{1}_B(x) = \begin{cases} 1 & \text{if } x \in B, \\ 0 & \text{if } x \notin B \end{cases}$.

[8]See the Appendix B for a proof.

$S \in \mathcal{F}$, $\gamma(\cdot, S)$ is $\mathcal{G}$-measurable, and (iii) for any $g : \Omega_C \times \Omega_F \to \mathbb{R}$ which is $\mathcal{F} \otimes \mathcal{G}$-measurable and $\mu_C \otimes \theta$-integrable,

$$\int_{\Omega_F} \left[\int_{\Omega_C} g(s,t)\gamma(t, ds) \right] \mu_F(dt) = \int_{\Omega_C \times \Omega_F} g \, d(\mu_C \otimes \theta)$$
$$= \int_{\Omega_C} \left[\int_{\Omega_F} g(s,t)\theta(s, dt) \right] \mu_C(ds) \qquad (3.3)$$

For every $t \in \Omega_F$, the probability $\gamma(t, \cdot)$ represents firm t's *distribution of shares* across consumers.

The probability space of consumers $(\Omega_C, \mathcal{F}, \mu_C)$, together with the measurable space of firms $(\Omega_F, \mathcal{G})$ and an ownership structure described by the kernel θ from Ω_C to Ω_F defines a *private ownership production economy* $\mathcal{E}$,

$$\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta) \, .$$

A *finite economy* is an economy for which the σ-algebras $\mathcal{F}$ and $\mathcal{G}$ are finite. An *atomless economy* is an economy for which the measure μ_F is atomless.[9] Equation (3.2) implies that μ_F is atomless if and only if for every $t \in \Omega_F$, $\theta(s, \mathcal{G}(t)) = 0$ for μ_C-a.e. $s \in \Omega_C$.[10]

The prototypical Arrow-Debreu production economy with I consumers and J firms, in which the i-th consumer owns $s(i,j)$ shares of j-th firm can be represented as an economy $\mathcal{E}_1 = ((\Omega_C, \mathcal{F}_1, \mu_C), (\Omega_F, \mathcal{G}_1), \theta_1)$ with $\mathcal{F}_1 := 2^{\mathcal{I}} \times [0,1]$, $\mathcal{G}_1 := 2^{\mathcal{J}} \times [0,1]$, $\mu_C = \lambda_{\mathcal{I}} \otimes \lambda$, and $\theta(\{i\} \times [0,1]), \{j\} \times [0,1])) = s(i,j)$, with $\lambda_{\mathcal{I}}$ being the uniform probability on $\mathcal{I}$ (i.e. $\lambda_{\mathcal{I}}(i) = 1/I, \ \forall i \in \mathcal{I}$) and λ being the Lebesque measure on $[0,1]$. Thus we identify the i-th consumer, respectively the j-th firm of the Arrow-Debreu economy with the atom $\{i\} \times [0,1]$ of $\mathcal{F}_1$, respectively the atom $\{j\} \times [0,1]$ of $\mathcal{G}_1$.

Using sequences of replica economies to draw inferences about (strategic)

[9] The measure μ_F on $(\Omega_F, \mathcal{G})$ is atomless, or nonatomic, if $\mathcal{G}$ has no μ_F-nonnull atoms (Dudley 2002, p.82).

[10] Throughout the paper, "a.e." means "almost every(where)" and "a.s." means "almost surely".

equilibrium behavior in large economies is a technique introduced by Debreu and Scarf (1963), for pure exchange economies, and also widely used in the literature for economies with production. An n-fold replica consists of n "clones" of each firm and each consumer of the prototype Arrow-Debreu economy. There are many ways to assign ownership of firms across consumers in replica economies. For example, a replica may be constructed such that each clone of a certain type holds the same number of shares in firms of the same industry (type); in the example of Section 2 we referred to this ownership structure as a "diffuse ownership" replication. This approach is advocated by Gabszewicz and Vial (1972), Roberts (1980), Mas-Colell (1982), and Allen (1994), among others. However this is not the only way one can construct replicas of a particular economy, even when similarity of the clones is a concern. Aliprantis, Brown, and Burkinshaw (1987), and Florenzano and Mercato (2004) assume that each clone of the prototype economy inherits the initial ownership structure. In this "concentrated ownership" replication (see Section 2), a clone of a consumer of type i owns $s(i,j)$ shares of the corresponding clone of firm j. The name captures the idea that ownership is segmented across the clones of the prototype economy, rather than being spread across multiple clones.

We can embed replicas with arbitrary ownership structure in our framework. For every $n \in \mathbb{N}$, let $H_n^1 := [0, 1/n]$ and for $k \in \{2, 3, \ldots, n\}$, let $H_n^k := \left(\frac{k-1}{n}, \frac{k}{n}\right]$. Denote by $\mathcal{H}_n$ the algebra generated by $\{H_n^1, \ldots, H_n^n\}$. For each $a \in [0,1]$, let $k(a) := \{k : a \in H_n^k\}$ and $\mathcal{H}_n(a) := H_n^{k(a)}$. Define $\mathcal{F}_n := 2^{\mathcal{I}} \otimes \mathcal{H}_n$, $\mathcal{G}_n := 2^{\mathcal{J}} \otimes \mathcal{H}_n$ and $\mu_C^n := \lambda_{\mathcal{I}} \otimes \lambda$. Thus consumers and firms in the *n-fold replica* are pairs of the form (i, H_n^k) and, respectively, (j, H_n^k), with $k = 1, ..., n$. By an abuse of notation we will often identify a point $a \in [0,1]$ with the interval $\mathcal{H}_n(a)$ and thus represent consumers and firms as pairs (i, a) and respectively (j, a). We will refer to the first component of such pair as the "type" and to the second as the "name" of the consumer/firm.

The *concentrated ownership* n-fold replica can be modeled as an economy

$$\mathcal{E}_n^c := \left((\Omega_C, 2^{\mathcal{I}} \otimes \mathcal{H}_n, \lambda_{\mathcal{I}} \otimes \lambda); (\Omega_F, 2^{\mathcal{J}} \otimes \mathcal{H}_n); \theta_n^c\right), \tag{3.4}$$

where, by letting $\delta_a(A) := \mathbf{1}_A(a)$,

$$\theta_n^c((i,a),(j,A)) = s(i,j) \cdot \delta_a(A), \forall (i,a) \in \Omega_C, \forall (j,A) \in 2^{\mathcal{J}} \otimes \mathcal{H}_n. \tag{3.5}$$

The *diffuse ownership* n-fold replica can be described as the economy

$$\mathcal{E}_n^d := \left((\Omega_C, 2^{\mathcal{I}} \otimes \mathcal{H}_n, \lambda_{\mathcal{I}} \otimes \lambda); (\Omega_F, 2^{\mathcal{J}} \otimes \mathcal{H}_n); \theta_n^d\right), \tag{3.6}$$

with

$$\theta_n^d((i,a),(j,A)) = s(i,j) \cdot \lambda(A), \forall (i,a) \in \Omega_C, \forall (j,A) \in \mathcal{G}_n. \tag{3.7}$$

Note that for every consumer (i,a) the total number of shares owned by (i,a) is the same in the concentrated ownership and diffuse replica economies $\mathcal{E}_n^c, \mathcal{E}_n^d$. Thus, the total mass of the ownership distribution of a consumer stays the same. However, under the diffuse ownership specification, the support of the distribution becomes larger as the size of the economy increases.

Intuitively, the sequence of economies $(\mathcal{E}_n^c)$ "converges" to the atomless economy

$$\mathcal{E}^c := \left((\Omega_C, 2^{\mathcal{I}} \otimes \mathcal{B}[0,1], \lambda_{\mathcal{I}} \otimes \lambda); (\Omega_F, 2^{\mathcal{J}} \otimes \mathcal{B}[0,1]); \theta^c\right), \tag{3.8}$$

where $\theta^c((i,a),(j,A)) = s(i,j) \cdot \delta_a(A)$, for any $A \in \mathcal{B}([0,1])$, and $\mathcal{B}([0,1])$ is the Borel σ-algebra on $[0,1]$. Similarly, the sequence of economies $(\mathcal{E}_n^d)$ "converges" to the atomless economy

$$\mathcal{E}^d := \left((\Omega_C, 2^{\mathcal{I}} \otimes \mathcal{B}[0,1], \lambda_{\mathcal{I}} \otimes \lambda); (\Omega_F, 2^{\mathcal{J}} \otimes \mathcal{B}[0,1]); \theta^d\right), \tag{3.9}$$

where $\theta^d((i,a),(j,A)) = s(i,j) \cdot \lambda(A)$, for any $A \in \mathcal{B}([0,1])$. We formalize the notion of convergence for a sequence of finite economies in Section 5.

4 Cournot S-equilibrium

This section defines our notion of equilibrium for production economies in which consumers are price takers when making their consumption decisions, and firms interact strategically via a Cournot-type quantity competition but, rather than maximizing profits, they follow an objective that is consistent with their shareholders' interests. We call this new concept a *Cournot S-equilibrium.*

To simplify exposition, we make no distinction here between the consumers who own the firm and those who control it. However all our results remain true if we assume that a firm's decisions are controlled by a (predetermined) group of consumers (e.g., the Board of Directors).

Consider a production economy $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$. An *allocation* for the economy $\mathcal{E}$ is a pair (c, y), such that $c : \Omega_C \to \mathbb{R}^L_+$ is $\mathcal{F}$-measurable, $y : \Omega_F \to \mathbb{R}^L$ is $\mathcal{G}$-measurable and $\theta(s, \cdot)$-integrable for μ_C-almost all $s \in \Omega_C$, and $y(j,a) \in Y_j$ for all $(j,a) \in \Omega_F$. Hence for any $s \in \Omega_C$ and $t \in \Omega_F$, $c(s)$ represents the consumption bundle of agent s, and $y(t)$ is the production per outstanding share of firm t. We call c a consumption allocation and y a production plan for the economy $\mathcal{E}$. The allocation (c, y) is called *feasible* if

$$\int_{\Omega_C} c\, d\mu_C = \int_{\Omega_C} e\, d\mu_C + \int_{\Omega_F} y\, d\mu_F,$$

where $e(s) := e^{\imath(s)}$, $s \in \Omega_C$ and $\mu_F = \mu_C \theta$. A given production plan y generates the *intermediate endowment* mapping $w_y : \Omega_C \to \mathbb{R}^L$ defined by

$$w_y(s) := e(s) + \int_{\Omega_F} y(t)\theta(s, dt), \quad s \in \Omega_C. \tag{4.1}$$

Note that (3.3) implies that w_y is μ_C-integrable and

$$\int_{\Omega_C} w_y \, d\mu_C = \int_{\Omega_C} e \, d\mu_C + \int_{\Omega_F} y \, d\mu_F.$$

When the market price vector is $p \in \Delta^{L-1}$ (with Δ^{L-1} denoting the unit simplex in $\mathbb{R}^L_+$), the budget constraint of a consumer $s \in \Omega_C$ is $\{x \in \mathbb{R}^L_+ | p \cdot x \leq p \cdot w_y(s)\}$ and his consumption choice is $D^{\imath(s)}(p, w_y(s))$, where

$$D^i(p, z) := \arg\max\{u^i(x) \mid x \in \mathbb{R}^L_+, \; px \leq pz\}, \quad i \in \mathcal{I}, p \in \Delta^{L-1}, z \in \mathbb{R}^L_+.$$

The utility of consumer s at his optimal consumption choice, when faced with prices p and a production plan y, is $V^{\imath(s)}(p, w_y(s))$, where $V^i(p, z) := u^i\left(D^i(p, z)\right), \; i \in \mathcal{I}, \; p \in \Delta^{L-1}, \; z \in \mathbb{R}^L_+$.

For any production plan y, denote by $\mathcal{E}(y)$ the associated pure-exchange economy in which consumers' endowments are given by w_y. The set of Walrasian equilibrium prices of the economy $\mathcal{E}(y)$ depends only on the *distribution of intermediate endowments across types* defined as $\mu_C \circ \widetilde{w}_y^{-1}$, where $\widetilde{w}_y : \Omega_C \to \mathcal{I} \times \mathbb{R}^L_+$, $\widetilde{w}_y(s) := (\imath(s), w_y(s))$. Even stronger, if the distribution of intermediate endowments across types for two different economies $\mathcal{E}(y)$ and $\mathcal{E}'(y')$ coincide, then the sets of Walrasian equilibrium prices for $\mathcal{E}(y)$ and $\mathcal{E}'(y')$ are identical (Hildenbrand 1970). Let $P(\mu_C \circ \widetilde{w}_y^{-1}) \subseteq \Delta^{L-1}$ be the set of Walrasian equilibrium price vectors of $\mathcal{E}(y)$. For convenience, we will let $P(y) := P(\mu_C \circ \widetilde{w}_y^{-1})$.

For some production sets, in particular for those that exhibit free disposal, the economy $\mathcal{E}_y$ may have no Walrasian equilibrium. Certain lower bounds, or capacity constraints, need to be imposed on the firms' strategy sets to avoid this occurrence and make the problem meaningful. It is sufficient, for example to restrict firms' choices to production plans that generate positive intermediate endowments.[11] For such production plans, the main theorem in

[11] This happens, for example, if each production set is contained in the set $\{y \in \mathbb{R}^L | y_l \geq -\frac{\min_{i \in \mathcal{I}} e^i_l}{M}, \forall l = 1, \ldots, L\}$, where M is the upper bound on the kernels θ.

McKenzie (1981) implies that $P(\cdot)$ is not empty-valued. However, positivity of intermediate endowments is not necessary for the existence of a Walrasian equilibrium in the associated pure-exchange economy and therefore a much larger set than the one described above may still generate non-empty values for P. For the remaining of the paper we are going to abstract from the difficulties posed by the possible empty-values of P by assuming that the production sets $(Y_j)_{j\in\mathcal{J}}$ contain some capacity constraints that are tight enough to guarantee the existence of a competitive equilibrium for *every* production plan y.[12] In particular, we assume that the production sets $(Y_j)_{j\in\mathcal{J}}$ are bounded, hence compact. Since the correspondence P is closed, has compact values,[13] and it is defined on a compact space,[14] P is weakly measurable and thus, according to Kuratowski-Ryll-Nardzewski theorem (Aliprantis and Border 1999, Theorem 14.86) it has a measurable selection $\mathbf{p}$ (i.e., $\mathbf{p}$ is measurable and $\mathbf{p}(y) \in P(y)$).

A pair $(\bar{p}, \bar{y})$ of prices $\bar{p} \in \Delta^{L-1}$ and production plan $\bar{y}$ is a *Walrasian equilibrium* for the economy $\mathcal{E}$ if and only if $\bar{p} \in P(\bar{y})$ and, for μ_F-almost every $t \in \Omega_F$,

$$\bar{p} \cdot \bar{y}(t) = \max_{z \in Y_{\jmath(t)}} \bar{p} \cdot z.$$

We introduce next the concept of a *Cournot S-equilibrium*, which captures the idea that, although an individual consumer cannot affect market prices through his consumption decisions, he is aware of the effect that a firm that he owns has on market prices.

[12]One can dispense of this assumption by restricting the firms' strategy sets to a subset on which existence of a competitive equilibrium is guaranteed. With due care, all the results of this paper can be derived under such restriction, but the details of the construction are beyond the scope of this paper.

[13]The standard reference is Hildenbrand and Mertens (1972). However, in our case an extension of the classical result is needed since the intermediate endowments may generate zero wealth (see, for example, Bejan 2008).

[14]Note that intermediate endowments must lie in a compact subset of $\mathbb{R}^L$, since the set of feasible production plans is bounded. The space of laws on $\mathbb{R}^L$ with support in a given compact is compact, when endowed with the weak convergence topology (Dudley 2002, Theorem 9.3.3).

Definition 4.1. *A production plan y^* is called S-efficient for firm $t = (j, a)$, given the measurable price selection $\mathbf{p}$ from P if and only if there does not exist $z \in Y_j$ such that:*

$$\gamma\left(t, \left\{s \mid V^{\imath(s)}(\mathbf{p}(\tilde{y}), w_{\tilde{y}}) \geq V^{\imath(s)}(\mathbf{p}(y^*), w_{y^*})\right\}\right) = 1, \qquad (4.2)$$
$$\gamma\left(t, \left\{s \mid V^{\imath(s)}(\mathbf{p}(\tilde{y}), w_{\tilde{y}}) \geq V^{\imath(s)}(\mathbf{p}(y^*), w_{y^*})\right\}\right) > 0,$$

where $\tilde{y} : \Omega_F \to \mathbb{R}^L$ is defined as $\tilde{y} := y^* + (z - y^*)\mathbf{1}_{\mathcal{G}(t)}$ and $\mathbf{1}_{\mathcal{G}(t)}$ is the indicator function of the set $\mathcal{G}(t)$. A pair $(\mathbf{p}, y^*)$ consisting of a measurable selection $\mathbf{p}$ from P and a production plan y^* is called a Cournot S-equilibrium if, given the selection $\mathbf{p}$, y^* is S-efficient for μ_F-almost every $t \in \Omega_F$.

We will simply refer to a production plan y^* as being a Cournot S-equilibrium whenever there exists a measurable price selection $\mathbf{p}$ such that $(\mathbf{p}, y^*)$ is a Cournot S-equilibrium. Thus for a fixed price selection, a production plan is S-efficient for a firm if, given the choices of the other firms, there does not exist another production plan such that every shareholder of the firm is better off in the new market equilibrium. It is important to note here that S-efficiency is a very weak condition, since different production choices made by a firm may generate equilibrium allocations for that firm's shareholders which are not Pareto comparable. It is therefore likely that the set of Cournot S-equilibria is large. Our example in section 5 supports this hypothesis.

We examine first the relationship between Walrasian equilibria and Cournot S-equilibria in atomless economies. We start by showing that a measure zero of firms cannot affect the equilibrium price in atomless economies. Since every single firm (atom) is of measure zero in an atomless economy, an individual firm's change in production has a negligible price effect.

Lemma 4.2. *Let $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$ be an atomless economy, and assume the production plans $y, y' : \Omega_F \to \mathbb{R}^L$ are equal μ_F-a.e, where $\mu_F = \mu_C\theta$. Then $P(y) = P(y')$.*

Proof. Since $\mu_F(\{y \neq y'\}) = 0$, for any set $S \in \mathcal{F}$ of consumers, (3.3) implies

$$\begin{aligned}\int_S \left[\int_{\Omega_F} y(t)\theta(s,dt)\right]\mu_C(ds) &= \int_{\Omega_F} y(t)\gamma(t,S)\mu_F(dt)\\ &= \int_{\Omega_F} y'(t)\gamma(t,S)\mu_F(dt)\\ &= \int_S \left[\int_{\Omega_F} y'(t)\theta(s,dt)\right]\mu_C(ds).\end{aligned}$$

Thus the intermediate endowments associated to the two production plans coincide μ_C-a.e., and therefore the sets of equilibrium prices associated to the two production plans coincide. □

In atomless economies, a Walrasian equilibrium is also a Cournot S-equilibrium, but the choice of an S-efficient production plan may not necessarily lead to profit maximization (at the Walrasian prices) for *some* ownership structures.

Proposition 4.3. *Let $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$ be an atomless economy.*

1. *If $(\bar{p}, \bar{y})$ is a Walrasian equilibrium for $\mathcal{E}$, then $(\bar{\mathbf{p}}, \bar{y})$ is a Cournot S-equilibrium for $\mathcal{E}$, where $\bar{\mathbf{p}}$ is any price selection such that $\bar{\mathbf{p}}(\bar{y}) = \bar{p}$.*

2. *If $(\bar{\mathbf{p}}, \bar{y})$ is a Cournot S-equilibrium for $\mathcal{E}$, then $\bar{y}$ is profit maximizing at prices $\bar{\mathbf{p}}(\bar{y})$ on the set of firms Ω_F^{max} defined by*

$$\Omega_F^{max} := \{t \mid \gamma\,(t, \{s \mid \theta(s, \mathcal{G}(t)) > 0\}) > 0\}\,. \tag{4.3}$$

   Moreover, $\left(\bar{p}, \bar{y}\mathbf{1}_{\Omega_F^{max}} + y\mathbf{1}_{\Omega_F \setminus \Omega_F^{max}}\right)$ is a Cournot S-equilibrium, for any production plan y.

Proof. 1. By Lemma 4.2, any deviation from $\bar{y}$ by a single firm will leave the price $\bar{p}$ unchanged, and the best choice of a production plan for each firm, from the perspective of its shareholders, is a profit maximizing plan. Hence the conclusion follows.

2. Using Lemma 4.2 and Definition 4.1, it follows that $(\bar{\mathbf{p}}, \bar{y})$ is a Cournot S-equilibrium if and only if for every firm t, the set of its shareholders whose wealth can be increased by a deviation to a profit maximization choice is negligible. Formally, for any $\hat{y}$ which is a profit maximizing at prices $\bar{p}$,

$$\gamma\left(t, \{s \mid (\bar{p} \cdot \hat{y}(t) - \bar{p} \cdot \bar{y}(t))\theta(s, \mathcal{G}(t)) > 0\}\right) = 0, \text{ for } \mu_F\text{-a.e. } t \in \Omega_F. \quad (4.4)$$

Condition (4.4) requires that for μ_F-a.e. $t \in \Omega_F$, either $\bar{p} \cdot \hat{y}(t) = \bar{p} \cdot \bar{y}(t)$ or $\gamma\left(t, \{s \mid \theta(s, \mathcal{G}(t)) > 0\}\right) = 0$ (or both), and the conclusion follows. □

The last part of the proposition points out that, in an atomless economy, any production plan is S-efficient for firms in $\Omega_F \backslash \Omega_F^{max}$. If $\mathcal{E}$ is atomless then, for every $t \in \Omega_F$, $\{s \,|\, \theta(s, \mathcal{G}(t)) > 0\}$ is a μ_C-measure zero set. Thus any firm $t \in \Omega_F^{max}$ has a positive share of it owned by a μ_C-measure zero set of consumers. Note that a firm belongs to the set Ω_F^{max} if it satisfies two conditions. One is to have some shareholders whose portfolios are not fully diversified (in the sense of putting a positive mass on at least one firm and thus being affected significantly by that firm's profits). The second requirement is that the set of those non fully-diversified shareholders is non-negligible relative to the set of all shareholders of the firm. These two conditions insure that the firm's choices have a non-negligible effect on the wealth of a significant subset of shareholders. Because the price effect is absent in an atomless economy, those shareholders unanimously approve profit maximization and thus any Cournot S-efficient production plan for a given firm has to be profit maximizing.

Note that $\Omega_F^{max} = \emptyset$ if the measures $\theta(s, \cdot)$ are atomless for every $s \in \Omega_C$. This happens, for instance, in the diffuse ownership economy $\mathcal{E}^d$ introduced in (3.9). At the other extreme is the case of the concentrated ownership economy $\mathcal{E}^c$ defined in (3.8), where $\Omega_F^{max} = \Omega_F$ and thus every Cournot S-equilibrium is profit maximizing.

5 Limit behavior of Cournot S-equilibrium

We investigate the behavior of Cournot S-equilibrium in large economies and establish under what conditions Cournot-S equilibrium production plans in a sequence of convergent economies approach profit maximizing production plans of the limit economy. First, we define a convergence notion on the space of private ownership economies. We say that a sequence of finite economies $(\mathcal{E}_n = ((\Omega_C, \mathcal{F}_n, \mu_C^n); (\Omega_F, \mathcal{G}_n); \theta_n))_{n\in\mathbb{N}}$ converges to an economy $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$ if each component in the description of $\mathcal{E}_n$ converges to the appropriate component of $\mathcal{E}$, in a sense made precise in the following definition.

Definition 5.1. *Finite economies $(\mathcal{E}_n)_{n\in\mathbb{N}}$ converge to the economy $\mathcal{E}$ if*

(i) $\mathcal{F}_n \nearrow \mathcal{F}$ and $\mathcal{G}_n \nearrow \mathcal{G}$,[15]

(ii) The ownership kernels θ_n converge to θ, in the sense that given any uniformly bounded sequence (X_n) of random variables on Ω_F such that X_n is $\mathcal{G}_n$-measurable and, for μ_C-a.e. $s \in \Omega_C$, $X_n \to X$ $\theta(s,\cdot)$-a.s., the following holds

$$\int_{\Omega_F} X_n(t)\theta_n(\cdot, dt) \to \int_{\Omega_F} X(t)\theta(\cdot, dt), \quad \mu_C\text{-a.s.}$$

(iii) μ_C^n has an extension to $\mathcal{F}$ that converges setwise to μ_C.[16]

Condition (i) requires that the sequence $(\mathcal{F}_n)$, respectively $(\mathcal{G}_n)$, asymptotically generates $\mathcal{F}$, respectively $\mathcal{G}$. Condition (ii) is satisfied if, e.g., for μ_C-almost all $s \in \Omega_C$, the kernel $\theta_n(s,\cdot)$ has an extension to $\mathcal{G}$ that converges setwise to $\theta(s,\cdot)$ (Lemma C.2), and sufficient conditions for the existence of

[15] If $(\mathcal{A}_n)_{n\in\mathbb{N}}, \mathcal{A}$ are σ-algebras on a set A, then $\mathcal{A}_n \nearrow \mathcal{A}$, if and only if $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ for all $n \in \mathbb{N}$, and $\mathcal{A} = \sigma(\cup_{n\in\mathbb{N}}\mathcal{A}_n)$.

[16] This means that there exist measures $\tilde{\mu}_C^n(\cdot)$ on $\mathcal{F}$ which coincide with μ_C^n when restricted to $\mathcal{F}_n$ (i.e., $\tilde{\mu}_C^n(\cdot)|_{\mathcal{F}_n} = \mu_C^n(\cdot)$), and $\tilde{\mu}_C^n(S) \to \mu_C(S)$ for all $S \in \mathcal{F}$. Lemma C.1 gives sufficient conditions for the existence of such extensions.

such an extension are given in Lemma C.1. Finally, Condition (iii), in conjunction with condition (ii) guarantees that the distribution of intermediate endowments in the finite economies will converge to the distribution of intermediate endowments in the limit economy (Proposition 5.3).

The convergence notion for finite economies introduced in Definition 5.1 is flexible enough to allow any economy to be approximated by finite economies. Notice that the ownership kernel θ_n^c defined in equation (3.5) can be viewed as a "conditional expectation" of the ownership kernel in the limit economy, in other words for any $T \in 2^{\mathcal{J}} \otimes \mathcal{H}_n$ and any $s \in \Omega_C$, $\theta_n^c(s,T) = E^{\mu_C}[\theta^c(\cdot,T)|2^{\mathcal{J}} \otimes \mathcal{H}_n](s)$. The same holds true for the θ_n^d defined in equation (3.7). This idea can be extended to general ownership structures.

Proposition 5.2. *Consider an economy $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$ such that $(\Omega_F, \mathcal{G})$ is Polish. Let $(\mathcal{F}_n)_{n\in\mathbb{N}}$, respectively $(\mathcal{G}_n)_{n\in\mathbb{N}}$ be finite σ-algebras on Ω_C, respectively Ω_F, such that $\mathcal{F}_n \nearrow \mathcal{F}$ and $\mathcal{G}_n \nearrow \mathcal{G}$. There exists a sequence of finite economies $(\mathcal{E}_n)_{n\in\mathbb{N}}$ with $\mathcal{E}_n = ((\Omega_C, \mathcal{F}_n, \mu_C); (\Omega_F, \mathcal{G}_n); \theta_n))$, converging to $\mathcal{E}$ and satisfying*

$$\theta_n(s,T) = E^{\mu_C}[\theta(\cdot,T)|\mathcal{F}_n](s),\ \mu_C - a.e.\ s \in \Omega_C, \forall T \in \mathcal{G}. \tag{5.1}$$

Proof. Proceeding as for the construction of the kernel γ done in Appendix B, we show that there exists a probability kernel η from Ω to $\mathcal{G}$, such that for all $T \in \mathcal{G}$, $\eta(\cdot,T)$ is $\mathcal{F}_n$-measurable, and

$$E^{\mu_C}[\theta(\cdot,T)|\mathcal{F}_n](s) = \eta(s,T) \cdot E^{\mu_C}[\theta(\cdot,\Omega_F)|\mathcal{F}_n](s),\ \mu_C - \text{a.e. } s. \tag{5.2}$$

Indeed, consider the probability space $(\Omega_C \times \Omega_F, \mathcal{F}_n \otimes \mathcal{G}, \mu_C \otimes \theta/\alpha)$, where $\mu_C \otimes \theta$ is defined in (2.1) and $\alpha = (\mu_C \otimes \theta)(\Omega_F \otimes \Omega_C)$. Since $(\Omega_F, \mathcal{G})$ is Polish, there exists a regular conditional distribution η of π_F given π_C, where π_F, respectively π_C, are the projections of $\Omega_C \times \Omega_F$ on Ω_F, respectively on Ω_C (Dudley 2002, Theorem 10.2.2). Thus η will be a probability kernel from $(\Omega_C, \mathcal{F}_n)$ to $(\Omega_F, \mathcal{G})$, such that $\eta(\cdot,T)$ is $\mathcal{F}_n$-measurable, for all $T \in \mathcal{G}$, and η

satisfies (5.2).

Define $\theta_n(s,T) := \eta(s,T) \cdot E^{\mu_C}[\theta(\cdot,\Omega_F)|\mathcal{F}_n](s)$. Therefore θ_n satisfies (5.1), and for each $s \in \Omega_C$, $\theta_n(s,\cdot)$ is a measure. Moreover, by construction, $\theta_n(\cdot,T)$ is $\mathcal{F}_n$-measurable for any $T \in \mathcal{G}$. Hence θ_n is a kernel from $(\Omega_C,\mathcal{F}_n)$ to $(\Omega_F,\mathcal{G})$.

Consider a sequence of random variables $(X_n)_n$ with $X_n : \Omega_F \to \mathbb{R}$ being $\mathcal{G}_n$-measurable, and let $X, Y : \Omega_F \to \mathbb{R}$, $\mathcal{G}$-measurable, such that $|X_n| \leq Y$, $\int_{\Omega_F} Y d\mu_F < \infty$ and $X_n \to X$, μ_C-a.s. Starting with simple functions and then extending the argument using a monotone class theorem (Kallenberg 2002, Theorem 1.1), it follows that

$$W_n(s) := \int_{\Omega_F} X_n d\theta_n(s,\cdot) = E^{\mu_C}\left[\int_{\Omega_F} X_n d\theta(s',\cdot)|\mathcal{F}_n\right](s). \tag{5.3}$$

By Lebesgue's dominated convergence theorem, the sequence of functions $f_n(s') := \int_{\Omega_F} X_n d\theta(s',\cdot)$ converges pointwise to $W(s') := \int_{\Omega_F} X d\theta(s',\cdot)$. Using an extension of the martingale convergence theorem due to Hunt (Kopp 1984, Theorem 2.8.5), it follows that $W_n \to W$, μ_C-a.s. Thus we proved that indeed θ_n converges to θ in the sense of Definition 5.1. □

Notice that, by the martingale convergence theorem, (5.1) implies that for all $T \in \mathcal{G}$, $\theta_n(\cdot,T) \to \theta(\cdot,T)$, μ_C-a.e. However the set of μ_C-measure zero where the convergence might fail depends on T, and thus the set of s where $\theta(s,T)$ does not converge to $\theta(s,T)$ for a $T \in \mathcal{G}$ might be large, even cover the whole Ω_C. Thus it might not be true that, for μ_C-a.e. $s \in \Omega_C$, $\theta_n(s,\cdot)$ converges setwise to $\theta(s,\cdot)$.

For a general sequence of convergent economies as in Definition 5.1 and a sequence of convergent production plans, we show that the distribution of intermediate endowments across types converges.

Proposition 5.3. *Let $(\mathcal{E}_n = ((\Omega_C,\mathcal{F}_n,\mu_C^n);(\Omega_F,\mathcal{G}_n);\theta_n))_{n\in\mathbb{N}}$ be a sequence of finite economies converging to an economy $\mathcal{E} = ((\Omega_C,\mathcal{F},\mu_C);(\Omega_F,\mathcal{G});\theta)$.*

Let y_n be a production plan in $\mathcal{E}_n$ and y a production plan in $\mathcal{E}$. If $y_n \to y$, μ_F-a.s., where $\mu_F = \mu_C\theta$, then $\mu_C^n \circ \widetilde{w}_{y_n}^{-1}$ converges weakly to $\mu_C \circ \widetilde{w}_y^{-1}$.

Proof. Using equation (3.3), for any $T \in \mathcal{B}(\Omega_F)$, with $\mu_F(T) = 0$ it follows that $\theta(s, T) = 0$ for μ_C-a.e. $s \in \Omega_C$. Thus the fact that $y_n \to y$ μ_F-a.s. implies that, for μ_C-a.e. $s \in \Omega_C$, $y_n \to y$, $\theta(s, \cdot)$-a.s. Since θ_n converges to θ, it follows that $w_{y_n} \to w_y$, μ_C-a.s. (see Definition 5.1,(ii)). For any $g : \mathcal{I} \times \mathbb{R}^L \to \mathbb{R}$ continuous and bounded, Lemma C.2 applied to the sequence μ_C^n having extensions converging setwise to μ_C gives

$$\int_{\mathcal{I}\times\mathbb{R}^L} g \; d\mu_C^n \circ \tilde{w}_{y_n}^{-1} = \int_{\Omega_C} g \circ \tilde{w}_{y_n} \; d\mu_C^n \to \int_{\Omega_C} g \circ \tilde{w}_y \; d\mu_C = \int_{\mathcal{I}\times\mathbb{R}^L} g \; d\mu_C \circ \tilde{w}_y^{-1}. \tag{5.4}$$

Thus the convergence of the distribution of intermediate endowments across types is established. □

Given an economy $\mathcal{E}$ and $\varepsilon > 0$, we denote the set of firms for which almost every shareholder owns more than ε shares by

$$\Omega_F^{\mathcal{E}}(\varepsilon) := \{t \in \Omega_F \mid \theta(\cdot, \mathcal{G}(t)) \geq \varepsilon, \; \gamma(t, \cdot)\text{-a.e}\}. \tag{5.5}$$

We show next that for each $\varepsilon > 0$, Cournot S-equilibrium plans converge to profit maximizing plans of the limit economy for those firms having all shareholders owning at least ε shares in them.

Theorem 5.4. *Let $(\mathcal{E}_n)_{n\in\mathbb{N}}$ with $\mathcal{E}_n = ((\Omega_C, \mathcal{F}_n, \mu_C^n); (\Omega_F, \mathcal{G}_n); \theta_n)$ be a sequence of finite economies converging to $\mathcal{E} = ((\Omega_C, \mathcal{F}, \mu_C); (\Omega_F, \mathcal{G}); \theta)$, and for each $n \in \mathbb{N}$, let y_n be a Cournot S-equilibrium of the economy $\mathcal{E}_n$, such that $y_n \to y$, μ_F-a.s.,*[17] *where $\mu_F = \mu_C\theta$. Assume that there exists a unique equilibrium price p associated with the production plan y in the limit economy*

[17]The μ_F-almost sure convergence of production plans can be replaced by convergence in μ_F-measure, since any sequence convergent in measure has a subsequence converging almost surely (Dudley 2002, Theorem 9.2.1).

$\mathcal{E}$ (i.e. $P(y)$ is a singleton). Then for any $\varepsilon > 0$, y is profit maximizing at prices p for μ_F-almost all firms belonging to $\Omega_F^*(\varepsilon) := \bigcap_{n\in\mathbb{N}} \Omega_F^{\mathcal{E}_n}(\varepsilon)$.

Proof. Assume, by contradiction, that y is not profit maximizing for a μ_F-positive measure of firms belonging to $\Omega_F^*(\varepsilon)$. Let $\bar{y}$ be a profit maximizing production plan at price p, i.e.,

$$p \cdot \bar{y}(t) = \max_{z\in Y_{\jmath(t)}} p \cdot z,$$

such that firms of the same type choose identical production plans, thus $\bar{y}$ is selected to be $2^{\mathcal{J}} \times [0,1]$-measurable. Notice that $\bar{y}$ is $\mathcal{G}_n$-measurable, for any $n \in \mathbb{N}$. The boundedness of the production sets implies that $\bar{y}$ is bounded as well. Construct the $\mathcal{G}$-measurable function $d : \Omega_F \to \mathbb{R}$ defined as $d(\cdot) := p \cdot \bar{y}(\cdot) - p \cdot y(\cdot) \geq 0$ and for all n, let $d_n : \Omega_F \to \mathbb{R}$, $d_n(\cdot) := p \cdot \bar{y}(\cdot) - p \cdot y_n(\cdot) \geq 0$, which is $\mathcal{G}_n$-measurable. Therefore there exists a set $T \in \mathcal{G}$ and $\delta > 0$ such that $T \subset \{d > 0\} \cap \Omega_F^*(\varepsilon)$ and $\mu_F(T) = \delta > 0$. Since $T \subset \cup_{k\in\mathbb{N}} \{d \geq 1/k\} \cap \Omega_F^*(\varepsilon)$, we can choose $k \in \mathbb{N}$ and $T' \subset T$ such that $T' \subset \{d \geq 1/k\} \cap \Omega_F^*(\varepsilon)$ and $\mu_F(T') > \delta/2$. Moreover, for any $n \in \mathbb{N}$, $\{d \geq 1/k\} \subset \{|d - d_n| \geq 1/(2k)\} \cup \{d_n \geq 1/(2k)\}$, and thus for all $m \in \mathbb{N}$,

$$\{d \geq 1/k\} \subset (\cup_{n\geq m} \{|d - d_n| \geq 1/(2k)\}) \cup (\cap_{n\geq m} \{d_n \geq 1/(2k)\}).$$

As $y_n \to y$ μ_F-a.s., it follows that $d_n \to d$ μ_F-a.s., and hence

$$\lim_{m\to\infty} \mu_F (\cup_{n\geq m} \{|d - d_n| \geq 1/(2k)\}) = 0.$$

Choose N large enough such that $\mu_F (\cup_{n\geq N} |d - d_n| \geq 1/(2k)) < \delta/4$. Thus there exists $T'' \subset T'$ with $\mu_F(T'') > \delta/4$ such that

$$T'' \subset \cap_{n\geq N}\{d_n \geq 1/(2k)\} \cap \Omega_F^*(\varepsilon). \tag{5.6}$$

As $\mathcal{G}_N$ is finite, it has an atom G_N such that $\mu_F(T'' \cap G_N) > 0$. Moreover

G_N is a finite union of atoms of $\mathcal{G}_{N+1}$, hence we can construct inductively a sequence $(G_n)_{n\geq N}$ with G_n being an atom of $\mathcal{G}_n$, such that

$$\forall n \geq N: \; G_n \cap T'' \subset \{d_n \geq 1/(2k)\} \cap \Omega_F^{\mathcal{E}_n}(\varepsilon), \; \mu_F(G_n \cap T'') > 0, \tag{5.7}$$

and, moreover, $G_{n+1} \subset G_n$ for all $n \geq N$. Since $\{d_n \geq 1/(2k)\} \cap \Omega_F^{\mathcal{E}_n}(\varepsilon) \in \mathcal{G}_n$, it follows that

$$G_n \subset \{d_n \geq 1/(2k)\} \cap \Omega_F^{\mathcal{E}_n}(\varepsilon), \forall n \geq N. \tag{5.8}$$

Define the alternative production plans

$$\hat{y}_n = y_n + (\bar{y} - y_n)\mathbf{1}_{G_n}.$$

Given that G_n is an atom of $\mathcal{G}_n$, $\cap_{n\geq N} G_n$ is either an atom of $\mathcal{G}$ or equals the empty set.[18] Since the limit economy is atomless, $\mu_F(G_n) \searrow 0$. This implies that $\hat{y}_n$ converges in measure to y, and hence it converges almost surely to y along a subsequence. Consider a price selection $\mathbf{p}_n$ associated to y_n (that is, $(\mathbf{p}_n, y_n)$ is Cournot S-equilibrium for $\mathcal{E}_n$). Let $p_n = \mathbf{p}_n(y_n)$ and $\hat{p}_n = \mathbf{p}_n(\hat{y}_n)$. By Proposition 5.3, $\mu_C^n \circ \widetilde{w}_{\hat{y}_n}^{-1} \to \mu_C \circ \widetilde{w}_y^{-1}$. Since the price correspondence $P(\cdot)$ has closed graph and $\hat{p}_n \in P\left(\mu_C^n \circ \widetilde{w}_{\hat{y}_n}^{-1}\right)$, it follows that, for any convergent subsequence $(\hat{p}_{n_r})$ of $(\hat{p}_n)$,

$$\lim_{r\to\infty} \hat{p}_{n_r} \in P\left(\lim_{r\to\infty} \mu_C^{n_r} \circ \widetilde{w}_{\hat{y}_{n_r}}^{-1}\right) = P(\mu_C \circ \widetilde{w}_y^{-1}) = \{p\}.$$

Repeating the reasoning for $(p_n)_n$ we can assume without loss of generality that $p_n, \hat{p}_n \to p$, rather than selecting a subsequence where convergence holds.

Equation (5.8) implies that for all $n \geq N$,

$$p(w_{\hat{y}_n}(s) - w_{y_n}(s)) = \theta_n(s, G_n) d_n(G_n) \geq \frac{\varepsilon}{2k}, \text{ for } \gamma_n(G_n, \cdot)\text{-a.e. } s \in \Omega_C. \tag{5.9}$$

[18]If $t \in \cap_{n\geq N} G_n$, it is shown in the Appendix A that $\mathcal{G}(t) = \cap\{G \mid t \in G, G \in \cup_n \mathcal{G}_n\}$, hence $\cap_{n\geq N} G_n = \mathcal{G}(t)$.

We show that there exists $\delta > 0$, such that for all $n \geq N$,

$$V^{\imath(s)}(\hat{p}_n, w_{\hat{y}_n}(s)) > V^{\imath(s)}(p_n, w_{y_n}(s)) + \delta, \text{ for } \gamma_n(G_n, \cdot)\text{-a.e. } s \in \Omega_C, \quad (5.10)$$

which contradicts the Cournot S-efficiency of the production plans y_n.

Indeed, if (5.10) is not satisfied, we can choose a subsequence $(n_r)_{r\in\mathbb{N}}$, a type $i \in \mathcal{I}$ and a sequence of consumers $(s_{n_r})_{r\in\mathbb{N}} \in \Omega_C$ such that $\imath(s_{n_r}) = i, \gamma(G_{n_r}, \mathcal{F}_{n_r}(s_{n_r})) > 0$ and $V^i(\hat{p}_{n_r}, w_{\hat{y}_{n_r}}(s_{n_r})) \leq V^i(p_{n_r}, w_{y_{n_r}}(s_{n_r})) + 1/r$, for all $r \in \mathbb{N}$. The sequence of production plans $(\hat{y}_{n_r})$ are uniformly bounded, thus the sequences of intermediate endowments $(w_{\hat{y}_{n_r}}(s_{n_r}))_{r\in\mathbb{N}}$ and $(w_{y_{n_r}}(s_{n_r}))_{r\in\mathbb{N}}$ are also bounded and therefore contain converging subsequences. We can thus assume, without loss of generality, that $w_{y_{n_r}}(s_{n_r}) \to w \in \mathbb{R}^L$ and $w_{\hat{y}_{n_r}}(s_{n_r}) \to \hat{w} \in \mathbb{R}^L$. Taking limits with $r \to \infty$ and using the continuity of the indirect utility function V^i, we obtain $V^i(p, \hat{w}) \leq V^i(p, w)$ and thus $p \cdot \hat{w} \leq p \cdot w$. Notice that, by (5.9),

$$p(w_{\hat{y}_{n_r}}(s_{n_r}) - w_{y_{n_r}}(s_{n_r})) \geq \frac{\varepsilon}{2k}, \ \forall r \in \mathbb{N}. \quad (5.11)$$

Taking the limit with $r \to \infty$ in (5.11), we obtain $p \cdot \hat{w} \geq p \cdot w + \frac{\varepsilon}{2k}$, and we reached a contradiction. □

The theorem relies heavily on the assumption that there is a unique equilibrium price corresponding to the limit production plan y in the atomless economy. This condition is needed to insure continuity of the price selection at the limit point. While we cannot dispense with it completely, the requirement can be considerably relaxed, with a construction as in Roberts (1980). That approach allows for multiplicity of equilibria at the limit point, but requires regularity of the limit equilibrium and thus its *local* uniqueness. Even so, it remains a strong condition since, as shown by Roberts himself, existence of critical equilibria is non-pathological.

Allen (1994) pointed out that this negative result is alleviated if, instead of simple price selections, one uses *randomized* price selections (i.e., selections from the correspondence coP instead of P; this amounts to saying that firms hold non-trivial beliefs over the possible market clearing prices). As opposed to the case of simple price selections, the existence of continuous randomized price selections is a generic result (see also Mas-Colell and Nachbar (1991)). Allen proves therefore that, if firms maximize their *expected* profits with respect to some non-trivial beliefs over prices, convergence of Cournot equilibria (in which firms maximize profits) to competitive equilibria does obtain generically. However, the problem is more complex here and Allen's approach cannot be directly applied. The reason is that, as opposed to the standard Cournot model in which the firms maximize profits, in our model S-efficiency requires firms to make pairwise comparisons between a status quo and an alternative. For that, a firm has to use its beliefs over two different equilibrium sets. To make this comparison meaningful, some global beliefs need to be defined. This was done in Bejan (2008). Whether allowing for firms' non-trivial global beliefs over prices does indeed improve the convergence result is an interesting question which remains open for now and will be subject of future research.

For the diffuse ownership economy $\mathcal{E}_n^d$ (see equations (3.6),(3.7)) $\Omega_F^{\mathcal{E}_n^d}(\varepsilon) = \emptyset$, for all $\varepsilon > 0$ and large enough n. Thus $\Omega_F^*(\varepsilon) = \emptyset$, and Theorem 5.4 holds trivially and is devoid of implications. For the concentrated ownership case described in (3.4),(3.5), $\Omega_F^{\mathcal{E}_n^c}(\varepsilon) = \Omega_F$ for every $0 < \varepsilon < \min_{i,j} s(i,j)$. Moreover, for this particular case, Theorem 5.4 goes through if we require just weak convergence (convergence in distribution) of the production plans. For a production plan $y : (\mathcal{J} \times [0,1], 2^{\mathcal{J}} \otimes \mathcal{B}([0,1]), \lambda_{\mathcal{J}} \otimes \lambda) \to \mathbb{R}^L$, we let $\mathcal{L}(y)$ be the distribution of $(y(1,\cdot),\dots,y(J,\cdot)) : [0,1] \to (\mathbb{R}^L)^J$ and refer to it as the *law* of y. Thus $\mathcal{L}(y) := \lambda \circ (y(1,\cdot),\dots,y(J,\cdot))^{-1}$.

Theorem 5.5. *For each $n \in \mathbb{N}$, let y_n be a Cournot S-equilibrium of the concentrated ownership economy $\mathcal{E}_n^c$ and y be a production plan in $\mathcal{E}^c$, such that the law of y_n converges weakly to the law of y. Assume that there exists a unique equilibrium price p associated with the production plan y in the limit* *economy $\mathcal{E}^c$ (i.e. $P(y)$ is a singleton). Then y is profit maximizing at prices p.*

Proof. By Skorohod's embedding theorem (Kallenberg 2002, Theorem 4.30), there are alternative production plans $(\tilde{y}_n)_{n\in\mathbb{N}}, \tilde{y}$, defined on $(\mathcal{J} \times [0,1], 2^{\mathcal{J}} \otimes \mathcal{B}([0,1]), \lambda_{\mathcal{J}} \otimes \lambda)$, such that for all $n \in \mathbb{N}$, $(\tilde{y}_n(1,\cdot), \ldots, \tilde{y}_n(J,\cdot))$ has the same distribution as $(y_n(1,\cdot), \ldots, y_n(J,\cdot))$, that is $\mathcal{L}(y_n) = \mathcal{L}(\tilde{y}_n))$, $\mathcal{L}(\tilde{y}) = \mathcal{L}(y)$, and $\tilde{y}_n \to \tilde{y}$ μ_F-a.s., where $\mu_F := (\lambda_{\mathcal{J}} \otimes \lambda)$.

The intermediate endowment of an agent $(i,\cdot)$ given the production plan $y' \in \{y_n, \tilde{y}_n\}$ is $w_{y'}(i,\cdot) = e^i + \sum_{j\in\mathcal{J}} s(i,j)y'(j,\cdot)$. The laws $\mathcal{L}(y_n), \mathcal{L}(\tilde{y}_n)$ coincide, thus by the continuous mapping theorem (Dudley 2002, Theorem 9.3.7), for all $i \in \mathcal{I}$ the distributions of $w_{y_n}(i,\cdot), w_{\tilde{y}_n}(i,\cdot)$ are identical, that is $\lambda \circ w_{y_n}^{-1}(i,\cdot) = \lambda \circ w_{\tilde{y}_n}^{-1}(i,\cdot)$. It follows that the distribution of intermediate endowments across types for the two plans coincide, $\mu_C \circ \widetilde{w}_{y_n}^{-1} = \mu_C \circ \widetilde{w}_{\tilde{y}_n}^{-1}$.

Let $z \in y_n(\{j\} \times [0,1]) \subset \mathbb{R}^L$, and $A := (\tilde{y}_n(j,\cdot))^{-1}(z)$. Since $\lambda(A) = \lambda((y_n(j,\cdot))^{-1}(z))$, there exists $k \in \{1,\ldots,n\}$ such that $\lambda(A) = k/n$. Using Liapunov's convexity theorem (λ is atomless) we can construct disjoint sets $A_1, \ldots, A_k \subset [0,1]$ such that $A = \cup_{l=1}^k A_l$ and $\lambda(A_l) = 1/n$ for all $l \in \{1,\ldots,k\}$. Repeating this process, for each $j \in \mathcal{J}$ we can partition $[0,1]$ into some sets $A_1^j, \ldots, A_n^j$ with $\lambda(A_l^j) = 1/n$ for all $l \in \{1,\ldots,n\}$, and such that $\tilde{y}_n$ is measurable with respect to the partition $\tilde{\mathcal{G}}_n := \{\{j\} \times A_l^j \mid j \in \mathcal{J}, l \in \{1,\ldots,n\}\}$ of $\mathcal{J} \times [0,1]$.

Notice that $\tilde{y}_n$ is a Cournot S-efficient production plan for the economy $\tilde{\mathcal{E}}_n := ((\Omega_C, \mathcal{F}_n^c, \lambda_{\mathcal{I}} \otimes \lambda), (\Omega_F, \tilde{\mathcal{G}}_n, \lambda_{\mathcal{J}} \otimes \lambda))$. Otherwise, Cournot S-efficiency of the plan y_n in the economy E_n^c would be contradicted. Moreover, $\tilde{y}$ satisfies also the singleton property because y and $\tilde{y}$ generate identical distributions of intermediate endowments in $\mathcal{E}^c$, hence $P(\tilde{y}) = P(y)$.

If $\tilde{\mathcal{G}}_n \nearrow 2^{\mathcal{J}} \otimes \mathcal{B}([0,1])$, the sequence of economies $(\tilde{\mathcal{E}}_n)$ converges to $\mathcal{E}^c$ and the Cournot S-efficient production plans $(\tilde{y}_n)$ converge almost surely to $\tilde{y}$. Theorem 5.4 applies and we conclude that y is profit maximizing on $\Omega_F^*(\varepsilon)$. For every $0 < \varepsilon < \min_{i,j} s(i,j)$ (see equation (3.4)), $\Omega_F^*(\varepsilon) = \Omega_F$.

Assume now that the sequence $(\tilde{\mathcal{G}}_n)_{n\in\mathbb{N}}$ is not necessarily monotonically increasing. The monotonicity of the sequence of σ-algebras $(\tilde{\mathcal{G}}_n)$, coupled with the setwise convergence of the ownership structures, was used to establish the convergence in distribution of the sequence of intermediate endowments. However we proved this fact directly here, using the strong symmetry properties of the ownership structure. Also the atoms (G_n), in equation (5.8) cannot be guaranteed to be monotonically decreasing. Nevertheless, all that is needed is that $\mu_F(G_n) \to 0$, which is automatically satisfied, since for any atom G_n of $\tilde{\mathcal{G}}_n$, $\mu_F(G_n) = 1/(n \cdot J)$. The proof of Theorem 5.4 can be replicated without any additional changes, obtaining the conclusion. □

A direct proof of Theorem 5.5, without the use of Theorem 5.4, was given in Bejan (2005). Convergence in distribution is too weak for Theorem 5.4, in which we allow for heterogeneity in the ownership of firms and consumers of identical type. Given an arbitrary production plan, one can permute the choices of identical type firms, resulting in two production plans with identical distributions; however, in the presence of asymmetric ownership the two plans induce different distributions over the space of intermediate endowments.

Based on the examples provided so far, one might think that a sequence of Cournot S-equilibria of a converging sequence of finite economies approaches a Cournot S-efficient equilibrium of the limit economy. If true, this property would imply, according to Proposition 4.3, that sequences of Cournot S-equilibria of converging finite economies approach the Walrasian equilibria of the limit economy if Ω_F^{max} defined in (4.3) is a full-measure set. The following example shows that this is not true and thus sequences of Cournot S-equilibria do not converge to a Cournot S-equilibrium of the limit economy.

Example 5.1
We modify slightly the example of section 2, by removing the consumer that does not own shares in the prototype economy, and letting the endowments of the unique agent of the prototype economy be $(2, 2)$. Assume that in the

finite n-fold replica $\mathcal{E}_n$ and in the continuum replica $\mathcal{E}_\infty$, half of each firm is owned exclusively by the agent with the same name and the rest is uniformly distributed across all agents (including the agent with the same name). We will refer to this way of assigning ownership of the firms in the replicas as the *hybrid ownership structure*.

Given a production plan $y = ((-\alpha_j, \alpha_j))_{j=1}^n$ in the n-fold replica $\mathcal{E}_n$, the resulting exchange equilibrium price vector, normalized to the unit simplex, is

$$(p_1, p_2) = \left(\frac{2+\kappa(y)}{4}, \frac{2-\kappa(y)}{4}\right),$$

with $\kappa(y)$ defined as in (2.1). For the continuum replica, prices have the same expression, with $\kappa(y)$ defined in (2.2). The Walrasian equilibrium in the finite and continuum replica economies are associated with prices $(\frac{1}{2}, \frac{1}{2})$ and $\kappa(y) = 0$, hence all firms choose the production plan $(0, 0)$ in a Walrasian equilibrium.

We start by determining the Cournot S-efficient production plans in the n-fold replica economy. The wealth of a consumer that is a majority shareholder in a firm choosing $(-\alpha, \alpha)$ is

$$w(\kappa(y), \alpha) = 2 + \frac{1}{2}(p_2 - p_1)(\alpha + \kappa(y)) = 2 - \frac{1}{4}\kappa^2(y) - \frac{1}{4}\kappa(y)\alpha,$$

and its utility is $u(\kappa(y), \alpha) = 2 \ln w(\kappa(y), \alpha) - \ln(2p_1) - \ln(2p_2)$. We let $\kappa := \kappa(y)$ for brevity. Notice that

$$\frac{\partial u(\kappa, \alpha)}{\partial \kappa} = \frac{2}{n}\frac{(\kappa^3 - 4\alpha)}{(\kappa\alpha + \kappa^2 - 8)(\kappa + 2)(\kappa - 2)}, \tag{5.12}$$

and it follows that the derivative of u with respect to α is negative:

$$\frac{du(\kappa, \alpha)}{d\alpha} = \frac{1}{n}\frac{\partial u(\kappa, \alpha)}{\partial \kappa} + \frac{\partial u(\kappa, \alpha)}{\partial \alpha} = \frac{2}{n}\frac{(\kappa^3 - 8n\kappa - 4\alpha + 2n\kappa^3)}{(\kappa\alpha + \kappa^2 - 8)(\kappa + 2)(\kappa - 2)} < 0.$$

Thus a firm that chooses $(-\alpha, \alpha)$ hurts its majority shareholder by switching to a production plan (α', α') with $\alpha' > \alpha$. Moreover, (5.12) shows that by switching to a production plan (α', α') with $\alpha' < \alpha$, the firm hurts a minority shareholder that owns half of a firm that chose a production plan $(-\beta, \beta)$ satisfying $\left(\kappa - \frac{\alpha}{n}\right)^3 \geq 4\beta$. This discussion enables us to construct a multitude of Cournot S-equlibria. In particular, for any $k \in \{1, 2, \ldots, n-1\}$, a production plan with k firms choosing $(0, 0)$ and $n-k$ firms choosing $(-1, 1)$ is always a Cournot S-equilibrium.

The economies $(\mathcal{E}_n)$ and $\mathcal{E}_\infty$ can be embedded in the general framework of section 3 as discussed there, by letting $\theta_n = \theta_n^c/2 + \theta_n^d/2$ and $\theta = \theta^c/2 + \theta^d/2$ (see (3.4)-(3.9)). Notice that for the limit economy, $\Omega_F^{max} = \Omega_F = [0, 1]$ (since $\mathcal{I} = \mathcal{J} = \{1\}$, we identify $\{1\} \times [0, 1]$ with $[0, 1]$) and thus by Proposition 4.3, the only Cournot S-equilibrium allocation of the continuum economy $\mathcal{E}_\infty$ coincides with the Walrasian equilibrium and corresponds to all firms choosing $(0, 0)$. This can be seen directly, also, since in the absence of price effects, any firm that chose $(-\alpha, \alpha)$ with $\alpha > 0$ will increase the wealth and hence the utility of its majority shareholder by switching to $(0, 0)$, while its minority shareholders are unaffected. Moreover, $\Omega_F^* = \emptyset$, hence Theorem 5.4 has no bite in this example, suggesting that a sequence of Cournot S-equilibrium plans does not converge necessarily to a profit maximization plan.

Indeed, for an arbitrary $\eta \in [0, 1]$, consider the production plan y^η in the continuum economy in which firms in $[0, \eta]$ choose the production plan $(-1, 1)$ and the firms in $(\eta, 1]$ choose $(0, 0)$. Let the production plan y_n^η in the economy $\mathcal{E}_n$ be such that the first $[n \cdot \eta]_*$ firms (i.e., firms in $[0, [n \cdot \eta]_*/n]$) choose $(-1, 1)$ and the the rest choose $(0, 0)$ ($[n \cdot \eta]_*$ denotes the largest integer smaller than $n \cdot \eta$). Clearly $y_n^\eta \to y^\eta$ almost surely and (y_n^η) is a sequence of Cournot S-equilibrium production plans, but y^η is not a profit maximizing plan unless $\eta = 0$. This shows that a convergent sequence of Cournot S-equilibrium production plans in converging economies does not have to approach a Cournot S-equilibrium in the limit.

6 Conclusions

This paper contributes to the literature on non-cooperative foundations of Walrasian equilibrium, by pointing out to the firms' ownership structure as a potential source of inefficiency in arbitrarily large economies. If (some) shareholders control a firm's production decisions, its objective is shaped by the interaction between the price and the income effects on those shareholders' welfare. Each of these effects, and therefore the dominance of one over the other, depends on the ownership structure.

In the light of Hart's (1979) results, one may argue that profit maximization (under a specific price normalization) is a justified objective for an oligopolistic firm in a large economy, since gains obtained by deviating to shareholders' welfare improving plans are modest, and might easily be outweighed by the inherent "costs" of finding and implementing such plans among a large group. While this may be a convincing argument for firms that are controlled by a large group of consumers, we find it less compelling for a firm controlled by a very small group of consumers (say, its Board of Directors). The extreme case here would be sole ownership firms or, by an extension, firms controlled by a small board of consumers with aligned interests. Our results suggest, for example, that if firms are controlled by a small board with insignificant ownership of shares, market power inefficiencies can persists in arbitrarily large economies. A sufficient condition for the elimination of those inefficiencies via increased competition is having board members who own a significant share (but not necessarily a majority) of the firm they control. In general, we prove that Cournot S-equilibria of a converging sequence of finite economies approaches a Walrasian equilibrium of the limit economy if for (almost) every firm, each of its (controlling) shareholders owns a significant (i.e., bounded away from zero) fraction of the firm. For arbitrary ownership structures, sequences of Cournot S-efficient equilibria may not converge to a Cournot S-equilibrium of the limit economy.

Although we do not model trade in shares, we do allow for *arbitrary* (fixed) distributions of shares in each finite economy along the converging sequence and identify the class of those ownership structures that are conducive to competitive behavior. Our results bear implications even for richer environments in which share trading is allowed. It shows, for example, that perfectly competitive behavior will prevail in any large economy model of security trade in which the (post-trade) equilibrium distribution of shares is concentrated. On the other hand, perfect diversification of individual portfolios across firms (as predicted, for example, by mean-variance portfolio selection models) might lead to inefficiencies.

On the technical side, the paper contributes to the literature by defining a suitable topology on the space of production economies, which generalizes previous results and allows for full generality on the ownership structure.

Appendix

A Atoms of a countably generated σ-algebra

Let $\mathcal{A}$ be a σ-algebra on Ω. Define a binary relation on Ω as: $x \sim y$ if and only if $x \in A, A \in \mathcal{A} \Rightarrow y \in A$. Equivalently, if for $x \in \Omega$, we define $\mathcal{A}(x) := \cap\{A \in \mathcal{A} : x \in A\}$, then $x \sim y$ if and only if $y \in \mathcal{A}(x)$. It is easy to see that "$\sim$" is an equivalence relation, and hence $\mathcal{A}(x)$ is the equivalence class containing x. One is tempted to call $\mathcal{A}(x)$ an *atom* of $\mathcal{A}$, in the sense of the definition in footnote 3. However, in general $\mathcal{A}(x) \notin \mathcal{A}$.

We show in what follows that if $\mathcal{A}$ is a *countably generated* σ-algebra, i.e., if $\mathcal{A}$ is generated by a countable subset of itself, then $\mathcal{A}(x) \in \mathcal{A}, \forall x \in \Omega$. This means that $\mathcal{A}(x)$ is an atom of $\mathcal{A}$, i.e., for all $B \in \mathcal{A}$, either $\mathcal{A}(x) \subset B$ or $\mathcal{A}(x) \cap B = \emptyset$. Let $\mathcal{C}$ be a countable subset generating $\mathcal{A}$, i.e. $\mathcal{A} = \sigma(\mathcal{C})$, and let $\bar{\mathcal{C}}$ be the algebra generated by $\mathcal{C}$, which consists exactly of all elements of $\mathcal{C}$ together with all sets obtainable from finite sequences of set theoretic operations on $\mathcal{C}$. Thus $\bar{\mathcal{C}}$ is also countable. Fix a point $x \in A$, and let

$\bar{\mathcal{C}}(x) := \cap\{C \in \bar{\mathcal{C}} | x \in C\}$. Define

$$\mathcal{D}_x := \{A \in \mathcal{A} \mid x \notin A\} \cup \{A \in \mathcal{A} \mid x \in A, \bar{\mathcal{C}}(x) \subset A\}.$$

It is easy to check that $\mathcal{D}_x$ is a λ-system, which means that it contains Ω and is closed under proper differences and increasing limits. Moreover $\bar{\mathcal{C}} \subset \mathcal{D}_x$, and $\bar{\mathcal{C}}$ is a $\pi-$system (i.e. is closed under finite intersections). The monotone class theorem (Kallenberg 2002, Th. 1.1) implies that

$$\mathcal{A} = \sigma(\bar{\mathcal{C}}) \subset \mathcal{D}_x \subset \mathcal{A},$$

and thus $\mathcal{D}_x = \mathcal{A}$. It follows that $\bar{\mathcal{C}}(x) = \mathcal{A}(x)$, but $\bar{\mathcal{C}}(x) \in \mathcal{A}$ since $\bar{\mathcal{C}}$ is countable.

B Construction of the γ-kernel

Let $\alpha := (\mu_C \otimes \theta)(\Omega_C \times \Omega_F) < \infty$ and thus we can write $\mu_C \otimes \theta = \alpha \cdot \Theta$, with Θ a probability on $\mathcal{F} \otimes \mathcal{G}$.

Define π_C, π_F to be the projection functions of $(\Omega_C \times \Omega_F, \mathcal{F} \otimes \mathcal{G})$ on Ω_C, respectively on Ω_F. Since $(\Omega_C, \mathcal{F})$ is a Polish space, there exists a regular conditional distribution of π_C given π_F, which will be a probability kernel γ from Ω_F to $\mathcal{F}$ (Dudley 2002, Theorem 10.2.2). The probability $\gamma(t, \cdot)$ represents the ownership distribution of firm $t \in \Omega_F$ over consumers. Let Θ_F be the marginal on Ω_F of Θ. By construction, and using the fact that $\mu_C \otimes \theta = \alpha \cdot \Theta$, it follows that for any $g : \Omega_C \times \Omega_F \to \mathbb{R}$, which is $\mathcal{F} \otimes \mathcal{G}$-

measurable and $\mu_C \otimes \theta$-integrable,

$$\int_{\Omega_C} \left[\int_{\Omega_F} g(s,t)\theta(s,dt) \right] \mu_C(ds) = \int_{\Omega_C \times \Omega_F} g(s,t)(\mu_C \otimes \theta)(ds,dt) \tag{B.1}$$
$$= \int_{\Omega_F} \left[\int_{\Omega_C} g(s,t)\gamma(t,ds) \right] (\alpha \cdot \Theta_F)(dt)$$
$$= \int_{\Omega_F} \left[\int_{\Omega_C} g(s,t)\gamma(t,ds) \right] \mu_F(dt),$$

and hence we obtained equation (3.3).

C Setwise convergence of measures on a filtration

For all $n \in \mathbb{N}$, let ν_n be a measure on $(\Omega, \mathcal{A}_n)$ where $\mathcal{A}_n$ is finite and $\mathcal{A}_n \nearrow \mathcal{A}$ (i.e. $\mathcal{A}_n \subset \mathcal{A}_{n+1}$ and $\mathcal{A} = \sigma(\cup_n A_n)$), and let ν be a finite measure on $(\Omega, \mathcal{A})$. The next result provides sufficient conditions for the existence of extensions $(\tilde{\nu}_n)$ of the measures (ν_n) to $\mathcal{A}$ that converge setwise to ν. This means that, for all $n \in \mathbb{N}$, the restriction of $\tilde{\nu}_n$ to $\mathcal{A}_n$ coincides with ν_n (i.e. $\tilde{\nu}_n|_{\mathcal{A}_n} = \nu_n$) and $\tilde{\nu}_n(A) \to \nu(A)$ for all $A \in \mathcal{A}$. Notice that if such extensions $(\tilde{\nu}_n)$ are to exist, then for any m and $A_m \in \mathcal{A}_m$, $\lim_{n\to\infty} \nu_n(A_m) = \nu(A_m)$. It turns out that this condition is also sufficient, in the presence of a uniform boundedness condition imposed on (ν_n).

Lemma C.1. *Assume that*

(i) For any $m \in \mathbb{N}$ and $A_m \in \mathcal{A}_m$, $\lim_{n\to\infty} \nu_n(A_m) = \nu(A_m)$,

(ii) There exists $L > 0$ such that $\nu_n \leq L \cdot \nu$ for all $n \in \mathbb{N}$, that is,

$$\nu_n(A) \leq L \cdot \nu(A), \quad \forall n \in \mathbb{N}, \forall A \in \mathcal{A}_n.$$

Then (ν_n) have extensions to $\mathcal{A}$ that converge setwise to ν.

Proof. For all n, label the atoms of $\mathcal{A}_n$ as $A_1^n, A_2^n, \ldots, A_{k(n)}^n$. Define

$$\tilde{\nu}_n(A) := \sum_{i=1}^{k(n)} \nu_n(A_i^n) \cdot \frac{\nu(A \cap A_i^n)}{\nu(A_i^n)}, \quad \forall A \in \mathcal{A}.$$

Thus $\tilde{\nu}_n$ is constructed by summing the measures obtained as the conditionals of ν with respect to each atom of $\mathcal{A}_n$, scaled so that the measure of each atom of $\mathcal{A}_n$ coincides under $\tilde{\nu}_n$ and ν_n. Clearly $\tilde{\nu}_n$ is a measure on $\mathcal{A}$ which is equal to ν_n when restricted to $\mathcal{A}_n$. Define

$$\mathcal{D} := \{A \in \mathcal{A} \mid \tilde{\nu}_n(A) \to \nu(A)\}.$$

Condition (i) implies that $\cup_n \mathcal{A}_n \subset \mathcal{D}$. In particular, $\Omega \in \mathcal{D}$. Moreover, $\mathcal{D}$ is closed under proper differences, since if $A, B \in \mathcal{D}$ with $A \subset B$, then

$$\tilde{\nu}_n(B \setminus A) = \tilde{\nu}_n(B) - \tilde{\nu}_n(A) \to \nu(B) - \nu(A) = \nu(B \setminus A).$$

We will show that $\mathcal{D}$ is closed under increasing limits. Let $A_1, A_2, \ldots$ disjoint sets in $\mathcal{D}$. Notice that $\tilde{\nu}_n(\cup_m A_m) = \sum_m \tilde{\nu}_n(A_m)$, since $\tilde{\nu}_n$ is a sum of a finite number of measures, and $\tilde{\nu}_n(A_m) \leq L \cdot \nu(A_m)$, while $\sum_m \nu(A_m) = \nu(\cup_m A_m) < \infty$. Lebesgue's dominated convergence theorem implies

$$\lim_{n\to\infty} \tilde{\nu}_n(\cup_m A_m) = \sum_m \lim_{n\to\infty} \tilde{\nu}_n(A_m) = \sum_m \nu(A_m) = \nu(\cup_m A_m).$$

It follows that $\cup_m A_m \in \mathcal{D}$. We proved that $\mathcal{D}$ is a λ-system containing the algebra $\cup_n \mathcal{A}_n$ which is a π-system, being closed under finite intersections. The $\pi - \lambda$ theorem (Kallenberg 2002, Theorem 1.1) implies that $\mathcal{D} = \mathcal{A}$, and hence we proved that, indeed, $\tilde{\nu}_n \to \nu$ setwise on $\mathcal{A}$. $\square$

If a sequence of measures $(\tilde{\nu}_n)$ on $\mathcal{A}$ converges setwise to ν, then $E^{\tilde{\nu}_n}(f) \to E^{\nu}(f)$ for any bounded function $f : \Omega \to \mathbb{R}$ which is $\mathcal{A}$-measurable (Stokey and Lucas 1989, p.335). This result is strengthened in the next Lemma.

Lemma C.2. *Assume that for all n, ν_n has an extension to $\mathcal{A}$ that converges setwise to ν. For all $n \in \mathbb{N}$, let $X_n : \Omega \to \mathbb{R}$ such that X_n is $\mathcal{A}_n$-measurable, $|X_n| < M$, and $X_n \to X$, ν-almost surely, where $X : \Omega \to \mathbb{R}$ is $\mathcal{A}$-measurable. Then* $\lim_{n\to\infty} \int_\Omega X_n d\nu_n = \int_\Omega X d\nu$.

Proof. Let $\tilde{\nu}_n$ be an extension of ν_n to $\mathcal{A}$ that converges setwise to ν. X_n is $\mathcal{A}_n$-measurable, therefore $\int_\Omega X_n d\nu_n = \int_\Omega X_n d\tilde{\nu}_n$. By the triangle inequality,

$$\left|\int_\Omega X_n d\tilde{\nu}_n - \int_\Omega X d\nu\right| \leq \int_\Omega |X_n - X| d\tilde{\nu}_n + \left|\int_\Omega X d\tilde{\nu}_n - \int_\Omega X d\nu\right|. \quad \text{(C.1)}$$

Pick $\varepsilon > 0$ arbitrary. Notice that

$$\int_\Omega |X_n - X| d\tilde{\nu}_n \leq \varepsilon \cdot \tilde{\nu}_n(\Omega) + 2M \cdot \tilde{\nu}_n(\{|X_n - X| \geq \varepsilon\}). \quad \text{(C.2)}$$

Define $A_m := \cup_{n\geq m}\{|X_n - X| \geq \varepsilon\}$. Since $X_n \to X$, ν-a.s., it follows that $A_m \searrow A$ with $\nu(A) = 0$. The triangle inequality implies

$$|\tilde{\nu}_n(A_n) - \nu(A)| \leq \tilde{\nu}_n(A_n \setminus A) + |\tilde{\nu}_n(A) - \nu(A)|.$$

Since $(A_n \setminus A) \searrow \emptyset$, by the Vitali-Hahn-Saks theorem (Kopp 1984, p.34), $\lim_{m\to\infty} \sup_n \tilde{\nu}_n(A_m \setminus A) \to 0$. As $\tilde{\nu}_n(A) \to \nu(A)$ and $\int_\Omega X d\tilde{\nu}_n \to \int_\Omega X d\nu$, we can choose $N_1(\varepsilon) \in \mathbb{N}$ such that for all $n \geq N_1(\varepsilon)$, $\tilde{\nu}_n(\{|X_n - X| \geq \varepsilon\}) \leq \varepsilon$ and $\left|\int_\Omega X d\tilde{\nu}_n - \int_\Omega X d\nu\right| \leq \varepsilon$. By the setwise convergence of $\tilde{\nu}_n$ to ν, we can choose $N_2(\varepsilon) \in \mathbb{N}$ such that $\tilde{\nu}_n(\Omega) \leq \nu(\Omega) + \varepsilon$, for all $n \geq N_2(\varepsilon)$. Equations (C.1) and (C.2) imply that for all $n \geq \max\{N_1(\varepsilon), N_2(\varepsilon)\}$,

$$\left|\int_\Omega X_n d\tilde{\nu}_n - \int_\Omega X d\nu\right| \leq \varepsilon \cdot (\nu(\Omega) + \varepsilon) + 2M\varepsilon + \varepsilon.$$

Since ε can be chosen arbitrarily small, the conclusion follows. □

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