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Extreme Risk and Fat-tails Distribution Model: Empirical Analysis

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Abstract

This paper investigates estimation of extreme risk in a number of stock markets in the Gulf Cooperation Council (GCC) countries¹, Saudi, Kuwait, and United Arab Emirates, in addition to S& P 500 stock index, using the Generalized Pareto Distribution (GPD) model. The estimated tails parameter values for stock returns of Kuwait, Saudi, and Dubai, markets show the likelihood of significant extreme losses as well as significant extreme gains, compared to the case of more mature S&P 500 stock returns, which exhibit possibility of significant extreme losses with insignificant gain prospects.

Keywords: Stock markets, VaR, Generalized Pareto Distribution, Expected Shortfall, Extreme Risk.

1- Introduction:

Stock markets are subject to all types of financial risks that affect business institutions, such as credit risk, operational risk, and liquidity risk². Monitoring financial risk is important for financial institutions as well as capital market regulators, because it helps devising and restructuring risk management strategies. In recent years, Value at Risk (VaR) has gained momentum and became the standard measure of financial risk analysis. VaR is defined as the maximum possible loss to a portfolio or a security with a given probability over a certain time

¹ GCC countries include Saudi Arabia, Kuwait, United Arab Emirates, Qatar, Sultanate Oman, and Bahrain.

² Credit risk reflect the risk of incurring loss due to a default on the part of a debtor to meet his/her obligations at a designated time. Operational risk includes the risk of fraud and regulatory risk which result from errors made when instructing payments or settling transactions. Liquidity risk is caused by an unexpected large negative cash flow over a short period of time, causing failure of a firm to meet its liquidity needs arising in the short runs, and thereby subject the firm to the sale of some of its assets at a discount.

horizon. In other words, VaR, reflects how much an investor can lose with a probability, say p over a certain period of time.

Though, there has been voluminous research on VaR in financial markets, the task of estimating VaR still remains challenging. The major challenge lies in modeling the return distribution which is characterized as fat tailed and skewed in empirical research. While there are different approaches of modeling financial risk, in recent years VaR method attracted more attention in financial literature. The main purpose in this paper to estimate VaR values of GCC stock markets using Extreme Value Theory (EVT). There are two alternative approaches of modeling VaR based on EVT. Hill's (1975) estimation approach, employed by Embrechts, Kluppelberg and Mikosch, 1997; Danielsson and de Vries, 1997; and the high threshold approach that employ fat-tailed distributions models such as the Generalized Pareto Distribution (GPD).

Extreme risk analysis using GPD model gained momentum in the past two decades, after high swings of stock prices volatility and stock markets crashes in a number of developed and emerging markets. McNeil (1997, 1998) investigates estimation of extreme risks in financial time series, using EVT. Embrechts (1999, 2000a) show robustness of EVT in risk estimates. McNeil and Frey (2000) extend the analysis of extreme risk using heteroskedastic financial time series. Mullar et al (1998), and Pictet et al (1998) study extreme risk in foreign exchange markets using GARCH models. Gencay and Selcuk (2004) investigate the relative performance of VaR models using EVT, in a number of emerging markets after the Asian financial crisis of 1998. Giot and Laurent (2003) model VaR using a number of parametric univariate and multivariate models of ARCH class with skewed Student density.

The main strength of employing Generalized Pareto Distribution model is parameterizing the tails of asset price returns, and its flexibility, as GPD embeds a number of commonly used distributions, such as Cauchy, Student-t, Pareto, and Log-gamma. However, the use of GPD is not without cost. The assumption of independent, identical distribution (iid) of observations seem to be at odds with the characteristics of financial data, although generalization to include the case of dependent observations has been proposed by Embrechts et al (1997). In addition, the selection of the cut-off point that determines the number of observations at the tail of the GPD is not established yet in theory, as there is no well defined statistical criteria to choose the threshold point.

Then a question to be answered is: Why extreme risk analysis is needed for risk estimates in GCC stock markets? A number of factors that have been occurring in the past few years highlight the relevance and importance of extreme distribution analysis in GCC stock markets. As GCC capital markets become increasingly open to the outside world during the past five years, they become more susceptible to shocks in international markets, and thus become more volatile. Also the increasing cointegration among GCC markets, increased the spillover effect of shocks among these markets, which in turn contributed towards increasing volatility in GCC markets. It is also to be noted that, due to lack of sound regulatory and transparent policy framework, in some of GCC markets hyping, dumping, and rumors are the main driving forces behind the frequent changes in stock price. In addition, as GCC economies depend largely on crude oil revenue, extreme oil price changes in the past few years instigated extreme stock price volatility in these markets.

The remaining parts of the paper includes the following. Section two illustrates descriptive statistics. Section three present the methodology of the research. Section four includes estimation results. The final section concludes the study.

2: Descriptive statistics

Table (1) present several descriptive statistics on stock returns defined as $100[(p_t) - (p_{t-1})]/(p_{t-1})$, where p_t is the log of weekly price index³. The sample period of the research covers from Jan/1/2004 to June/7/2008. After excluding the weekend holiday periods the sample size constitutes 223 observations⁴. The weekly stock return series presented in the table display positive mean returns, but with negatively skewed distribution for all markets except the two UAE markets, which yield insignificant skewness coefficients. The maximum and minimum statistics show Saudi and Dubai stock returns exhibit highest range of variation among the group, as Saudi stock returns vary between (0.97 percent) as maximum gain and (-2.31 percent) maximum loss, and for Dubai market the maximum gain is (1.60 percent), and the maximum loss is (-1.37). The Ljung-Box Q-statistics of order 15 on the squared residual series reflect a high serial correlation in the second moments or variance. The excess kurtosis coefficient indicate significant leptokurtotic (fat-tailedness) for

³ The price series are daily prices indexes, but the aggregation on weekly basis is computed based on computation of the average of each five trading days of each week. The aggregation on weekly basis and log transformation of the price series aim to remove systematic day-of-the-week effect . The stock returns, as defined here, is not a total market return since dividends are not included. However, in empirical work on the S&P 500 index, by Gallant, Rossi, and Tauchen (1992) indicate results are invariant to inclusion or exclusion of dividends in stock returns.

⁴ This sample period excludes the sharp downfall of GCC stock markets due to the international financial crisis which impacted on them from Mid-September-2009.

Kuwait, Saudi and Abu-Dhabi markets⁵. However, descriptive plots in the appendix, indicate the QQ-plots of returns against the two thin tailed distributions of the Exponential and Normal distributions, show departure of the return quantiles, from the thin tailed Normal distribution for all markets⁶.

Table (1): Descriptive Statistics

	Kuwait	Saudi	Ab.Dhabi	Dubai	S&P 500
Mean	0.054	0.04	0.059	0.10	0.013
St.deviation	0.19	0.44	0.36	0.45	0.18
minimum	-0.74	-2.31	-1.01	-1.37	-0.50
maximum	0.56	0.97	1.70	1.60	0.39
Jarque-Bera (p-value)	18.14* (0.00)	469* (0.00)	66.4* (0.00)	4.75 (0.09)	9.41* (0.01)
skewness	-0.35	-1.63	0.08	0.065	-0.51
Ex.kurtosis	1.26	6.53	2.76	0.75	0.21
$Q^2(15)$ (p-value)	76.8* (0.00)	19.8 (0.17)	55.04* (0.00)	27.17* (0.02)	74.3* (0.00)

* Significant at 5% significance level.

Notes: $Q^2(15)$ is the Ljung-Box Q-statistic of order 15 on the squared series.

3. Methodology:

In financial literature, it is widely believed that returns of high frequency data, characterized with fatter tails compared to the Normal distribution

⁵These statistics calculated using the formulas $sk = \frac{m_3}{(m_2)^{3/2}}$, and $k = \frac{m_4}{(m_2)^2} - 3$,

where m_j stand for the jth moment around the mean. Under the null-hypothesis of

normality, the two statistics are normally distributed with standard errors, $\sigma_{sk} = \frac{\sqrt{6}}{N}$,

and $\sigma_k = \frac{\sqrt{24}}{N}$, where N is the sample size.

⁶ If the data is from an exponential distribution, the points on the graph would lie along positively sloped straight line. If there is a concave presence, it is an indication that of fat-tailed distribution, whereas a convex shape indicates short-tailed distribution.

returns. The fat tailedness phenomena of stock returns have popularized use of Extreme Value Theorem (EVT) which justify use of Generalized Pareto Distribution (GPD) to capture the influence of extreme returns on stock markets risk. The basic assumption underlying EVT is that the tails of every fat tailed distribution converge asymptotically (as the sample size increases) to the tails of Pareto distribution⁷. The tails estimate based on GPD can be expressed as:

$$F(x) = [1 - F(u)]G_{\alpha,\beta}(x - u) + F(u) \quad (1)$$

Where x is all points of returns above a threshold point, u , so that $x > u$, and $G_{\alpha,\beta}(x)$ is the two parameters GPD distribution function:

$$G_{\alpha,\beta}(x) = \begin{cases} 1 - (1 + \alpha x / \beta)^{-1/\alpha} & \alpha \neq 0 \\ 1 - e^{(-x/\beta)} & \alpha = 0 \end{cases} \quad (2)$$

Where α is the tail index, and $\beta > 0$ is the scale parameter. The case of $\alpha > 0$ is relevant for extreme risk modeling of stock returns. For $\alpha > 0$, $E(x^k)$ is infinite for $k \geq 1/\alpha$. For example, the GPD has an infinite variance when $\alpha = 0.5$, and for $\alpha = 0.25$, it has an infinite fourth moment. Usually, for high frequency asset returns, the estimates of α can be less than 0.5, implying that returns have finite variance (Dacorogna et al., 2001). In the case of $\alpha = 0$, the GPD corresponds to the Exponential distribution, and when $\alpha < 0$ it is known as Pareto II type distribution.

Following McNeil (1999), setting $F(u) = \frac{N-n}{N}$, where N is the total sample observations, n is the number of observations above the threshold level, and substituting $G_{\alpha,\beta}(x)$ from (2) into equation (1), for $\alpha > 0$, the tail estimate can be stated as:

⁷In this paper we apply the "Peaks – Over – Threshold" (POT) model following McNeil and Frey, (2000). The POT model is based on the "Pickands – Balkema-de Haan Theorem" which state that the distribution of observations in excess of certain high threshold can be approximated by a Generalized Pareto Distribution (GPD).

$$\hat{F}(x) = 1 - \frac{n}{N} \left(1 + \hat{\alpha} \frac{x - \mu}{\hat{\beta}}\right)^{-1/\hat{\alpha}} \quad (3)$$

for a given probability q , then VaR estimate is computed by inverting (3) to get:

$$\hat{VaR}_q = u + \frac{\hat{\beta}}{\hat{\alpha}} \left\{ \left(\frac{N}{n} (1 - q) \right)^{-\hat{\alpha}} - 1 \right\} \quad (4)$$

vigilant specification of the threshold value is considered necessary for reliable estimates of GPD parameters. There is a trade-off between setting a high threshold value that reduce the sample size to insufficient level to meet the asymptotic properties of ETV, and setting a low threshold level that end up with sizable sample size but with more of the non-extreme values in the estimation process.

Given that VaR represents a high quantile of the distribution of losses, i.e., 95th or 99th percentile, it stand for the maximum loss that is only exceeded on a small proportion of occasions.

Artzner, Delbaen, Eber and Heath (1997), have criticized VaR as a measure of risk on the basis that it fails to capture the potential losses that exceeds VaR value. They propose the use of Expected Shortfall (ES) as a measure of the expected size of a loss that exceeds VaR. It should be realized that ES is not an alternative estimation method for VaR, but only useful when we want to answer a question like: When VaR values underestimate risk, what is the size of the expected loss ? Thus, ES is a measure of the likelihood of high unusual loss. The Expected Shortfall can be estimated as:

$$ES_q = VaR_q + E(X - VaR_q) \mid x > VaR_q \quad (5)$$

Equation (5) can be simplified into (see McNeil (1998)):

$$\hat{ES}_q = \frac{V\hat{a}R}{1-\hat{\alpha}} + \frac{\hat{\beta} - \hat{\alpha} u}{1-\hat{\alpha}} \quad (6)$$

Following McNeil (1998), dynamic VaR and dynamic Expected shortfall values, can be determined as:

$$VaR_k^t = u_{t+k} + \sigma_{t+k} VaR(e_t)_q \quad (7)$$

$$ES_k^t = u_{t+k} + \sigma_{t+k} ES(e_t)_q \quad (8)$$

where $k = 1, 2, \dots$

Where $VaR(e_t)_q$ denotes the qth quantile of a noise variable e_t and $ES(e_t)_q$ is the corresponding expected shortfall. The simplest approach to estimating a dynamic VaR_k^t from equations (7) and (8) is to estimate conditional expected return u_{t+k} and conditional volatility σ_{t+k} , k-periods ahead using GARCH model and then apply GPD method on the residuals to estimate $VaR(e)_q$ and $ES(e)_q$ as shown in the previous section⁸.

4. Results

The maximum likelihood estimates of the lower and upper tail index values, α_L and α_U , with corresponding standard errors are presented in table (2). The estimated values of the right and left tails parameters of Kuwait, Saudi, and Dubai stock returns indicate extreme losses and extreme gains are both significant and of equal probability of occurrence. However, such equal chances of extreme losses and gains are not apparent in the more mature S&P 500 stock returns, which show higher likelihood of extreme losses compared to chances of extreme gains. It is

⁸ This approach may not yield reliable results if the conditional variance of GARCH model is not stationary and therefore it can produce inconsistent results. Another alternative approach of estimating multi-period VaR and ES can be specified as:

$$R(k) = \begin{cases} \sqrt[k]{k} R(1) \\ \sqrt{k} R(1) \end{cases} .$$

where $R(k)$ stand for VaR, or ES for k holding period. But this latest approach also suffer from the implicit assumption that risk is uniformly distributed across the holding multi-period, k.

also to be noted that the equal chances of extreme losses and extreme gains observed in the three GCC markets are unaffected by the holding period of the assets in these markets. Unlike the other GCC markets, Abu-Dhabi stock market show the likelihood of significant extreme losses compared to extreme gain prospects.

Table (3) present estimation results of VaR and ES based on a single day, one week, and five weeks holding periods⁹. Estimated risk values of VaR and ES indicate GCC markets in general exhibit higher risk than S&P 500, stock returns. Among the four GCC markets, Kuwait is the least subject to extreme losses, and Saudi is the most susceptible to extreme losses.

To check the accuracy of estimated VaR values an often employed criteria of back-testing, is comparing k-period ahead VaR estimates with the actual loss values computed from the data set. Given that VaR estimates indicate one-day holding period with 95% confidence level, for example, we expect only around 5 failure (actual losses exceed estimated VaR values) in every 100 trading days. If the number of failures (or violations) exceed significantly such a limit, then the model under estimates VaR values, and the opposite is true when the number of violations is significantly smaller than the expected level. In general, the ideal model yield estimates of failure rates close to stipulated significance level to pass the back-testing requirement. Table (4), report results of back-testing for VaR and ES estimates, and indicate that at 95% confidence level, the VaR and ES values reported in table (3) yield violation rate equal or less than the expected 5% tolerance level.

⁹ Infact, VaR and ES values provided in table (3) are in percentage form, and thus they can be termed as the relative VaR and ES numbers, as they refer to the percentage of a portfolio value that can be lost after h-holding period with a probability of 0.05, (or confidence level of 0.95). It is not straight forward to deduce the absolute values of VaR and ES from the log transformed relative values.

Table (2): Maximum likelihood estimates (MLE) of the parameters of lower and upper tails of the Generalized Pareto Distribution (GPD)

	1-day holding		1-week holding		5-weeks holding	
	$\hat{\alpha}_L$	$\hat{\alpha}_U$	$\hat{\alpha}_L$	$\hat{\alpha}_U$	$\hat{\alpha}_L$	$\hat{\alpha}_U$
Kuwait (std. error)	0.50* (0.09)	0.58* (0.08)	0.37* (0.21)	0.48* (0.19)	0.36* (0.21)	0.51* (0.14)
Saudi (std. error)	0.37* (0.36)	0.45* (0.13)	0.53* (0.18)	0.51* (0.17)	0.53* (0.18)	0.51* (0.17)
Dubai (std. error)	0.45* (0.20)	0.43* (0.19)	0.53* (0.14)	0.57* (0.12)	0.53* (0.14)	0.57* (0.12)
Abu-Dhabi (std. error)	0.44* (0.17)	0.44* (0.12)	0.52* (0.21)	0.36 (5.25)	0.52* (0.21)	0.36 (5.2)
S&P 500 (std. error)	0.44* (0.17)	0.30 (17.7)	0.45* (0.20)	0.38 (0.40)	0.45* (0.20)	0.38 (0.40)

*significant at 5% significance level.

Note: $\hat{\alpha}_L$ is lower tail, and $\hat{\alpha}_U$ is the upper tail index. GPD parameters estimated using shazam programming procedure. The MLE estimation procedure carried out using the nonlinear David-Fletcher-Powell algorithm.

Table (3): Risk estimates

	VaR (GPD)			Expected shortfall		
	1-day holding	1-week holding	5-weeks holding	1-day holding	1-week holding	5-weeks holding
Kuwait	0.078	0.22	0.11	0.15	0.23	0.12
Saudi	0.18	0.63	0.28	0.26	0.63	0.28
Dubai	0.16	0.42	0.29	0.24	0.42	0.29
Abu-Dhabi	0.12	0.36	0.24	0.18	0.37	0.25
S&P 500	0.09	0.17	0.17	0.13	0.17	0.17

Note: VaR and ES values estimated using nonlinear MLE method, using the nonlinear David-Fletcher-Powell algorithm. Shazam programming procedure employed to carry-out computations.

Table (4): Back-Testing Results

	VaR (GPD)		Expected shortfall	
	Daily series	Weekly series	Daily series	Weekly series
Kuwait	0.001	0.035	0.003	0.017
Saudi	0.005	0.026	0.004	0.022
Dubai	0.008	0.058	0.006	0.031
Abu-Dhabi	0.01	0.040	0.004	0.022
S&P 500	0.02	0.035	0.004	0.022

Note: Number in each cell indicate the percentage number of days and weeks, in which actual loss exceeds estimated VaR values, at a given significance level of 0.05.

5: Concluding remarks:

This paper investigates estimation of risk in the major GCC stock markets, Saudi, Kuwait, Abu-Dhabi, and Dubai, beside S&P 500 index, using the Generalized Pareto Distribution (GPD) model.

The estimated values of the right and left tails parameters of Kuwait, Saudi, and Dubai stock returns indicate extreme losses and extreme gains are both significant and of equal likelihood of occurrence. However, such equal chances of extreme losses and extreme gains are not apparent in the more mature S&P 500 stock returns, which reveal higher chances of extreme losses compared to chances of extreme gains. The paper also indicate, the symmetry of extreme losses and extreme gains observed in the three GCC markets is unaffected by the holding period of the assets in these markets. Unlike the other GCC markets, Abu-Dhabi stock market show the likelihood of significant extreme losses with insignificant extreme gain prospects.

Estimated risk values of VaR and ES indicate GCC markets in general exhibit higher risk than S&P 500, stock returns. Among the four GCC

markets, Kuwait is the least subject to extreme losses, and Saudi is the most susceptible to extreme losses.

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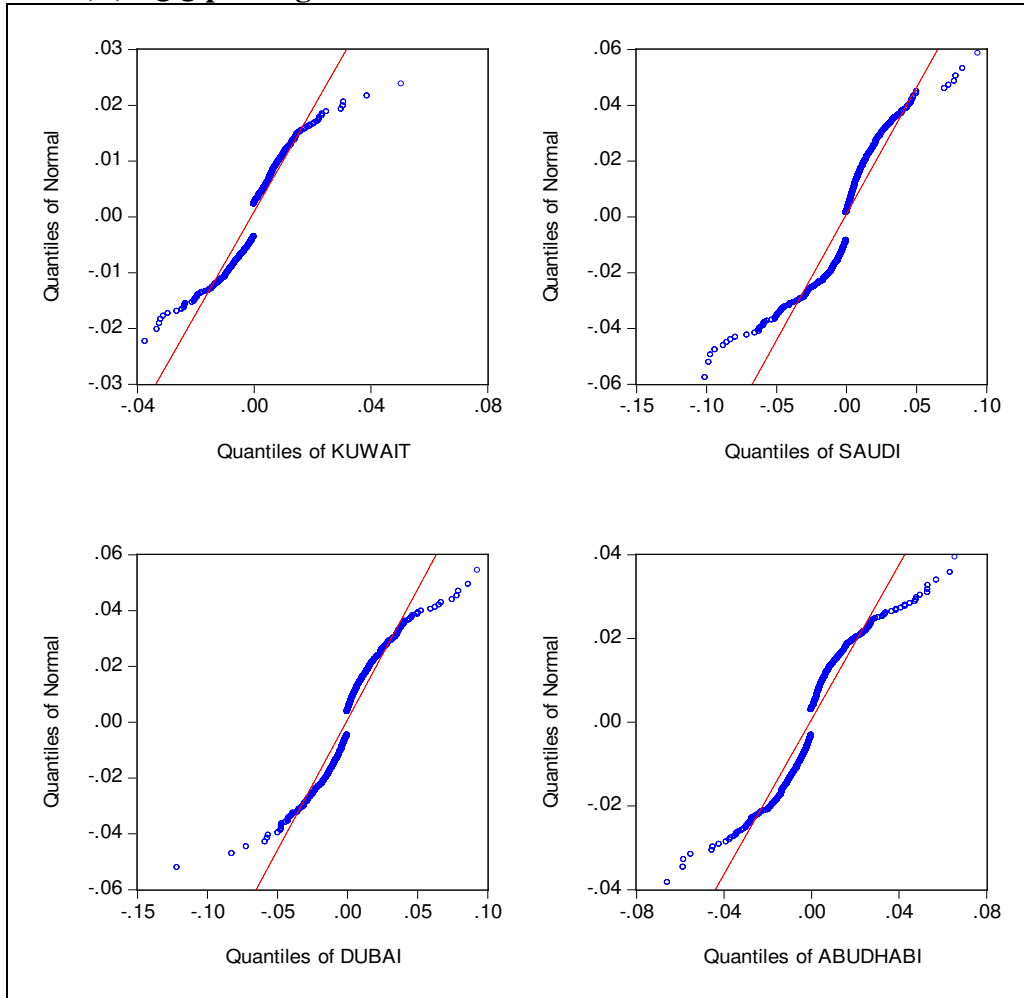
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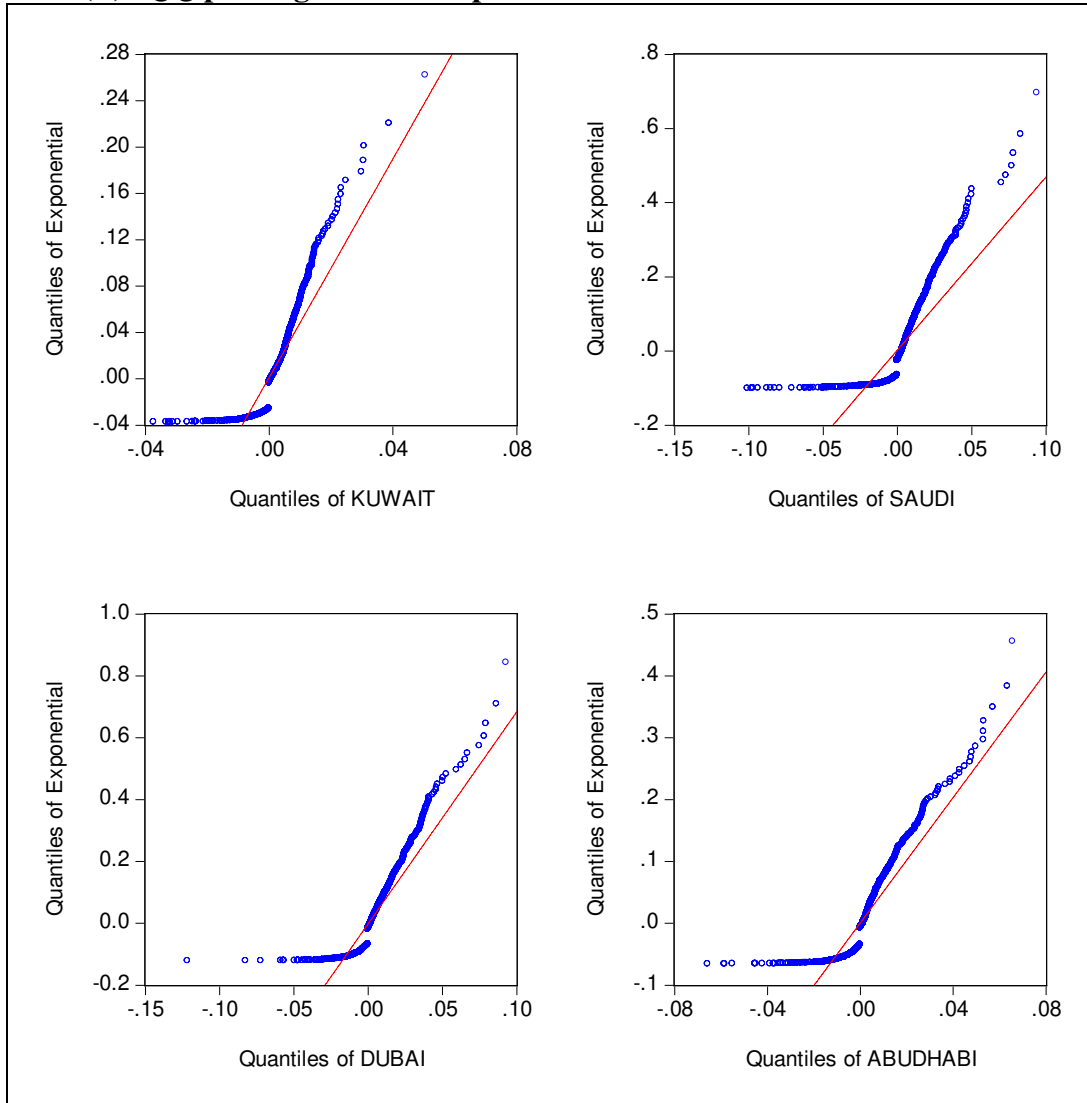
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Plots (A): QQ-plots against the Normal distribution



Plots (B): QQ-plots against the Exponential distribution



Plots (C): QQ-plots against the Normal and exponential distribution for S&P 500

