Family Institution and Filial Attention Contract

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Abstract

In this paper, we examine the pure exchange motive for intergenerational transfers within the family. We consider a model where a selfish parent offers a financial transfer in exchange for the services of the child. Using a Stackelberg game, we study the optimal attention-money contract between the generations. We prove that the amount of gift received may be either positively or negatively related with the child's income. In addition, the relationship between the two variables is non linear and affected by the parent's degree of risk aversion. This non-linearity, which has been largely neglected to date in empirical analyses, may explain why the exchange transfer motive has received little support in developed countries.

JEL classification: D1; J1; J2

Keywords: Family ; Inter generational transfers ; Filial exchange ; attention ; care
1 Introduction

Since the pioneering work conducted by Becker (1991) on altruism within the family, economists have been widely interested in the study of the motives for redistributive behaviors between generations. To date, the literature on private inter vivos transfers between parents and their children has differentiated two main strands regarding the motivation for such transfers (see Laferrère, 1999, Laferrère and Wolff, 2002). According to the altruistic hypothesis, parents derive satisfaction from the children's well-being. It follows that financial transfers are expected to flow from the least to the most financially needy generation, independently of any present or future reciprocating transfer (Becker, 1991). In the competing exchange model, financial transfers bestowed by parents correspond to a payment for non-market services provided by children (Cox, 1987, Cox and Rank, 1992). In this framework of reciprocity, monetary transfers can be rendered later as part of a long-term contract, so that parents can extract a greater amount of attention from their children (Bernheim et alii, 1985). In another exchange model, transfers are familial loans between non-altruistic family members. Parents first help their liquidity-constrained children early in the life-cycle, and then are paid back during their old-age years (Cox, 1990, Cigno, 1993).

The empirical exploration of motives for family transfers matters for the effectiveness of public safety nets, since the effects of public income redistribution depend in part on private responses to them and especially on family assistance decisions. Different outcomes for public policies that redistribute income are expected under altruism and under exchange. On the one hand, households responding to altruistic feelings will completely neutralize the effects of a government redistribution by perfectly adjusting the level of their family assistance, at least when there exists interior solutions for such private transfers (Barro, 1974)
(Hence, attempts by a central authority to affect the distribution of intergenerational resources through welfare programs are doomed to failure with benevolent transfers.)

On the other hand, under exchange motives, a public income redistribution is not necessarily neutralized by modifications of family transfers. For instance, Cox and Jakubson (1995) demonstrate that the redistributional effects of a welfare program can be reinforced by such private exchange-motivated transfers.

Thus, the question worth related to the altruism neutrality result is whether one can distinguish from an empirical viewpoint between the two competing motives for inter vivos transfers inside the family. An answer to this interrogation may be found in the pioneering paper of Cox (1987), who was the first to introduce a unified model of private assistance carrying either altruism or exchange motives. These two regimes yield a different prediction about the effect of the recipient's income on transfers, and this testable restriction can determine which theoretical hypothesis is consistent with the data. Under altruism, the amount bestowed is expected to decrease in response to the child's higher income. But in the case of an exchange, the effect of the child's income cannot be signed since it strongly depends on the elasticities of supply and demand for services (Cox, 1987). When the demand of services from children is inelastic, the gift value is expected to rise with the child's level income (Cox and Rank, 1992). Parents have then to pay a higher price to get the same amount of attention from their children. So, while a positive relationship between the child's income and the gift value holds only under exchange, a negative relationship reveals itself little about transfer motives since it consistent both with altruism and exchange.

Recently, two criticisms have stressed the inability to infer the main motives behind inter-vivos transfers from this previous empirical test. First, Stark and Falk (1998) propose a model where financial gifts directly affect preferences through a recipient's empathy function. In
this setting, a child's lower income may be positively correlated with an exchange transfer and hence the altruism and exchange hypotheses can give rise to behaviors that can be observationally equivalent. Second, Altonji et alii (1997) demonstrate that the gift value can be positively correlated with a child's higher income even under altruistic feelings. Indeed, the distribution of the altruism parameter has to be taken into account, since it influences the existence of positive gifts (interior solutions). In particular, a parent has to be comparatively more altruistic to transfer resources to a richer offspring, thus leading to an increase in the gift value.

Then, Altonji et alii (1997) suggest a different way to test which behavior is consistent with the data. In the altruism model, a change in the distribution of income between the generations does not affect the optimal levels of consumptions. This well-known result, which is called the income-pooling property, yealds the following test corresponding to a unitary difference in transfer income derivatives. Conditional on positive transfers, offsetting a given change in both the child's and parent's income is completely neutralized by a same change in the transfer value given by the parent to the child (Altonji et alii, 1997, Cox, 1987). Nevertheless, this test of intra-familial neutrality is only able to provide support for the relevance of the altruism model. When the difference in transfer income derivatives is not equal to one, the test conveys in itself no information about the possibility of exchange-motivated transfers.

A more subtle and insightful approach to test the motives for transfers inside the family has recently been followed by Cox et alii (1998, 2000). Using a model that nests the beckerian altruism hypothesis, Cox et alii (2000) show that substantial non-linearities are likely to occur between the child's income and the amount of financial transfer from parent. This relationship
between the two variables arises because either altruistic or exchange assistance can predominate depending on the recipient's level of resources. As the child's income rises to a certain threshold, the regime of transfer motive is expected to switch from altruism to exchange. This test is much informative since it suggests that one or the other motive may be more appropriate to characterize the behavior of particular families. Even in the exchange regime, Cox et alii (1998) prove using a Nash bargaining model that transfers can first rise, then fall with the recipient's pre-transfer income. This result occurs because an increase in the child's income also leads to a rise of the recipient's threat-point utility, defined by his well-being with no exchange. Hence, from an empirical viewpoint, one should account for a non-linear specification in the recipient's income concerning the transfer function.

In this paper, we attempt to go one step further by examining in greater detail this relationship between the two key variables associated with the transfer hypotheses, i.e. the recipient's income and the gift value. We use a model of transfers relying on an exchange motive which is observationally equivalent to the one proposed by Cox (1987): the child provides services and the parent buys this attention. However, in contrast with the previous literature, we focus on a pure exchange model in which parents derive no satisfaction from the well-being of their children. The situation is like a Stackelberg game since the parent just wants the child to participate in the exchange contract, where the willingness to supply services depends on the financial payment. We provide a theoretical characterization of the filial attention contract. Our theoretical analysis puts forward the role of the parental degree of risk aversion when explaining family transfers, since this parameter influences the relationship between the child's income and the gift value.
The remainder of the paper is organized as follows. In section 2, we introduce a pure exchange model of transfer where a selfish parent buys the services of the child. We describe the corresponding Stackelberg game in which the parent is a dominant player. In section 3, we study the optimal pattern of intergenerational transfers. We derive the solutions for the two transfers currencies, services and money, and focus on the issues of non-linearity and risk-aversion. In section 4, we present some concluding remarks dealing with empirical considerations for transfer behavior.

2 A Pure Exchange Model of Transfer

Let us consider a model of family transfer whose motives correspond to a pure exchange between two generations. The underlying mechanism of reciprocity is organized as follows (see Cox, 1987). Children provide services and attention to their parents, and the latter buy these services by giving them some money. In an intertemporal framework, transfers may also be seen as loans between the generations (Cox, 1990). Parents are expected to grant loans to their children and are reimbursed latter by financial transfers. The two variants (static and dynamic) of the exchange model are observationally equivalent as long as the repayment from children occurs through filial attention.

Following Cox (1996), we suppose that the family exchange involves time-related transfers that have no close substitutes on the market. An interpretation is that parents attach an important emotional value specific to the amount of attention received from their own children (Lafferère, 1999). We use a two-generational setting where each generation is represented by only one person. For the presentation, the parent and the child are respectively denoted by subscript $p$ and $k$. The model includes two currencies of transfers, money $T$ which is given by the parent to the child and attention $A$ which flows from the child to the parent. In contrast with the model of Cox (1987) and Cox et alii (1998), the parent is purely selfish and
he does not care for the well-being of the child. Cigno (1993, 2002) also examines a non-altruistic model of transfer, where family acts as a substitute or a complement to the credit market and transfers are a means of improving the inter temporal allocation of resources. However, the mechanism is different from the exchange that we present. Transfers are no longer a substitute for private consumption, but they are instead a form of investment (transfers are like a portfolio choice operation. This implies that his level of satisfaction is an increasing function of his own consumption \( C_p \) and of the amount of upstream services \( A \) received from the child. Without loss of generality, we restrict our analysis to the case of separable utility functions. Thus, the utility function \( U \) for the parent can be expressed as:

\[
U = u(C_p) + \phi(A) \tag{1}
\]

where \( u \) and \( \phi \) are utility functions defined respectively over personal consumption and attention. We make the standard assumptions that the two utility \( u \) and \( \phi \) are continuous, two times differentiable and strictly quasi-concave. This implies that we have \( u' > 0, u'' < 0 \) and \( \phi' > 0, \phi'' < 0 \) with the normalization \( u(0) = 0 \). Concerning the budget constraint, we suppose that the parent is characterized by an exogenous level of income \( Y_p \). The parental revenue is devoted to the personal consumption \( C_p \) and to the provision of a selfish financial gift \( T \) to the child (\( T \geq 0 \)). Therefore, the budget constraint is simply given by \( C_p = Y_p - T \).

We now turn to the situation to the child, which may either accept or reject the filial attention contract. When the child decides to refuse the exchange contract proposed by the parent, for instance because the financial counterpart is not high enough, the child devotes all his time on the labor market. If we denote by \( w \) is the child's market wage and by \( h \) the work time for a salary job, the child's income is given by \( wh \). This amount of resources is affected to the child's consumption \( C_k \), so that the budget constraint is \( C_k = wh \). Let \( v(\cdot) \) be the child's utility function which is defined over the consumption \( C_k = wh \) , where \( v(\cdot) \) is
continuous and strictly quasi-concave \((v' > 0, v'' < 0)\). The level of satisfaction \(V_0\) when the child chooses not to spare time with his parent may be expressed as:

\[
V_0 = v(wh)
\]  

What happens when the child accepts the parental the familial contract? In this situation, the child is expected to receive two types of financial resources. First, the child still receives an income on the labor market. However, the provision of upstream services decreases the amount of time that the child can devote to the job market. Thus, the salary income is lowered. In addition, it is more expensive for the child to visit his parent when the two generations are distant. Several studies have shown that the child's supply of services is strongly affected by the geographical distance to the parent (see Hiedemann and Stern, 1999, Sloan et alii, 1997, Wolff, 2001. Let \(x\) be a variable that indicates the distance between the two generations. We suppose that the opportunity cost related to the wage loss involved by the provision of attention is proportional to the distance from the parent. It follows that the earned income of a child who accepts to care for the parent given by \(w[h - (1 + x)A]\)

Second, the parent is expected to make a financial transfer to compensate for the loss in the child's level of income. The gift value \(T\) is an increasing function of the volume of upstream services. The budget constraint is now \(C_k = T + w[h - (1 + x)A]\). So, the child's level of satisfaction \(V_1\) when attention is bought by the parent becomes:

\[
V_1 = v(T + w[h - (1 + x)A])
\]  

In our framework, the supply of upstream attention basically depends on the opportunity cost for the child's time. Let \(c = w(1 + x)\) be a variable corresponding to the time value for the recipient's child. The two components of this opportunity cost are the wage rate \(w\) which indicates the capacities of the agents on the labor market and the geographical distance \(x\). The implicit cost related to the supply of services is all the more low as the child is badly paid in the labor market and/or lives near his parent. These two variables are supposed to belong to
closed intervals, so that we can write $w \in [0, \bar{w}]$ and $x \in [0, \bar{x}]$. Given these two restrictions, the opportunity cost $c$ also belongs to the closed interval $c \in [0, \bar{c}]$. Hence, a child faces a specific time value $c$ and the supply of upstream services is subordinated to this cost heterogeneity.

We can now characterize the optimal intergenerational contract. We first define the rules of the game and then present the corresponding maximization program.

**Definition 1**

The optimal family contract is such that:

i) the parent sets the transfers $(T, A)$ of the contract and the child plays according to this transfer rule;

ii) the child accepts the contract if and only if $V \geq 0$, where $V$ is the rent obtained by the child when he takes part in the family contract.

Thus, in the context of our model, the situation is like a Stackelberg game in which the parent is the dominant player. The parent is absolutely not concerned with the child's utility, he just wants the child to participate in the exchange contract provided that the willingness to supply $A$ depends on $T$. Clearly, the parent sets the rule and the child plays according to them. From the child's perspective, we assume that the familial exchange contract is desirable if and only if the following condition holds:

$$ V = V_1 - V_0 \quad (4) $$

which corresponds to the individual rationality constraint for the child. The rent for the child is simply the difference between the child's levels of well-being according to whether he accepts or not the money-services exchange. It is in the interest of the child to accept the family contract only when the parent proposes a sufficiently high remuneration to compensate the child for his disutility (When the compensation for the wage loss is sufficiently raised by the gift value, the child derives satisfaction from the money-exchange reciprocity).
Therefore, the problem for the child is given by the following program:

$$\max_{A \geq 0} V = v(T(A) + wh - cA) - v(wh)$$

(5)

The solution of this maximization program gives some kind of a reaction function for the child. Now, the problem for the parent is to maximize his utility function $U$ given the individual rationality constraint of the child. Hence, in this Stackelberg game, the optimal family contract is solution of:

$$\max_{A, V} U = u(C_p) + \phi(A)$$

(6)

s.t $V \geq 0$, $0 \leq A \leq h$

It is important to note that in our setting, the parent receives no utility from the child's rent when taking part in the family contract. The Stackelberg equilibrium gives solutions for the two currencies $A$ and $T$ at the same time: the child plays according to the rules set by the parent. Again, there is in effect no altruism involved. In the case of effective altruism, the utility function $U$ for the parent would be instead $U = u(C_p) + \phi(A) + \beta V_1$, where the parameter $\beta \in ]0,1[$ indicates the degree of parental benevolence. Such an expression would be similar to the utility function in Cox (1987), and then it would be appropriate to separate the money and services decisions (in presence of altruism, one gets more complicated results, where the parent transfers more to the poor child than motivated by the level of attention.)

3 The Optimal Family Contract

To find the optimal solutions for the family contract, we consider a setting of perfect information between the two generations. This implies that the parent knows with certainty the value of his child opportunity cost $c = c(w, x)$. The parent is perfectly aware of his child's geographical location and of the wage rate on the labor market. We also make an additional assumption concerning the supply of services.

Assumption 1 The condition $\frac{\phi'}{u'} < \bar{c}$ holds.
Where $\phi' = \frac{\partial \phi}{\partial A}$ and $u' = \frac{\partial u}{\partial A}$. Clearly, $\frac{\phi'}{u'}$ is the marginal rate of substitution between private consumption (or financial transfer) and services. The interpretation of this assumption is that there exist some children whose opportunity cost is so high that it appears too costly and not very desirable for the parent to make them take part in the family exchange. For example, a child who is remunerated at a very high wage rate and who lives far away will not provide attention to his parent. Thus, his supply of upstream services is expected to be null. We first calculate the optimal solutions for $A$ and $T$, and then examine how the gift value $T$ depends on the child's income.

3.1 The Pattern of Intergenerational Transfers

Since the parent is perfectly aware of the child's opportunity cost and given the Stackelberg equilibrium for the game, the family contract of exchange consists in determining jointly the supply of services by the child and the amount of financial transfer devoted to the purchase of this attention.

**Proposition 1**

The optimal allocation of transfers for the family contract is such that:

\begin{align*}
\text{i)} A(c) &= \begin{cases} 
  h & \text{for } 0 \leq c \leq c_1 \\
  \Phi^{-1}(c) & \text{for } c_1 \leq c \leq c_2 \\
  0 & \text{for } c_2 \leq c \leq \hat{c}
\end{cases}

\text{ii)} T(c) &= \begin{cases} 
  ch & \text{for } 0 \leq c \leq c_1 \\
  cA(c) & \text{for } c_1 \leq c \leq c_2 \\
  0 & \text{for } c_2 \leq c \leq \hat{c}
\end{cases}
\end{align*}

with $\Phi = \frac{\phi'}{u'}$, $c_1 = \Phi(h)$ and $c_2 = \Phi(0)$.

Proof:

Let us first solve the maximization problem (5) for the child. From the corresponding first-order condition: $(T'(c) - c)u'(T(A) + wh - cA) = 0$, we deduce that the optimal financial
transfer satisfies $T'(cA) = c$, which is the reaction function for the child. Since $T(A = 0) = 0$, it follows that: $T(A) = cA(c)$. Hence, from the definition of the rent $V = v(T(A) + wh - cA) - v(wh)$, we obtain that the rent for the child is such that $V = 0$ at the equilibrium. To take part in the family exchange does not affect the child's well-being.

Now, for the parent, the problem is to maximize the selfish utility $U$ subject to the constraint $V = 0$ since the parent is the dominant player. Thus, the Lagrangian of the parental maximization program is:

$$
\mathcal{L} = u(C_p) + \phi(A) + \lambda V \quad \text{where the inequalities } V \geq 0, \ 0 \leq A(c) \leq h \ \text{and } \lambda > 0 \text{ hold.}
$$

Using $C_p = Y_p - T, \ T(A) = cA(c)$ and $V = 0$, the solution for the parent is to maximize $U = u(Y_p - cA(c)) + \phi(A(c))$. From the first-order condition for an interior solution, we get: $-cu'(Y_p - cA(c)) + \phi'(A(c)) = 0$ which can also be expressed as: $\frac{\phi'(A(c))}{u'(Y_p - cA(c))} = c$. Let $\Phi(A)$ be the function such that $\Phi(A) = \frac{\phi'(A(c))}{u'(Y_p - cA(c))}$. Then, the optimal level of services satisfies $\Phi(A(c)) = c$ and hence $A(c) = \Phi^{-1}(c)$. By differentiating with respect to $c$ the following quantity $\frac{\phi'(A(c))}{u'(Y_p - cA(c))} = c$, the derivative $A'(c)$ is such that $A'(c) = \frac{(u')^2 - \phi' Au'^n}{\phi' u' + c\phi' u'^n}$.

Clearly, the sign of $A'(c)$ is negative from the concavity of $u$ and $\phi$.

By definition, we have $c \in [0, \bar{c}]$. For $c = 0$ we have: $\partial \mathcal{L} / \partial A = \phi'(A(0)) > 0$ which implies $A(0) = h$. Since $A'(c) < 0$, one can find a value $c_1$ such that $\Phi(h) = c_1$. In the same way, the inequality $\Phi(0) < 0$ holds from assumption 1 and thus there exists a cost level $c_2$ such that $c_2 = \Phi'(0)$. Finally, using $T(Ac)) = cA(c)$ from the child's reaction function, we find the different optimal financial transfers. QED

Let us interpret this proposition. One can distinguish three regimes of assistance for the child's supply of services.
When the child is characterized by a low value for his opportunity cost $c$, whether his wage rate remains low or he leaves near his kin, the parent asks the child to allocate the maximum of his time to the provision for upstream transfers. For $c \leq c_1$, the level of attention is thus $A(c) = h$. Given the concavity of $u$, the corresponding cost level $c_1$ is all the more low as the variable $h$ is high. With many children, this first regime where $c \in [0, c_1]$ implies that the same volume of services is expected from the children. The latter are then compensated for their attention by the same amount of financial assistance, so that there may exist equal sharing in an exchange model of family transfers.

For intermediate values for the opportunity cost such that $c \in [c_1, c_2]$, the parent expects a specific level of attention from his child in accordance with the time value $c$. Since the condition $A'(c) < 0$ holds for $c \in [c_1, c_2]$, a child with an increased opportunity cost (because of a higher wage rate or a greater geographical distance) will be comparatively less appealed by the parent.

When the child faces a very high value for his opportunity cost (high wage rate and/or geographical distance), the parent does not wish to receive upstream time related transfers. Indeed, the gift value related to the purchase of the child's attention would be too expensive for the parent, so that $A(c) = 0$ for $c \geq c_2$.

It seems important to note that the design of our theoretical framework is very closed to the one proposed by Cox (1987), but within a different framework. Cox (1987) assumes that the child accepts to take part in a money-services exchange only if his level of satisfaction is not lowered. Thus, there is an additional constraint given by the difference between the child's level of utility with transfers and the child's threat point defined when no exchange takes place. Another way to model the exchange model is to consider a Nash equilibrium, which leads to an increase in satisfaction for the two generations (see Cox, 1987, Cox and Rank, 1992). This utility constraint leads to two regimes of motives. When the child's constraint is
not binding, transfers are relevant from altruism. But when the child's constraint is binding, the marginal gift amount does not equalize the marginal utilities of consumptions between the two generations and the transfers are only motivated by an exchange between money and services. Relying on a Stackelberg equilibrium as we do in our model of reciprocity leads to a similar conclusion, since the child has the same utility level whether he participates in the exchange or not.

3.2 Financial Transfer and Parental Risk Aversion

Given the previous levels of attention, the parent is involved to propose a compensation to the child using a financial transfer. According to the optimal filial attention contract, we find that the gift value is defined as the product of the amount of attention and the price of services which corresponds to the value of the child's opportunity cost. In the exchange model of Cox (1987), the financial gift may also be expressed as the product of services and an implicit average price of services in the exchange regime. It follows that the optimal provision of financial gift is a piecewise function of the amount of services.

Given the profile of gift value $T$ which is a function of the type of child $c \in [0, \bar{c}]$, we can study how the child's opportunity cost $c = c(h,x)$ affects the optimal transfer amount. Indeed, one has to remind that the empirical tests carried out to find the motive for transfer behaviors inside the households depends on the relationship between the child's income and the amount of financial gift bestowed by the parent. We now indicate how the parent's degree of risk aversion influences transfer decisions.
Proposition 2

The optimal amount of parental financial gift is affected by the child's opportunity cost in the following way:

i) for \(0 \leq c \leq c_1\), \(\text{sgn } T'(c) > 0\)

ii) for \(c_1 \leq c \leq c_2\), \(\text{sgn } T'(c) = \text{sgn } [\sigma_r - 1]\)

where \(\sigma_r = \frac{-A\Phi'(A)}{\Phi(A)}\) indicates the parental degree of relative risk aversion.

Proof

The sign of \(T'(c) > 0\) depends on the value of \(c\). First, when the condition \(0 \leq c \leq c_1\) holds, the transfer is \(T(c) = ch\). We obtain \(T'(c) = h\) and hence the derivative is positive. Second, for \(c_1 \leq c \leq c_2\), the gift value is defined by \(T(c) = cA(c)\). Hence, the derivative \(T_0(c)\) may be calculated as: \(T'(c) = A(c) + cA'(c)\)

From the first-order condition we know that the equality \(-cu'(Y_p - cA(c)) + \Phi'(A(c)) = 0\) holds. Using the function \(\Phi(A) = \frac{\Phi'(A(c))}{u'(Y_p - cA(c))}\), it can also be expressed as \(\Phi(A(c)) = c\). By differentiation, we find that \(A'(c)\Phi'(A(c)) = 1\), which implies that \(A'(c) = \frac{1}{\Phi'(A(c))}\).

By replacing \(c\) and \(A'(c)\) by their values, we deduce that: \(\text{sgn } (T'(c) = A(c) + \frac{\Phi(A(c))}{\Phi'(A(c))} = sgn (A(c)(1 - \frac{1}{\sigma_r}))\), where \(\sigma_r = \frac{-A\Phi'(A)}{\Phi(A)}\).

Thus, we have \(T'(c) > 0\) (respectively \(T'(c) < 0\)) when the condition \(\sigma_r = \frac{-A\Phi'(A)}{\Phi(A)} > 1\) (respectively \(\sigma_r < 1\)) holds for the parental degree of relative risk aversion. \(QED\)

Let us interpret this result, which indicates the existence of different effects for the child's income according to value of \(c\) considered. For a child with a low opportunity cost, the parent proposes a transfer comparatively more favorable for a wealthy child since one has \(T'(c) > 0\). Indeed, the transfer is an increasing function of the opportunity cost, which is itself
positively correlated with the level of income \( w \) for the child. In other words, for \( 0 \leq c \leq c_1 \), a child devotes the totality of his time to care for the parent and he receives in return a higher gift value when he earns a high wage rate on the job market. This anti compensatory effect of intergenerational family redistribution has been put forward as the main characteristic of the money-services exchange model (Cox, 1987, Cox and Rank, 1992).

However, in the second regime where \( c_1 \leq c \leq c_2 \), the effect of the recipient's income basically depends on the degree of relative risk aversion for the parent. The justification of this result is due to the fact that the volume of services made by the child varies according to his opportunity cost. So, the full cost for a parent to buy the child's attention is \( T(A(c)) = \Phi(A(c))A(c) \) with \( c = \Phi(A(c)) \), i.e. the product of the unit price of attention and the quantity of upstream services \( A(c) \). Consequently, the derivative \( T'(A(c)) \) is as follows:

\[
T'(A(c)) = \Phi'(A(c)) + A(c)\Phi''(A(c))
\]

so that one finds that the :

\[
\text{sgn} T'(A(c)) = \text{sgn} [1 - \sigma_r] \quad \text{with} \quad \sigma_r = \frac{-A\Phi'(A)}{\Phi(A)}
\]

Thus, the impact of the supply of services on the financial transfer clearly depends on the parental measure for relative risk aversion. We have to examine the two following cases:

First, when the parent has a low value for risk aversion \( (\sigma_r < 1) \) , he agrees to pay a higher gift value to receive more attention from his child since \( T'(c) > 0 \). But we have previously shown that an increased supply of services could only emanate from a child characterized by a low opportunity cost \( A'(c) < 0 \). Thus, for \( \sigma_r < 1 \), one finds a negative derivative \( T'(c) < 0 \). A parent is expected to transfer more financial resources to his child when the latter earns a low income. This compensatory redistribution of resources between the generations does not result here from altruistic motivations, but it is explained by the lower parental risk aversion.

Second, when the parent is characterized by a high value for risk aversion \( (\sigma_r > 1) \), the condition \( T'(A(c)) < 0 \) holds. The parent is expected to receive a low volume of upstream
services. Such a limited attention is necessarily provided by a child with a high opportunity cost, whether his wage on the labor market is high or he lives far away from the parent. Thus, for $\sigma_r > 1$, the derivative $T'(c)$ is positive and the parent seeks to offer a greater compensation for a richer child. Therefore, this theoretical framework of filial contract puts forward an essential characteristic of the exchange-motivated model. The relationship between the amount of financial transfer to the child and the volume of services received in return is largely nonlinear. In the case of a parent having a preference for risky behavior, one always expects Anti compensatory effects for the redistribution of resources between the generations. A richer child receives a greater amount of financial gift from the parent. Conversely, in the probable case where the relative dislikes risky attitudes, the amount of financial transfer is first an increasing function of the child's income for a low value of opportunity cost, but it is then a decreasing function as one considers intermediate levels of opportunity cost for the child. Hence, one expects an inverted U-shaped relationship between the child's income and the gift value for a risk-averse parent.

Finally, in the intermediary regime ($c_1 \leq c \leq c_2$), we can easily prove that the parental income exerts a positive impact on the gift value to the child, which is a standard result in exchange models. Using $T(c) = cA(c)$, we deduce that $\frac{\partial T}{\partial Y_p} = c \frac{\partial A}{\partial Y_p}$. Now, from the first-order condition : $-cu'(Y_p - cA(c)) + \phi'(A(c)) = 0$, it follows that $sgn \frac{\partial A}{\partial Y_p} = sgn (-cu") > 0$. This derivative is clearly positive given the concavity of $u(.)$. Thus, while the gift value may either increase or decrease with the child's income depending on the parental risk aversion, the exchange model predicts that a richer parent always offers a greater compensation for the services received from the child.
4 Conclusion

In this paper, we have developed a model of exchange-motivated transfers between a parent and a child using a Stackelberg game. There is no utility gain for the child when entering into the transfer-services arrangement, and the financial gift provides perfect compensation for the child's attention. Each child is characterized by an opportunity cost for his time value. Looking at the optimal exchange contract, we prove that both types of transfers are likely to follow a non-linear pattern. The gift value from parent may be positively or negatively related with the child's level of income, depending on the parent's degree of risk aversion. When the parent is risk-lover, a positive relationship between the recipient's income and the transfer amount is always expected. But when the parent is risk-averse, the gift value is expected to first rise, then fall at higher levels of income. This characterization of family exchange contract seems important with regard to the determination of the intrinsic motives for inter vivos transfers. In order to find whether private redistributive behavior between generations are consistent with altruism or with exchange according to the data, various empirical works completed to date have focused on the sign of the relationship between the recipient's income and the transfer amount bestowed by parents. We show here that such a test provides little information concerning the family motives, at least until the implemented specifications neglect both the possibility of non-linearities for the child's income and the donor's attitudes towards risk. Anyway, theoretical analyses on the topic are unanimous to recognize that one meets with difficulties to infer the motives for inter-vivos transfers inside the families (see the discussion in Laferrère and Wolff, 2002). Indeed, the other tests which have been proposed in the literature seem hardly more convincing. On the one hand, the unitary difference in transfer-income derivatives predicted by the beckerian model does not necessarily hold when the parent is altruistic. For example, the derivative restriction fails to hold in a dynamic setting when an altruistic parent uses observations on his child's current income to update his
expectations about future income (McGarry, 2000). On the other hand, attempts to estimate the sign of the correlation between services and financial transfers also convey few information about transfer motives. A positive pattern is of course expected under exchange, but it is also consistent with an altruistic behavior (see Cox, 1996). In particular, it might be that altruism is determined endogenously by attention and visits between the two generations as suggested by Becker and Murphy (1988). From an empirical viewpoint, there is also a problem with studies that seek to examine the correlation between upstream time-intensive services rather than relying on contact and visits which have no market substitutes (Altonji et alii, 2000). Indeed, the availability of substitutes strongly affects the theoretical properties of the money-services exchange model (Cox, 1987).

Finally, our theoretical results point out that caution should be used when drawing conclusions from previous empirical studies that have examined the sign of the effect of the recipient's income on the transfer amount. Nevertheless, we suggest that the ability to distinguish between the two explanations is perhaps not so severely limited. Empirical specifications should be refined in two directions, by including the parental degree of risk-aversion and by allowing for a non-linear relationship between the child's income and the gift value received from parents. These two key factors, which have been largely neglected to date in empirical analyses of family redistribution behavior, may probably explain why the exchange-transfer motive has received so little support. In developing countries, the introduction of non-linearities has proved especially fruitful in explaining family transfer behavior (Cox et alii, 1998, 2000). It is certainly premature to close the door to the exchange hypothesis in developed countries without further empirical evidence that control jointly for parental risk-aversion and non-linearities.
References


