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ABSTRACT

The way monoline insurers estimate the FAS 157 credit value adjustments (CVA) on their ABS CDO insurance portfolios vastly overstates the benefits. We propose a simple method that is more accurate, especially when the counterparty default risk is high. The counterparty default recovery rate is also a critical input.

Counterparty credit risk is an important topic today with credit crunch affecting increasing number of financial firms. Firms holding super senior piece of ABS CDO would normally hedge their position with a monoline insurer in the form of credit default swap (CDS). Under the stress market conditions for monoline insurers, the effectiveness of the hedge has to be reevaluated based on the credit quality of the counterparty. Credit value adjustment (CVA) is precisely the measurement of counterparty credit risk on OTC derivative transactions. Under the new FAS 157 requirement, counterparties can also adjust their liabilities based on their own credit quality, which leads to the CVA benefits shown on monoline insurers' quarterly reports when their credit spreads widened a lot due to downgrading below AAA rating.

In the usual framework for credit value adjustment calculation, potential future exposure has to be simulated before the adjustment can be calculated (see reference [1]). With all the structural complexities of ABS CDOs, it is not realistic to carry out the simulation in order to calculate CVA. One simple way people would use is to adjust the discount rate curve by the counterparty CDS spread curve. This method is reasonable only when the counterparty CDS spread is small. It is the market convention to quote spread for IG names, but quote price for HY names, because using CDS spread adjusted discount curve is no longer accurate for HY names. The recent mind boggling results of monoline insurers' quarterly reports on CVA benefits have proven that this simple method has to be improved for stressed counterparties. Here we propose a simplified approach with reasonable accuracy even for stressed names, which will make the calculation easy to implement in the ABS CDO valuation frame work.

We make a few assumptions about the model as follows. First, most of the CDS negative basis trades on super senior CDOs were entered at very low spread, which will have a negative mark-to-market value for the protection seller in the foreseeable future. So the counterparty risk will be unilateral on the protection seller, or the monoline insurers. Second, we assume the credit quality of the counterparty is independent of the ABS CDO collateral performance. In reality, this assumption is not necessarily true, in the light of the fact that monoline insurers have been dragged down by the MTM losses

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due to their subprime exposures. Assuming all the stress has already been reflected in the CDS spread curves, this is still reasonable. Third, we assume recovery rate R is constant, thus independent of counterparty default or market conditions. This is currently the standard market practice, especially R is normally set to 40%.

Under these three assumptions, the standard formula for CVA (see reference [1]) is

$$CVA = (1 - R)\int_0^T EE(t)dPD(0,t) \tag{1}$$

where $EE(t)$ is the risk-neutral discounted expected exposure given by

$$EE(t) = E^Q[\frac{B_0}{B_t}E(t)] \tag{2}$$

where $E(t)$ is the net present value of future cash flows valued at time t and is independent of counterparty default, B_t is the future value of one unit of base currency invested in the money market account. $PD(0,t)$ is the risk-neutral probability of counterparty default between time 0 and t with $PD(0,0) = 0$. $PD(0,t)$ can be calibrated from the term structure of counterparty CDS spreads. The calibration is based on the assumption of 40% recovery rate for corporate bond.

Normally firms would have a cash flow model or a correlation model to value their ABS CDO insurance portfolio. We assume essentially $EE(t)$ is the sum of risk-neutral discounted expected future cash flows at or after time t under the CDS contract. So $-dEE(t)$ is the risk-neutral discounted value of the expected cash flow at time t. We have

$$\begin{aligned} CVA &= (1 - R)EE(t)PD(0,t)\Big|_0^T - (1 - R)\int_0^T PD(0,t)dEE(t) \\ &= (1 - R)\int_0^T PD(0,t)(-dEE(t)) \\ &= (1 - R)\int_0^T (1 - PS(0,t))(-dEE(t)) \\ &= (1 - R)EE(0) - (1 - R)\int_0^T PS(0,t)(-dEE(t)) \end{aligned} \tag{3}$$

where we choose T to be slightly after the maturity of the deal so that the exposure at T will always be zero, and $PS(0,t)$ is the survival probability between time 0 and time t with $PS(0,t) = 1 - PD(0,t)$. $EE(0)$ is the current value of the ABS CDO insurance portfolio without counterparty default risk.

An intuitive way to understand equation (3) is to separate the future cash flows into two parts, the loss and the recovery. The recovery part will be independent of counterparty default risk, while the loss part is contingent on counterparty default before the time of the future cash flows. One easy way to implement the CVA calculation is to multiply the discount factors by $(1-R)*PD(0,t)$ and rerun the CDS on ABS CDO valuation. To facilitate the calculation, it is required to separate discount curve from the forward Libor rate calculation.

Unlike the interest rate swaps, credit default swaps could terminate earlier when there is a credit event. In fact, there are several forms of CDS contracts: cash settlement, physical settlement and Pay-As-You-Go. The first two will terminate when credit event happens, but the latter has the same maturity as the underlying ABS/CDO bond. Monoline insurers normally have the Pay-As-You-Go settlement on their insurance contracts. The method described above is suitable for this case. Since the timing of credit event could be scenario dependent and the counterparty risk no longer exists after settlement of default loss, the above framework has to be expanded to handle the first two cases.

Assume for a scenario the credit event happens at time t_0, then the CVA for that scenario is calculated as follows

$$\begin{aligned} CVA_0 &= (1-R)\int_0^{t_0} \frac{B_0}{B_t} E(t)dPD(0,t) \\ &= (1-R)\frac{B_0}{B_{t_0}} E(t_0)PD(0,t_0) - (1-R)\int_0^{t_0} PD(0,t)d(\frac{B_0}{B_t}E(t)) \end{aligned} \tag{4}$$

Assume C(0,t) is the cumulative cash flow from time 0 to time t. Then

$$\frac{B_0}{B_t}E(t) = \frac{B_0}{B_t}\int_t^T \frac{B_t}{B_s} dC(0,s) = \int_t^T \frac{B_0}{B_s} dC(0,s) \tag{5}$$

and

$$\begin{aligned} CVA_0 &= (1-R)\int_{t_0}^{T} PD(0,t_0)\frac{B_0}{B_t} dC(0,t) + (1-R)\int_0^{t_0} PD(0,t)\frac{B_0}{B_t} dC(0,t) \\ &= (1-R)E(0) - (1-R)(\int_0^{t_0} PS(0,t)\frac{B_0}{B_t} dC(0,t) + \int_{t_0}^{T} PS(0,t_0)\frac{B_0}{B_t} dC(0,t)) \end{aligned} \tag{6}$$

This suggests that, for that scenario, the default probability curve should be set to constant after the credit event and then be multiplied to the discount curve. So we have to make the adjustment to discount curve based on the timing of credit event in each scenario. The final CVA value will be the average over all scenarios.

The method currently used by the monoline insurers is equivalent to setting the recovery rate to zero when calibrating the CDS default probability and calculating the CVA. The market standard recovery rate assumption is 40% when quoting the CDS spreads. When

CDS spreads are not high, CVA valuation is not very sensitive to the recovery rate assumption. However, when CDS spreads are wide, the recovery rate assumption is very important. A quick numerical evaluation on a sample portfolio of ABS CDO insurance portfolio reveals that, when counterparty CDS spreads are high (over 2000bps), which is typical for the monoline insurers, the difference between the two methods (recovery rate 0% vs 40%) is huge (mostly over 100%). Of course, actual results may vary depending on the underlying CDO portfolios. But it is safe to say that the monoline insurers' quarterly results are hugely overstated compared with using 40% recovery rate.

Before making any conclusion, we would like to review the assumptions underlying our method and see if they need revision to be more precise. The assumption of negative potential future exposures for the CDS insurance on super senior ABS CDO is reasonable, since the insurance premium is so low that market spread will not be lower in foreseeable future. The rationale is that if the premium on the deal is very low then the method can be used. Next, as mentioned before, the assumption of independence between counterparty default and the CDS valuation is questionable. For single name CDS, there is discussion in the literature on how to add correlation to CVA calculation (see reference [2]). In the case of ABS CDO affected by the mortgage crisis, we can assume the correlation is through a systematic factor, like the housing price appreciation (HPA). If the CDO valuation is driven by HPA scenarios imbedded in a cash flow model, this should not be difficult to implement. The third assumption is also debatable. Research on historical data suggests recovery rate is negatively correlated with default rate. Recent work has also demonstrated that stochastic recovery rate model might be helpful for calibrating the Gaussian copula model on the super senior tranches of CDX and iTraxx indices (see reference [3]). It is also arguable to use the same recovery rate for both corporate bonds and OTC derivative contracts.

In this note, we showed a more accurate method to calculate CVA for CDS on super senior ABS CDO, especially for stressed counterparties. The way is to adjust the discount curve by counterparty default probability multiplied by the loss given default. The key point is that, when counterparty credit spread is wide, CVA calculation is very sensitive to recovery rate assumption. Adjusting the discount curve by the counterparty credit spread curve is equivalent to assuming zero recovery rate, which could vastly overstate the CVA benefits for stressed counterparties. The fair value CVA benefits reported by the monoline insurers in their quarterly reports should be taken with a grain of salt.

References

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