Effects of market sentiment in index option pricing: a study of CNX NIFTY index option

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Abstract

This paper provides evidence of the role of sentiments in pricing Indian CNX Nifty index call Option during the period from April 2002 to December 2008. It also shows that Black-Scholes option pricing model using the implied volatility of previous day is pricing the Index options much closer to the actual price compared to Modified Black-Scholes pricing model incorporating non-normal skewness and kurtosis suggested by Corrado & Sue [1996]. The market is pricing the call option higher than Black-Scholes price during bullish period compared to that of bearish period even though sentiments are incorporated in the underlying asset which in this case is the Nifty Index. The index call options are priced about 1.5 percent more than Black-Scholes price during Bullish period compared to that of Bearish period during the period of observation.

Keywords: Option Pricing, Black-Scholes option pricing model, Modified Black- Scholes by Corrado & Sue, Put call ratio, Sentiment indicators

1. Introduction

Black-Scholes model is the most widely used model for pricing options. According to this model option price is a function of spot price, strike price, volatility of underlying asset, risk free rate and the time to maturity. But sentiments plays a major role in markets and Black-Scholes pricing is independent of market sentiments. It is generally believed that, out of two call options with the same spot price, strike price, volatility, risk free rate and time to maturity, the one during bullish period will generally be priced higher compared to that of the other in a bearish market. The reasons stated are,

- In a bullish market, the market expects the spot price to go up much faster so that the expected pay-off (S_T-K) will be much higher compared to that of a bearish market. Similarly a put option will be priced higher in a bearish market.
- Consider a rational investor (market maker- with a limited risk-bearing capacity) who provides liquidity in option trading by risking his own capital and takes the opposite side of public orders. Now assume a market condition where bearish sentiment dominates and customers demand more put protection. Then in this scenario put option will be relatively more expensive because, as the market maker writes more options, he faces higher hedging costs and exposure to upward movement. Due to his risk aversion, he charges higher premium per contract as the number of contracts he writes increases. Consistent with this idea, an option becomes more expensive as its net buying pressure increases. Bollen and Whaley (2004)

This research was primarily carried out with the aim to study the effects of market sentiments in pricing CNX Nifty call option.

Though Black-Scholes pricing has been widely used to price options, Manvendra Tiwari & Rritu Saurabha [2007] in their paper suggested that the modified Black-Scholes pricing suggested by Corrado & Sue [1996] incorporating non-normal skewness and kurtosis (let us call it the CS model) is more accurate in predicting the CNX Nifty option price. So an initial study was carried out to compare Black-Scholes option pricing model using the implied volatility of previous day with CS option pricing model using implied volatility, skewness and kurtosis of previous day. We found that, for options traded from April 2002 to December 2008 Black-Scholes model was very efficient in pricing compared to CS model. So the further study was carried out using Black Scholes pricing with previous day implied volatility.

To find out the effects of Sentiments we carried out the following analysis

- Initially call options from April 2002 to December 2008 are selected based on criteria described in section 3
- Implied volatility in Black-Scholes formula for each day is found
- Using this implied volatility Black-Scholes call price for options traded on the next day is calculated
- Based on increase or decrease in put call ratio Call options are segregated in to calls during bearish and bullish period respectively
- The percentage price difference between the actual price and the Black-Scholes price (let us call it as percentage excess Price) is calculated for each option
- Frequency distribution of the percentage excess price is plotted and 2.5% of the outliers on both the sides of the distribution is removed for both bullish and bearish period calls
- Finally hypothesis testing is carried out to find whether there is any significant difference between the percentage excess price during bullish and bearish period.

The results concluded that there is a significant difference between the percentage excess price during bullish and bearish period and calls options during Bullish period are priced higher than Black-Scholes price compared to call options during bearish period.

2. Literature Review

2.1 Black Scholes Option Pricing

According to Black-Scholes option pricing model the price of a call option is given by

$$C_{BS} = S_o N(D_1) - \ Ke^{-rt} N(D_2)$$

Where, So is the current asset price
K is the Strike Price
r is the risk free rate of interest
σ is the standard deviation of the returns for the underlying asset
t is the time to expiry

$$D_1 = \frac{\ln\left(\frac{S_o}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$D_2 = D_1 - \sigma\sqrt{t}$$

N (d) = cumulative probability distribution function of standard normal distribution at d.

2.2 NIFTY returns follow Normal Distribution with Skewness and Kurtosis

According to Misra, Kannan and Misra [2006] daily returns of the NSE NIFTY have been found to follow normal distribution with some Skewness and Kurtosis. The result suggested that the volatility smile observed in the NSE NIFTY options can be explained in some measure by the observed Skewness and Kurtosis. So for pricing Nifty options Manvendra Tiwari & Rritu Saurabha [2007] used CS model.

2.3 CS Model to incorporate Skewness and Kurtosis

Corrado and Su [1996] developed a method to incorporate the effects of non-normal skewness and kurtosis of asset returns into an expanded Black-Scholes option pricing formula. According to them the modified option price formula is the sum of Black–Scholes option price plus two adjustment terms one for non-normal skewness and another for kurtosis.

$$C_{cs} = C_{BS} + \mu_3 * Q_3 + (\mu_4 - 3) * Q_4$$

Where, C_{CS} is the call price using CS model

C_{BS} is the Call price using Black –Scholes described in 2.1

$$Q_3 = \frac{1}{3!} S_o \sigma \sqrt{t}((2\sigma\sqrt{t} - D_1)N(D_1) + \sigma^2 t N(D_1))$$

$$Q_4 = \frac{1}{4!} S_o \sigma \sqrt{t}(D_1{}^2 - 1 - 3\sigma\sqrt{t}(D_1 - \sigma\sqrt{t})N(D_1) + \sigma^3 t^{3/2} N(D_1))$$

μ_3 is the measure of Skewness of underlying asset returns

μ_4 is the measure of Kurtosis of underlying asset returns

2.4 Sentiment Indicators

Bing Han[2006] and Yaw-huei Wang, Aneel Keswani and Stephen J Taylor [2005] suggest four sentiment indicators. They are

1. Put-call trading volume or put call ratio
2. ARMS index
3. Sharpe's (2002) valuation errors
4. Investors Intelligence's weekly surveys

Buraschi and Jiltsov (2006) estimate a heterogeneous-belief model using index option prices and option trading volume. They found that that option trading volume is an excellent proxy for the dispersion of beliefs. According to this we will use option trading volume as a proxy for market sentiments.

2.4.1 Put-call trading volume or put call ratio

The put-call trading volume or put call ratio is a measure of market sentiments which is equal to the trading volume of put options divided by the trading volume of call options. When the market is bearish, people buy more put options either to hedge their spot positions or to speculate bearishly. Therefore the trading volume of put options becomes large in relation to the trading volume of call options and the put call ratio goes up and vice versa. So an increase in put call ratio is an indication of bearish sentiments and a decrease is an indication of bullish sentiments.

3. Data Collection

The complete options data from April 2002 to December 2008 was downloaded from NSE website. The call price on any day is taken as the average of the opening and the closing price and the same is applied for spot price as well which in this case is the CNX NIFTY Index. Then the retrieved data is filtered with the following standard criteria followed in most of the research papers on options

- Options with less than 8 and more than 90 trading days to expiration was excluded as it may induce liquidity related biases. Options with small time to expiry will have relatively small time premium, hence the estimation of volatility is extremely sensitive to any possible measurement errors, particularly if option are not at-the-money.
- Options trading at a value below its intrinsic value will be excluded as their implied volatility is undefined.
- Options with moneyness between 0.90 and 1.10 are only considered. Moneyness is defined as the ratio of futures price ($S_0 * e^{rt}$) to strike price for call options. The moneyness criterion is applied because deep out-of-the money and in-the-money options tend to be thinly traded.
- Thinly traded options, less than 100 contracts on a given day are excluded from the study.

91 days T Bill interest rate for every month quoted per year compounded annually is converted into per year compounded continuously and used as the risk free interest rate.

4. Research

The entire research has been divided into two parts. In the first part Black-Scholes option price (using previous day implied volatility) and CS option price (using previous day implied volatility, skewness and kurtosis) is compared to find out which is pricing the option closer to the actual market price. In the next part the percentage excess price (as described in section 1) during Bullish and Bearish period is compared to find out whether there is any significant difference between them. If there is any significant difference then a series of t test (assuming samples with unequal variances) for various hypothesised mean difference between percentage excess price during Bearish and Bullish period is carried out till the null hypothesis is accepted. This is to find the average percentage excess price paid for sentiments in Indian Index option market.

4.1 Implied volatility and Black-Scholes option price

The filtered options data as mentioned in section 3.0 is taken. The Black-Scholes implied volatility for each day is found by minimizing the function

$$MIN_{ISD} \sum_{j=1}^{N} \left[C_{O,j} - C_{BS,j}(ISD)\right]^2$$

Where, $C_{O,j}$ – Observed Call Price for J^{th} observation

$C_{BS,j}$ – Call Price using Black Scholes for J^{th} observation

ISD – Implied Volatility

J = 1 to N, where N is the total no of observations made on that particular day

After finding the implied volatility, Black-Scholes price (theoretical or predicted) for each day is calculated using previous day implied volatility.

4.2 Implied volatility, skewness, kurtosis and CS option price

The volatility, skewness and kurtosis implied by CS model, for each day is found by minimizing the function

$$MIN_{ISD,ISK,IKT} \sum_{j=1}^{N} \left[C_{O,j} - \left(C_{BS,j}(ISD) + ISK * Q_3 + (IKT - 3) * Q_4\right)\right]^2$$

Where, $C_{O,j}$ – Observed Call Price for J^{th} observation

$C_{BS,j}$ – Call Price using Black Scholes for J^{th} observation

ISD – Implied volatility

ISK – Implied Skewness

IKT – Implied Kurtosis

J = 1 to N, where N is the total no of observations made on that particular day

After finding the implied volatility, skewness and kurtosis, CS option price (theoretical or predicted) for each day is calculated using previous day implied volatility, skewness and kurtosis.

4.3 Comparison of Black-Scholes and CS model

4.3.1 Hypothesis:

The total error of theoretical option price for various strike prices given by CS model is less than that of the theoretical option price given by Black Scholes model

4.3.2Testing Parameter:
The theoretical option prices given by CS model and Black-Scholes model is compared with the actual call price. The testing parameter used is the Error Sum of Squares (ESS) which is the sum of the square of the difference between the theoretical and the actual price. The ESS of both the models is then compared for statistically significant difference using paired t-test. Another t test (samples with unequal variances) is carried out to find which model is having less ESS among the two models.

4.3.3 Results:
The paired t- test of ESS of the two models resulted in a t statistic value of 9.740832 with 95% confidence level which overwhelmingly rejects the Null hypothesis that the errors in the prediction of option prices by the two methods are not significantly different.
The t-test assuming unequal variances resulted in a t statistic value of 6.618471 with 95% confidence level which again overwhelmingly rejects the null hypothesis that CS model ESS is less than the ESS of Black Scholes.

Table 1: Results of paired t-test and t-test of two sample assuming unequal variances

t-Test: Paired Two Sample for Means			t-Test: Two-Sample Assuming Unequal Variances		
	CS model	Black-Scholes		CS model	Black-Scholes
Mean	171.79	117.17	Mean	171.79	117.17
Variance	617356.97	207496.17	Variance	617356.97	207496.17
Hypothesized Mean Difference	0.00		Hypothesized Mean Difference	0.00	
t Stat	9.74		t Stat	6.62	
P(T<=t) one-tail	0.00		P(T<=t) one-tail	0.00	
t Critical one-tail	1.64		t Critical one-tail	1.64	
P(T<=t) two-tail	0.00		P(T<=t) two-tail	0.00	
t Critical two-tail	1.96		t Critical two-tail	1.96	

4.3.4 Conclusion:
From the above test it can be concluded that Black-Scholes pricing using implied volatility of previous day is pricing the call option much closer to the actual value compared to CS model, which is contradictory to the results of Manvendra Tiwari & Rritu Saurabha [2007]. From this we conclude that Black-Scholes option pricing model is pricing NIFTY call options better and this model will be used for further research.

4.4 Comparison of Call option during Bullish and Bearish period
Based on the increase or decrease in put call ratio, call options from April 2002 to December 2008 is divided into two groups namely Bearish and Bullish period call respectively. Then the percentage excess price for each option in the two groups is found.

$$\text{Percentage Excess Price} = \frac{(C_{OB} - C_{BS})}{C_{OB}} * 100$$

Where, C_{OB} – Observed call price on that day
C_{BS} – Black-Scholes price using previous day implied volatility

Then the outliers, that is 2.5% of the data on both sides of the distribution of Percentage Excess price for bullish and bearish period is removed and the comparison of percentage Excess Price between the two groups is carried with the remaining 95% of the call options.

4.4.1 Hypothesis

The percentage Excess Price of options with various strike prices in Bullish period is less than that of the price during the Bearish period.

H_o (Null Hypothesis): The percentage Excess Price during Bullish period is lesser that of the Percentage Excess Price during Bearish Period.

H_a (Alternate Hypothesis): The percentage Excess price during Bullish period is greater than the Percentage Excess Price during Bearish Period.

4.4.2 Testing Parameter

The testing parameter used is the Percentage Excess Price. The two Percentage excess Price one during bullish period and the other during bearish period is compared using t-test assuming unequal variances.

4.4.3 Results

The t-test assuming unequal variances resulted in a t statistic value of 7.725485 with 95% confidence level which again overwhelmingly rejects the null hypothesis that the percentage Excess Price during Bullish period is lesser than that of the Percentage Excess Price during Bearish Period. And it is found that null hypothesis is rejected with 95% confidence till a hypothesized mean difference of around 1.5 to 1.6.

Table 1: Results of t-test of two sample assuming unequal variances with different hypothesized mean differences

t-Test: Two-Sample Assuming Unequal Variances						
Hypothesized Mean Difference	0		1.5		1.6	
	Bull calls	*Bear calls*	*Bull calls*	*Bear calls*	*Bull calls*	*Bear calls*
Mean	2.075	0.069	2.075	0.069	2.075	0.069
Variance	255.332	242.194	255.332	242.194	255.332	242.194
t Stat	**7.72549**		**1.94879**		**1.56367**	
P(T<=t) one-tail	5.93E-15		2.57E-02		5.90E-02	
t Critical one-tail	1.64496		1.64496		1.64496	
P(T<=t) two-tail	1.19E-14		5.13E-02		1.18E-01	
t Critical two-tail	1.96013		1.96012		1.96012	

4.4.4 Conclusion:

- From the above test it can be concluded the percentage Excess Price during Bullish period is greater than that of the Percentage Excess Price during Bearish Period which means the actual call price is greater than Black-Scholes price during the bullish period compared to Bearish period
- With 95% confidence interval we can say the percentage excess price over the Black-Scholes price between the two periods is around 1.5 to 1.6 percent on an average.

5. Summary and Concluding Remarks

The results of the above research can be summarized as follows

- Black-Scholes is pricing the CNX NIFTY call option much closer to the actual price compared to CS model which is a modified Black-Scholes model to incorporate non normal skewness and kurtosis
- The index call options are priced at a much higher price than Black-Scholes price during bullish period compared to that of a call option during bearish period
- On and average the percentage excess price over Black-Scholes price paid for market sentiments when pricing CNX NIFTY call option is around 1.5 percent.
- There is a concrete evidence to prove that sentiments play an important role in option pricing

The above study can be extended by observing how the call price varies with the amount of increase/decrease in put call ratio, so that we can arrive at an empirical formula to include the sentiment indicator (put-call ratio) in the Black-Scholes model. This will act as an approximate model for the traders and help them to price the option with more accuracy.

Also the same study can be extended to find how other sentiment indicators affect the call price. A comparison of different sentiment indicators will give us a holistic picture so that we can choose the best indicator among them.

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