The Unemployment Volatility Puzzle: The Role of Matching Costs Revisited

José Ignacio Silva and Manuel Toledo

Universitat de Girona

25. May 2009

Online at http://mpra.ub.uni-muenchen.de/17719/
MPRA Paper No. 17719, posted 8. October 2009 13:51 UTC
The Unemployment Volatility Puzzle: The Role of Matching Costs Revisited

José I. Silva∗ Manuel Toledo†

September 30, 2009

Abstract

Recently, Pissarides (2009) has argued that the standard search model with sunk fixed matching costs increases unemployment volatility without introducing an unrealistic wage response in new matches. We revise the role of matching costs and show that when these costs are not sunk and, therefore, can be partially passed on to new hired workers in the form of lower wages, the amplification mechanism of fixed matching costs is considerably reduced and wages in new hired positions become more sensitive to productivity shocks.

Keywords: unemployment volatility puzzle, search and matching, matching costs

JEL Classifications: E32 J32 J64

∗Universitat de Girona, Spain. Email: jose.silva@udg.edu
†Universidad Carlos III de Madrid. Email: matoledo@eco.uc3m.es
1 Introduction

The Mortensen-Pissarides (MP) search and matching model (Mortensen and Pissarides, 1994; Pissarides, 1985, 2000) studies the dynamics of unemployment in an environment where jobs are continuously created and destroyed. A sequence of papers by Costain and Reiter (2008), Hall (2005) and Shimer (2005) have questioned the model’s ability to match the observed cyclical fluctuations of the unemployment rate in the U.S. For example, Shimer shows that under a reasonable calibration strategy, the MP model predicts that the vacancy-unemployment ratio and the average labor productivity should have nearly the same volatility. In contrast, the standard deviation of the vacancy-unemployment ratio in the U.S. is almost 20 times as large as the standard deviation of average labor productivity. This large discrepancy between the volatility implied by the model and the data constitutes an empirical puzzle, known as the unemployment volatility puzzle.

Pissarides (2009) shows that introducing fixed matching costs into the model (e.g., training costs) can significatively increase the volatility of labor-market outcomes, such as tightness and the job finding rate. He points out that this result is obtained without inducing a counterfactually low volatility in the wages of new jobs. In his quantitative exercise, Pissarides only considers sunk fixed matching costs. That is, “they are sunk once the wage bargain is concluded and the worker takes up the position”. He shows that when these costs increase from zero to 40 percent of average labor productivity, the volatility of the vacancies-unemployment ratio (measured by its elasticity) increases almost twofold, and it matches the observed volatility in the U.S. labor market. He also argues that non-sunk fixed training costs play a similar role.

In this paper we evaluate the amplification mechanism of non-sunk fixed matching costs, and examine whether the cyclical volatility predicted by the model is substantially augmented. We show that when these costs are not sunk
and, therefore, can be partially passed on to workers through lower wages, the 
volatility of the vacancy-unemployment ratio is approximately an order of 
magnitude less responsive to variations in these costs. Thus, from a quantita-
tive standpoint, the contribution of fixed matching costs in explaining labor 
market volatility depends not only on the level, but also on what proportion 
of these costs is sunk. Moreover, we observe that non-sunk fixed matching 
costs may also introduce a significative change in the volatility of wages of 
new hired workers.

2 The model

Given that our model is essentially the same as Pissarides’ (2009), its present-
atation is reduced to a minimum. In this economy, there is a continuum of 
risk-neutral, infinitely-lived workers and firms which discounts future payoffs 
at a common rate $r$; capital markets are perfect; and time is continuous.

There is a time-consuming and costly process of matching workers and job 
vacancies, captured by a standard constant-returns-to-scale matching function 
$m(u, v) = m_o u^n v^{1-n}$, where $u$ denotes the unemployment rate, $v$ is the vacancy 
rate, and $\eta$ and $m_o$ are the function parameters. Unemployed workers find 
jobs at the rate $f(\theta) = m(u, v)/u$, and vacancies are filled at the rate $q(\theta) = 
m(u, v)/v$, where $\theta = v/u$ denotes labor market tightness. From the properties 
of the matching function, the higher the number of vacancies with respect to 
the number of unemployed workers, the easier it is to find a job, $f'(\theta) > 0$, 
and the more difficult it is to fill up vacancies, $q'(\theta) < 0$.

A job can be either filled or vacant. Before a position is filled, the firm has 
to open a job vacancy with a flow cost $c$. Firms have a linear technology with 
labor as the only production factor. Each filled job yields instantaneous profit 
equal to the difference between labor productivity $p$ and the wage. When the
worker arrives, the firm pays fixed costs $H$ which is sunk. Moreover, it pays non-sunk fixed costs $T$ right after both the firm and the worker agree to start a working relationship. A job remains “new” until a shock with arrival rate $\lambda$ hits the match and changes its status to a continuing job. In that case, the worker and the firm renegotiate wages. Notice that $T$ becomes sunk after the initial negotiation. Therefore, new and continuing jobs will have different wages $w^n$ and $w^c$, respectively. Thus, the value of vacancies $V$, the value of a new job $J^n$, and the value of a continuing job $J^c$ are represented by the following Bellman equations:

$$rV = -c + q(\theta)(J^n - H - T - V), \quad (1)$$

$$rJ^n = p - w^n + s(V - J^n) + \lambda(J^c - J^n), \quad (2)$$

$$rJ^c = p - w^c + s(V - J^c), \quad (3)$$

When finding a job, the unemployed worker first belongs to a new job. At rate $\lambda$, it becomes a continuing job. All employed workers separate from their firm at the constant rate $s$. Unemployed and employed workers’ Bellman equations are given by

$$rU = z + f(\theta)(W^n - U), \quad (4)$$

$$rW^n = w^n + s(U - W^n) + \lambda(W^c - W^n), \quad (5)$$

$$rW^c = w^c + s(U - W^c), \quad (6)$$

where $z$ represents the flow utility from leisure.

As is standard, we assume that there is free entry for vacancies. Therefore, in equilibrium:

$$V = 0. \quad (7)$$

We also assume that wages in new jobs are determined through bilateral Nash bargaining between the worker and the firm. The first-order conditions
for entrant employees yield the following equation:

\[(1 - \beta)(W^n - U) = \beta(J^n - T),\]  

(8)

where \(\beta \in (0, 1)\) denotes the workers’ bargaining power relative to firms’. Note that the Nash condition depends on matching costs \(T\) but not \(H\) because the former are not sunk to new jobs, and therefore they are explicitly considered in the wage negotiation with new entrants.

This sharing rule implies that \(J^n - T = (1 - \beta)S^n\), where \(S^n = J^n + W^n - U - T\) is the surplus of a new job (net of sunk cost \(H\)). Using all the value functions (1)-(6) and the zero-profit condition (7), we obtain the equilibrium job creation condition

\[\frac{(1 - \beta)(p - z) - \beta(c\theta + f(\theta)H)}{r + s} = \frac{c}{q(\theta)} + H + (1 - \beta)T.\]  

(9)

As Pissarides (2009) points out, this job creation condition is independent of the specific wage determination scheme for continuing jobs. If, in particular, we assume a Nash wage rule for continuing matches as well, we obtain the following equilibrium wages:

\[w^n = (1 - \beta)z + \beta(c\theta + p + f(\theta)H - (r + s + \lambda)T),\]  

(10)

\[w^c = (1 - \beta)z + \beta(c\theta + p + f(\theta)H).\]  

(11)

Since \(H\) are sunk, they increase the implicit bargaining power of all workers and, therefore, their wages. In contrast, firms can pass on part of the non-sunk matching costs \(T\) to new employees in the form of lower wages.

A steady-state equilibrium in this economy is a triplet of labor market tightness and wage rates \((\theta^*, w^{n*}, w^{n*})\) that solves equations (9), (10), and (11) for the steady-state productivity level \(p^*\).
3 Parameter values and elasticities

For comparative purposes, we use the same targets and parameter values as in Pissarides (2009), and calibrate the model at monthly frequency without fixed matching costs, $T = H = 0$ (benchmark). We calibrate the job conversion rate $\lambda$ by assuming that “new” jobs are converted to continuing jobs at the end of the training period. According to Barron Berger and Black (1997), a new hired worker becomes fully trained after 20.2 weeks on average. Thus, the average duration of a new job is 5.1 months, so $\lambda = 0.196$ (i.e., $1/\lambda = 5.1$). Notice that the value of $\lambda$ is irrelevant when $T = 0$. See Table 1 for all the parameter values of our benchmark calibration.

The quantitative exercise we carry out in this section is very simple. We increase either the sunk ($H$) or non-sunk ($T$) matching costs and adjust the free vacancy parameter $c$ in order to maintain the same steady-state value for the labor market tightness $\theta^*$ and, therefore, the equilibrium unemployment rate $u^* = \frac{s}{s + f(\theta^*)}$.\(^1\)

The central question in this paper is whether this extended MP matching model with fixed matching costs can explain the size of the business cycle fluctuations in labor-market tightness and unemployment given the separation rate. To explore this issue, we find the elasticities of the vacancy-unemployment ratio, $\varepsilon_\theta$, and wages in new jobs, $\varepsilon_{w^*}$, with respect to labor productivity $p$. Thus, from the job creation condition (9) and the wage equations (10), we obtain

$$\varepsilon_\theta = \frac{1}{\eta} \left[ \frac{(1 - \beta)p^*}{(1 - \beta)(p^* - z) + \beta \frac{1 - \eta}{\eta} c \theta^* - [r + s + \beta \frac{1 - 2\eta}{\eta} f(\theta^*)]H - (r + s)(1 - \beta)T} \right],$$

and

$$\varepsilon_{w^*} = \beta \left[ \frac{p^* + \varepsilon_\theta (c \theta^* + f(\theta^*)(1 - \eta)H)}{u^*} \right].$$

\(^1\)As robustness check of this experiment, later on we calibrate $c$ and $\beta$ differently.
Table 2 shows these elasticities for different values of $H$ and $T$. We find that the volatility of the vacancies-unemployment ratio $\theta$ is much higher when sunk fixed matching costs $H$ are increased. For example, the elasticity of the vacancies-unemployment ratio is multiplied almost by two (from 3.67 to 7.24) when these costs increase from 0 to 40 percent of the average labor productivity. In contrast, this elasticity increases only by 5.72 percent (from 3.67 to 3.88) for the same variation in the non-sunk matching costs $T$.

To understand this result, notice that there are two effects. There is a direct effect associated with the terms that depend on $H$ and $T$ in the denominator of (12). It is easy to see that if $\eta > 1/(r + s + 2)$, as in our parametrization, then $r + s + \beta \frac{1-2\eta}{\eta} f(\theta^*) > (r + s)(1 - \beta)$ and, consequently, an increase in $H$ has a larger positive impact on $\varepsilon_\theta$. Furthermore, we have an indirect effect through the recalibration of parameter $c$ as explained above. Note that an increase in $H$ causes $\theta^*$ to fall more compared to the impact of $T$. Therefore, in order to keep $\theta^*$ constant, $c$ has to fall more when $H$ increases. Clearly, $\varepsilon_\theta$ is decreasing in $c$. Thus, the indirect effect of a change in fixed matching costs on $\varepsilon_\theta$ through $c$ is larger for $H$. Provided that $\eta > 1/(r + s + 2)$, both effects are bigger in the case of a change in $H$, which explains why $\varepsilon_\theta$ increases more when we raise $H$.

The question now is to what extent each effect contributes to this result. To assess the size the direct effect we let $c$ at its benchmark value and adjust $\theta^*$ in order to satisfy equilibrium condition (9). The difference between both resulting elasticities (again, one recalibrating $c$ and the other letting $\theta$ adjust) can be interpreted as the magnitude of the indirect effect. After all, $\theta^*$ would inevitably change if we keep the remaining parameters constant when fixed costs change. In our alternative exercise, it turns out that the resulting elasticities are in most cases very close to the ones where we change $c$ (see last column of Table 2). For instance, for $H = 0.1$ or $T = 0.1$, we obtain $\varepsilon_\theta|_{c=.356}$
equal to 4.191 and 3.719, respectively. In this case, the real indirect effect is quite small, actually negative. This remains to be the case for other values of $T$. For $H \geq 0.2$, this indirect effect actually becomes positive but it is arguably small relative to the overall change in $\varepsilon_\theta$. Therefore, we can say that our results do not hinge on our particular way of recalibrating the model.

Notice that in Table 2 we obtain very different values of $c$. In particular, when we change $H$. One question one might want to ask is whether this calibrated values of $c$ are reasonable. We argue that lower values of $c$ are in line with the empirical evidence. Using information reported by Barron, Berger and Black (1997) that comes from the 1982 Employer Opportunity Pilot Project (EOPP), a cross-sectional firms-level survey containing detailed information on these labor turnover costs, and Dolfin (2006), Silva and Toledo (2009) shows that hiring costs represent 12.9% of the monthly wage of a new hired worker. In the benchmark calibration without fixed costs, hiring costs represents about 35% of the wage.

Table 3 presents the simulated results when the hiring costs parameter $c$ is calibrated so that it represents 12.9% of the monthly wage of a new hired worker. This exercise requires to adjust a different parameter to satisfy the same equilibrium unemployment rate. We choose the worker bargaining power $\beta$. The simulated elasticities are somewhat similar to the ones shown in Table 2. Thus, the conclusions we draw from this alternative experiment are fundamentally the same.

Now that we understand that sunk fixed costs are crucial to amplify the shocks, the next question should be focused on the size of sunk and non-sunk fixed costs from the data. Under the assumption that $H$ and $T$ only capture training costs, we find that this source of labor turnover costs is able to match the unemployment volatility if about 36% of them are sunk. We reach this figure as follows. First, we look at some empirical findings regarding on-
the-job training in the U.S. Barron et al. (1997), using the EOPP survey, find that 95% of new hired workers received some kind of training and spent, on average, 142 hours in training activities during the first quarter in the firm.\textsuperscript{2} When adding the contribution of incumbent workers and supervisors in training new employees, which is placed at 87.5 hours on average, the resulting cost amounts to 55% of the quarterly wage of a new hire.\textsuperscript{3} Thus, given our assumption that $H$ and $T$ only reflect training costs, $H + T = 0.55 \times 3 \times w^{n*}$. Then, we set $H$ and $T$ such that this last equation is satisfied as well as the model matches the observed $\theta$ elasticity of 7.56 in the U.S. reported by Pissarides (2009). As is shown in the last row of Table 3, we get $H = 0.524$ and $T = 0.921$ (i.e., $H/H + T = 0.363$), which implies $w^{n*} = 0.873$. However, under this scenario, wages in new matches are about 12.3% more sensitive to labor productivity shocks than in the data. This elasticity is somewhat above the near-proportionality between wages in new matches and labor productivity estimated in Haefke, Sonntag, and van Rens (2007) as well as in Pissarides (2009).

Now we ask whether this ratio of sunk to non-sunk training costs is consistent with the empirical evidence. This ratio can be approximated indirectly by looking at the wage response of new hired workers to training costs. When these costs are non-sunk ($T$), the worker pays the costs by accepting a lower wage during the training period as it can be shown in equation (10). In contrast, when these costs are sunk, the firm pays for the training process and the wage of the new hired worker is increased proportionally to $H$. Most of the empirical studies have either failed to find a significant negative relation-

\textsuperscript{2} Using a more recent survey, the 1992 Small Business Administration survey, they report a similar number of hours spent on on-the-job training during the first three months of employment (150 hours).

\textsuperscript{3} For more information, see Table 1 in Silva and Toledo (2009).
ship between training and starting wages or find a very small negative effect.  
Along this line, in order for the model to be consistent with this empirical findings, and predict a (nearly) neutral effect of training costs on the wage (10), it is necessary that \( f(\theta^*) H = (r + s + \lambda) T \). Thus, given \( f(\theta^*) = 0.594 \), \( r = 0.004 \), \( s = 0.036 \) and \( \lambda = 0.196 \), then \( \frac{H}{T} = \frac{(r + s + \lambda)}{f(\theta)} = 0.397 \). This ratio is somewhat similar to the ratio of 0.363 found above which, again, is required to reproduce the observed elasticity in labor market tightness when vacancy and training costs account for 13 and 55 percent of the quarterly wage of a new hire, respectively. Summarizing, from an empirical perspective, it seems that not only the magnitude of the fixed costs are important but also the size of sunk costs relative to non-sunk costs as an amplification mechanism of shocks.

4 Conclusion

In a recent paper, Pissarides (2009) argues that the presence of fixed matching costs can improve the volatility of unemployment maintaining the one-to-one response of wages to productivity fluctuations observed in the data. In his model, the matching costs are sunk, so new matched workers take actions directed to extract the quasi-rents created by them. We show that when these fixed matching costs can be partially passed on to workers through lower wages, the volatility of the vacancy-unemployment ratio is significantly reduced. Therefore, it is crucial not only the size of these fixed matching costs but also the proportion of this costs that are sunk. Moreover, we also observe that non-sunk fixed matching costs introduce changes in the elasticity of wages of new hired workers and may violate its proportionality with respect to labor productivity shocks.

---

\(^4\text{See for example, Barron et al., 1989; Barron et al., 1999a; Barron et al., 1999b; Holzer, 1990; Loewenstein and Spletzer, 1998; and Veum, 1999.}\)
References


Table 1: Calibrated parameter values for the U.S. economy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor productivity, $p^*$</td>
<td>1</td>
<td>Normalization</td>
</tr>
<tr>
<td>Exogenous separation probability, $s$</td>
<td>0.036</td>
<td>Data (Shimer, 2005)</td>
</tr>
<tr>
<td>Interest rate, $r$</td>
<td>0.004</td>
<td>Data</td>
</tr>
<tr>
<td>Employment opportunity cost, $z$</td>
<td>0.71</td>
<td>Hall &amp; Milgrom (2008)</td>
</tr>
<tr>
<td>Matching function elasticity, $\eta$</td>
<td>0.5</td>
<td>Petrongolo &amp; Pissarides (2001)</td>
</tr>
<tr>
<td>Matching function scale, $m_o$</td>
<td>0.7</td>
<td>To match the job finding prob.</td>
</tr>
<tr>
<td>Workers’ bargaining power, $\beta$</td>
<td>0.5</td>
<td>$\beta = \eta$ (efficiency)</td>
</tr>
<tr>
<td>Cost of vacancy, $c$</td>
<td>0.356</td>
<td>Solves (9)</td>
</tr>
<tr>
<td>Sunk fixed matching costs, $H$</td>
<td>0</td>
<td>Benchmark</td>
</tr>
<tr>
<td>Non sunk fixed matching costs, $T$</td>
<td>0</td>
<td>Benchmark</td>
</tr>
<tr>
<td>Job conversion rate, $\lambda$</td>
<td>0.196</td>
<td>To match the average duration of training</td>
</tr>
</tbody>
</table>

**Variable**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor market tightness, $\theta^*$</td>
<td>0.72</td>
<td>JOLTS</td>
</tr>
<tr>
<td>Job finding probability, $f(\theta^*)$</td>
<td>0.594</td>
<td>Shimer (2005)</td>
</tr>
</tbody>
</table>
Table 2: Short-run effects of sunk vs. non-sunk fixed matching costs

| $H$ | $T$ | $c$ | $\varepsilon_\theta$ | $\varepsilon_{w^n}$ | $\varepsilon_{\theta|c=0.356}$ |
|-----|-----|-----|----------------------|---------------------|-------------------------------|
| 0.00| 0   | 0.356| 3.666                | 0.985               | -                             |
| 0.10| 0   | 0.273| 4.183                | 0.989               | 4.191                         |
| 0.20| 0   | 0.191| 4.867                | 0.995               | 4.757                         |
| 0.30| 0   | 0.108| 5.821                | 1.000               | 5.369                         |
| 0.40| 0   | 0.026| 7.238                | 1.013               | 6.003                         |
| 0   | 0.00| 0.356| 3.666                | 0.985               | -                             |
| 0   | 0.10| 0.351| 3.717                | 0.999               | 3.719                         |
| 0   | 0.20| 0.346| 3.770                | 1.013               | 3.773                         |
| 0   | 0.30| 0.343| 3.824                | 1.023               | 3.829                         |
| 0   | 0.40| 0.336| 3.880                | 1.043               | 3.887                         |

Table 3: Short-run effects of sunk vs. non-sunk fixed matching costs with targeted hiring costs ($\frac{c}{w^n} = 0.129$)

<table>
<thead>
<tr>
<th>$H$</th>
<th>$T$</th>
<th>$\beta$</th>
<th>$\varepsilon_\theta$</th>
<th>$\varepsilon_{w^n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0</td>
<td>0.743</td>
<td>3.598</td>
<td>0.995</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.635</td>
<td>4.564</td>
<td>0.996</td>
</tr>
<tr>
<td>0.20</td>
<td>0</td>
<td>0.552</td>
<td>5.201</td>
<td>0.998</td>
</tr>
<tr>
<td>0.30</td>
<td>0</td>
<td>0.487</td>
<td>5.670</td>
<td>1.000</td>
</tr>
<tr>
<td>0.40</td>
<td>0</td>
<td>0.434</td>
<td>6.041</td>
<td>1.003</td>
</tr>
<tr>
<td>0</td>
<td>0.00</td>
<td>0.743</td>
<td>3.598</td>
<td>0.995</td>
</tr>
<tr>
<td>0</td>
<td>0.10</td>
<td>0.744</td>
<td>3.648</td>
<td>1.014</td>
</tr>
<tr>
<td>0</td>
<td>0.20</td>
<td>0.745</td>
<td>3.700</td>
<td>1.034</td>
</tr>
<tr>
<td>0</td>
<td>0.30</td>
<td>0.746</td>
<td>3.753</td>
<td>1.054</td>
</tr>
<tr>
<td>0</td>
<td>0.40</td>
<td>0.747</td>
<td>3.807</td>
<td>1.075</td>
</tr>
<tr>
<td>0.524</td>
<td>0.921</td>
<td>0.352</td>
<td>7.560</td>
<td>1.123</td>
</tr>
</tbody>
</table>