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Abstract:

Households in real cities are heterogeneous regarding their size and composition. An aspect usually neglected in urban models used to study economic and policy issues that arise in today's cities. We develop an urban general equilibrium model that takes a more complex household structure explicitly into account. The model is based on the single consumer type model of Anas and Xu (1999) or Anas and Rhee (2006) and treats the interactions of urban product, labor and land markets as well as linkages between city firms and different consumer types living in different household structures. Households differ not only in endowments, preferences and their valuation in regard to different travel modes, but also in size and the composition regarding their members. The implementation of a more complex household structure then allows studying a broad range of further urban economic issues, which treat different household structures differently.

JEL-Classification: R12; R13; R14; R20

Keywords: Urban Economics, General Equilibrium, Household structure, Location choice

1 Introduction

The modeling of urban spatial structure originated in the monocentric city model by William Alonso (1964). In this model, it is a priori assumed that all production activities within an urban area take place in the Central Business District (CBD), and all residents reside in the surrounding area and commute to the CBD. One of the contributions of the monocentric city model is to give insight into the effect of income on location decisions. Hence, it is able to explain various location patterns observed in real cities. Since then, the model has been generalized in various ways. However, in recent decades the process of decentralization has taken a more polycentric urban structure (Anas et al., 1998). Therefore, in order to study economic and policy issues that arise in today's cities, various urban economists have developed partial as well as general equilibrium urban models that are not restricted to the assumption of the CBD as the place where all employment is concentrated (see for instance Sullivan, 1986; Wieand, 1987; Anas and Xu, 1999). However, most of these advanced urban models often incorporate a homogeneous consumer type as the basic economic agent in the urban economy. Even if different consumer types (for example rich and poor persons) are considered, they are assumed to live in a homogeneous household structure. But, households in real cities are heterogeneous regarding their size and composition. This aspect is usually neglected in the literature.

Therefore, the purpose of this paper is to develop an urban general equilibrium model that explicitly takes a more complex household structure into account. The implementation of a more complex household structure then allows studying a broad range of further urban economic issues, which treat different household structures differently.[1]

Urban models used to examine the more complex process of location decision concerning households with two working members were developed by Curran et al. (1982), White (1977), and Hotchkiss and White (1993). But these models ignore the interactions between different

[1] For instance fiscal policies, gender issues, urban transportation.

markets, households and firms in the city. For instance, these models ignore shopping trips required to buy the consumption goods in the city. Apart from that, these models do not incorporate non-working households, although in reality the number of non-working households is substantial. Hence, these models ignore several interdependencies within an urban economy and thus a priori exclude the treatment of various interesting economic issues.

In contrast, we develop an urban general equilibrium model with multiple homogeneous as well as heterogeneous household types, based on the single consumer type model of Anas and Xu (1999) or Anas and Rhee (2006). Our model treats the interactions of product, labor and land markets as well as linkages between city firms and different consumer types living in different household structures. Households differ not only in endowments and preferences, but also in size and the composition regarding their members.

Referring to real-world observations, we implement the following different consumer and household types: non-working single-person households and non-working two-person households, i.e. non-working couples; lower-skilled and higher-skilled single-worker households; lower-skilled and higher-skilled homogeneous two-worker households each composed of two potentially employed adults each with the same skill level; and heterogeneous two-worker households each composed of one potentially employed adult with a lower skill level and one potentially employed adult with a higher skill level.

The persons are potentially employed because the work decision is endogenous in the model and, as we will see later, depends also on the household type the person belongs to. Households decide where to reside, where to work (if working), where and how much to shop, how much labor to supply and how much land to rent in the urban area, bearing in mind full economic travel costs. All prices (commodity prices, wages, rents) as well as location decisions are determined endogenously in the model.

Since households can vary in idiosyncratic tastes for locations within the urban area, decisions of households create mixed land use and commuting patterns as is commonly observed in real cities. The crucial aspect in the case of two-worker households is that their household members make a joint decision regarding the residential location and the potential work location of both members. These decisions are interdependent in real decision processes, as observed, for instance, by Freedman and Kern (1997) using an empirical approach. In addition, the members of these household types have to decide not only where to shop in the urban area, but also who shall execute the shopping trips. Hence, internal economies of scale in shopping can be realized by a two-worker household compared to the usually assumed single-worker household. There are no predetermined residential or employment locations in the city, so the spatial pattern can exhibit a polycentric structure.

The paper is organized as follows: In Section 2, we describe the setting of the urban general equilibrium model. Besides the behavior of the economic agents, equilibrium conditions are described to close the model. In Section 3 we offer some ideas of possible model applications. As is shown, there is a broad range of economic issues that can be highlighted by using a model which takes into account a heterogeneous household structure. Section 4 concludes.

2 The model

2.1 The general setting

The urban area is partitioned into I zones.[2] The zones are linked via an exogenously given transport network with distance d_{ij}. At each zone i ($i \in I$), a homogeneous and fixed land area A_i is available for the development of residences and establishments. At each zone i land rent is endogenously determined[3] and firms produce a composite commodity using labor supplied by differently skilled residents and land. Commodities produced in the same zone

[2] In the following, the index i ($i \in I$) is used to denote a home location, the indices j ($j \in I$) and l ($l \in I$) are used to denote a work location and the index k ($k \in I$) is used to denote a shopping location. A complete description of indices used is shown in Appendix A.

[3] One can imagine that the urban area has already reached its natural boundary or the urban area is surrounded by land that is not convertable into urban land due to political restrictions of bordering regions. Hence, land rent at the edge of the city can differ from land rent beyond the city boundary.

are not differentiated, but commodities produced in different zones are product varieties, that is there is a spatial product differentiation. The zone specific markets for the composite commodity and the production factors are competitive.

Households with working members are differentiated in regard to the skill levels of their members either as high skilled, low skilled or mixed skilled households. Household members are free to choose home and work zones within the urban area. They derive utility from consumption of the spatially differentiated commodities, housing and leisure. Household members might have to commute to work and make shopping trips to the selling points of the commodities.

It is assumed that the urban economy is closed in the sense that the total population in the urban area is fixed and exogenously given, so there is no interurban migration and utility levels of households are endogenously determined.[4] Apart from this, it is assumed that the city is partly open in the sense that some share of the urban production will not be consumed in the urban area, but will be exported in exchange for some monetary expenditures for goods or services produced (e.g. fuel produced by an external transport sector) or owned (e.g. land owned by absentee landowners) outside the urban economy but consumed by urban residents. Travel times from zone i to zone j are exogenously given but depend on transport mode m ($m \in M$) that is used to travel from i to j.

2.2 Households

There are 4 different household types y ($y \in Y$) : non-working households ($y = 1$), single-worker households ($y = 2$), homogeneous two-worker households ($y = 3$), and heterogeneous two-worker households ($y = 4$). In addition, non-working households are differentiated in regard to the number of household members, where $g = 1$ denotes a non-working single

[4] The assumption that no interurban migration occurs is appropriate if it is assumed that consequences arising in the city, for instance based on policy or demographic changes, also affect the other cities in the (national) economy.

household and $g = 2$ denotes a non-working couple household. Furthermore, households with working members are differentiated in regard to the composition of different skill types h ($h \in H$), where $h = 1$ denotes a lower skill level and $h = 2$ denotes a higher skill level. That means, there are two different non-working household types, two different single-worker household types, two different homogeneous two-worker household types each composed of two employed persons each with the same skill level, and one heterogeneous (mixed) two-worker household type composed of one employed lower-skilled person and one employed higher-skilled person.[5] Let N be the number of households belonging to a specific household type, total number of households in the urban area is $\sum_{\forall g} N^{g,1} + \sum_{\forall h} N^{h,2} + \sum_{\forall h} N^{h,3} + N^4$.

Each household resides in some zone i. In the case of single-worker households, the household member s ($s \in S$) is employed in zone j. In the case of homogeneous and heterogeneous two-worker households, each household member is employed in zone j (the one household member) and in zone l (the other household member). Therefore, type 1 household's location choice set is $\{i\}$, type 2 household's choice set is $\{i, j\}$ and the choice set of household types 3 and 4 is $\{i, j, l\}$. Each household type has to pay a rent r_i [\$/m^2 lot size] for a residence in zone i. When working in zone j (zone l), household members of household types 2-4 earn a hourly wage w_j [\$/hour] (w_l [\$/hour]). In addition, since travel costs, travel time, rents and wages depend on the specific location choice set, the utility U of each household type also depends on $\{i\}$ (Type $y = 1$), $\{i, j\}$ (Type $y = 2$) and $\{i, j, l\}$ (Type $y = 3$ and $y = 4$).

In the following we present the two-stage decision process of a typical homogeneous two-worker household (Type $y = 3$) with location choice set $\{i, j, l\}$. All other household types face equivalent decision problems, depending on the specific location choice set.[6] In the first stage, the household decides on consumption quantities, i.e. commodities, housing and leisure,

[5] We have already extended the model in a way that allows us to consider an even more detailed household structure. Then, another index is needed to denote additional household subtypes. Therewith it is possible to make further differentiations between households within one specific household type, for example in regard to preferences and tastes, endowments, car availability and children in the household. With such an even more detailed household structure it is possible to deal with interesting urban economic issues, as we will describe in Section 3.

[6] The utility maximization problems of other household types are provided in Appendix B.

given its location choice set. In the second stage, the two-worker household chooses its joint home location and the work locations of both household members considering deterministic utility levels associated with each location choice set as well as idiosyncratic tastes reflected by a stochastic utility component.

<u>NESTED UTILITY APPROACH</u>

For a given location choice set $\{i, j, l\}$, the homogeneous two-worker household's (Type $y = 3$) utility function can be written as follows:

$$U_{ijl}^{h,3} = U_{ijl}^{h,3}(Z) + U_{ijl}^{h,3}(q) + U_{ijl}^{h,3}(\ell) + \epsilon_{ijl}^{h,3}, \tag{1}$$

where

$$U_{ijl}^{h,3}(Z) = \frac{\alpha^{h,3}}{\omega^{h,3}} \ln \sum_{s=1}^{2} \left(\sum_{k=1}^{I} \Gamma_k^{h,3,s} \left(Z_{ijlk}^{h,3,s} \right)^{\eta^{h,3}} \right)^{\frac{\omega^{h,3}}{\eta^{h,3}}} \tag{2a}$$

is the subutility function for the consumption of the composite commodity with $0 < \alpha^{h,3} < 1$,

$$U_{ijl}^{h,3}(q) = \beta^{h,3} \ln q_{ijl}^{h,3} \tag{2b}$$

is the housing (lot size in i as a proxy for housing) subutility function with $0 < \beta^{h,3} < 1$, and

$$U_{ijl}^{h,3}(\ell) = \frac{\gamma^{h,3}}{\rho^{h,3}} \ln \left(\sum_{s=1}^{2} \mathit{\Gamma}^{h,3,s} \left(\ell_{ijl}^{h,3,s} \right)^{\rho^{h,3}} \right) \tag{2c}$$

is the leisure subutility function with $0 < \gamma^{h,3} < 1$ and $\alpha^{h,3} + \beta^{h,3} + \gamma^{h,3} = 1$. The $\epsilon_{ijl}^{h,3}$ are idiosyncratic taste constants and represent the stochastic part of the random utility function, whereas the deterministic or systematic part is given by the subutility functions. The idiosyncratic taste constants vary among the households within the homogeneous two-worker household type for each location choice set $\{i, j, l\}$.[7]

[7] One can imagine that households differ in tastes for specific attributes regarding the choice set $\{i, j, l\}$ and these attributes are not observable by the researchers. Hence, they can determine only a choice probability of the household's decision on $\{i, j, l\}$.

The overall utility is of the Cobb-Douglas form, whereas the leisure (shopping) subutility function is a (two-stage nested) C.E.S. utility function. Note that there is a lower-skilled ($h = 1$) and a higher-skilled ($h = 2$) homogeneous two-worker household. In both households, there are two members s each with the same skill level.

The household members residing at i travel from zone i to every zone k where production takes place to purchase the composite commodity Z_k produced there, taking into account full economic shopping costs. The constant elasticity of substitution $1/(1-\eta^{h,3})$, $\eta^{h,3} < 1$ reflects spatial taste variety in shopping. As $\eta^{h,3} \to 1$, shopping locations and therefore consumption goods sold in zone k are perfect substitutes. Hence, the household members shop only at that zone where full economic shopping costs are the lowest. As $\eta^{h,3} \to -\infty$, the household members prefer to shop at each zone where shopping is possible, regardless of the price, travel costs and travel time of making such a trip. We assume that separate trips are made to each production (shopping) zone, purchasing one unit of the local good per shopping trip. Hence, for this time being, we ignore trip chaining. Besides the fact that both household members value spatial variety in shopping, they have a taste for an internal task sharing concerning shopping trips, reflected by the constant elasticity of substitution $1/(1 - \omega^{h,3})$, $\omega^{h,3} < 1$. As $\omega^{h,3} \to 1$, shopping trips within the household are perfect substitutes, so the household member with the lowest full economic shopping costs is doing all shopping in the respective zone. Hence, full economic shopping costs per capita in a two-worker household can be lower compared to an identical single-worker household. This implies that the two-worker household might realize internal economies of scale in shopping. As $\omega^{h,3} \to -\infty$, there is an extreme taste to spread shopping trips over both household members, regardless of their specific full economic shopping costs.[8] The constants $\Gamma_k^{h,3,s} \geqq 0$ measure the relative attractiveness of shopping location k to household member s compared to the other locations.

Furthermore, the homogeneous two-worker household derives utility from joint demand of lot

[8] As usual, in reality the truth lies somewhere in the middle. Hence, $-\infty < \sigma^{h,3} < 1$. One can imagine that in some cases, the physical presence of a specific household member s is necessary to by a certain commodity such that s buys the consumption good in zone k despite higher full economic shopping costs.

size $q_{ijl}^{h,3}$ and leisure $\ell_{ijl}^{h,3,s}$ of both household members s. The constant elasticity of substitution between leisure of both household members is $1/(1-\rho^{h,3})$, $\rho^{h,3} < 1$ and reflects the preference to spread leisure over both household members. The constants $F^{h,3,s} \geqq 0$ measure the relative preference of household member s for leisure.

MONETARY BUDGET CONSTRAINT

Now let $c_{ik}^{h,3,s}$ be the expected monetary two-way shopping trip costs from home zone i to shopping zone k of household member s in a homogeneous two-worker household where each member has skill level h. Furthermore, let $c_{ij}^{h,3,1}$ and $c_{il}^{h,3,2}$ be the expected monetary two-way commuting costs from household‘s home zone i to work zone j (household member $s = 1$) and work zone l (household member $s = 2$) and $D_{ijl}^{h,3,s}$ be the number of working days supplied by household member s. The total monetary household expenditures per period are

$$\sum_{s=1}^{2} \sum_{k=1}^{I} \left(p_k + c_{ik}^{h,3,s} \right) Z_{ijlk}^{h,3,s} + r_i q_{ijl}^{h,3} + c_{ij}^{h,3,1} D_{ijl}^{h,3,1} + c_{il}^{h,3,2} D_{ijl}^{h,3,2}, \tag{3}$$

where $\sum_{s=1}^{2} \sum_{k=1}^{I} \left(p_k + c_{ik}^{h,3,s} \right) Z_{ijlk}^{h,3,s}$ denotes total consumption expenditures including the price of the composite commodity p_k and monetary two-way shopping trip costs, $r_i q_{ijl}^{h,3}$ is the total rent paid for a residence in zone i and $c_{ij}^{h,3,1} D_{ijl}^{h,3,1} + c_{il}^{h,3,2} D_{ijl}^{h,3,2}$ are total monetary two-way commuting costs per period.

Monetary household income per period can be written as follows:

$$L \left(w_j^h D_{ijl}^{h,3,1} + w_l^h D_{ijl}^{h,3,2} \right) + R^{h,3}, \tag{4}$$

where L is an exogenously given number of hours each household member works per day. Note that as long as $D_{ijl}^{h,3,1} = D_{ijl}^{h,3,2} = 0$, there is no wage income earned by work for that household in the city. In this case, the only income results from a rent dividend $R^{h,3} = 1/N^{h,3} \sum_{i=1}^{I} \Theta_i^{h,3} r_i A_i$, where $\Theta_i^{h,3} \leqq 1$ is a share of the entire land in zone i that is owned by the homogeneous two-worker household type and $N^{h,3}$ is the number of such households residing in the urban area.[9]

TIME CONSTRAINTS

Besides the monetary budget constraint, each member s of the homogeneous two-worker household is subject to a time constraint which can be written as follows:

$$D_{ijl}^{h,3,s}L + \ell_{ijl}^{h,3,s} + T_{ijl}^{h,3,s} = E, \tag{5}$$

where E is the total time endowment [hours/period],

$$T_{ijl}^{h,3,1} = \sum_{k=1}^{I} t_{ik}^{h,3,1} Z_{ijlk}^{h,3,1} + t_{ij}^{h,3,1} D_{ijl}^{h,3,1} \tag{6a}$$

is the expected total travel time per period over all available travel modes for household member $s = 1$, conditional on the specific location choice set $\{i, j, l\}$ and

$$T_{ijl}^{h,3,2} = \sum_{k=1}^{I} t_{ik}^{h,3,2} Z_{ijlk}^{h,3,2} + t_{il}^{h,3,2} D_{ijl}^{h,3,2} \tag{6b}$$

is the expected total travel time over all available travel modes of household member $s = 2$, also conditional on the specific choice set $\{i, j, l\}$. The expected two-way travel time for a shopping trip from i to k for household member s is denoted by $t_{ik}^{h,3,s}$ and the expected two-way travel time for commuting is denoted by $t_{ij}^{h,3,1}$ for household member $s = 1$ and by $t_{il}^{h,3,2}$ for household member $s = 2$. So the total time endowment can be allocated to work, leisure and travel. As we will see in Section 2.3, travel times and costs are determined as expected values over travel modes m. Note that the expected total travel time $T_{ijl}^{h,3,1}$ of household member $s = 1$ is conditional on l, the work location of household member $s = 2$ (and vice versa). The reason for this is that the demand for $Z_{ijlk}^{h,3,1}$ depends on joint monetary household income and that the working days $D_{ijl}^{h,3,1}$ supplied by household member $s = 1$ depend on

[9] Taking into account all the other households within the urban area, then $\Theta_i \equiv \sum_{\forall g} \Theta_i^{g,1} + \sum_{\forall h} \Theta_i^{h,2} + \sum_{\forall h} \Theta_i^{h,3} + \Theta_i^4 \leqq 1 \; \forall i$. If $\Theta_i < 1$ for some i, some share of land in zone i is owned by absentee landowners.

the labor supply decision of household member $s = 2$ (and vice versa). In turn, household income and labor supply decisions are conditional on the location choice set $\{i, j, l\}$.

FULL ECONOMIC INCOME CONSTRAINT

Besides the monetary expressions described above, it is useful to take a look at full economic expressions. First, substituting the expected total travel time $T_{ijl}^{h,3,1}$ of household member $s = 1$ and the expected total travel time $T_{ijl}^{h,3,2}$ of household member $s = 2$ into the corresponding total time constraints yields

$$D_{ijl}^{h,3,1} L + \ell_{ijl}^{h,3,1} + \sum_{k=1}^{I} t_{ik}^{h,3,1} Z_{ijlk}^{h,3,1} + t_{ij}^{h,3,1} D_{ijl}^{h,3,1} = E, \tag{7a}$$

$$D_{ijl}^{h,3,2} L + \ell_{ijl}^{h,3,2} + \sum_{k=1}^{I} t_{ik}^{h,3,2} Z_{ijlk}^{h,3,2} + t_{il}^{h,3,2} D_{ijl}^{h,3,2} = E. \tag{7b}$$

Rearranging to $D_{ijl}^{h,3,s}$, the total working days supplied by household member s and then substituting into household's total monetary income per period yields full economic income and expenditures. Full economic expenditures consist of full economic shopping costs of household member $s = 1$ and household member $s = 2$:

$$\sum_{k=1}^{I} \left(p_k + c_{ik}^{h,3,1} + VOT_{ij}^{h,3,1} t_{ik}^{h,3,1} \right) Z_{ijlk}^{h,3,1}, \tag{8a}$$

$$\sum_{k=1}^{I} \left(p_k + c_{ik}^{h,3,2} + VOT_{il}^{h,3,2} t_{ik}^{h,3,2} \right) Z_{ijlk}^{h,3,2}, \tag{8b}$$

joint full economic expenditures $r_i q_{ijl}^{h,3}$ for a residence in zone i and full economic expenditures for leisure of household member $s = 1$ and household member $s = 2$:

$$VOT_{ij}^{h,3,1} \ell_{ijl}^{h,3,1} \qquad VOT_{il}^{h,3,2} \ell_{ijl}^{h,3,2}. \tag{9}$$

In contrast to the monetary income, full economic income of the homogeneous two-worker household can be written as follows:

$$VOT_{ij}^{h,3,1}E + VOT_{il}^{h,3,2}E + R^{h,3}. \tag{10}$$

In full economic expressions, $VOT_{ij}^{h,3,1}$ is the Value of Time of household member $s = 1$ and $VOT_{il}^{h,3,2}$ is the Value of Time of household member $s = 2$, where

$$VOT_{ij}^{h,3,1} \equiv \frac{Lw_j^h - c_{ij}^{h,3,1}}{L + t_{ij}^{h,3,1}} \qquad VOT_{il}^{h,3,2} \equiv \frac{Lw_l^h - c_{il}^{h,3,2}}{L + t_{il}^{h,3,2}}. \tag{11}$$

The Value of time, which is the effective hourly wage rate, of both household members can also be derived by dividing the marginal utility of time by the marginal utility of income. These effective hourly wage rates decrease with an increase in monetary travel costs and travel time. Hence, as monetary travel costs and/or travel time differ between the household members, effective hourly wage rates can differ even though gross wage rates paid by city firms are equal.

$$\frac{\partial VOT_{ij}^{h,3,1}}{\partial c_{ij}^{h,3,1}} < 0 \qquad \frac{\partial VOT_{ij}^{h,3,1}}{\partial t_{ij}^{h,3,1}} < 0$$

$$\frac{\partial VOT_{il}^{h,3,2}}{\partial c_{il}^{h,3,2}} < 0 \qquad \frac{\partial VOT_{il}^{h,3,2}}{\partial t_{il}^{h,3,2}} < 0$$

Now, for a given location choice set $\{i, j, l\}$, the homogeneous two-worker household maximizes the utility function $U_{ijl}^{h,3}$ subject to the joint monetary budget constraint and time constraints of household members $s = 1$ and $s = 2$ with respect to $Z_{ijlk}^{h,3,1}$, $Z_{ijlk}^{h,3,2}$, $q_{ijl}^{h,3}$, $\ell_{ijl}^{h,3,1}$, $\ell_{ijl}^{h,3,2}$, $D_{ijl}^{h,3,1}$, $D_{ijl}^{h,3,2}$. Note that the household members optimally choose the number of working days, so intra-household savings in full economic commuting costs can arise when household members adjust their labor decisions. Using the Lagrangian, one can derive the first-order conditions for the maximization problem. Then, substituting the optimized $Z_{ijlk}^{h,3,1}$, $Z_{ijlk}^{h,3,2}$, $q_{ijl}^{h,3}$, $\ell_{ijl}^{h,3,1}$, $\ell_{ijl}^{h,3,2}$ into the utility function $U_{ijl}^{h,3}$ yields optimized utility levels

$$\tilde{U}_{ijl}^{h,3} = U_{ijl}^{h,3}\left(\tilde{Z}_{ijlk}^{h,3,1}, \tilde{Z}_{ijlk}^{h,3,2}, \tilde{q}_{ijl}^{h,3}, \tilde{\ell}_{ijl}^{h,3,1}, \tilde{\ell}_{ijl}^{h,3,2}\right)$$

and finally the complete indirect random utility function

$$\overset{*}{U}{}^{h,3}_{ijl} = \tilde{U}^{h,3}_{ijl} + \epsilon^{h,3}_{ijl}\,, \tag{12}$$

where $\tilde{U}^{h,3}_{ijl}$ is the optimized deterministic part and $\epsilon^{h,3}_{ijl}$ is the stochastic part of the random utility function.

<u>LOCATION DECISION</u>

Given this optimization for each $\{i, j, l\}$, each homogeneous two-worker household compares the complete location choice set $\{i, j, l\}$ and chooses the most preferred choice set $\{i, j, l\}$ which offers the highest utility, taking into account idiosyncratic tastes. Due to the fact that these idiosyncratic tastes are distributed among the households for each $\{i, j, l\}$, choices are described probabilistically in the form of a discrete choice model. The probability that the homogeneous two-worker household chooses a specific location choice set $\{i, j, l\}$ is

$$\Psi^{h,3}_{ijl} = \text{Prob}\left[\overset{*}{U}{}^{h,3}_{ijl} > \overset{*}{U}{}^{h,3}_{abc}\ \forall\ (a,b,c) \neq (i,j,l)\right] \tag{13a}$$

$$= \text{Prob}\left[\tilde{U}^{h,3}_{ijl} + \epsilon^{h,3}_{ijl} > \tilde{U}^{h,3}_{abc} + \epsilon^{h,3}_{abc}\ \forall\ (a,b,c) \neq (i,j,l)\right], \tag{13b}$$

where $\Psi^{h,3}_{ijl}$ is the probability that a randomly selected homogeneous two-worker household prefers the choice set $\{i, j, l\}$. Assuming that each $\epsilon^{h,3}_{ijl}$ is independently, identically (i.i.d.) Gumbel distributed with $E[\epsilon^{h,3}_{ijl}] = 0$, variance σ^2 and dispersion parameter $\Lambda^{h,3} = \pi/\sigma\sqrt{6}$, the choice probabilities are given by the multinomial logit model (MNL):

$$\Psi^{h,3}_{ijl} = \frac{\Delta^{h,3}_{ijl} \exp\left(\Lambda^{h,3}\tilde{U}^{h,3}_{ijl}\right)}{\sum\limits_{a=1}^{I}\sum\limits_{b=1}^{I}\sum\limits_{c=1}^{I} \Delta^{h,3}_{abc} \exp\left(\Lambda^{h,3}\tilde{U}^{h,3}_{abc}\right)}, \qquad \sum_{i=1}^{I}\sum_{j=1}^{I}\sum_{l=1}^{I} \Psi^{h,3}_{ijl} = 1. \tag{14}$$

The logit probabilities exhibit several desirable properties (Train, 2003). First, $\Psi^{h,3}_{ijl}$ is necessarily between zero and one, as required for a probability. On the one hand, when $\tilde{U}^{h,3}_{ijl}$ rises, reflecting an improvement in the observed attributes of the location choice set $\{i, j, l\}$, with

$\tilde{U}^{h,3}_{abc}$ $\forall$ $abc \neq ijl$ held constant, $\Psi^{h,3}_{ijl}$ approaches one. On the other hand, $\Psi^{h,3}_{ijl}$ approaches zero when $\tilde{U}^{h,3}_{ijl}$ decreases, since the exponential in the numerator of (14) approaches zero as $\tilde{U}^{h,3}_{ijl}$ approaches $-\infty$. The logit probability for an alternative is never exactly zero. If one believes that a choice set has actually no chance of being chosen by the homogeneous two-worker household, one can exclude that certain choice set $\{i, j, l\}$ by making $\Delta^{h,3}_{ijl} = 0$. A probability of exactly 1 is obtained only if the location choice set $\{i, j, l\}$ is the only choice. Second, the choice probabilities for all alternatives sum to one: $\sum_{\forall(i,j,l)} \Psi^{h,3}_{ijl} = 1$. The household necessarily chooses a location choice set. The denominator in (14) is simply the sum of the numerator over all choice sets, which gives this summing-up property automatically.

Further, the dispersion parameter is important (Anas, 1990). At one extreme, as $\Lambda^{h,3} \to \infty$ $\left(\sigma^{h,3} \to 0\right)$, taste idiosyncrasies vanish and all households within the homogeneous two-worker household type choose identically. In this case, the $\Psi^{h,3}_{ijl}$ corresponding to the highest $\tilde{U}^{h,3}_{ijl}$ approaches one and all others converge to zero (if there are some $\tilde{U}^{h,3}_{ijl}$ with the same level, the choice sets $\{i, j, l\}$ corresponding to these highest $\tilde{U}^{h,3}_{ijl}$ are chosen with the same probability). At the other extreme, as $\Lambda^{h,3} \to 0$ $\left(\sigma^{h,3} \to \infty\right)$, idiosyncrasies swamp the deterministic part of utility and homogeneous two-worker households choose randomly ($\Psi^{h,3}_{ijl} = 1/I^3$). Figure 1 shows the density of the Gumbel distribution for different values of $\Lambda^{h,3}$.

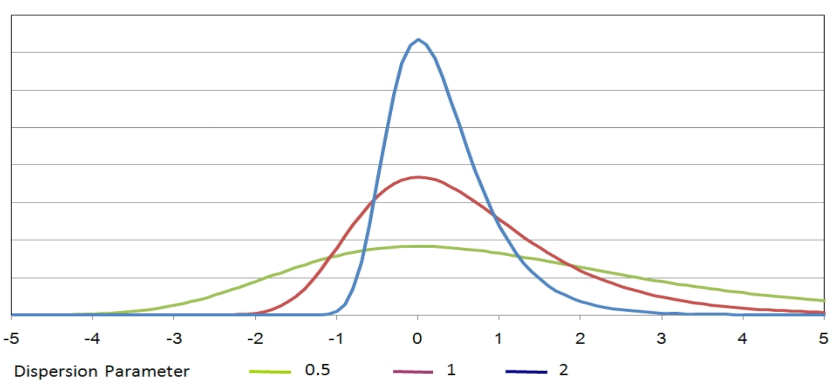

Figure 1: Density of the Gumbel distribution

The case of finite Λ has empirical validity and is in line with the hypothesis of wasteful commuting (first noted by Hamilton, 1982). In reality, many possible commuting patterns can be observed (see e.g. Anas and Rhee, 2007; Glaeser et al., 2001).[10] Such patterns are explained by assuming idiosyncratic tastes, but cannot be explained using the assumption of uniform tastes (which means $\Lambda = \infty$ or $\sigma = 0$). Hence, if $\Lambda < \infty$, at equilibrium different household types can choose the same location choice set, as is observed in real cities.

The optimization problem for the non-working household type ($y = 1$), the single-worker household type ($y = 2$) and the heterogeneous two-worker household type ($y = 4$) is similar. However, utility functions, monetary and full economic budget constraints and time constraints are slightly different. For example, non-working households do not optimally choose their number of working days supplied, so there is no wage income for non-working households a priori. Nevertheless, leisure enters the utility function of non-working households. Hence, besides monetary shopping costs, the members of these households taking into account opportunity costs of travel time for shopping trips. Subsequent Table 1 contains a description of main decision variables.

Table 1: Household types and main decision variables

Household type	Number	Consumption (Shopping)	Housing (lot size)	Leisure	Work-days
NW ($y = 1$)	$N^{g,1}$	$Z_{ik}^{1,1}$ / $Z_{ik}^{2,1,s}$	$q_i^{g,1}$	$\ell_i^{1,1}$ / $\ell_i^{2,1,s}$	–
SW ($y = 2$)	$N^{h,2}$	$Z_{ijk}^{h,2}$	$q_{ij}^{h,2}$	$\ell_{ij}^{h,2}$	$D_{ij}^{h,2}$
HoTW ($y = 3$)	$N^{h,3}$	$Z_{ijlk}^{h,3,s}$	$q_{ijl}^{h,3}$	$\ell_{ijl}^{h,3,s}$	$D_{ijl}^{h,3,s}$
HeTW ($y = 4$)	N^4	$Z_{ijlk}^{4,h}$	q_{ijl}^4	$\ell_{ijl}^{4,h}$	$D_{ijl}^{4,h}$

NW: Non-working household
SW: Single-worker household
HoTW: Homogeneous two-worker household
HeTW: Heterogeneous-two-worker household

Note that there are two different subtypes (single household $g = 1$ and couple household $g = 2$) within the non-working household type. In the case of the single household $g = 1$, there is only one adult household member. In the case of the couple household $g = 2$,

[10] For example, commuters who live in the suburbs and work in central cities and commuters who live in central cities and work in the suburbs (known as reverse commuting).

there are two household members ($s = 1$ and $s = 2$). It is assumed that the household members of the non-working household type are not differentiated with respect to skill level h. The number of these households is denoted by $N^{g,1}$. In addition, there are two different single-worker household types ($h = 1$ and $h = 2$) and two different homogeneous two-worker household types ($h = 1$ and $h = 2$). Hence, the number of these households is denoted by $N^{h,2}$ and $N^{h,3}$, respectively. In the case of the homogeneous two-worker household type, 2 household members ($s = 1$ and $s = 2$) are considered, each with the same skill level. In contrast, in heterogeneous two-worker households, the differentiation with regard to the skill level arises directly within the household. Hence, the number of heterogeneous two-worker households in the urban area is denoted by N^4 (without superscript h).

2.3 Travel mode choice

Individual expected travel times and travel costs for all residents in the city depend on the travel mode m used to travel from zone i to zone j. Assuming that an exogenously given specific average speed of a transport mode m is given by $\bar{v}^m$ $[km/h]$, two-way travel time t_{ij}^m $[h]$ from zone i to zone j with travel mode m can be determined as follows:

$$t_{ij}^m = \frac{2d_{ij}}{\bar{v}^m}. \tag{15}$$

Further, let $c_{ij}^{v,m}$ be the mode specific average variable travel costs per km for a trip (shopping, commuting) from zone i to zone j (e.g. fuel) and let $c_{ij}^{o,m}$ be other mode specific travel costs for a two-way trip from zone i to zone j (e.g. ticket for public transport, parking fee). Then, aggregate two-way travel costs from zone i to zone j with travel mode m are defined as follows:

$$c_{ij}^m = c_{ij}^{v,m} 2d_{ij} + c_{ij}^{o,m} \tag{16}$$

These travel mode specific two-way travel times t_{ij}^m and travel costs c_{ij}^m can be transformed into household member specific expected travel times and travel costs, which, as already seen, enter the budget and time constraints of all households. See Table 2 for a complete notation of household member specific expected travel times and travel costs.

Table 2: Household types, travel times and travel costs

Household type	Number	Travel times		Travel costs	
		Shopping	Commuting	Shopping	Commuting
NW $(y = 1)$	$N^{g,1}$	$t_{ik}^{1,1}/t_{ik}^{2,1,s}$	–	$c_{ik}^{1,1}/c_{ik}^{2,1,s}$	–
SW $(y = 2)$	$N^{h,2}$	$t_{ik}^{h,2}$	$t_{ij}^{h,2}$	$c_{ik}^{h,2}$	$c_{ij}^{h,2}$
HoTW $(y = 3)$	$N^{h,3}$	$t_{ik}^{h,3,s}$	$t_{ij}^{h,3,1}/t_{il}^{h,3,2}$	$c_{ik}^{h,3,s}$	$c_{ij}^{h,3,1}/c_{il}^{h,3,2}$
HeTW $(y = 4)$	N^{4}	$t_{ik}^{4,h}$	$t_{ij}^{4,1}/t_{il}^{4,2}$	$c_{ik}^{4,h}$	$c_{ij}^{4,1}/c_{il}^{4,2}$

Again on closer examination of the homogeneous two-worker household type, household member s specific expected two-way travel time and cost from i to j can be determined as follows:

$$t_{ij}^{h,3,s} = \sum_{\forall m} \pi_{ij}^{m,h,3,s} t_{ij}^{m}, \tag{17}$$

$$c_{ij}^{h,3,s} = \sum_{\forall m} \pi_{ij}^{m,h,3,s} c_{ij}^{m}, \tag{18}$$

where $\pi_{ij}^{m,h,3,s}$ is the probability that household member s chooses travel mode m for a trip from zone i to zone j. That means, it is assumed that over a certain period, household member s will choose each available mode m with some probability. These mode choice probabilities can be computed by using a mode choice model in multinomial logit form:

$$\pi_{ij}^{m,h,3,s} = \frac{\Xi_{ij}^{m,h,3,s} \exp\left(-\upsilon^{h,3,s}\left(VOT_{ij}^{m,h,3,s} t_{ij}^{m} + c_{ij}^{m}\right) + b_{o,ij}^{m}\right)}{\sum_{n=1}^{M} \Xi_{ij}^{n,h,3,s} \exp\left(-\upsilon^{h,3,s}\left(VOT_{ij}^{n,h,3,s} t_{ij}^{n} + c_{ij}^{n}\right) + b_{o,ij}^{n}\right)}, \qquad \sum_{m=1}^{M} \pi_{ij}^{m,h,3,s} = 1, \tag{19}$$

where $\Xi_{ij}^{m,h,3,s} = 0$ if transport mode m is not available for trips from i to j for household member s belonging to a homogeneous two-worker household and $\Xi_{ij}^{m,h,3,s} = 1$ if transport mode m is available for trips from i to j.[11] Note that, to determine $\pi_{ij}^{m,h,3,s}$, the exogenously given transport mode specific travel time t_{ij}^{m} from i to j is valued at the transport mode specific value of time $VOT_{ij}^{m,h,3,s} = \left(Lw_E^h - c_{ij}^m\right) / \left(L + t_{ij}^m\right)$, where $w_E^h = \sum_{\forall j} w_j^h / I$ is the wage rate household member s with skill level h can expect in the case of being employed in the city.[12] The mode choice dispersion parameter is denoted by $\upsilon^{h,3,s}$ and $b_{o,ij}^m$ denotes a mode specific constants. Figure 2, illustrates how household member s specific expected travel times $t_{ij}^{h,3,s}$ (left panel) and travel costs (right panel) $c_{ij}^{h,3,s}$ from i to j are determined.

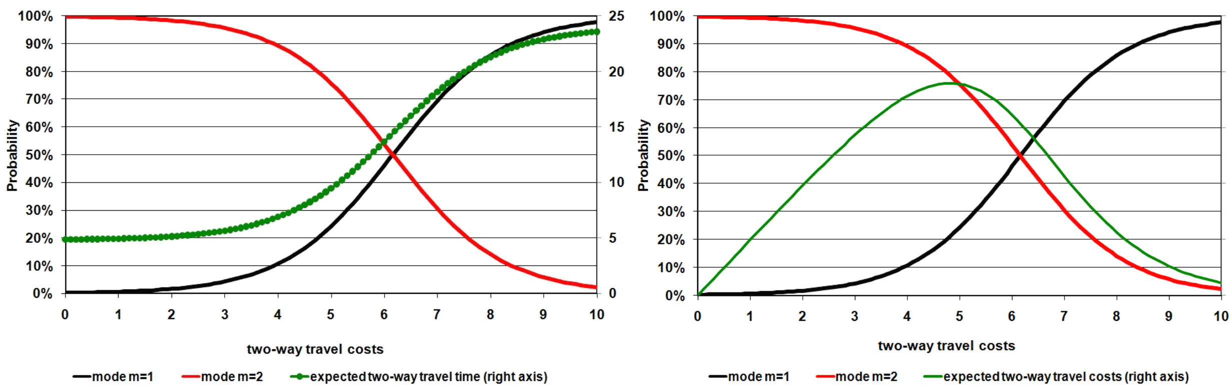

Figure 2: Expected travel time and travel costs

$m = 1$: walking $\quad m = 2$: public transport $\quad w_E^h = 10$ [\$/$hour$] $\quad c_{ij}^1 = 0$

$d_{ij} = 2$ [km] $\quad \bar{v}^1 = 5$ [km/h] $\quad \bar{v}^2 = 25$ [km/h] $\quad b_{o,ij}^m = 0 \ \forall i, j, m \quad \upsilon^{h,3,s} = 1$

Assuming a certain fixed travel time t_{ij}^m from zone i to zone j, the probability of choosing $m = 1$ (walking) increases with an increase in travel mode $m = 2$ (public transport) specific travel costs c_{ij}^2 compared to c_{ij}^1 ($c_{ij}^1 = 0$). At the same time, the probability of choosing $m = 2$ decreases, where $\sum_{\forall m} \pi_{ij}^{m,h,3,1} = 1$.

[11] Assuming for example $\Xi_{ij}^{m,h,3,1} = 0$ and $m = 3$ denotes private car, $\Xi_{ij}^{3,h,3,1} = 0 \ \forall i, j$ means that for household member $s = 1$ a private car is generally not available.

[12] Instead of using $w_E^h = \sum_{\forall j} w_j^h / I$ to determine $VOT_{ij}^{m,h,3,s}$, one can use the actual wage rate w_j^h. In this case, the transport mode specific value of time $VOT_{ij}^{m,h,3,s}$ must be differentiated additionally in regard to the work location of household member s. As a consequence, also the probability $\pi_{ij}^{m,h,3,s}$ that household member s chooses travel mode m for a trip from zone i to zone j as well as the household member s specific expected travel times $t_{ij}^{h,3,s}$ and travel costs $c_{ij}^{h,3,s}$ must be differentiated in regard to the work location of household member s.

Household member s specific expected travel time $t_{ij}^{h,3,s}$ from zone i to zone j increases as travel mode specific travel costs c_{ij}^{2} increase. As travel costs c_{ij}^{2} are getting higher, household member s specific expected travel time $t_{ij}^{h,3,1}$ is getting higher because the probability of choosing $m = 1$ increases compared to $m = 2$, and, on the one hand, using $m = 1$ is for free in monetary terms, but, on the other hand, $m = 1$ is the slower mode.

In contrast, household member s specific expected travel costs $c_{ij}^{h,3,s}$ from zone i to zone j increase as travel mode specific travel costs c_{ij}^{2} increase up to a certain value of c_{ij}^{2}. Then, $c_{ij}^{h,3,s}$ decreases because the probability of choosing $m = 1$ dominates the probability of choosing $m = 2$ extensively. As travel costs c_{ij}^{2} are getting higher, household member s specific expected travel costs $c_{ij}^{h,3,s}$ are getting lower because the probability of choosing $m = 1$ increases compared to $m = 2$ and using $m = 1$ is for free in monetary terms.

2.4 Producers (Firms)

Within each zone i competitive firms in the input and output markets employ a constant-returns-to-scale Cobb–Douglas production function that combines land and labor to produce a zone specific composite commodity. Each composite commodity is sold at a shop located in the zone in which it is produced. Firms producing at the same zone are identical, but households differentiate firms based on their location. Let M_i^h be the aggregate labor input of skill level h [hours/period] in zone i and let Q_i be the aggregate land input [m^2] in zone i, the production function of the zone specific aggregate output X_i can then be written as follows:

$$X_i = B_i \prod_{h=1}^{H} \left(M_i^h\right)^{\delta_i^h} Q_i^{\phi_i}, \tag{20}$$

where B_i is the productivity (scale-) parameter in zone i, δ_i^h is the output elasticity with respect to labor (skill level h) in zone i, ϕ_i is the output elasticity with respect to land in zone i and $\sum_{h=1}^{H}\delta_i^h+\phi_i=1$. Given the production technology, profit maximization

$$\max_{M_i^h(\forall h),Q_i}\pi_i=p_iX_i-\sum_{h=1}^{H}w_i^hM_i^h-r_iQ_i=p_iB_i\prod_{h=1}^{H}\left(M_i^h\right)^{\delta_i^h}Q_i^{\phi_i}-\sum_{h}^{H}w_i^hM_i^h-r_iQ_i \tag{21}$$

yields profit-maximizing input demands $\tilde{M}_i^h$ and $\tilde{Q}_i$.

The zone specific commodity prices are determined from the zero profit condition, since free entry in each zone ensures that profit maximizing firms make zero economic profit in the competitive market. Hence, the condition that price equals marginal (and average) cost yields

$$p_i=\frac{\partial C_i(w_i^h\forall h,r_i,X_i)}{\partial X_i}=\frac{C_i(w_i^h\forall h,r_i,X_i)}{X_i}\quad\forall i, \tag{22}$$

where $C_i(w_i^h\forall h,r_i,X_i)$ is the cost function of each city firm located in zone i.

2.5 Equilibrium Conditions

In addition to the utility and profit maximization conditions, several other conditions are necessary to close the model. At general equilibrium, the factor markets for land and labor as well as the market for the locally produced composite commodity must clear in each zone i. Furthermore, firms in each zone i must make zero economic profits. Since individual land and commodity demand as well as individual labor supply (except for households of the non-working household type) and thus overall demand and supply are influenced by the location decisions (home and work locations), it is useful to see how these location decisions can be determined. In Table 3, the number of households within each household type is combined with the probability of choosing a specific location choice set $\{i\}$, $\{i,j\}$ and $\{i,j,l\}$, respectively.

Keeping in mind these location decisions, equilibrium in the land market in zone i requires

$$\sum_{\forall g}\Psi_i^{g,1}N^{g,1}\tilde{q}_i^{g,1}+\sum_{\forall(h,j)}\Psi_{ij}^{h,2}N^{h,2}\tilde{q}_{ij}^{h,2}+\sum_{\forall(h,j,l)}\Psi_{ijl}^{h,3}N^{h,3}\tilde{q}_{ijl}^{h,3}+\sum_{\forall(j,l)}\Psi_{ijl}^{4}N^{4}\tilde{q}_{ijl}^{4}+\tilde{Q}_i=A_i. \tag{23}$$

Table 3: Household types and location decisions

Household type	Number	Probability	Location Home in i	Location Work in j, l
			Number	Number
NW ($y=1$)	$N^{g,1}$	$\Psi_i^{g,1}$	$\Psi_i^{g,1}N^{g,1}$	-
SW ($y=2$)	$N^{h,2}$	$\Psi_{ij}^{h,2}$	$\sum_{\forall j}\Psi_{ij}^{h,2}N^{h,2}$	$\sum_{\forall i}\Psi_{ij}^{h,2}N^{h,2}$
HoTW ($y=3$)	$N^{h,3}$	$\Psi_{ijl}^{h,3}$	$\sum_{\forall (j,l)}\Psi_{ijl}^{h,3}N^{h,3}$	$\sum_{\forall (i,l)}\Psi_{ijl}^{h,3}N^{h,3}$ $\sum_{\forall (i,j)}\Psi_{ijl}^{h,3}N^{h,3}$
HeTW ($y=4$)	N^{4}	Ψ_{ijl}^{4}	$\sum_{\forall (j,l)}\Psi_{ijl}^{4}N^{4}$	$\sum_{\forall (i,l)}\Psi_{ijl}^{4}N^{4}$ $\sum_{\forall (i,j)}\Psi_{ijl}^{4}N^{4}$

Note: First row of the work location decision of the homogeneous two-worker household corresponds with the work location decision of household member $s=1$ and second row corresponds with the work location decision of household member $s=2$. First row of the work location decision of the heterogeneous two-worker household corresponds with the work location decision of household member $h=1$ and second row corresponds with the work location decision of household member $h=2$.

The left-hand side is the sum of lot size demand of all households of all household types residing in zone i and commuting to all zones plus land demands of all the firms in zone i. The right-hand side is the available developable land in zone i.

Equilibrium in the labor market regarding skill level h in zone i requires

$$\sum_{\forall a}\Psi_{ai}^{h,2}N^{h,2}\tilde{D}_{ai}^{h,2}L+\sum_{\forall (a,c)}\Psi_{aic}^{h,3}N^{h,3}\tilde{D}_{aic}^{h,3,1}L+\sum_{\forall (a,b)}\Psi_{abi}^{h,3}N^{h,3}\tilde{D}_{abi}^{h,3,2}L+\{\cdot\}=\tilde{M}_i^h, \tag{24}$$

where $\{\cdot\}=\begin{cases}\sum_{\forall (a,c)}\Psi_{aic}^{4}N^{4}\tilde{D}_{aic}^{4,h}L & \text{if } h=1\\ \sum_{\forall (a,b)}\Psi_{abi}^{4}N^{4}\tilde{D}_{abi}^{4,h}L & \text{if } h=2\end{cases}.$

The left-hand side is the supply of labor by all household members (household types 2-4) working in zone i and the right-hand side is the demand for labor by all the firms producing in zone i. Note that in the case of the heterogeneous two-worker household (Type $y=4$), only one household member supplies labor in a specific labor market h.

In the local market i for the composite commodity, market clearing requires

$$\begin{aligned} & \sum_{\forall(g,a)} \Psi_a^{g,1} N^{g,1} \tilde{Z}_{ai}^{g,1} + \sum_{\forall(h,a,b)} \Psi_{ab}^{h,2} N_{abi}^{h,2} \tilde{Z}_{abi}^{h,2} + \sum_{\forall(h,s,a,b,c)} \Psi_{abc}^{h,3} N^{h,3} \tilde{Z}_{abci}^{h,3,s} + \sum_{\forall(h,a,b,c)} \Psi_{abc}^{4} N^{4} \tilde{Z}_{abci}^{4,h} \\ & + EX_i \qquad (25) \\ = \; & \tilde{X}_i \end{aligned}$$

The left-hand side is the quantity of the composite commodity purchased in zone i by all household members of all household types y who live and work in all the zones in the urban area plus the quantity of the composite commodity that must be exported in exchange for the monetary expenditures for goods or services produced (e.g. fuel) or owned (e.g. land owned by absentee landowners) outside the urban economy. It is assumed that the composite commodity produced in the urban area can be exported at price p_i at zero transport costs. Therefore, the following condition must be met such that the urban economy is in equilibrium:

$$p_i EX_i = \frac{1}{I} ATC + \frac{1}{I} \sum_{\forall i} ALR_i, \qquad (26)$$

where ATC are total monetary transport costs paid by all city residents for two-way shopping trips and two-way commuting, $ALR_i = (1 - \Theta_i) A_i r_i$ is the aggregate land rent generated in zone i but not owned by city households with $\Theta_i \equiv \sum_{\forall g} \Theta_i^{g,1} + \sum_{\forall h} \Theta_i^{h,2} + \sum_{\forall h} \Theta_i^{h,3} + \Theta_i^4$. The local zone specific production not exported $X_i - EX_i$ is consumed locally. It is assumed that an equal share of total monetary transport costs and aggregate land rent is distributed to zones. Note that if $\Theta_i = 1$ $\forall i$ and $ATC = 0$, it follows that $EX_i = 0$ for each zone.[13]

According to the conditions described above, general equilibrium finds for each zone i rents, wages with respect to skill level h, commodity prices, firm outputs, export quantities and, based on it, the entire set of endogenous variables, where relative prices can be determined.

[13] Alternatively one can assume that $\Theta_i = 1$ and $ATC > 0$, but travel costs paid by city residents are fully redistributed to city households.

3 Model applications

The model implementation of a more complex household structure allows studying a broad range of further urban economic issues, which treat different household types differently.

Generally, the impacts of a different household structure on an urban economy can be examined. The household size can change over time, as realty shows. Therefore, the impacts on urban spatial structure and the economic conditions in the city can be analyzed.

Further, since real-world income taxation schemes treat different household types differently, the effects of fiscal policies on an urban economy and its spatial structure can be examined. In the model, wages are endogenously determined. Hence, income tax rates can influence labor decisions of households and thus wages in the city. As a result, location decisions can change and depend on the share of urban single and non-single households.

The mode choice model allows the consideration of different mode availability within households. This can influence full economic commuting and shopping costs. As a result, for instance, location decisions, commuting and shopping patterns, and labor supply decisions of household members living in non-single households can differ from those members living in single households.

In addition, the modeling of children offers the opportunity to examine various interesting issues. Children influence labor force participation of parents and, probably also the location decision of the household. For instance, the locations of private schools and child care facilities might influence the joint residential as well as the work location decisions of household members in the urban area.

4 Summary and conclusions

This paper has developed an urban general equilibrium model with a multiple household structure. Besides the usually assumed single-worker household, we have implemented non-working households as well as homogeneous and heterogeneous two-worker households. Based on these different structures, the households in the city differ not only in endowments and preferences, but also in size and the composition regarding their members. Applying the model, various economic issues and policy arrangements that arise in today's cities can be examined. However, various issues can only be examined in the case of appropriate model extensions. For instance, trip chaining is ignored in this model version, as in most urban models so far (excepting Anas, 2007). But, especially in the case of two-worker households, trip chaining can increase household utility. Assuming that both household members are employed at different locations. Then, they can satisfy their taste for spatial product variety by shopping trips that directly originate at work location. These and further aspects are left for future research.

Appendix-A: Notation

Dimensions

- Y : Number of household types
- H : Number of skill levels
- S : Number of household members (non-working couple-/homogeneous two-worker household)
- I : Number of zones in the urban area
- M : Number of different travel modes

Indices

- y : Household type ($y \in Y$)
- h : Skill level ($h \in H$)
- s : Household member (non-working couple-/homogeneous two-worker household) ($s \in S$)
- i : Home location ($i \in I$)
- j : Work location ($j \in I$) household member
  - $s = 1$ homogeneous two-worker household (Type 3)
  - $h = 1$ heterogeneous two-worker household (Type 4)
- l : Work location ($l \in I$) household member
  - $s = 2$ homogeneous two-worker household (Type 3)
  - $h = 2$ heterogeneous two-worker household (Type 4)
- k : Shopping location, ($k \in I$)
- m : Travel Mode ($m \in M$)

Appendix-B: Household optimization

Non-working household (Type $y = 1$, Subtype $g = 1$)

$$\max_{Z_{ik}^{1,1}(\forall k), q_i^{1,1}, \ell_i^{1,1}} U_i^{1,1} = U_i^{1,1}(Z) + U_i^{1,1}(q) + U_i^{1,1}(\ell) + \epsilon_i^{1,1}$$

$$\begin{aligned} U_i^{1,1}(Z) &= \frac{\alpha^{1,1}}{\eta^{1,1}} \ln\left(\sum_{k=1}^{I} \Gamma_k^{1,1}\left(Z_{ik}^{1,1}\right)^{\eta^{1,1}}\right) \\ U_i^{1,1}(q) &= \beta^{1,1} \ln q_i^{1,1} \\ U_i^{1,1}(\ell) &= \gamma^{1,1} \ln \ell_i^{1,1} \end{aligned}$$

subject to

$$\sum_{k=1}^{I}\left(p_k + c_{ik}^{1,1}\right) Z_{ik}^{1,1} + r_i q_i^{1,1} = R^{1,1}$$

$$\ell_i^{1,1} + T_i^{1,1} = E$$

$$T_i^{1,1} = \sum_{k=1}^{I} t_{ik}^{1,1} Z_{ik}^{1,1}$$

Non-working household (Type $y = 1$, Subtype $g = 2$)

$$\max_{Z_{ik}^{2,1,s}(\forall k,s),q_i^{2,1},\ell_i^{2,1,s}(\forall s)} U_i^{2,1} = U_i^{2,1}(Z) + U_i^{2,1}(q) + U_i^{2,1}(\ell) + \epsilon_i^{2,1}$$

$$\begin{aligned}
U_i^{2,1}(Z) &= \frac{\alpha^{2,1}}{\omega^{2,1}} \ln \sum_{s=1}^{2} \left(\sum_{k=1}^{I} \Gamma_k^{2,1,s} \left(Z_{ik}^{2,1,s} \right)^{\eta^{2,1}} \right)^{\frac{\omega^{2,1}}{\eta^{2,1}}} \\
U_i^{2,1}(q) &= \beta^{2,1} \ln q_i^{2,1} \\
U_i^{2,1}(\ell) &= \frac{\gamma^{2,1}}{\rho^{2,1}} \ln \left(\sum_{s=1}^{2} F^{2,1,s} \left(\ell_i^{2,1,s} \right)^{\rho^{2,1}} \right)
\end{aligned}$$

subject to

$$\sum_{s=1}^{2} \sum_{k=1}^{I} \left(p_k + c_{ik}^{2,1,s} \right) Z_{ik}^{2,1,s} + r_i q_i^{2,1} = R^{2,1}$$

$$\ell_i^{2,1,s} + T_i^{2,1,s} = E$$

$$\begin{aligned}
T_i^{2,1,1} &= \sum_{k=1}^{I} t_{ik}^{2,1,1} Z_{ik}^{2,1,1} \\
T_i^{2,1,2} &= \sum_{k=1}^{I} t_{ik}^{2,1,2} Z_{ik}^{2,1,2}
\end{aligned}$$

Single-worker household (Type $y = 2$)

$$\max_{Z_{ijk}^{h,2}(\forall k), q_{ij}^{h,2}, \ell_{ij}^{h,2}} U_{ij}^{h,2} = U_{ij}^{h,2}(Z) + U_{ij}^{h,2}(q) + U_{ij}^{h,2}(\ell) + \epsilon_{ij}^{h,2}$$

$$\begin{aligned}
U_{ij}^{h,2}(Z) &= \frac{\alpha^{h,2}}{\eta^{h,2}} \ln \left(\sum_{k=1}^{I} \Gamma_k^{h,2} \left(Z_{ijk}^{h,2} \right)^{\eta^{h,2}} \right) \\
U_{ij}^{h,2}(q) &= \beta^{h,2} \ln q_{ij}^{h,2} \\
U_{ij}^{h,2}(\ell) &= \gamma^{h,2} \ln \ell_{ij}^{h,2}
\end{aligned}$$

subject to

$$\sum_{k=1}^{I} \left(p_k + c_{ik}^{h,2} \right) Z_{ijk}^{h,2} + r_i q_{ij}^{h,2} + c_{ij}^{h,2} D_{ij}^{h,2} = L w_j^h D_{ij}^{h,2} + R^{h,2}$$

$$D_{ij}^{h,2} L + \ell_{ij}^{h,2} + T_{ij}^{h,2} = E$$

$$T_{ij}^{h,2} = \sum_{k=1}^{I} t_{ik}^{h,2} Z_{ijk}^{h,2} + t_{ij}^{h,2} D_{ij}^{h,2}$$

Homogeneous two-worker household (Type $y = 3$)

$$\max_{Z_{ijlk}^{h,3,s}(\forall k,s), q_{ijl}^{h,3}, \ell_{ijl}^{h,3,s}(\forall s), D_{ijl}^{h,3,s}(\forall s)} U_{ijl}^{h,3} = U_{ijl}^{h,3}(Z) + U_{ijl}^{h,3}(q) + U_{ijl}^{h,3}(\ell) + \epsilon_{ijl}^{h,3}$$

$$\begin{aligned}
U_{ijl}^{h,3}(Z) &= \frac{\alpha^{h,3}}{\omega^{h,3}} \ln \sum_{s=1}^{2} \left(\sum_{k=1}^{I} \Gamma_k^{h,3,s} \left(Z_{ijlk}^{h,3,s} \right)^{\eta^{h,3}} \right)^{\frac{\omega^{h,3}}{\eta^{h,3}}} \\
U_{ijl}^{h,3}(q) &= \beta^{h,3} \ln q_{ijl}^{h,3} \\
U_{ijl}^{h,3}(\ell) &= \frac{\gamma^{h,3}}{\rho^{h,3}} \ln \left(\sum_{s=1}^{2} \digamma^{h,3,s} \left(\ell_{ijl}^{h,3,s} \right)^{\rho^{h,3}} \right)
\end{aligned}$$

subject to

$$\begin{aligned}
& \sum_{s=1}^{2} \sum_{k=1}^{I} \left(p_k + c_{ik}^{h,3,s} \right) Z_{ijlk}^{h,3,s} + r_i q_{ijl}^{h,3} + c_{ij}^{h,3,1} D_{ijl}^{h,3,1} + c_{il}^{h,3,2} D_{ijl}^{h,3,2} \\
= \; & L \left(w_j^h D_{ijl}^{h,3,1} + w_l^h D_{ijl}^{h,3,2} \right) + R^{h,3}
\end{aligned}$$

$$D_{ijl}^{h,3,s} L + \ell_{ijl}^{h,3,s} + T_{ijl}^{h,3,s} = E$$

$$\begin{aligned}
T_{ijl}^{h,3,1} &= \sum_{k=1}^{I} t_{ik}^{h,3,1} Z_{ijlk}^{h,3,1} + t_{ij}^{h,3,1} D_{ijl}^{h,3,1} \\
T_{ijl}^{h,3,2} &= \sum_{k=1}^{I} t_{ik}^{h,3,2} Z_{ijlk}^{h,3,2} + t_{il}^{h,3,2} D_{ijl}^{h,3,2}
\end{aligned}$$

Heterogeneous two-worker household (Type $y = 4$)

$$\max_{Z^{4,h}_{ijlk}(\forall k,h), q^4_{ijl}, \ell^{4,h}_{ijl}(\forall h), D^{4,h}_{ijl}(\forall h)} U^4_{ijl} = U^4_{ijl}(Z) + U^4_{ijl}(q) + U^4_{ijl}(\ell) + \epsilon^4_{ijl}$$

$$\begin{aligned}
U^4_{ijl}(Z) &= \frac{\alpha^4}{\omega^4} \ln \sum_{h=1}^{2} \left(\sum_{k=1}^{I} \Gamma^{4,h}_k \left(Z^{4,h}_{ijlk} \right)^{\eta^4} \right)^{\frac{\omega^4}{\eta^4}} \\
U^4_{ijl}(q) &= \beta^4 \ln q^4_{ijl} \\
U^4_{ijl}(\ell) &= \frac{\gamma^4}{\rho^4} \ln \left(\sum_{h=1}^{2} F^{4,h} \left(\ell^{4,h}_{ijl} \right)^{\rho^4} \right)
\end{aligned}$$

subject to

$$\sum_{h=1}^{2} \sum_{k=1}^{I} \left(p_k + c^{4,h}_{ik} \right) Z^{4,h}_{ijlk} + r_i q^4_{ijl} + c^{4,1}_{ij} D^{4,1}_{ijl} + c^{4,2}_{il} D^{4,2}_{ijl} = L \left(w^1_j D^{4,1}_{ijl} + w^2_l D^{4,2}_{ijl} \right) + R^4$$

$$D^{4,h}_{ijl} L + \ell^{4,h}_{ijl} + T^{4,h}_{ijl} = E$$

$$\begin{aligned}
T^{4,1}_{ijl} &= \sum_{k=1}^{I} t^{4,1}_{ik} Z^{4,1}_{ijlk} + t^{4,1}_{ij} D^{4,1}_{ijl} \\
T^{4,2}_{ijl} &= \sum_{k=1}^{I} t^{4,2}_{ik} Z^{4,2}_{ijlk} + t^{4,2}_{il} D^{4,2}_{ijl}
\end{aligned}$$

References

Alonso, W., 1964. Location and Land Use. Harvard University Press, Cambridge, MA.

Anas, A., 2007. A unified theory of consumption, travel and trip chaining. *Journal of Urban Economics* 62(2), 162-186.

Anas, A., 1990. Taste heterogeneity and urban spatial structure: The logit model and monocentric theory reconciled. *Journal of Urban Economics* 28(3), 318–335.

Anas, A., Liu, Y., 2007. A regional economy, land use, and transportation model. *Journal of Regional Science* 47(3), 415–455.

Anas, A., Arnott, R., Small, K. A., 1998. Urban spatial structure. *Journal of Economic Literature* 36(3), 1426–1464.

Anas, A., Rhee, H.-J., 2007. When are urban growth boundaries not second-best policies to congestion tolls? *Journal of Urban Economics* 61(2), 263-286.

Anas, A., Rhee, H.-J., 2006. Curbing excess sprawl with congestion tolls and urban boundaries. *Regional Science and Urban Economics* 36(4), 510–541.

Anas, A., Xu, R., 1999. Congestion, land use, and job dispersion: A general equilibrium model. *Journal of Urban Economics* 45(3), 451–473.

Anderson, S. P., de Palma, A., Thisse, J.-F., 1988. A representative consumer theory of the logit model. *International Economic Review* 29(3), 461–66.

Becker, G.S., 1965. A theory of the allocation of time. *The Economic Journal* 75(299), 493–517.

Brueckner, J. K., Selod, H., 2006. The political economy of urban transport-system choice. *Journal of Public Economics* 90, 983-1005.

Curran, C., Carlson, L. A., Ford, D. A., 1982. A theory of residential location decisions of two-worker households. *Journal of Urban Economics* 12(1), 102-114.

DeSalvo, J. S., Huq, M., 2005. Mode choice, commuting cost, and urban household behavior. *Journal of Regional Science* 45(3), 493–517.

Dixit, A. K., Stiglitz, J. E., 1977. Monopolistic competition and optimum product diversity. *American Economic Review* 67(3), 297–308

Freedman, O., Kern, C. R., 1997. A model of workplace and residence choice in two-worker households. *Regional Science and Urban Economics* 27(3), 241-260.

Glaeser, E. L., Kolko, J. Saiz, A., 2001. Consumer city. *Journal of Economic Geography* 1(1), 27-50.

Hamilton, B. W., 1982. Wasteful Commuting. *Journal of Political Economy* 90(5), 1035-51.

Hotchkiss, D., White, M. J., 1993. A simulation model of a decentralized metropolitan area with two-worker, traditional, and female-headed households. *Journal of Urban Economics* 34(2), 159–185.

Jara-Diaz, S. R., Guevara, C. A., 2003. Behind the Subjective Value of Travel Time Savings. *Journal of Transport Economics and Policy* 37 Part 1, 29-46.

Jara-Diaz, S. R., Munizaga, M.A., Greeven, P., Guerra, R., Axhausen, K., 2008. Estimating the value of leisure from a time allocation model. *Transportation Research Part B: Methodological* 42(10), 946–957.

Madden, J. F., 1980. Urban land use and the growth in two-earner households. *American Economic Review* 70(2), 191–97.

McFadden, D., 1973. Conditional logit analysis of qualitative choice behavior. In: Zarembka, P. (Ed.), Frontiers of Econometrics, Academic Press, New York.

O´Sullivan, A., 2007. Urban Economics. McGraw-Hill, 6. Edition.

Small, K. A., Song, S., 1992. "Wasteful" Commuting: A Resolution. *Journal of Political Economy* 100(4), 888-98.

Sullivan, A., 1986. A general equilibrium model with agglomerative economies and decentralized employment, *Journal of Urban Economics* 20, 55-74.

Train, K. E., 2003. Discrete choice methods with simulation, Cambridge University Press.

Wheaton, W. C., 2004. Commuting, congestion, and employment dispersal in cities with mixed land use. *Journal of Urban Economics* 55(3), 417-438.

Wieand, K., 1987. An extension of the monocentric urban spatial equilibrium model to a multicenter setting: The case of the two-center city, *Journal of Urban Economics* 21, 259-271.

White, M. J., 1999. Urban areas with decentralized employment: Theory and empirical work. *Handbook of Regional and Urban Economics* 3, 1375–1412.

White, M. J., 1988a. Location choice and commuting behavior in cities with decentralized employment. *Journal of Urban Economics* 24(2), 129-152.

White, M. J., 1988b. Urban Commuting Journeys Are Not "Wasteful". *Journal of Political Economy* 96(5), 1097-110.

White, M. J., 1977. A model of residential location choice and commuting behavior by men and women workers. *Journal of Regional Science* 17(1), 41-52.

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