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November 2006

Online at <http://mpa.ub.uni-muenchen.de/1795/>  
MPRA Paper No. 1795, posted 15. February 2007

# The Maastricht convergence criteria and optimal monetary policy for the EMU accession countries\*

Anna Lipińska<sup>†</sup>

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## Abstract

The EMU accession countries are obliged to fulfill the Maastricht convergence criteria prior to entering the EMU. What should be the optimal monetary policy satisfying these criteria? To answer this question, the paper proposes a DSGE model of a two-sector small open economy.

First, I derive the micro founded loss function that represents the objective function of the optimal monetary policy not constrained to satisfy the criteria. I find that the optimal monetary policy should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Second, I show how the loss function changes when the monetary policy is constrained to satisfy the Maastricht criteria. The loss function of such a constrained policy is characterized by additional elements penalizing fluctuations of the CPI inflation rate, the nominal interest rate and the nominal exchange rate around the new targets which are different from the steady state of the unconstrained optimal monetary policy.

Under the chosen parameterization, the optimal monetary policy violates two criteria: concerning the CPI inflation rate and the nominal interest rate. The constrained optimal policy is characterized by a deflationary bias. This results in targeting the CPI inflation rate and the nominal interest rate that are 0.7% lower (in annual terms) than the CPI inflation rate and the nominal interest rate in the countries taken as a reference. Such a policy leads to additional welfare costs amounting to 30% of the optimal monetary policy loss.

JEL Classification: F41, E52, E58, E61.

Keywords: Optimal monetary policy, Maastricht convergence criteria, EMU accession countries

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\*I would like to thank Kosuke Aoki for his excellent supervision and encouragement. I also thank Evi Pappa, Hugo Rodriguez, Gianluca Benigno and Michael Woodford for helpful comments and suggestions. Part of this work was done while I was visiting the European Central Bank. I would like to thank the members of the Monetary Policy Strategy Division for valuable discussions and hospitality.

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# 1 Introduction

On May 1, 2004 eight countries from Central and Eastern Europe (i.e. Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia) together with Cyprus and Malta entered the European Union (EU). Importantly, the Accession Treaty signed by all these countries includes an obligation to participate in the third stage of the economic and monetary union, i.e. an obligation to enter the European Monetary Union (EMU) in the near future. Moreover, in order to enter the EMU, these countries are required to satisfy the Maastricht convergence criteria (Treaty of Maastricht, Article 109j(1)). The criteria are designed to guarantee that prior to joining the European Monetary Union, countries attain a high degree of economic convergence not only in real but also in nominal terms. To this end, the Article 109j(1) of the Maastricht Treaty lays down the following criteria as a prerequisite for entering the EMU:<sup>1</sup>

- *the achievement of a high degree of price stability* which means that a Member State (of the EU) has a sustainable price performance and an average rate of inflation (the Consumer Price Index (CPI) inflation), observed over a period of one year before the examination, which does not exceed that of the three best performing Member States in terms of price stability by more than 1.5% points (the CPI inflation rate criterion);
- *the durability of the convergence ... reflected in the long term interest rate levels* which means that, over a period of one year before the examination, a Member State has an average nominal long-term interest rate that does not exceed that of the three best performing Member States in terms of price stability by more than 2% points (the nominal interest rate criterion);
- *the observance of the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System ( $\pm 15\%$  bound around the central parity), for at least two years, without devaluing against the currency of any other Member State (the nominal exchange rate criterion).*

By setting constraints on the monetary variables, these criteria affect the way monetary policy should be conducted in the EMU accession countries. Importantly, monetary policy plays a crucial role in the stabilization process of an economy exposed to shocks. The stochastic environment of the EMU accession countries has both domestic and external origins. As far as the domestic environment is concerned, a strong productivity growth has been observed in the EMU accession countries in the last years (see Figure 1 in Appendix A). Moreover, all these countries are small open economies which also makes them vulnerable to external shocks (see Table 1 in Appendix A).

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<sup>1</sup>Importantly, the Maastricht Treaty also imposes the criterion on the fiscal policy, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article 104c(6) of the Maastricht Treaty). However, in this paper, we focus on the monetary aspects of the Maastricht convergence criteria.

An obligation to fulfill the Maastricht convergence criteria by the EMU accession countries can restrict the stabilization role of the monetary policy. At the moment, many EMU accession countries do not satisfy some of the Maastricht convergence criteria. Estonia, Hungary, Latvia, Lithuania and Slovakia fail to fulfill the CPI inflation rate criterion (see Figures 2 and 3 in Appendix A). Moreover, Hungary also violates the nominal interest rate criterion (see Figures 4 and 5 in Appendix A). On the other hand, the nominal exchange rate fluctuations versus the euro for all EMU accession countries remain within the band set by the nominal exchange rate criterion (see Figure 6 in Appendix A).

Keeping this in mind, two natural questions arise. Are the Maastricht convergence criteria compatible with the optimal monetary policy in the EMU accession countries? What are the characteristics of the optimal policy that satisfies the Maastricht convergence criteria? The goal of this paper is to answer these questions. To this purpose, we develop a DSGE model of a small open economy with nominal rigidities exposed to both domestic and external shocks.

The production structure of the economy is composed of two sectors: a nontraded good sector and a traded good sector. There are several reasons why we decide to impose such a structure in our model. According to the literature, the existence of the nontraded sector helps us explain international business cycle fluctuations and especially real exchange rate movements (e.g. Benigno and Thoenisen (2003), Corsetti et al. (2003), Stockman and Tesar (1994)). Moreover, the empirical studies regarding the OECD countries find that a major part of the aggregate fluctuations rather have their source in sector-specific than country-wide shocks (e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998)). Finally, we want to match our model with the empirical literature on the EMU accession countries that emphasizes the role of sector productivity shocks in shaping inflation and real exchange rate patterns in these countries (e.g. Mihaljek and Klau (2004)).

In this framework, we characterize the optimal monetary policy from a timeless perspective (Woodford (2003)). We derive the micro founded loss function using the second-order approximation methodology developed by Rotemberg and Woodford (1997) and Benigno and Woodford (2005). We find that the optimal monetary policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. Since the Maastricht convergence criteria are not easily implementable in our model, we reformulate them using the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem of the nominal interest rate. This method enables us to verify whether a given criterion is satisfied by only computing first and second moments of a variable for which the criterion is set. We focus on the criteria imposed on the CPI inflation rate, the nominal interest rate and the nominal exchange rate as we do not explicitly model the fiscal policy. Moreover, we present how the loss function changes when the monetary policy is constrained by the Maastricht convergence criteria. Finally, we derive the optimal monetary policy that satisfies all Maastricht convergence criteria (constrained policy).

Under the chosen parameterization (which aims at reflecting the Czech Republic economy), the optimal monetary policy violates two Maastricht convergence criteria: concerning the CPI inflation

rate and the nominal interest rate. The optimal policy which satisfies these two criteria also guarantees satisfaction of the nominal exchange criterion. Both the stabilization component and the deterministic component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a smaller variability of the CPI inflation, the nominal interest rate and the nominal exchange rate than under optimal monetary policy. Moreover, it is also characterized by a deflationary bias which results in targeting a CPI inflation rate and a nominal interest rate that are 0.7% lower (in annual terms) than the CPI inflation rate and the nominal interest rate in the reference countries. As a result, the policy constrained by the Maastricht convergence criteria induces additional welfare costs which amount to 30% of the initial deadweight loss associated with the optimal monetary policy.

The literature has so far concentrated on two aspects of monetary policy in the EMU accession countries: the appropriate monetary regime in the light of the future accession to the EMU and also the ability of the alternative monetary regimes to comply with the Maastricht convergence criteria. The first stream of literature represented by, among others, Buiters and Grafe (2003), Coricelli (2002), calls for adopting the peg regime to the euro in these countries, as it enhances the credibility of the monetary policy and also strengthens the links with the EU and the EMU. Moreover, using a DSGE model with nominal rigidities and imperfect credibility, Ravenna (2005) finds that the gain from a credible adoption of the fixed regime towards the euro can outweigh the loss of resignation from the independent monetary policy. Nevertheless, Buiters and Grafe (2003) also claim that an adoption of the fixed regime can seriously endanger the fulfillment of the CPI inflation criterion and therefore call for a change in this criterion. Their reasoning is based on the empirical studies regarding sources of the CPI inflation and real exchange rate developments in the EMU accession countries. A majority of the studies<sup>2</sup> concentrate on the Balassa–Samuelson effect (Balassa (1964)), which predicts that countries experiencing a higher productivity growth in the traded sector are also characterized by a higher CPI inflation rate and real exchange rate appreciation. Others (e.g. Mihaljek and Klau (2004)) also highlight the role of productivity shocks in the nontraded sector in affecting the CPI inflation rate and the real exchange rate appreciation in the EMU accession countries.<sup>3</sup>

The second stream of the literature builds an analysis in the framework of open economy DSGE models. Devereux (2003) and Natalucci and Ravenna (2003) find that the monetary regime characterized by flexible inflation targeting with some weight on exchange rate stability succeeds in fulfilling the Maastricht criteria. Two other studies are also worth noting: Laxton and Pesenti (2003) and Ferreira (2006). The authors of the first paper study how different interest rate rules perform in stabilizing variability of inflation and output in a small open economy. The second paper focuses on calculation of the welfare loss that the EMU accession countries will face when they join the EMU. However,

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<sup>2</sup>We can list the following empirical studies that analyze CPI inflation and real exchange rate developments in the EMU accession countries: Cipriani (2001), de Broeck and Slok (2001), Egert et al. (2002), Fisher (2002), Halpern and Wyplosz (2001), Coricelli and Jazbec (2001), Arratibel et al. (2002), Mihaljek and Klau (2004).

<sup>3</sup>This study goes in line with a recent paper by Altissimo et al (2004) on the sources of inflation differentials in the euro area. The authors find that the nontraded sector (proxied as the service sector) contributes the most to price dispersion among member countries.

contrary to our study, it does not provide the micro founded welfare criterion.

In contrast to previous studies, our analysis is characterized by the normative approach. We construct the optimal monetary policy for a small open economy and contrast it with the optimal policy that is also restricted to satisfy the Maastricht convergence criteria. Therefore, our framework enables us to set guidelines on the way in which monetary policy should be conducted in the EMU accession countries.

The rest of the paper is organized as follows. The next section introduces the model and derives the small open economy dynamics. Section 3 describes derivation of the optimal monetary policy. Section 4 presents the way we reformulate the Maastricht convergence criteria in order to implement them in our framework. Section 5 is dedicated to the derivation of the optimal policy constrained by the Maastricht convergence criteria. Section 6 compares the optimal monetary policy with the optimal monetary policy constrained by the Maastricht convergence criteria under the chosen parameterization of the model. Section 7 concludes.

## 2 The model

Our modelling framework is based on a one-sector small open economy model of de Paoli (2004) where all goods, i.e. home and foreign ones, are tradable. We extend this model by incorporating two domestic sectors, i.e. a nontraded and a traded sector. Our model is also closely related to the studies of Devereux (2003) and Natalucci and Ravenna (2003). However, we relax an assumption present in their studies regarding perfect competition and homogeneity of goods in the traded sector, which enables us to discuss a role of terms of trade in the stabilization process of a small open economy. In that way our modelling framework is similar to a two-country model with two production sectors of Liu and Pappa (2005).

Following de Paoli (2004), we model a small open economy as the limiting case of a two-country problem, i.e. where the size of the small open economy is set to zero. In the general framework, the model represents two economies of unequal size: a small open home economy and a foreign large economy (which is proxied as the euro area). We consider two highly integrated economies where asset markets are complete. In each of the economies, there are two goods sectors: nontraded goods and traded goods. Moreover, we assume that labour is mobile between sectors in each country and immobile between countries. We assume the existence of home bias in consumption which, in turn, depends on the relative size of the economy and its degree of openness. This assumption enables us to consider a limiting case of the zero size of the home economy and concentrate on the small open economy.

Purchasing power parity (PPP) is violated for two reasons: existence of the nontraded sector and home bias in consumption. Furthermore, in order to study the role of monetary policy in this framework, we introduce monopolistic competition and price rigidities with staggered Calvo contracts in all goods sectors. However, we abstract from any monetary frictions by assuming cashless limiting

economies.<sup>4</sup> The stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks and foreign consumption shocks.

## 2.1 Households

The world economy consists of a continuum of agents of unit mass:  $[0, n)$  belonging to a small country (home) and  $[n, 1]$  belonging to the rest of the world, i.e. the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval  $[0, n)$  and foreign traded goods on the interval  $[n, 1]$ , respectively. The same applies to nontraded goods. In order to simplify the exposition of the model, we explain in detail only the structure and dynamics of the domestic economy. Thus, from now on, we assume the size of the domestic economy to be zero, i.e.  $n \rightarrow 0$ .

Households are assumed to live infinitely and behave according to the permanent income hypothesis. They can choose between three types of goods: nontraded, domestic traded and foreign traded goods.  $C_t^i$  represents consumption at period  $t$  of a consumer  $i$  and  $L_t^i$  constitutes his labour supply. Each agent  $i$  maximizes the following utility function:<sup>5</sup>

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} [U(C_t^i, B_t) - V(L_t^i)] \right\}, \quad (1)$$

where  $E_{t_0}$  denotes the expectation conditional on the information set at date  $t_0$ ,  $\beta$  is the intertemporal discount factor and  $0 < \beta < 1$ ,  $U(\cdot)$  stands for flows of utility from consumption and  $V(\cdot)$  represents flows of disutility from supplying labour.<sup>6</sup>  $C$  is a composite consumption index. We define consumers' preferences over the composite consumption index  $C_t$  of traded goods ( $C_{T,t}$ ) (domestically produced and foreign ones) and nontraded goods ( $C_{N,t}$ ):

$$C_t \equiv \left[ \mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (2)$$

where  $\phi > 0$  is the elasticity of substitution between traded and nontraded goods and  $\mu \in [0, 1]$  is the share of the nontraded goods in overall consumption. Traded good consumption is a composite of the domestically produced traded goods ( $C_H$ ) and foreign produced traded goods ( $C_F$ ):

$$C_{T,t} \equiv \left[ (1-\lambda)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

<sup>4</sup>See Woodford (2003).

<sup>5</sup>In general, we assume  $U$  to be twice differentiable, increasing and concave in  $C_t$  and  $V$  to be twice differentiable, increasing and convex in  $L_t$ .

<sup>6</sup>We assume specific functional forms of consumption utility  $U(C_t^i)$ , and disutility from labour  $V(L_t^i)$ :  $U(C_t^i) \equiv \frac{(C_t^i)^{1-\rho} B_t^\rho}{1-\rho}$ ,  $V(L_t^i) \equiv \varphi_l \frac{(L_t^i)^{1+\eta}}{1+\eta}$  with  $\rho$  ( $\rho > 0$ ), the inverse of the intertemporal elasticity of substitution in consumption and  $\eta$  ( $\eta \geq 0$ ), the inverse of labour supply elasticity and  $B_t$ , preference shock.

where  $\theta > 0$  is the elasticity of substitution between home traded and foreign traded goods, and  $\lambda$  is the degree of openness of the small open economy ( $\lambda \in [0, 1]$ ).<sup>7</sup> Finally,  $C_j$  (where  $j = N, H, F$ ) are consumption sub-indices of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \quad (4)$$

where  $\sigma > 1$  represents elasticity of substitution between differentiated goods in each of the sectors. Based on the above presented preferences, we derive consumption-based price indices expressed in the units of currency of the domestic country:

$$P_t \equiv \left[ \mu P_{N,t}^{1-\phi} + (1-\mu) P_{T,t}^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (5)$$

$$P_{T,t} \equiv \left[ \nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \quad (6)$$

with

$$P_{j,t} \equiv \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n p_t(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}. \quad (7)$$

Although we assume the law of one price in the traded sector (i.e.  $p(h) = Sp^*(h)$  and  $p(f) = Sp^*(f)$  where  $S$  is the nominal exchange rate), both the existence of the nontraded goods and the assumed home bias cause deviations from purchasing power parity, i.e.  $P \neq SP^*$ . The real exchange rate can be defined in the following manner:  $RS \equiv \frac{SP^*}{P}$ . Moreover, we define the international terms of trade as  $T \equiv \frac{P_F}{P_H}$  and the ratio of nontraded to traded goods' prices (domestic terms of trade) as  $T^d \equiv \frac{P_N}{P_T}$ .

From consumer preferences, we can derive total demand for the generic goods –  $n$  (home nontraded ones),  $h$  (home traded ones),  $f$  (foreign traded ones):

$$y^d(n) = \left[ \frac{p(n)}{P_N} \right]^{-\sigma} \left[ \frac{P_N}{P} \right]^{-\phi} \mu C, \quad (8)$$

$$y^d(h) = \left[ \frac{p(h)}{P_H} \right]^{-\sigma} \left[ \frac{P_H}{P_T} \right]^{-\theta} (1-\lambda) C_T + \left[ \frac{p^*(h)}{P_H^*} \right]^{-\sigma} \left( \frac{P_H^*}{P_T^*} \right)^{-\theta} \lambda C_T^*, \quad (9)$$

$$y^d(f) = \left[ \frac{p^*(f)}{P_F^*} \right]^{-\sigma} \left[ \frac{P_F^*}{P_T^*} \right]^{-\theta} C_T^* \quad (10)$$

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<sup>7</sup>Following de Paoli (2004) and Sutherland (2002), we assume home bias ( $\nu$ ) of the domestic households to be a function of the relative size of the home economy with respect to the foreign one ( $n$ ) and its degree of openness ( $\lambda$ ) such that  $(1-\nu) = (1-n)\lambda$  where  $\lambda \in [0, 1]$ . Importantly, the higher is the degree of openness, the smaller is the degree of home bias. Since  $n \rightarrow 0$ , we obtain that  $\nu = 1 - \lambda$ .



where variables with an asterisk represent the foreign equivalents of the domestic variables. Importantly, since the domestic economy is a small open economy, demand for foreign traded goods does not depend on domestic demand. However, at the same time, demand for domestic traded goods depends on foreign demand.

Households get disutility from supplying labour to all firms present in each country. Each individual supplies labour to both sectors, i.e. the traded and the nontraded sector:

$$L_t^i = L_t^{i,H} + L_t^{i,N}. \quad (11)$$

We assume that consumers have access to a complete set of securities-contingent claims traded internationally. Each household faces the following budget constraint:

$$P_t C_t^i + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + TR_t^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \frac{\int_0^n \Pi_{N,t}^i di}{n} + \frac{\int_0^n \Pi_{H,t}^i di}{n}, \quad (12)$$

where at date  $t$ ,  $D_{t+1}$  is nominal payoff of the portfolio held at the end of period ( $t$ ),  $Q_{t,t+1}$  is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household,  $\Pi_{H,t}$  and  $\Pi_{N,t}$  are nominal profits from the domestic firms and  $TR_t^i$  are nominal lump-sum transfers from the domestic government to household  $i$ . Moreover, consumers face no Ponzi game restriction.

The short-term interest rate ( $R_t$ ) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs in the next period:<sup>8</sup>

$$R_t^{-1} = E_t\{Q_{t,t+1}\}. \quad (13)$$

The maximization problem of any household consists of maximizing the discounted stream of utility (1) subject to the budget constraint (12) in order to determine the optimal path of the consumption index, the labour index and contingent claims at all times. The solution to the household decision problem gives a set of first-order conditions.<sup>9</sup> Optimization of the portfolio holdings leads to the following Euler equations for the domestic economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\}. \quad (14)$$

There is a perfect sharing in this setting, meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times.<sup>10</sup> Subsequently, appropriately choosing the distribution of initial wealth, we obtain the risk sharing condition:

<sup>8</sup>Following the literature, we assume one period to be one quarter.

<sup>9</sup>We here suppress subscript  $i$  as we assume that in equilibrium, all agents are identical. Therefore, we represent optimality conditions for a representative agent.

<sup>10</sup>We have to point out here that although the assumption of complete markets conveniently simplifies the model, it neglects a possibility of wealth effects in response to different shocks (Benigno (2001)).

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*, B_t^*)} = v \frac{P_t}{S_t P_t^*} = v R S_t^{-1}, \quad (15)$$

where  $v > 0$  and depends on the initial wealth distribution. The risk sharing condition implies that the real exchange rate is equal to the marginal rate of substitution between domestic and foreign consumption.

The optimality condition for labour supply in the domestic economy is the following:

$$\frac{W_t^k}{P_t} = \frac{V_L(L_t)}{U_C(C_t, B_t)}, \quad (16)$$

where  $W^k$  is the nominal wage of the representative consumer in sector  $k$  ( $k = H, N$ ).<sup>11</sup> So the real wage is equal to the marginal rate of substitution between labour and consumption.

## 2.2 Firms

All firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. The production function is linear in labour which is the only input. Consequently, its functional form for firm  $i$  in sector  $k$  ( $k = N, H$ ) is the following:

$$Y_{k,t}(i) = A_t^k L_t^k(i). \quad (17)$$

Price is set according to the Calvo (1983) pricing scheme. In each period, a fraction of firms  $(1 - \alpha_k)$  decides its price, thus maximizing the future expected profits. The maximization problem of any firm in sector  $k$  at time  $t_0$  is given by:

$$\begin{aligned} \max_{P_{k,t_0}(i)} E_{t_0} \sum_{t=t_0}^{\infty} (\alpha_k)^s Q_{t_0,t} [(1 - \tau_k) P_{k,t_0}(i) - MC_t^k(i)] Y_{k,t_0:t}^d(i) \\ \text{subject to } Y_{k,t_0:t}^d(i) = \left( \frac{P_{k,t_0}(i)}{P_{k,t}} \right)^{-\sigma} Y_{k,t}, \end{aligned} \quad (18)$$

where  $Y_{k,t_0:t}^d(i)$  is demand for the individual good in sector  $k$  produced by producer  $i$  at time  $t$  conditional on keeping the price  $P_{k,t_0}(i)$  fixed at the level chosen at time  $t_0$ ,  $MC_t^k = \frac{W_t^k(i)}{A_t^k}$  is the nominal marginal cost in sector  $k$  at time  $t$ , and  $\tau_k$  are revenue taxes in sector  $k$ .

Given this setup, the price index in sector  $k$  evolves according to the following law of motion:

$$(P_{k,t})^{1-\sigma} = \alpha_k (P_{k,t-1})^{1-\sigma} + \alpha_k (P_{k,t_0}(i))^{1-\sigma}. \quad (19)$$

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<sup>11</sup>Notice that wages are equalized between sectors inside each of the economies, due to perfect labour mobility and perfect competition in the labour market.

## 2.3 Fiscal and monetary policies

The government in the domestic economy is occupied with collecting revenue taxes from firms that are later redistributed to households in the form of lump-sum transfers in such a way that each period, there is a balanced budget:

$$\int_0^n (\tau_N P_{N,t}(i) Y_{N,t}(i) + \tau_H P_{H,t}(i) Y_{H,t}(i)) di = \int_0^n TR_t^j dj. \quad (20)$$

A role for the monetary policy arises due to existing nominal and real rigidities in the economy: price stickiness (together with monopolistic competition), home bias and the nontraded good sector, which lead to deviations from PPP. The system is therefore closed by defining appropriate monetary rule for the domestic economy.

## 2.4 A loglinearized version of the model

This section presents a system of the equilibrium conditions for the small open economy in the loglinear form, which is derived through the first-order approximation around the deterministic steady state with zero inflation defined in Appendix B. Here, we characterize the dynamic features of this model where the variables with a hat stand for the log deviations from the steady state. Additionally, the variables with an asterisk represent the foreign equivalents of the domestic variables.

The supply-side of the economy is given by two Phillips curves, one for the nontraded and one for the domestic traded sector, respectively, which are derived from (18):

$$\hat{\pi}_{N,t} = k_N(\rho \hat{C}_t + \eta \hat{L}_t - \hat{A}_{N,t} - \rho \hat{B}_t - \hat{p}_{N,t}) + \beta \hat{\pi}_{N,t+1}, \quad (21)$$

$$\hat{\pi}_{H,t} = k_H(\rho \hat{C}_t + \eta \hat{L}_t - \hat{A}_{H,t} - \rho \hat{B}_t - \hat{p}_{H,t}) + \beta \hat{\pi}_{H,t+1} \quad (22)$$

where  $\hat{\pi}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_{N,t-1}})$ ;  $\hat{\pi}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_{H,t-1}})$ ;  $k_N \equiv \frac{(1-\alpha_N)(1-\alpha_N\beta)}{\alpha_N}$ ;  $k_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H}$  and aggregate labour supply ( $\hat{L}_t$ ) is defined through the labour market clearing condition ((11), (17)):

$$\hat{L}_t = \tilde{d}_{Y_N}(\hat{Y}_{N,t} - \hat{A}_{N,t}) + \tilde{d}_{Y_H}(\hat{Y}_{H,t} - \hat{A}_{H,t}), \quad (23)$$

where  $\tilde{d}_{Y_N} \equiv \frac{\bar{Y}_N}{\bar{Y}_N + \bar{Y}_H}$ ,  $\tilde{d}_{Y_H} \equiv \frac{\bar{Y}_H}{\bar{Y}_N + \bar{Y}_H}$  are ratios evaluated in the steady state (see Appendix B).

It is worth underlining that inflation dynamics in both domestic sectors do not only depend on the real marginal costs in a given sector, but also on the relative prices of goods. In particular, a higher relative price of goods in one sector in relation to other goods induces a substitution away effect and leads to deflationary pressures in this sector.

The demand side of the small open economy is represented by the market clearing conditions in both nontraded and domestic traded sectors ((8), (9)):

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi \widehat{p}_{N,t}, \quad (24)$$

$$\begin{aligned} \widehat{Y}_{H,t} = & d_{CH} \widehat{C}_t - \theta \widehat{p}_{H,t} + b(\phi - \theta) d_{CH} \widehat{T}_t^d + (1 - d_{CH}) \theta \widehat{R}S_t + \\ & + (1 - d_{CH}) \widehat{C}_t^* + b^*(\phi - \theta)(1 - d_{CH}) \widehat{T}_t^{d*} \end{aligned} \quad (25)$$

where  $d_{CH} \equiv (1 - \lambda)(1 - \mu) \frac{\bar{C}}{\bar{Y}_H} \bar{p}_H^{-\theta} \bar{p}_T^{-\theta - \phi}$ ,  $b \equiv \mu(\bar{p}_N)^{1 - \phi}$ ,  $b^* \equiv \mu^*(\bar{p}_N^*)^{1 - \phi}$  are ratios evaluated in the steady state (see Appendix B). Additionally, we define aggregate output as the sum of sector outputs:

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}), \quad (26)$$

where  $d_{YN} \equiv \frac{\bar{p}_N \bar{Y}_N}{\bar{Y}}$  and  $d_{YH} \equiv \frac{\bar{p}_H \bar{Y}_H}{\bar{Y}}$  are ratios evaluated in the steady state (see Appendix B).

The complete asset market assumption (15) gives us the following risk sharing condition:

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho} \widehat{R}S_t + \widehat{C}_t^* - \widehat{B}_t^*. \quad (27)$$

From the definition of price indices ((5), (6)), we obtain the following relations between relative prices, terms of trade, domestic terms of trade and real exchange rate:

$$(a - 1) \widehat{p}_{H,t} = b \widehat{T}_t^d + a \widehat{R}S_t - b^* a \widehat{T}_t^{d*}, \quad (28)$$

$$\widehat{p}_{N,t} = (1 - b) \widehat{T}_t^d, \quad (29)$$

$$\widehat{p}_{H,t} = -b \widehat{T}_t^d - a \widehat{T}_t^{d*}, \quad (30)$$

where  $a \equiv \lambda \left( \frac{\bar{R}S \bar{p}_F^*}{\bar{p}_T} \right)^{1 - \theta}$  is the ratio evaluated in the steady state (see Appendix B). We also derive the laws of motion for the international terms of trade and the domestic terms of trade from their definitions:

$$\widehat{T}_t = \widehat{\pi}_{F,t} - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \quad (31)$$

$$\widehat{T}_t^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}_{t-1}^d, \quad (32)$$

where  $\widehat{\pi}_{T,t} = (1 - a) \widehat{\pi}_{H,t} + a \widehat{\pi}_{F,t}$  and  $\widehat{\pi}_{F,t} = \widehat{\pi}_{F,t}^* + (\widehat{S}_t - \widehat{S}_{t-1})$  with  $\widehat{\pi}_{T,t} \equiv \ln\left(\frac{P_{T,t}}{P_{T,t-1}}\right)$ ,  $\widehat{\pi}_{F,t} \equiv \ln\left(\frac{P_{F,t}}{P_{F,t-1}}\right)$ ,  $\widehat{\pi}_{F,t}^* \equiv \ln\left(\frac{P_{F,t}^*}{P_{F,t-1}^*}\right)$ .

Finally, we present equations defining the Maastricht variables: the CPI inflation rate ( $\widehat{\pi}_t$ ), the

nominal interest rate ( $\widehat{R}_t$ ) and the nominal exchange rate ( $\widehat{S}_t$ ). First, the nominal interest rate can be derived from the loglinearized version of the Euler condition (14):

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \quad (33)$$

where  $\widehat{\pi}_t \equiv \ln(\frac{P_t}{P_{t-1}})$ . CPI aggregate inflation is a weighted sum of the sector inflation rates:

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)\widehat{\pi}_{F,t}^* + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \quad (34)$$

Notice that CPI aggregate inflation does not only depend on the domestic sector inflation rates, but also on the foreign traded inflation rate and changes in the nominal exchange rate. For example, a nominal exchange rate depreciation puts an upward pressure on the CPI inflation rate.

The nominal exchange rate can be derived from the definition of the real exchange rate:

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{R}\widehat{S}_t - \widehat{R}\widehat{S}_{t-1}. \quad (35)$$

The law of motion of the nominal exchange rate depends on the real exchange rate fluctuations and differences in the aggregate inflation rates between the home and the foreign economy. Additionally, by combining the international risk sharing condition (27) and Euler conditions for the domestic and foreign economy (33), we obtain a relation between the nominal interest rate and the nominal exchange rate:

$$\widehat{S}_t = \widehat{R}_t^* - \widehat{R}_t + \widehat{S}_{t+1}. \quad (36)$$

This equation represents a version of the uncovered interest rate parity, which implies that changes in the nominal exchange rate result from differences between the domestic and foreign monetary policy. Let us point out that although very intuitive, this equation does not constitute an independent equilibrium condition.

The system is closed by specifying a monetary rule. In this paper, we derive the optimal monetary policy rule which maximizes welfare of the society subject to the structural equations of the economy. The optimal rule is specified as a rule where the monetary authority stabilizes the target variables in order to minimize the welfare loss of society and provide the most efficient allocation.<sup>12</sup> Apart from the optimal monetary derivation in this framework, we also consider the optimal monetary policy which is additionally constrained by the Maastricht convergence criteria.

Summing up, the dynamics of the small open economy are summarized by the following variables,  $\widehat{\pi}_{N,t}, \widehat{\pi}_{H,t}, \widehat{C}_t, \widehat{L}_t, \widehat{Y}_{H,t}, \widehat{Y}_{N,t}, \widehat{p}_{N,t}, \widehat{p}_{H,t}, \widehat{Y}_t, \widehat{R}\widehat{S}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{S}_t, \widehat{\pi}_t, \widehat{R}_t$  which are determined by equations (21)–(35), given the evolution of the stochastic shocks  $\widehat{A}_{N,t}, \widehat{A}_{H,t}, \widehat{B}_t$  and the foreign variables  $\widehat{C}_t^*, \widehat{T}_t^{d*}, \widehat{\pi}_t^*, \widehat{\pi}_{F,t}^*$ .<sup>13</sup>

<sup>12</sup>Giannoni and Woodford (2003) call these type of rules flexible inflation targeting rules.

<sup>13</sup>For simplicity, we choose to consider only one type of external shocks, foreign consumption shocks ( $\widehat{C}_t^*$ ). As a

### 3 The optimal monetary policy

This section characterizes the optimal monetary policy, i.e. the policy maximizing welfare of society subject to the structural equations of an economy. The micro foundations of our model give us a natural welfare measure, i.e. a discounted sum of expected utilities for the agents in the economy (see equation (1)).

We use a linear quadratic approach (Rotemberg and Woodford (1997, 1999)) and define the optimal monetary policy problem as a minimization problem of the quadratic loss function subject to the loglinearized structural equations (presented in the previous section). First, we present the welfare measure derived through a second-order Taylor approximation of equation (1):

$$W_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_v \hat{v}_t - \frac{1}{2} \hat{v}'_t Z_v \hat{v}_t - \hat{v}'_t Z_{\xi} \hat{\xi}_t] + tip + O(3), \quad (37)$$

where  $\hat{v}'_t = \begin{bmatrix} \hat{C}_t & \hat{Y}_{N,t} & \hat{Y}_{H,t} & \hat{\pi}_{N,t} & \hat{\pi}_{H,t} \end{bmatrix}$ ;  $\hat{\xi}'_t = \begin{bmatrix} \hat{A}_{N,t} & \hat{A}_{H,t} & \hat{B}_t & \hat{C}_t^* \end{bmatrix}$ ;  
 $z'_v = \begin{bmatrix} 1 & -s_C \tilde{d}_{Y_N} & -s_C \tilde{d}_{Y_H} & 0 & 0 \end{bmatrix}$  and matrices  $Z_v, Z_{\xi}$  are defined in Appendix B; *tip* stands for *terms independent of policy* and  $O(3)$  includes terms that are of a higher order than the second in the deviations of variables from their steady state values.

Notice that the welfare measure (37) contains the linear terms in aggregate consumption and sector outputs. These linear terms result from the distortions in the economy. First, monopolistic competition in both domestic sectors leads to inefficient levels of sector outputs and subsequently, an inefficient level of aggregate output. Second, since the domestic economy is open, domestic consumption and aggregate output are not equalized. Importantly, their composition depends on the domestic and international terms of trade. Third, there exists a(n) (international) terms of trade externality (see Corsetti and Pesenti (2001)) according to which monetary policy has an incentive to generate a welfare improving real exchange rate appreciation which leads to a lower disutility from labour without a corresponding decline in the utility of consumption.

The presence of linear terms in the welfare measure (37) means that we cannot determine the optimal monetary policy, even up to first order, using the welfare measure subject to the structural equations (21)–(35) that are only accurate to first order. Following the method proposed by Benigno and Woodford (2005) and Benigno and Benigno (2005), we substitute the linear terms in the approximated welfare function (37) by second moments of aggregate output, domestic and international terms of trade using a second-order approximation to the structural equations of the economy.<sup>14</sup> As a result, we obtain the fully quadratic loss function which can be represented as a function of aggregate output ( $\hat{Y}_t$ ), domestic and international terms of trade ( $\hat{T}_t^d, \hat{T}_t$ ) and domestic sector inflation rates ( $\hat{\pi}_{H,t}, \hat{\pi}_{N,t}$ ). Its general expression is given below:

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result,  $\hat{T}_t^{d*}, \hat{\pi}_t^*, \hat{\pi}_{F,t}^*$  are assumed to be zero. Moreover, all shocks follow an AR(1) process with normally distributed innovations.

<sup>14</sup>Details of the derivation can be found in Appendix B.

$$\begin{aligned}
L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} & \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_{T^d} (\hat{T}_t^d - \hat{T}_t^{dT})^2 + \right. \\
& \frac{1}{2} \Phi_T (\hat{T}_t - \hat{T}_t^T)^2 + \Phi_{TT^d} \hat{T}_t^d \hat{T}_t + \Phi_{YT^d} \hat{Y}_t \hat{T}_t^d + \Phi_{YT} \hat{Y}_t \hat{T}_t + \\
& \left. + \frac{1}{2} \Phi_{\pi_H} \hat{\pi}_{H,t}^2 + \frac{1}{2} \Phi_{\pi_N} \hat{\pi}_{N,t}^2 \right] + tip + O(3), \quad (38)
\end{aligned}$$

where  $\hat{Y}_t^T$ ,  $\hat{T}_t^{dT}$ ,  $\hat{T}_t^T$  are target variables which are functions of the stochastic shocks and, in general, are different from the flexible price equilibrium processes of aggregate output, domestic terms of trade and international terms of trade.<sup>15</sup> The coefficients  $\Phi_Y$ ,  $\Phi_{T^d}$ ,  $\Phi_T$ ,  $\Phi_{TT^d}$ ,  $\Phi_{YT^d}$ ,  $\Phi_{YT}$ ,  $\Phi_{\pi_H}$ ,  $\Phi_{\pi_N}$  are functions of the structural parameters of the model. The term *tip* stands for *terms independent of policy*.

Our loss function can be seen as a generalization of the previous studies encompassing both the closed (Aoki (2001), Benigno (2004), Rotemberg and Woodford (1997)) and open economy frameworks (Gali and Monacelli (2005), de Paoli (2004)).<sup>16</sup> Notice that if we set the size of the nontraded sector to zero and therefore obtain a one-sector small open economy, the loss function becomes identical to the loss function derived by de Paoli (2004).<sup>17</sup> In this case, the loss function is a function of the variances of aggregate output, terms of trade<sup>18</sup> and home traded inflation. On the other hand, if we set the degree of openness to zero, we obtain the case of a two-sector closed economy which was studied by Aoki (2001) and Benigno (2004). Here, the loss function is a function of the variances of aggregate output, domestic terms of trade, the covariance term between the two and variances of the sector inflation rates. Additionally, our loss function is closely related to the loss function derived for a national policymaker in a two-country model with two sectors of Liu and Pappa (2005). Interestingly, since their model is absent from the monopolistic competition distortion, trade imbalances and also expenditure switching effect, the loss function of a national policymaker depends only on the sector inflation rates and the sector output fluctuations around the flexible price targets.

We characterize the optimal plan under commitment where the policy maker chooses the set of variables

$$\{\hat{\pi}_{N,t}, \hat{\pi}_{H,t}, \hat{C}_t, \hat{L}_t, \hat{Y}_{H,t}, \hat{Y}_{N,t}, \hat{p}_{N,t}, \hat{p}_{H,t}, \hat{Y}_t, \hat{R}\hat{S}_t, \hat{T}_t^d, \hat{T}_t, \hat{S}_t, \hat{\pi}_t, \hat{R}_t\}_{t=t_0}^{\infty}$$

<sup>15</sup>As previously shown in papers by Gali and Monacelli (2005) and de Paoli (2004), in the small open economy framework the target variables will be identical to the flexible price allocations only in some special cases, i.e. an efficient steady state, no markup shocks, no expenditure switching effect (i.e.  $\rho\theta = 1$ ) and no trade imbalances.

<sup>16</sup>Rotemberg and Woodford (1998) present a one-sector closed economy model. Aoki (2001) presents a two-sector closed economy model with sticky prices only in one of the sectors. Benigno (2004) analyses the case of a monetary union comprised of two countries, which can be interpreted as a two-sector closed economy with totally segmented labour markets (interpretation as in Woodford (2003)). Gali and Monacelli (2005) and de Paoli (2004) study the case of a one-sector small open economy.

<sup>17</sup>In our representation, there is a covariance term between terms of trade and aggregate output which can be represented as the weighted sum of the variances of aggregate output and terms of trade.

<sup>18</sup>In the analysis of de Paoli (2004), it is actually the variance of the real exchange rate. However, it must be kept in mind that in a one-sector small open economy model, terms of trade and real exchange rate are proportional.

function (38) subject to constraints (21)–(35), given the initial conditions on this set of variables.<sup>19</sup> To simplify the exposition of the optimal plan, we reduce the number of variables to the set of five domestic variables which determine the loss function (38), i.e.  $\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{N,t}, \widehat{\pi}_{H,t}$ . Therefore, we represent the structural equations of the two-sector small open economy (21)–(35) in terms of these variables. The coefficients are defined in Appendix B.

The supply side of the economy is represented by two Phillips curves which are derived from equations (21) and (22) through a substitution of aggregate consumption, aggregate labour and relative prices:

$$\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}_t^d + m_{N,T}\widehat{T}_t + m_{N,A_N}\widehat{A}_{N,t} + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t) + \beta\widehat{\pi}_{N,t+1}, \quad (39)$$

$$\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}_t^d + m_{H,T}\widehat{T}_t + m_{H,A_N}\widehat{A}_{N,t} + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t) + \beta\widehat{\pi}_{H,t+1}. \quad (40)$$

The equation describing the demand side of the economy is derived from the market clearing conditions ((24), (25)) and the risk sharing condition (27):

$$\widehat{C}_t^* = \widehat{Y}_t + n_{T^d}\widehat{T}_t^d + n_T\widehat{T}_t + n_B\widehat{B}_t, \quad (41)$$

where aggregate consumption, relative prices and real exchange rate were substituted out.

The last structural equation represents the law of motion of the domestic and international terms of trade:

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}). \quad (42)$$

Finally, the policy maker following the optimal plan under commitment chooses  $\{\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{H,t}, \widehat{\pi}_{N,t}\}_{t=t_0}^\infty$  in order to minimize the loss function (38) subject to the constraints (39)–(42), given the initial conditions on  $\widehat{Y}_{t_0}, \widehat{T}_{t_0}^d, \widehat{T}_{t_0}, \widehat{\pi}_{H,t_0}, \widehat{\pi}_{N,t_0}$ . The first-order conditions of the problem are the following (where  $\gamma_{i,t}$  with  $i = 1, 2, 3, 4$  are accordingly the Lagrange multipliers with respect to (39)–(42)):

- with respect to  $\widehat{\pi}_{N,t}$  :

$$\Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} = 0, \quad (43)$$

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<sup>19</sup>The initial conditions guarantee the timeless perspective of the problem and make the first-order conditions of the problem time invariant (see Woodford (2003)).



- with respect to  $\widehat{\pi}_{H,t}$  :

$$\Phi_{\pi_H} \widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} = 0, \quad (44)$$

- with respect to  $\widehat{Y}_t$  :

$$\Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d} \widehat{T}_t^d + \Phi_{YT} \widehat{T}_t - k_N m_{N,Y} \gamma_{1,t} - k_H m_{H,Y} \gamma_{2,t} - \gamma_{3,t} = 0, \quad (45)$$

- with respect to  $\widehat{T}_t^d$  :

$$\begin{aligned} \Phi_{T^d} (\widehat{T}_t^d - \widehat{T}_t^{dT}) + \Phi_{TT^d} \widehat{T}_t + \Phi_{YT^d} \widehat{Y}_t - k_N m_{N,T^d} \gamma_{1,t} - k_H m_{H,T^d} \gamma_{2,t} - \\ + n_{T^d} \gamma_{3,t} + \gamma_{4,t} - \beta \gamma_{4,t+1} = 0, \end{aligned} \quad (46)$$

- with respect to  $\widehat{T}_t$  :

$$\begin{aligned} \Phi_T (\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d} \widehat{T}_t^d + \Phi_{YT} \widehat{Y}_t - k_N m_{N,T} \gamma_{1,t} - k_H m_{H,T} \gamma_{2,t} - \\ + n_T \gamma_{3,t} + a \gamma_{4,t} - \beta a \gamma_{4,t+1} = 0. \end{aligned} \quad (47)$$

Equations (43)–(47) and constraints (39)–(42) fully characterize the behaviour of the economy under the optimal monetary policy.

## 4 The Maastricht convergence criteria – a reinterpretation

Including the Maastricht criteria in their original form as additional constraints of the optimal monetary policy requires computationally demanding techniques. In particular, it results in solving the minimization problem of the loss function (38) subject to additional nonlinear constraints. On the other hand, the linear quadratic approach has two important advantages that make us decide to reformulate the criteria. First, it provides us with the analytical and intuitive expression for the loss function which can also serve as a welfare measure to rank alternative suboptimal policies. Second, the linear quadratic approach makes it easy to check second-order conditions (which would otherwise be quite difficult) for local optimality of the derived policy.

Therefore, the purpose of this section is to describe the way in which we reformulate the Maastricht criteria in order to implement them as additional constraints faced by the monetary policy in our linear quadratic framework.

First, we summarize the criteria (described in the introduction) by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \quad (48)$$

where  $B_\pi = 1.5\%$ ,  $\pi_t^A$  is annual CPI aggregate inflation in the domestic economy,  $\pi_t^{A,*}$  is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \quad (49)$$

where  $C_R = 2\%$ ,  $R_t^L$  is the annual interest rate for ten-year government bond in the domestic economy,  $R_t^{L,A*}$  is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \quad (50)$$

where  $D_S = 15\%$  and  $S$  is the central parity between euro and the domestic currency and  $S_t$  is the nominal exchange rate.

In order to adjust the criteria to the structure of the model, we assume that the variables  $\pi_t^{A,*}$  and  $R_t^{L,A*}$ , respectively, represent foreign aggregate inflation and the foreign nominal interest rate, i.e.  $\widehat{\pi}_t^*$ ,  $\widehat{R}_t^*$  (which are proxied to be the euro area variables). Here, we implicitly assume that the aggregate inflation rate and the nominal interest rate of the euro area do not differ to any great extent from the average of the three lowest inflation countries of the European Union.<sup>20</sup>

Second, we impose some simplifying assumptions regarding the criteria to adjust them to the quarterly nature of the model. The CPI inflation rate criterion is stated in annual terms. We decide to reformulate this criterion into the criterion on the quarterly CPI inflation rate with an appropriately changed upper bound, i.e.  $B_\pi \equiv ((1.015)^{0.25} - 1)$ . Notice that the criterion on the quarterly CPI inflation rate is stricter than the criterion set on the annual CPI inflation rate.<sup>21</sup> As far as the nominal interest rate criterion is concerned, we also decide to reformulate it into the criterion on the quarterly nominal interest rate. So, our reformulated criterion with the adjusted upper bound, i.e.  $C_R \equiv ((1.02)^{0.25} - 1)$ , is stricter than the original criterion.<sup>22</sup> Still, to keep the exposition of both criteria simple, we decide to use the reformulated criteria.

Moreover, the nominal exchange rate criterion is stated in terms of the quarterly nominal exchange rate movements. Additionally, we define the central parity of the nominal exchange rate as the steady state value of the nominal exchange rate ( $S = \bar{S}$ ).

<sup>20</sup>We are aware of the CPI inflation rate dispersion among the EMU member countries. Still the framework of the model does not allow us to consider the criteria strictly in their original form.

<sup>21</sup>This means that it is possible that the original criterion can be still satisfied, even though the quarterly CPI inflation rate violates the reformulated criterion. On the other hand, if the quarterly CPI inflation satisfies the criterion, the original criterion is also satisfied.

<sup>22</sup>If we assume that the expectations hypothesis holds, an upper bound restriction on the quarterly nominal interest rate implies an upper bound criterion on the ten-year government bond yield. However, the reverse is not true.

In order to implement the already adjusted criteria into the linear quadratic framework, we take advantage of the method proposed by Rotemberg and Woodford (1997, 1999) and Woodford (2003) which is applied to the zero bound constraint for the nominal interest rate. The authors propose to approximate the zero bound constraint for the nominal interest rate by restricting the mean of the nominal interest rate to be at least  $k$  standard deviations higher than the theoretical lower bound, where  $k$  is a sufficiently large number to prevent frequent violation of the original constraint. The main advantage of this alternative constraint over the original one is that it is much less computationally demanding and it only requires computation of the first and second moments of the nominal interest rate, while the original one would require checking whether the nominal interest rate is negative in any state which, in turn, depends on the distribution of the underlying shocks.

Importantly, to further simplify the exposition of the criteria, we assume that the foreign economy is in the steady state, so that foreign CPI inflation and the nominal interest rate ( $\widehat{\pi}_t^*$ ,  $\widehat{R}_t^*$ ) are zero. We are aware that if we relax this assumption and allow for a departure from the steady state of the foreign economy and possibly also a suboptimal foreign monetary policy, the nature of the optimal policy constrained by the Maastricht criteria and the associated welfare loss will be different than in our benchmark case.<sup>23</sup>

Similarly to Woodford (2003), we redefine the criteria using discounted averages in order to conform with the welfare measure used in our framework. Below, we show the reformulated Maastricht convergence criteria.<sup>24</sup> Each criterion is presented as a set of two inequalities:

- CPI aggregate inflation criterion:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq 0, \quad (51)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \right)^2; \quad (52)$$

- nominal interest rate criterion:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \geq 0 \quad (53)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \right)^2 \quad (54)$$

- nominal exchange rate criterion must be decomposed into two systems of the inequalities, i.e. the upper bound and the lower bound:

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<sup>23</sup>We discuss the consequences of relaxing this assumption in more detail in Section 6.

<sup>24</sup>The detailed derivation of the Maastricht convergence criteria can be found in Appendix B.

– upper bound

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t) \geq 0 \quad (55)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S - \widehat{S}_t) \right)^2 \quad (56)$$

– lower bound

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t) \geq 0 \quad (57)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (D_S + \widehat{S}_t) \right)^2 \quad (58)$$

where  $K = 1 + k^{-2}$  and  $D_S = 15\%$ ,  $B_\pi = (1.015)^{0.25} - 1$ ,  $C_R = (1.02)^{0.25} - 1$ ,  $k = 1.96$ .

The first inequality means that the average values of the CPI inflation rate, the nominal interest rate and the nominal exchange rate, respectively, should not exceed the bounds,  $B_\pi$ ,  $C_R$  and  $D_S$ . The second inequality further restrains fluctuations in the Maastricht variables by setting an upper bound on their variances. This upper bound depends on the average values of the Maastricht variables and the bounds,  $B_\pi$ ,  $C_R$  and  $D_S$ . Importantly, it also depends on parameter  $K$  which guarantees that the original constraints on the Maastricht variables ((48)–(50)) are satisfied with a high probability. Under a normality assumption, by setting  $K = 1 + 1.96^{-2}$ , we obtain that fulfillment of inequalities (51)–(58) guarantees that each of the original constraints should be met with a probability of 95%.

Summing up, the set of inequalities (51)–(58) represent the Maastricht convergence criteria in our model.

## 5 Optimal monetary policy constrained by the Maastricht criteria

This section presents how to construct the loss function of the optimal monetary policy constrained by the Maastricht convergence criteria summarized by inequalities (51)–(58) (constrained optimal policy). In this respect, we follow Woodford (2003). Specifically, the loss function of the constrained optimal monetary policy is augmented by the new elements which describe fluctuations in CPI aggregate inflation, the nominal interest rate and the nominal exchange rate.

We state the following proposition, which is based on Proposition 6.9 (p. 428) in Woodford (2003):

**Proposition 1** *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1 - \beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\}, \quad (59)$$

subject to (51) - (58). Let  $m_{1,\pi}$ ,  $m_{1,R}$ ,  $m_{1,S}^U$ ,  $m_{1,S}^L$  be the discounted average values of  $(B_\pi - \hat{\pi}_t)$ ,  $(C_R - \hat{R}_t)$ ,  $(D_S - \hat{S}_t)$ ,  $(D_S + \hat{S}_t)$  and  $m_{2,S}^U$ ,  $m_{2,S}^L$ ,  $m_{2,\pi}$ ,  $m_{2,R}$  be the discounted means of  $(B_\pi - \hat{\pi}_t)^2$ ,  $(C_R - \hat{R}_t)^2$ ,  $(D_S - \hat{S}_t)^2$ ,  $(D_S + \hat{S}_t)^2$  associated with the optimal policy. Then, the optimal policy also minimizes a modified discounted loss criterion of the form (59) with  $L_t$  replaced by:

$$\tilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \hat{\pi}_t)^2 + \Phi_R(R^T - \hat{R}_t)^2 + \Phi_{S,U}(S^{T,U} - \hat{S}_t)^2 + \Phi_{S,L}(S^{T,L} - \hat{S}_t)^2, \quad (60)$$

under constraints represented by the structural equations of an economy. Importantly,  $\Phi_\pi \geq 0$ ,  $\Phi_R \geq 0$ ,  $\Phi_{S,U} \geq 0$ ,  $\Phi_{S,L} \geq 0$  and take strictly positive values if and only if the respective constraints (52), (54), (56), (58) are binding. Moreover, if the constraints are binding, the corresponding target values  $\pi^T$ ,  $R^T$ ,  $S^{T,U}$ ,  $S^{T,L}$  satisfy the following relations:

$$\pi^T = B_\pi - Km_{1,\pi} < 0 \quad (61)$$

$$R^T = C_R - Km_{1,R} < 0 \quad (62)$$

$$S^{T,U} = D_S - Km_{1,S} < 0 \quad (63)$$

$$S^{T,L} = -D_S + Km_{1,S} > 0. \quad (64)$$

Proof can be found in Appendix B.

In the presence of binding constraints, the optimal monetary policy constrained by the Maastricht convergence criteria do not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the steady state of the optimal monetary policy.

In particular, if the constraints on the nominal interest rate or CPI inflation are binding, the target values for these variables are negative. This means that the constrained optimal monetary policy should target the CPI inflation rate or the nominal interest rate that is actually lower than the foreign CPI inflation or the foreign nominal interest rate, respectively. Therefore, this policy results in a deflationary bias. Finally, the deflationary bias together with a decrease in the nominal interest rate lead to a nominal exchange rate appreciation. Notice that if the upper bound criterion on the nominal exchange rate is binding, the constrained optimal policy is also characterized by a nominal exchange rate appreciation and negative averages of the nominal interest rate and the CPI inflation rate.

## 6 Numerical exercise

The purpose of this section is twofold, to characterize the optimal monetary policy for the EMU accession countries, given their obligation to satisfy the Maastricht convergence criteria and analyze whether and how it differs from the optimal monetary policy not constrained by the criteria (the

unconstrained optimal monetary policy). To this end, in the first step, we present the optimal monetary policy and identify whether such a policy violates any of the Maastricht convergence criteria. Second, based on the results, we construct the optimal policy that satisfies all the criteria (the constrained optimal monetary policy). Third, we compare both policies by studying their welfare costs and analyzing their response pattern to the shocks.

## 6.1 Parameterization

Following the previous literature on the EMU accession countries (i.e. Laxton and Pesenti (2003), Natalucci and Ravenna (2003)) we decide to calibrate the model to match the moments of the variables for the Czech Republic economy.

The discount factor,  $\beta$ , equals 0.99, which implies an annual interest rate of around four percent. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. As far as labour supply elasticity ( $\frac{1}{\eta}$ ) is concerned, the micro data estimates of  $\eta$  consider [3, 20] as a reasonable range. We decide to set  $\eta$  to 4. The elasticity of substitution between nontradable and tradable consumption,  $\phi$ , is set to 0.5 as in Stockman and Tesar (1994) and the elasticity of substitution between home and foreign tradable consumption,  $\theta$ , is set to 1.5 (as in Chari et al. (2002) and Smets and Wouters (2004)). The elasticity of substitution between differentiated goods,  $\sigma$ , is equal to 10, which together with the revenue tax of 0.1<sup>25</sup> implies a markup of 1.23.<sup>26</sup>

The share of nontradable consumption in the aggregate consumption basket,  $\mu$ , is assumed to be 0.42, while the share of foreign tradable consumption in the tradable consumption basket,  $\lambda$ , is assumed to be 0.4. These values correspond to the weights in CPI reported for the Czech Republic over the period 2000–2005.<sup>27</sup> As far as the foreign economy is concerned, we set the share of nontradable consumption in the foreign aggregate consumption basket,  $\mu^*$ , to be 0.6, which is consistent with the value chosen by Benigno and Thoenisen (2003) regarding the structure of euro area consumption.

Following Natalucci and Ravenna (2003), we set the degree of price rigidity in the nontraded sector,  $\alpha_N$ , to 0.85. The degree of price rigidity in the traded sector,  $\alpha_H$ , is slightly smaller and equals 0.8. These values are somewhat higher than the values reported in the micro and macro studies for the euro area countries.<sup>28</sup> Still, Natalucci and Ravenna (2003) justify them by a high share of the government regulated prices in the EMU accession countries.

<sup>25</sup>This value represents the average share of Taxes less Subsidies in the Gross Domestic Product at 1995 constant prices in the Czech Republic for the years 1995-2006 (source: Eurostat).

<sup>26</sup>Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

<sup>27</sup>Source: Eurostat.

<sup>28</sup>Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is nine months (which corresponds to the degree of price rigidity: 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find the overall degree of price rigidity for these countries to be 0.78.

All shocks that constitute the stochastic environment of the small open economy follow the AR(1) process. The parameters of the shocks are chosen to match the historical moments of the variables (see Table 3 in Appendix B). Similarly to Natalucci and Ravenna (2003) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of the productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector). These values roughly reflect the values chosen by Natalucci and Ravenna (2003), 1.8% (nontraded sector) and 2% (traded sector). Moreover, the productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.<sup>29</sup> All other shocks are independent of each other. Parameters defining the preference shock are, 0.72% (standard deviation) and 0.95 (persistence parameter). These values are similar to the values chosen by Laxton and Pesenti (2003), 0.4% (standard deviation) and 0.7 (persistence parameter). Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85.

Following Natalucci and Ravenna (2003), we parametrize the monetary policy rule, i.e. the nominal interest rate follows the rule described by:  $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$ , where  $\widehat{\varepsilon}_{R,t}$  is the monetary policy innovation with a standard deviation equal to 0.45%. Such a parametrization of the monetary policy rule enables us to closely match the historical moments of the Czech economy.

We summarize all parameters described above in Table 1 (Structural parameters) and Table 2 (Stochastic environment) in Appendix B. Moreover Table 3 (Matching the moments) in Appendix B compares the model moments with the historical moments for the Czech Republic economy.

## 6.2 Unconstrained optimal monetary policy

Now, we characterize the optimal monetary policy under the chosen parameterization. First, we analyze what the main concern of the optimal monetary policy is by studying the coefficients of the loss function given by (38).<sup>30</sup> In Table 1, we present these coefficients.

$\Phi_{\pi_N}$	$\Phi_{\pi_H}$	$\Phi_Y$	$\Phi_{T^d}$	$\Phi_T$	$\Phi_{TT^d}$	$\Phi_{YT^d}$	$\Phi_{YT}$
117.81	28.62	3.51	0.11	0.11	-0.05	-0.31	-0.28

The highest penalty coefficient is assigned to fluctuations in nontradable sector inflation and home tradable inflation. Therefore, the optimal monetary policy mainly stabilizes domestic inflation. This finding is in line with the literature on core inflation targeting (Aoki (2001)). Apart from that, the

<sup>29</sup>Empirical evidence shows that productivity shocks are highly persistent and positively correlated (see Backus et al (1992)).

<sup>30</sup>Following Benigno and Woodford (2005), we check whether the second-order conditions of the policy problem are satisfied in order to guarantee that there is no alternative random policy that could improve the welfare of society. This consists in checking whether all eigenvalues of the matrix representing the loss function (38) are nonnegative.

optimal monetary policy faces a trade off between stabilizing the output gap and the sector inflations which is reflected in the positive values of the penalty coefficients assigned to fluctuations in domestic and international terms of trade.

Next, we check whether the optimal monetary policy satisfies the Maastricht convergence criteria. Since the means of all variables under the optimal monetary policy are zero, we can reduce constraints (51)–(58) to the following set of inequalities:

$$\widehat{var}(\widehat{\pi}_t) \leq (K - 1)B_\pi^2 \quad (65)$$

$$\widehat{var}(\widehat{R}_t) \leq (K - 1)C_R^2 \quad (66)$$

$$\widehat{var}(\widehat{S}_t) \leq (K - 1)D_S^2, \quad (67)$$

where  $\widehat{var}(x_t) = E_{t_0} \sum_{t=t_0}^{\infty} \beta^t x_t^2$  and  $x_t = \widehat{\pi}_t, \widehat{R}_t, \widehat{S}_t$ . Notice that these constraints set the upper bounds on the variances of the Maastricht variables. In Table 2, we present variances of these variables under optimal monetary policy and the respective upper bounds that represent the right-hand side of equations (65)–(67). We write that a criterion is violated (satisfied) when the variance of the respective Maastricht variable is higher (smaller) than the upper bound.

	CPI inflation	nominal interest rate	nominal exchange rate
variance (in (%) <sup>2</sup> )	0.2638	0.3525	16.6195
mean (in%)	0	0	0
bound (in (%) <sup>2</sup> )	0.0356	0.0651	58.57
criterion	<i>violated</i>	<i>violated</i>	<i>satisfied</i>

The optimal monetary policy violates two of the Maastricht convergence criteria, the CPI inflation criterion and the nominal interest rate criterion. The nominal exchange rate criterion is satisfied.<sup>31</sup> Therefore, the loss function of optimal monetary policy for the EMU accession countries must be augmented by additional terms.

### 6.3 Constrained optimal policy

Now, we construct the optimal monetary policy that satisfies all Maastricht criteria. First, we augment the loss function of the optimal monetary policy with additional terms reflecting fluctuations of CPI inflation and the nominal interest rate and solve the new policy problem.<sup>32</sup> Second, we check whether such a policy also satisfies the nominal exchange rate criterion.

<sup>31</sup>Note that currently, the Czech Republic economy satisfies the Maastricht criteria regarding CPI inflation, the nominal interest rate and the nominal exchange rate. See Figures 3 (the CPI inflation criterion), 5 (the nominal interest rate criterion) and 6 (the nominal exchange rate criterion) in Appendix A.

<sup>32</sup>First-order conditions of this policy are presented in Appendix B.



The loss function of the optimal policy that satisfies two additional constraints on CPI inflation and the nominal interest rate is given below:

$$\tilde{L}_t = L_t + \frac{1}{2}\Phi_\pi(\pi^T - \hat{\pi}_t)^2 + \frac{1}{2}\Phi_R(R^T - \hat{R}_t)^2, \quad (68)$$

where  $\Phi_\pi > 0$ ,  $\Phi_R > 0$  and  $\pi^T < 0$ ,  $R^T < 0$ . Values of the penalty coefficients ( $\Phi_\pi$ ,  $\Phi_R$ ) and targets ( $\pi^T$ ,  $R^T$ ) can be obtained from the solution to the minimization problem of the original loss function constrained by structural equations (39)–(42) and also the additional constraints on the CPI inflation rate (51)–(52) and the nominal interest rate (53)–(54).<sup>33</sup> These values are presented in Table 3:

$\Phi_\pi$	$\Phi_R$	$\pi^T$ (in %)	$R^T$ (in %)
42.65	23.87	-0.1779	-0.1877

Notice that values of the penalty coefficients on the CPI inflation rate and nominal interest rate fluctuations are of the same magnitude as the penalty coefficients on the domestic inflation rates. The negative target value for the CPI inflation rate means that now, monetary policy targets the CPI inflation rate and the nominal interest rate that in annual terms are 0.7% smaller than their foreign counterparts.

Finally, we check whether this policy also satisfies the nominal exchange rate criterion. In Table 4, we present the first and second discounted moments of all Maastricht variables and evaluate whether each of the criteria is satisfied. A criterion is satisfied when the respective set of inequalities that describes this criterion holds. In particular, the CPI inflation criterion is described by the set of inequalities (51)–(52), the nominal interest criterion is explained by (53)–(54) and the nominal exchange rate criterion by (55)–(58).

	CPI inflation	nominal interest rate	nominal exchange rate
variance (in (%) <sup>2</sup> )	0.0475	0.0809	14.6207
mean (in%)	-0.0572	-0.0576	-5.7226
criterion	<i>satisfied</i>	<i>satisfied</i>	<i>satisfied</i>

Importantly, the nominal exchange rate criterion is satisfied. Not surprisingly, variances of the CPI inflation rate and the nominal interest rate are smaller than under the optimal policy. Notice that the variance of the nominal exchange rate is smaller than the one under the optimal monetary policy. This is due to the fact that the nominal exchange rate changes are, apart from the domestic sector inflation rates, one of the components of the CPI inflation rate (see (34)). So the policy that

<sup>33</sup>Special thanks to Michael Woodford for explaining the algorithm to find the parameters of the constrained policy problem (see p. 427–435 in Woodford (2003)).

targets domestic inflation rates and the CPI aggregate inflation rate at the same time also decreases the nominal exchange rate variability. Let us remark that the negative targets for the nominal interest rate and the CPI aggregate inflation lead to negative means of all Maastricht variables. Therefore, a central bank choosing such a policy commits itself to a policy resulting in the average CPI inflation rate and the nominal interest rate being 0.2% smaller in annual terms than their foreign counterparts. Additionally, this policy is characterized by an average nominal exchange rate appreciation of nearly 6%.

Summing up, the optimal monetary policy constrained by additional criteria on the CPI inflation and the nominal interest rate is the policy satisfying all Maastricht convergence criteria.

## 6.4 Comparison of the constrained and unconstrained optimal policy

Now, we focus on the comparison of the optimal monetary policy and the optimal policy constrained by the convergence criteria. First, we calculate the welfare losses associated with each policy and second, we analyze differences between the policies in their stabilization pattern when responding to the shocks.

In Table 5, we present the expected discounted welfare losses for both policies:

Table 5: Welfare losses for the unconstrained and constrained optimal policy		
	UOP	COP
loss (in $(\%)^2$ )	7.1533	9.2956

where UOP is the unconstrained optimal policy and COP is the constrained optimal monetary policy.

The obligation to comply with the Maastricht convergence criteria induces additional welfare costs equal to 30% of the optimal monetary policy loss. These welfare costs are mainly explained by the deterministic component of the constrained policy. Although the constrained optimal policy reduces variances of the Maastricht variables, it must also induce negative targets for the CPI inflation rate and the nominal interest rate to satisfy the criteria. These negative targets result in the negative means of all variables.

The welfare loss associated with the constrained optimal policy crucially depends on the foreign economy and the way its monetary policy is conducted. In our benchmark case, we assume the foreign economy to be in the steady state. This helps us simplify the exposition of the constrained optimal monetary policy problem. However, by allowing the foreign economy to be hit by stochastic shocks and, moreover, its monetary policy to be suboptimal, we obtain different targets and also penalty coefficients for domestic CPI inflation and the nominal interest rate. It can be shown that in such a situation, the targets on the CPI inflation rate and the nominal interest rate will not only depend on the average values of their foreign counterparts, but also on their fluctuations. However,

a deflationary bias feature of the constrained policy is preserved.<sup>34</sup> These different values of targets and penalty coefficients will alter the welfare loss associated with the constrained optimal monetary policy. Importantly, the more volatile is the foreign economy (due to suboptimal policy or a volatile stochastic environment of the foreign economy) the smaller is the welfare loss associated with the constrained optimal policy.

Now, we investigate how the two policies, constrained optimal monetary policy and unconstrained optimal monetary policy, differ when responding to the shocks. First, we analyze which shocks are most important in creating fluctuations of the Maastricht variables. In the table below, we present variance decomposition results for CPI aggregate inflation, the nominal interest rate and the nominal exchange rate. Since the variance decomposition structure does not change to any considerable extent with the chosen policy, we report results for the constrained policy.

Table 6: Variance decomposition of the Maastricht variables under the constrained policy				
	shocks:			
variables:	$A_N$	$A_H$	$B$	$C^*$
CPI inflation	80%	2%	11%	7%
nominal interest rate	86%	7%	4%	3%
nominal exchange rate	75%	3%	20%	2%

Around 80% of the total variability of CPI aggregate inflation, the nominal interest rate and the nominal exchange rate are explained by domestic nontradable productivity shocks. This result is consistent with the literature on the sources of inflation differentials in the euro area (Altissimo et al (2004)). Notice that although parameters describing productivity shocks are similar in our setup, each of the productivity shocks has a different impact on the real exchange rate and therefore, on the Maastricht variables. This can easily be understood by analyzing the following equation, which relates the real exchange rate to domestic and international terms of trade (see (28), (30)):

$$\widehat{RS}_t = b^* \widehat{T}_t^{d*} - b \widehat{T}_t^d + (1 - a) \widehat{T}_t. \quad (69)$$

Both domestic productivity shocks result in real exchange rate depreciation. However, the magnitude of the real exchange rate depreciation differs between the two shocks. Nontradable productivity shocks lead to a decline in the domestic terms of trade and a rise in the international terms of trade. Both changes have a depreciation effect on the real exchange rate. On the other hand, domestic tradable productivity shocks result in a rise of both types of terms of trade. From equation (69) we see that increases in both types of terms of trade cancel out and lead to a small change in the real exchange rate. As a result, domestic nontradable productivity shocks lead to a stronger real exchange rate depreciation and therefore, larger changes in the nominal interest rate and the CPI inflation rate.

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<sup>34</sup>See Proposition 3 in Appendix B.

Having all this in mind, we decide to study the stabilization pattern of both policies in response to domestic nontradable productivity shocks.

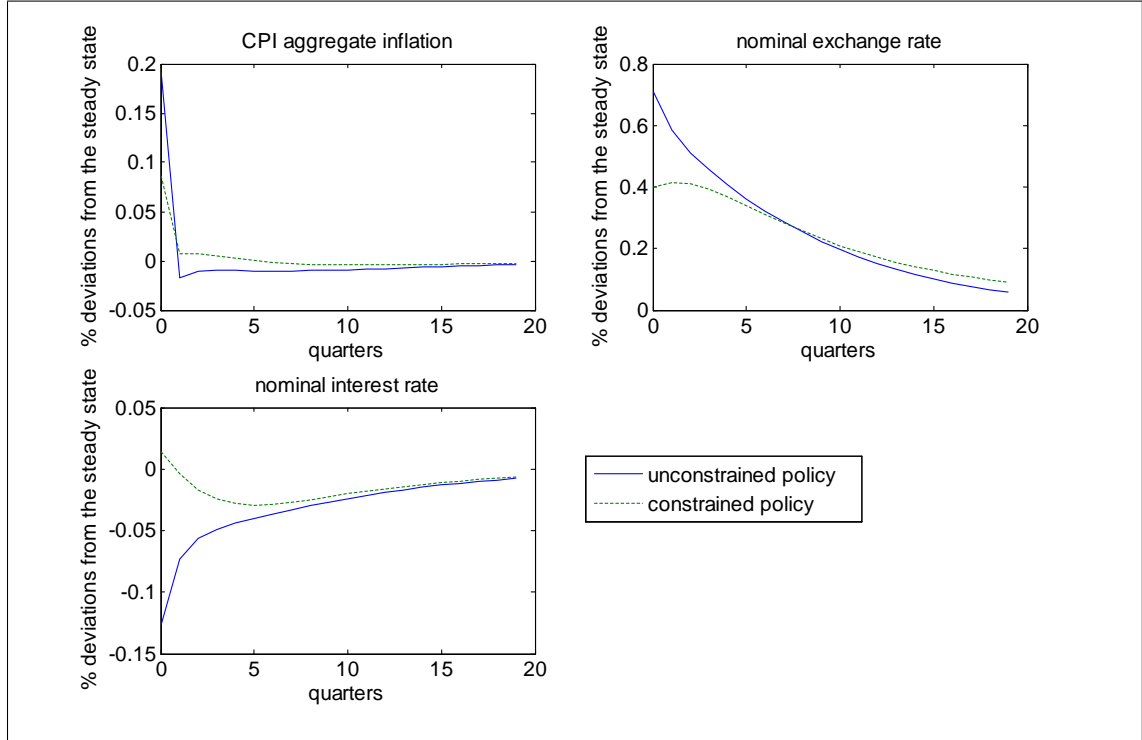


Figure 1: Impulse responses of the Maastricht variables to a positive domestic nontradable productivity shock

Under the unconstrained optimal policy, a positive domestic nontradable productivity shock leads to a fall in the nominal interest rate. This decrease of the nominal interest partially stabilizes deflationary pressures in the domestic nontraded sector and supports an increase in domestic aggregate output and consumption (not shown here). Since the foreign nominal interest rate remains constant, the uncovered interest parity induces a nominal exchange rate depreciation followed by an expected appreciation. The initial nominal exchange rate depreciation results in a strong initial increase of CPI inflation, which declines in subsequent periods, reverting to its mean.

The constrained policy is characterized by both CPI targeting and nominal interest rate targeting. To reduce the initial CPI increase (observed under the unconstrained policy), such a policy induces a more muted response of the real exchange rate and a stronger fall in domestic nontraded prices. These two effects are achieved through a more contractionary policy, i.e. a higher nominal interest rate as compared with the unconstrained optimal policy. Such behaviour of the nominal interest rate is in

line with the nominal interest rate targeting feature of the constrained optimal policy. As a result, an initial increase of the CPI inflation is smaller. Moreover, a higher domestic nominal interest rate leads to a smaller depreciation of the nominal exchange rate through the uncovered interest rate parity.

Summing up, in response to domestic nontradable productivity shocks, the constrained optimal policy leads to smaller fluctuations in all three Maastricht variables than unconstrained optimal monetary policy. However, it must be kept in mind that the constrained optimal policy commits to the inflation rate and the nominal interest rate that are lower than their foreign counterparts which results in substantial welfare costs.

## 6.5 Robustness analysis

The characteristics of the unconstrained and constrained optimal policy critically depend on the structural parameters of an economy and also the volatility of the stochastic environment. The purpose of this section is to investigate how changes in values of the parameters describing the structure and the stochastic environment of the small open economy affect our main findings.

As far as the structure of the small open economy is concerned, we identify two crucial parameters: share of nontradables ( $\mu$ ) and degree of openness ( $\lambda$ ). We derive the unconstrained and constrained optimal policy for different values of these parameters. Our findings can be summarized as follows:

- for all possible combinations of  $(\mu, \lambda)$ ,<sup>35</sup> the nominal exchange rate criterion is satisfied under the unconstrained optimal policy,
- for all possible combinations of  $(\mu, \lambda)$ , the nominal interest rate criterion is not satisfied under the unconstrained optimal policy,
- the CPI inflation rate criterion is satisfied under the unconstrained optimal policy for small values of  $\lambda$  and/or high values of  $\mu$ , i.e. for economies that are relatively closed and have a high share of nontradables (see Table 4 in Appendix B),
- for small values of  $\lambda$  and high values of  $\mu$ , the constrained policy that satisfies the CPI inflation rate criterion and the nominal interest rate criterion fails to satisfy the nominal exchange rate criterion (the lower bound constraint is not satisfied, i.e. the nominal exchange rate appreciates too much, see Table 5 in Appendix B).

Under our chosen parameterization of the stochastic environment, the productivity shocks are characterized by the highest standard deviation. Not surprisingly, elimination of the preference and foreign consumption shocks does not alter our results, i.e. the unconstrained optimal policy fails to satisfy the CPI inflation rate criterion and the nominal interest rate criterion and the optimal policy constrained by these two criteria also satisfies the nominal exchange rate criterion. The results do not

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<sup>35</sup>All combinations of  $(\mu, \lambda)$  for which the second-order conditions of the unconstrained policy problem are satisfied.

change, even if we eliminate one of the productivity shocks, i.e. in the traded or nontraded sector. Finally, the unconstrained optimal policy satisfies all Maastricht convergence criteria provided that the standard deviations of the productivity shocks in both sectors are reduced by at least 80% of the original values (see Table 6 in Appendix B)

Summing up, both the structure and the stochastic environment of the small open economy affect the characteristics of the unconstrained and constrained optimal policy. In relatively closed economies and/or with a high share of nontradables, there is a trade off between complying with the CPI inflation rate and the nominal interest rate criteria and the nominal exchange rate criterion. Moreover, volatility of productivity shocks plays a crucial role in determining whether the unconstrained optimal monetary policy is compatible with the Maastricht convergence criteria.

## 7 Conclusions

This paper characterizes the optimal monetary policy for the EMU accession countries, taking into account their obligation to meet the Maastricht convergence criteria. We perform our analysis in the framework of a two-sector small open economy DSGE model.

First, we derive the micro founded loss function which represents the policy objective function of the optimal monetary policy using the second-order approximation method (to the welfare function and all structural equations of the economy). We find that the optimal monetary policy should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. The main intuition for this result consists of understanding the effects of distortions present in the economy: monopolistic competition that implies inefficient sector outputs, price stickiness in both sectors that leads to an inefficient path of the domestic terms of trade and the international terms of trade externality that can affect the wedge between marginal disutility from labour and utility of consumption. All these distortions lead to the introduction of new elements in the loss function: domestic and international terms of trade.

Second, we reformulate the Maastricht convergence criteria taking advantage of the method developed by Rotemberg and Woodford (1997, 1999) to address the zero bound nominal interest rate problem. Subsequently, we show how the loss function of the monetary policy changes when the monetary policy is subject to the Maastricht convergence criteria: the CPI inflation rate criterion, the nominal interest rate criterion and the nominal exchange rate criterion. The loss function of such a constrained policy is characterized by additional elements that penalize fluctuations of the CPI inflation rate, the nominal interest rate and the nominal exchange rate around the new targets different from the steady state of the optimal monetary policy.

Under the chosen parameterization (which roughly represents the Czech Republic), optimal monetary policy violates two Maastricht criteria, the CPI inflation criterion and the nominal interest rate criterion. The optimal policy that instead satisfies these two criteria also satisfies the nominal exchange rate criterion. Both the deterministic component and the stabilization component of the

constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a lower variability of CPI inflation, the nominal interest rate and the nominal exchange rate. At the same time, this policy targets the CPI inflation rate and the nominal interest rate that are 0.7% lower (in annual terms) than their counterparts in the reference countries. This produces additional welfare costs that amount to 30% of the optimal monetary policy loss.

The tools developed in this paper can be used to describe the optimal policy which faces additional constraints that are exogenously decided and do not form part of the structural constraints of an economy. Importantly, the Maastricht Treaty also sets restrictions on the debt and deficit policy of the EMU accession countries. Therefore, a natural extension of the analysis involves the introduction of the fiscal policy by endogeneizing tax and debt decisions. Including all the restrictions faced by the fiscal and monetary policies in the EMU accession countries would enable us to investigate the effects of these restrictions on the interaction between the two policies.

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## **8 Appendix A**

### **8.1 Data on the EMU accession countries**

This section presents figures and data regarding the EMU accession countries. All the data were collected from the Eurostat database and the European Commission webpage.

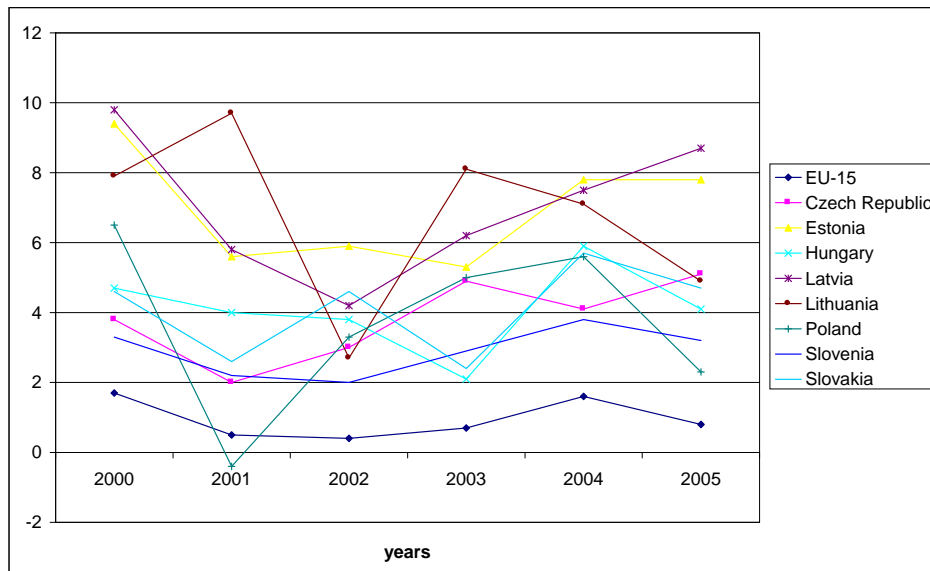


Figure 1: Total annual labour productivity growth in the EMU accession countries and the EU - 15 (annual rates in %)

countries	share of nontradables in consumption*	share of imports in GDP#
Czech Republic	42%	68%
Estonia	39%	86%
Hungary	44%	71%
Latvia	37%	55%
Lithuania	33%	58%
Poland	37%	35%
Slovenia	49%	59%
Slovakia	41%	78%
average in the EU - 15	51%	63%

note: \* – average value for the period 2000 - 2005; # – average value for the period 2000 - 2007

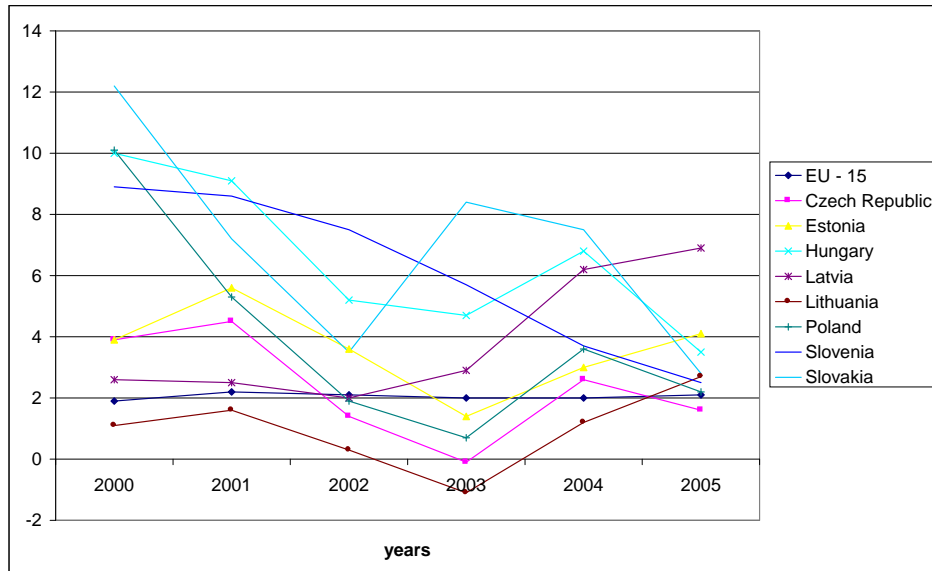


Figure 2: CPI inflation in the EMU accession countries and the EU - 15 in 2000 - 2005 (annual % rates)

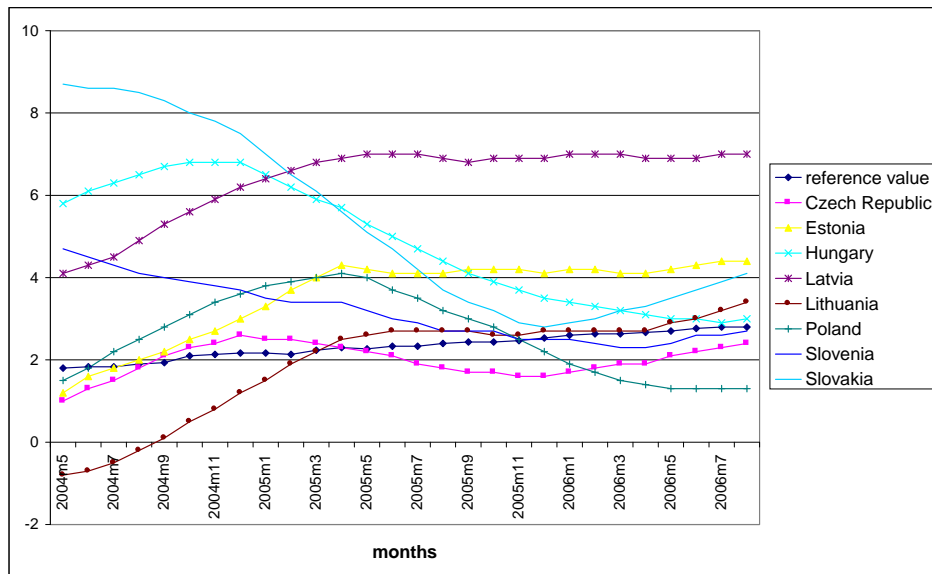


Figure 3: CPI inflation rates in the EMU accession countries since their accession to the EU (annual rates in %)

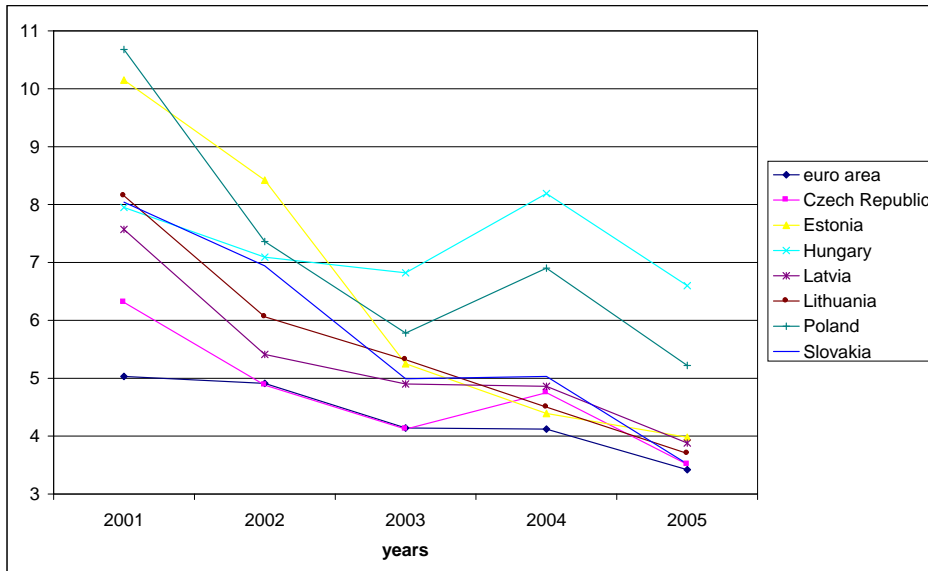


Figure 4: EMU convergence criterion bond yields for the EMU accession countries and the euro area in 2001 - 2005 (annual % rates)

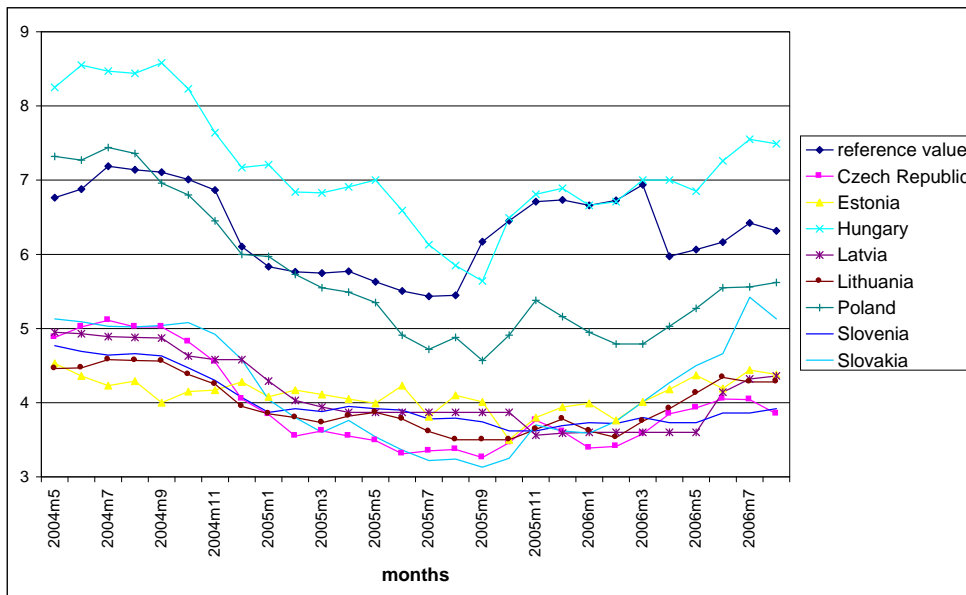


Figure 5: EMU convergence criterion bond yields for the EMU accession countries since their accession to the EU (annual rates in %)

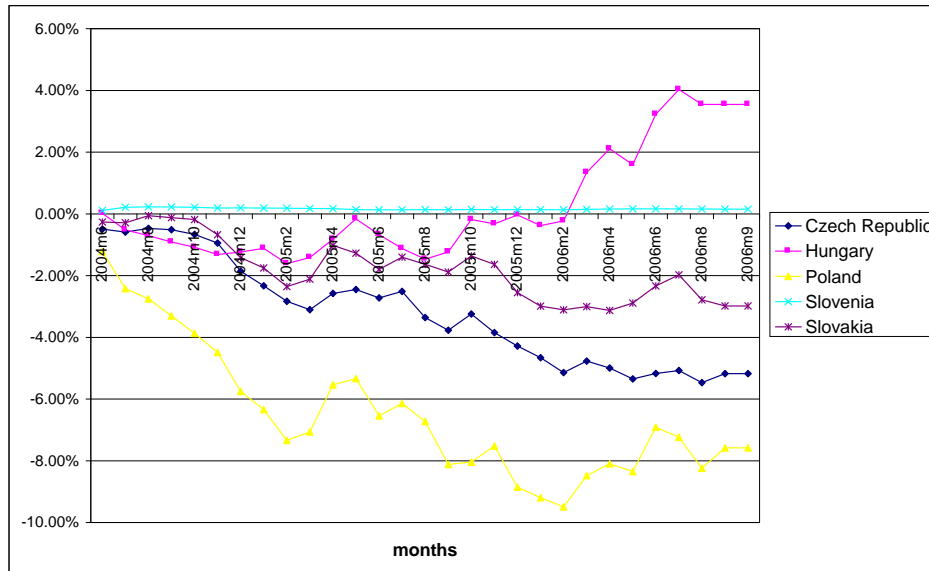


Figure 6: Nominal exchange rate fluctuations vs. euro of the EMU accession countries since the accession to the EU (average monthly changes since the EU accession date)

## 9 Appendix B

### 9.1 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

Table 1: Structural parameters		
The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	$\rho$	2
inverse of the labour supply elasticity	$\eta$	4
discount factor	$\beta$	0.99
intra-temporal elasticity between variety of the goods	$\sigma$	10
elasticity of substitution between home and foreign tradables	$\theta$	1.5
elasticity of substitution between tradables and nontradables	$\phi$	0.5
share of nontradables	$\mu$	0.42
degree of openness	$\lambda$	0.4
price rigidity in the nontradable sector	$\alpha_N$	0.85
price rigidity in the home tradable sector	$\alpha_H$	0.8
steady state share of taxes in the nontradable sector	$\tau_N$	0.1
steady state share of taxes in the tradable sector	$\tau_H$	0.1
share of nontradables in the foreign economy	$\mu^*$	0.6

Table 2: Stochastic environment		
shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity ( $A_N$ )	0.85	1.6
tradable productivity ( $A_H$ )	0.85	1.8
preference ( $B$ )	0.95	0.72
foreign consumption ( $C^*$ )	0.85	0.23
$corr(\hat{A}_{N,t}, \hat{A}_{H,t}) = 0.7$ where <i>corr</i> - correlation coefficient		

Note: The policy rule is calibrated following Natalucci and Ravenna (2003):  $\hat{R}_t = 0.9\hat{R}_{t-1} + 0.1(\hat{\pi}_t + 0.2\hat{Y}_t + 0.3\hat{S}_t) + \hat{\varepsilon}_{R,t}$ , where  $SD(\hat{\varepsilon}_{R,t}) = 0.45$ .

Statistics	My framework		Natalucci and Ravenna (2003)	
	Model	Historical	Model	Historical
Output:	1.87	1.68	1.53	1.74
nontraded sector	1.95	1.56	2.72	1.55
traded sector	3.23	4.32	2.87	2.25
Consumption	1.94	1.93	2.28	2.29
Nominal interest rate	0.52	0.52	0.48	0.47
Nominal exchange rate	2.84	2.59	2.79	3.04
Real exchange rate	2.35	3.62	2.48	2.75
CPI inflation rate:	0.82	0.91	1.1	1.08
nontraded sector	0.59	0.97	0.79	2.61
traded sector	0.94	0.74	2.4	0.99

Note: For comparison purposes the table shows also the results of the paper by Natalucci and Ravenna (2003). The model moments are theoretical.

As far as the historical statistics are concerned our data sample for the Czech Republic is 1995:1 - 2006:2 (Natalucci and Ravenna (2003) database is 1994:1 - 2003:1). CPI inflation rate in the traded and nontraded sector data sample is 2000:1 - 2006:2. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly.

All data were collected from the Eurostat webpage (the data in Natalucci and Ravenna (2003) were collected from the OECD publication Statistical Compendium (2003) and the Czech Republic National Accounts (July 2003)). Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency. Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: three months T - bill interest rate. Nominal exchange rate: Bilateral Koruny/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (6 trading partners, quarterly average). CPI inflation rate: Harmonised Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.



## 9.2 Steady state characterization

We define a deterministic steady state with zero inflation rate. We present a small open economy as the limiting case of a two country model, i.e.  $n = 0$  and  $\nu = 1 - \lambda$ . All variables in the steady state are denoted with a bar. All the shocks take the constant values, in particular:  $\overline{A}_N = \overline{A}_H = 1$ ,  $\overline{B} = 1$ .<sup>36</sup>

Moreover discount factors are:

$$Q_{t_0,t} = Q_{t_0,t}^* = \beta^{t-t_0} \quad (70)$$

First order conditions of the domestic firms are the following:

$$\overline{p}_N = \frac{\sigma_N}{(\sigma_N - 1)(1 - \overline{\tau}_N)} \frac{\overline{W}^N}{\overline{P}}, \quad (71)$$

$$\overline{p}_H = \frac{\sigma_H}{(\sigma_H - 1)(1 - \overline{\tau}_H)} \frac{\overline{W}^H}{\overline{P}} \quad (72)$$

where  $\overline{p}_N = \frac{\overline{P}_N}{\overline{P}}$ ,  $\overline{p}_H = \frac{\overline{P}_H}{\overline{P}}$ . We define markups in the domestic nontraded and home traded sector:  $\overline{\mu}_N = \frac{\sigma_N}{(\sigma_N - 1)(1 - \overline{\tau}_N)}$ ,  $\overline{\mu}_H = \frac{\sigma_H}{(\sigma_H - 1)(1 - \overline{\tau}_H)}$ .

Labour supply optimality conditions are presented below:

$$\overline{C}^{-\rho} \frac{\overline{W}^N}{\overline{P}} = (\overline{L})^\eta, \quad (73)$$

$$\overline{C}^{-\rho} \frac{\overline{W}^H}{\overline{P}} = (\overline{L})^\eta. \quad (74)$$

These two conditions imply that real wages are equalized in the domestic sectors:

$$\frac{\overline{W}^N}{\overline{P}} = \frac{\overline{W}^H}{\overline{P}} = \overline{\omega}. \quad (75)$$

Moreover substituting first order conditions of the firms ((71), (72)) into the labour supply optimality conditions ((73), (74)) we obtain the following relation:<sup>37</sup>

$$\overline{p}_N \overline{\mu}_N^{-1} = \overline{p}_H \overline{\mu}_H^{-1}. \quad (76)$$

From the market clearing condition in the domestic labour market we know that:

$$\overline{L} = \overline{L}_N + \overline{L}_H. \quad (77)$$

<sup>36</sup>Foreign consumption is derived from the steady state relations of the foreign economy.

<sup>37</sup>Notice that if markups in the nontraded and home traded sector are equal, i.e.  $\overline{\mu}_N = \overline{\mu}_H$  then also the domestic relative prices of nontraded and home traded goods are equal, i.e.  $\overline{p}_N = \overline{p}_H$ .

Moreover from the production function we get that:

$$\overline{L}_N = \overline{Y}_N, \quad (78)$$

$$\overline{L}_H = \overline{Y}_H. \quad (79)$$

Demands for domestic traded and nontraded goods are presented below:

$$\overline{Y}_N = \overline{C}_N = \overline{p}_N^{-\phi} \mu \overline{C}, \quad (80)$$

$$\overline{Y}_H = \overline{C}_H + \overline{C}_H^* = \overline{p}_H^{-\theta} \overline{p}_T^{-\theta-\phi} (1-\lambda)(1-\mu) \overline{C} + \overline{p}_H^{*-\theta} \overline{p}_F^{*\theta-\phi} \lambda (1-\mu^*) \overline{C}^* \quad (81)$$

where  $\overline{p}_H^* = \frac{\overline{P}_H^*}{\overline{P}^*}$ ,  $\overline{p}_F^* = \frac{\overline{P}_F^*}{\overline{P}^*}$ .

We define aggregate output in the following way:

$$\overline{Y} = \overline{p}_N \overline{Y}_N + \overline{p}_H \overline{Y}_H. \quad (82)$$

Notice that since the law of one price holds for the traded goods:  $\overline{p}_H^* = \overline{p}_H \overline{RS}^{-1}$ .

We define the following steady state ratios for the home economy:

$$d_{CH} = (1-\lambda)(1-\mu) \frac{\overline{C}}{\overline{Y}_H} \overline{p}_H^{-\theta} \overline{p}_T^{-\theta-\phi}, \quad (83)$$

$$s_C = \overline{\omega} \frac{\overline{L}}{\overline{C}} = \frac{\overline{p}_N}{\mu_N} \frac{\overline{Y}_N + \overline{Y}_H}{\overline{C}}, \quad (84)$$

$$d_{Y_N} = \frac{\overline{p}_N \overline{Y}_N}{\overline{Y}}, \quad (85)$$

$$d_{Y_H} = \frac{\overline{p}_H \overline{Y}_H}{\overline{Y}}, \quad (86)$$

$$\tilde{d}_{Y_N} = \frac{\overline{Y}_N}{\overline{Y}_N + \overline{Y}_H}, \quad (87)$$

$$\tilde{d}_{Y_H} = \frac{\overline{Y}_H}{\overline{Y}_N + \overline{Y}_H}. \quad (88)$$

From the complete asset market assumption and the assumption that the initial wealth distribution is such that  $v = RS_0 \left( \frac{C_0}{C_0^*} \right)^\rho = 1$  we obtain:

$$\overline{C} = \overline{RS}^{\frac{1}{\rho}} \overline{C}^*. \quad (89)$$

Lastly from the definition of price indices and the assumption that the law of one price holds in the traded sector we get the following relations between relative prices:

$$\overline{p_T}^{1-\theta} \overline{p_H}^{\theta-1} = (1-\lambda) + \lambda (\overline{RS} \overline{p_H}^{-1} \overline{p_F}^*)^{1-\theta}, \quad (90)$$

$$1 = \mu \overline{p_N}^{1-\phi} + (1-\mu) \overline{p_T}^{1-\phi}. \quad (91)$$

The set of the following conditions solves for the steady state values of domestic variables:  $\overline{p_N}$ ,  $\overline{p_H}$ ,  $\overline{p_T}$ ,  $\overline{C}$ ,  $\overline{RS}$  ( $\overline{C}^*$  and  $\overline{p_F}^*$  are treated as exogenous and are obtained from a similar set of the conditions for the foreign economy):<sup>38</sup>

$$\overline{C} = \overline{RS}^{\frac{1}{\rho}} \overline{C}^*, \quad (95)$$

$$\overline{p_N} \overline{\mu_N}^{-1} = \overline{p_H} \overline{\mu_H}^{-1}, \quad (96)$$

$$\overline{C}^{-\rho} \overline{p_H} \overline{\mu_H}^{-1} = \left( \overline{p_N}^{-\phi} \mu \overline{C} + \overline{p_H}^{-\theta} \overline{p_T}^{\theta-\phi} (1-\lambda)(1-\mu) \overline{C} + \overline{p_H}^{-\theta} \overline{RS}^{\theta} \overline{p_F}^{*\theta-\phi} \lambda (1-\mu^*) \overline{C}^* \right)^\eta, \quad (97)$$

$$\overline{p_T}^{1-\theta} = (1-\lambda) \overline{p_H}^{1-\theta} + \lambda (\overline{RS} \overline{p_F}^*)^{1-\theta}, \quad (98)$$

$$1 = \mu \overline{p_N}^{1-\phi} + (1-\mu) \overline{p_T}^{1-\phi}. \quad (99)$$

### 9.3 A loglinearized version of the model

We loglinearize the model around the above presented steady state. We present the structural equations that describe dynamics of the domestic economy. All the variables with hat represent the log deviations from the steady state. The system is closed by specifying the monetary rules for each of the economies.

$$\widehat{\pi}_{N,t} = k_N (\rho \widehat{C}_t + \eta \widehat{L}_t - \widehat{A}_{N,t} - \rho \widehat{B}_t - \widehat{p}_{N,t}) + \beta \widehat{\pi}_{N,t+1}, \quad (100)$$

---

<sup>38</sup>The set of optimality conditions for the foreign economy which determines the steady state values of  $\overline{C}^*$ ,  $\overline{p_F}^*$ ,  $\overline{p_N}^*$  is the following:

$$\overline{C}^{*-\rho-\eta} \overline{p_N}^* \overline{\mu_N}^{*-1} = \left( \overline{p_N}^{*-\phi} \mu^* + \overline{p_F}^{*-\phi} (1-\mu^*) \right)^\eta \quad (92)$$

$$\mu^* \overline{p_N}^{*1-\phi} + (1-\mu^*) \overline{p_F}^{*1-\phi} = 1 \quad (93)$$

$$\overline{p_N}^* \overline{\mu_N}^{*-1} = \overline{p_F}^* \overline{\mu_F}^{*-1} \quad (94)$$

$$\widehat{\pi}_{H,t} = k_H(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{H,t} - \rho\widehat{B}_t - \widehat{p}_{H,t}) + \beta\widehat{\pi}_{H,t+1}, \quad (101)$$

$$\widehat{L}_t = \widetilde{d}_{Y_N}(\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H}(\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \quad (102)$$

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi\widehat{p}_{N,t}, \quad (103)$$

$$\begin{aligned} \widehat{Y}_{H,t} = d_{CH}\widehat{C}_t - \theta\widehat{p}_{H,t} + b(\phi - \theta)d_{CH}\widehat{T}_t^d + (1 - d_{CH})\theta\widehat{RS}_t + \\ + (1 - d_{CH})\widehat{C}_t^* + b^*(\phi - \theta)(1 - d_{CH})\widehat{T}_t^{d*}, \end{aligned} \quad (104)$$

$$\widehat{Y}_t = d_{Y_N}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{Y_H}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}) \quad (105)$$

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho}\widehat{RS}_t + \widehat{C}_t^* - \widehat{B}_t^*, \quad (106)$$

$$(a - 1)\widehat{p}_{H,t} = b\widehat{T}_t^d + a\widehat{RS}_t - b^*a\widehat{T}_t^{d*}, \quad (107)$$

$$\widehat{p}_{N,t} = (1 - b)\widehat{T}_t^d, \quad (108)$$

$$\widehat{p}_{H,t} = -b\widehat{T}_t^d - a\widehat{T}_t, \quad (109)$$

$$\widehat{T}_t = \widehat{\Delta S}_t - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \quad (110)$$

$$\widehat{T}_t^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}_{t-1}^d, \quad (111)$$

$$\widehat{\pi}_{T,t} = \widehat{\pi}_{H,t} + a(\widehat{T}_t - \widehat{T}_{t-1}). \quad (112)$$

The Maastricht variables can be derived from the following equations:

$$\widehat{S}_t = \widehat{\Delta S}_t - \widehat{S}_{t-1}, \quad (113)$$

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{RS}_t - \widehat{RS}_{t-1}, \quad (114)$$

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \quad (115)$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \quad (116)$$

## 9.4 Quadratic representation of the optimal loss function

### 9.4.1 The second order approximation of the welfare function

We present a second order approximation to the welfare function (1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_v \widehat{v}_t - \frac{1}{2} \widehat{v}'_t Z_v \widehat{v}_t - \widehat{v}'_t Z_\xi \widehat{\xi}_t] + tip + O(3) \quad (117)$$

where  $\widehat{v}'_t = \left[ \widehat{C}_t \quad \widehat{Y}_{N,t} \quad \widehat{Y}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{H,t} \right]$ ;  $\widehat{\xi}'_t = \left[ \widehat{A}_{N,t} \quad \widehat{A}_{H,t} \quad \widehat{B}_t \quad \widehat{C}_t^* \right]$ ; *tip* stands for *terms independent of policy* and  $O(3)$  includes terms that are of order higher than the second in the deviations of variables from their steady state values. The matrices  $z_v$ ,  $Z_v$ ,  $Z_\xi$  are defined below:

$$z'_v = \left[ 1 \quad -s_C \widetilde{d}_{Y_N} \quad -s_C \widetilde{d}_{Y_H} \quad 0 \quad 0 \right], \quad (118)$$

$$Z_v = \begin{bmatrix} \rho - 1 & 0 & 0 & 0 & 0 \\ 0 & s_C \widetilde{d}_{Y_N} (1 + \eta \widetilde{d}_{Y_N}) & \eta s_C \widetilde{d}_{Y_N} \widetilde{d}_{Y_H} \widehat{Y} & 0 & 0 \\ 0 & \eta s_C \widetilde{d}_{Y_N} \widetilde{d}_{Y_H} \widehat{Y} & s_C \widetilde{d}_{Y_H} (1 + \eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & s_C \widetilde{d}_{Y_N} \frac{\sigma}{k_N} & 0 \\ 0 & 0 & 0 & 0 & s_C \widetilde{d}_{Y_H} \frac{\sigma}{k_H} \end{bmatrix}, \quad (119)$$

$$Z_\xi = \begin{bmatrix} 0 & 0 & -\rho & 0 \\ -s_C \widetilde{d}_{Y_N} (1 + \eta \widetilde{d}_{Y_N}) & -\eta s_C \widetilde{d}_{Y_H} \widetilde{d}_{Y_N} & 0 & 0 \\ -\eta s_C \widetilde{d}_{Y_H} \widetilde{d}_{Y_N} & -s_C \widetilde{d}_{Y_H} (1 + \eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (120)$$

### 9.4.2 Elimination of the linear terms

This section describes in detail how we eliminate the linear terms in the second order approximation to the welfare function in order to obtain a quadratic loss function. Moreover we reduce the number of structural variables that represent the policy problem by appropriate substitutions.

The optimal monetary policy solves the welfare maximization problem with the constraints given by the structural equations of the economy (their loglinearized versions are (100) - (112)). The matrix representation of the second order approximation to the welfare function is the following:

$$W = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z'_x \hat{x}_t - \frac{1}{2} \hat{x}'_t Z_x \hat{x}_t - \hat{x}'_t Z_{\xi} \hat{\xi}_t] + tip + O(3). \quad (121)$$

Similarly we present a second order approximation to all the structural equations in the matrix form:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \begin{array}{c} A_1 \hat{x}_t \\ A_2 \hat{x}_t \\ \dots \\ A_{13} \hat{x}_t \end{array} \right] + \frac{1}{2} \left[ \begin{array}{c} \hat{x}'_t B_1 \hat{x}_t \\ \hat{x}'_t B_2 \hat{x}_t \\ \dots \\ \hat{x}'_t B_{13} \hat{x}_t \end{array} \right] + \left[ \begin{array}{c} \hat{x}'_t C_1 \hat{\xi}_t \\ \hat{x}'_t C_2 \hat{\xi}_t \\ \dots \\ \hat{x}'_t C_{13} \hat{\xi}_t \end{array} \right] + tip + O(3) = 0 \quad (122)$$

with

$$\hat{x}'_t = \left[ \hat{Y}_t \quad \hat{L}_t \quad \hat{C}_t \quad \hat{Y}_{N,t} \quad \hat{Y}_{H,t} \quad \hat{p}_{N,t} \quad \hat{p}_{H,t} \quad \hat{T}_t^d \quad \hat{T}_t \quad \widehat{RS}_t \quad \widehat{\Delta S}_t \quad \widehat{\pi}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{T,t} \right], \quad (123)$$

$$\hat{\xi}'_t \equiv \left[ \hat{A}_{N,t} \quad \hat{A}_{H,t} \quad \hat{B}_t \quad \hat{C}_t^* \right]$$

where *tip* means terms independent of policy.

Following the methodology of Benigno and Woodford (2005) in order to eliminate the linear terms in the welfare function we solve the system of linear equations:

$$\zeta A = z'_x \quad (124)$$

$$\text{where } A_{(13 \times 14)} = \begin{bmatrix} A_1 \\ A_2 \\ \dots \\ A_{13} \end{bmatrix};$$

$$\zeta_{(1 \times 13)} = \left[ \zeta_1 \quad \zeta_2 \quad \zeta_3 \quad \zeta_4 \quad \zeta_5 \quad \zeta_6 \quad \zeta_7 \quad \zeta_8 \quad \zeta_9 \quad \zeta_{10} \quad \zeta_{11} \quad \zeta_{12} \quad \zeta_{13} \right] \text{ and } z_x_{(14 \times 1)}.$$

As a result we obtain the loss function:

$$L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} \hat{x}'_t L_x \hat{x}_t + \hat{x}'_t L_{\xi} \hat{\xi}_t \right] + tip + O(3) \quad (125)$$

where

$$L_x = Z_x + \zeta_1 B_1 + \zeta_2 B_2 + \zeta_4 B_4 + \zeta_6 B_6 + \zeta_9 B_9 + \zeta_{10} B_{10} + \zeta_{11} B_{11} + \zeta_{12} B_{12} + \zeta_{13} B_{13}, \quad (126)$$

$$L_{\xi} = Z_{\xi} + \zeta_1 C_1 + \zeta_2 C_2 + \zeta_4 C_4 + \zeta_6 C_6 + \zeta_{13} C_{13}. \quad (127)$$

### 9.4.3 Substitution of the variables

We want to represent the loss function (125) and also the whole model just in terms of the following variables:

$$\widehat{y}'_t = \left[ \widehat{Y}_t \quad \widehat{T}_t^d \quad \widehat{T}_t \quad \Delta \widehat{S}_t \quad \widehat{\pi}_{H,t} \quad \widehat{\pi}_{N,t} \quad \widehat{\pi}_{T,t} \right]. \quad (128)$$

In order to do this we define matrices  $N_x(14 \times 5)$  and  $N_\xi(14 \times 6)$  that map all the variables in the vector  $y'_t$  in the following way:

$$\widehat{x}_t = N_x \widehat{y}'_t + N'_\xi \widehat{\xi}_t \quad (129)$$

where:

$$N_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & ltd & lt & 0 & 0 & 0 & 0 \\ 1 & ctd & ct & 0 & 0 & 0 & 0 \\ 1 & yntd & ynt & 0 & 0 & 0 & 0 \\ 1 & yhtd & yht & 0 & 0 & 0 & 0 \\ 0 & pntd & 0 & 0 & 0 & 0 & 0 \\ 0 & phtd & pht & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & rstd & rst & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (130)$$

$$N_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \mathit{lan} & \mathit{lah} & \mathit{lb} & 0 \\ 0 & 0 & \mathit{cb} & 0 \\ 0 & 0 & \mathit{ynb} & 0 \\ 0 & 0 & \mathit{yhb} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (131)$$

with parameters defined below:



$$pntd = 1 - b \quad (132)$$

$$phtd = -b \quad (133)$$

$$pht = -a \quad (134)$$

$$rstd = -b \quad (135)$$

$$rst = 1 - a \quad (136)$$

$$cb = (1 - d_{CH})d_{YH} \quad (137)$$

$$ct = ad_{YH}(1 - \theta) + d_{YH}(1 - d_{CH})\left(\frac{1}{\rho} - \theta\right)(1 - a) \quad (138)$$

$$ctd = d_{YN}(1 - b)(\phi - 1) + bd_{YH}(1 - \theta) + b(\theta - \phi)d_{CH}d_{YH} + \quad (139)$$

$$+(1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)d_{YH}b \quad (140)$$

$$yntd = ctd - \phi * (1 - b) \quad (141)$$

$$ynb = cb \quad (142)$$

$$ynt = ct \quad (143)$$

$$yhtd = d_{YN}b\theta - d_{YN}d_{CH}b(\theta - \phi) - (1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)bd_{YN} + \quad (144)$$

$$-d_{YN} + b + d_{YN}\phi(1 - b) \quad (145)$$

$$yht = d_{YN}\theta a - d_{YN}(1 - d_{CH})\left(\theta - \frac{1}{\rho}\right)(a - 1) + d_{YH}a \quad (146)$$

$$yhb = -d_{YN}(1 - d_{CH}) \quad (147)$$

$$ltd = \tilde{d}_{YN} * yntd + \tilde{d}_{YH} * yhtd \quad (148)$$

$$lt = \tilde{d}_{YN} * ynt + \tilde{d}_{YH} * yht \quad (149)$$

$$lan = -\tilde{d}_{YN} \quad (150)$$

$$lab = -\tilde{d}_{YH} \quad (151)$$

$$lb = \tilde{d}_{YN} * ynb + \tilde{d}_{YH} * yhb \quad (152)$$

The loss function can be expressed now as:

$$L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} \hat{y}'_t L_y \hat{y}_t + \hat{y}'_t L_{\xi,y} \hat{\xi}_t \right] + tip + O(3) \quad (153)$$

where:

$$L_y = N'_x L_x N_x, \quad (154)$$

$$L_{\xi,y} = N'_x L_x N_{\xi} + N'_x L_{\xi}. \quad (155)$$

Since variables  $[\Delta S_t, \pi_{T,t}]$  do not appear in the original welfare objective function and in the second order terms of the structural equations we can further reduce the set of the variables which appear in the loss function to:

$$\hat{y}'_t = \left[ \hat{Y}_t \quad \hat{T}_t^d \quad \hat{T}_t \quad \hat{\pi}_{H,t} \quad \hat{\pi}_{N,t} \right] \quad (156)$$

The final set of the structural equations which represent the constraints of the maximization problem is:

$$\hat{\pi}_{N,t} = k_N \hat{m}c_t^{N,r} + \beta \hat{\pi}_{N,t+1}, \quad (157)$$

$$\hat{\pi}_{H,t} = k_H \hat{m}c_t^{H,r} + \beta \hat{\pi}_{H,t+1}, \quad (158)$$

$$\hat{C}_t^* = \hat{Y}_t + (ctd - \frac{1}{\rho} rstd) \hat{T}_t^d + (ct - \frac{1}{\rho} rst) \hat{T}_t + (cb - 1) \hat{B}_t, \quad (159)$$

$$\hat{T}_t^d - \hat{T}_{t-1}^d = \hat{\pi}_{N,t} - \hat{\pi}_{H,t} - a(\hat{T}_t - \hat{T}_{t-1}) \quad (160)$$

where:

$$\hat{m}c_t^{N,r} = (\rho + \eta) \hat{Y}_t + (\rho * ctd + \eta * ltd - pntd) \hat{T}_t^d + \quad (161)$$

$$+ (\rho * ct + \eta * lt) \hat{T}_t - (1 + \eta * \tilde{d}_{YN}) \hat{A}_{N,t} + \quad (162)$$

$$- \eta \tilde{d}_{YH} \hat{A}_{H,t} + (\rho * (cb - 1) + \eta * lb) \hat{B}_t \quad (163)$$

$$\hat{m}c_t^{H,r} = (\rho + \eta) \hat{Y}_t + (\rho * ctd + \eta * ltd - phtd) \hat{T}_t^d + \quad (164)$$

$$+ (\rho * ct + \eta * lt - pht) \hat{T}_t - \eta \tilde{d}_{YN} \hat{A}_{N,t} + \quad (165)$$

$$- (1 + \eta * \tilde{d}_{YH}) \hat{A}_{H,t} + (\rho * (cb - 1) + \eta * lb) \hat{B}_t \quad (166)$$

with:

$$m_{N,Y} = \rho + \eta \quad (167)$$

$$m_{N,T^d} = \rho * ctd + \eta * ltd - pntd \quad (168)$$

$$m_{N,T} = \rho * ct + \eta * lt \quad (169)$$

$$m_{N,A_N} = -(1 + \eta * \tilde{d}_{YN}) \quad (170)$$

$$m_{N,A_H} = -\eta \tilde{d}_{YH} \quad (171)$$

$$m_{N,B} = \rho * (cb - 1) + \eta * lb \quad (172)$$

$$m_{H,Y} = \rho + \eta \quad (173)$$

$$m_{H,T^d} = \rho * ctd + \eta * ltd - phtd \quad (174)$$

$$m_{H,T} = \rho * ct + \eta * lt - pht \quad (175)$$

$$m_{H,A_N} = -\eta \tilde{d}_{YN} \quad (176)$$

$$m_{H,A_H} = -(1 + \eta * \tilde{d}_{YH}) \quad (177)$$

$$m_{H,B} = \rho * (cb - 1) + \eta * lb \quad (178)$$

$$n_{T^d} = ctd - \frac{1}{\rho} rstd \quad (179)$$

$$n_T = ct - \frac{1}{\rho} rst \quad (180)$$

$$n_B = cb - 1 \quad (181)$$

Structural equations defining the Maastricht variables:

$$\hat{R}_t = \hat{\pi}_{t+1} - \rho(1 - cb)(\hat{B}_{t+1} - \hat{B}_t) + \rho(\hat{Y}_{t+1} - \hat{Y}_t) + \rho * ctd(\hat{T}_{t+1}^d - \hat{T}_t^d) + \rho * ct(\hat{T}_{t+1} - \hat{T}_t), \quad (182)$$

$$\hat{\pi}_t = \hat{\pi}_{H,t} + b(\hat{T}_t^d - \hat{T}_{t-1}^d) + a(\hat{T}_t - \hat{T}_{t-1}), \quad (183)$$

$$\hat{S}_t = \hat{S}_{t-1} + \hat{\pi}_t + rstd(\hat{T}_t^d - \hat{T}_{t-1}^d) + rst(\hat{T}_t - \hat{T}_{t-1}). \quad (184)$$

## 9.5 Reinterpretation of the Maastricht convergence criteria

We show how to reinterpret each of the Maastricht criteria in order to be able to use the method of Rotemberg and Woodford (1997, 1999).

### 9.5.1 Exchange rate criterion

We reinterpret the criterion on the nominal exchange rate (50) into two inequalities given below:<sup>39</sup>

$$E(\hat{S}_t) - k * SD(\hat{S}_t) \geq -15\%, \quad (185)$$

---

<sup>39</sup> $E$  stands for the expectation operator and  $SD$  stands for the standard deviation operator.

$$E(\widehat{S}_t) + k * SD(\widehat{S}_t) \leq 15\%. \quad (186)$$

where  $k$  is large enough to prevent from violating the criterion (50) and  $SD$  refers to the standard deviation statistic.

These two inequalities can be represented as the following two sets of inequalities (to conform with the welfare measure we use discounted statistics):

$$\left\{ \begin{array}{l} (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (\widehat{S}_t - (-15\%)) \geq 0 \\ (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (\widehat{S}_t - (-15\%))^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (\widehat{S}_t - (-15\%)) \right)^2, \end{array} \right. \quad (187)$$

$$\left\{ \begin{array}{l} (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (15\% - \widehat{S}_t) \leq 0 \\ (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (15\% - \widehat{S}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (15\% - \widehat{S}_t) \right)^2, \end{array} \right. \quad (188)$$

where  $K = 1 + k^{-2}$ .

### 9.5.2 Inflation criterion

We redefine the condition (48). We assume that the average inflation in the domestic economy should be at least  $k$  standard deviations smaller than the average inflation in the foreign economy plus a margin summarized by  $B_\pi$  (where  $B_\pi = \sqrt[4]{1,015} - 1$ ):

$$E(\widehat{\pi}_t) \leq E(\widehat{\pi}_t^*) + B_\pi - kSD(\widehat{\pi}_t) \quad (189)$$

where  $\widehat{\pi}_t, \widehat{\pi}_t^*$  are treated as deviations from the zero inflation steady state in the domestic economy and the foreign one accordingly (i.e.  $\bar{\pi} = \bar{\pi}^* = 0$ ) and  $k$  large enough to prevent from violating criterion (48). We assume that the foreign economy is in the steady state so  $\widehat{\pi}_t^* = 0 \forall t$ . As a result our restriction (189) becomes:

$$E(\widehat{\pi}_t) \leq B_\pi - kSD(\widehat{\pi}_t). \quad (190)$$

Since  $B_\pi$  is a constant we can use the following property of the variance:  $Var(\widehat{\pi}_t) = Var(B_\pi - \widehat{\pi}_t)$ . Our restriction becomes:

$$kSD(B_\pi - \widehat{\pi}_t) \leq E(B_\pi - \widehat{\pi}_t). \quad (191)$$

This restriction can be represented as a set of two restrictions:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \pi_t) \geq 0, \quad (192)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t) \right)^2. \quad (193)$$

### 9.5.3 Nominal interest rate criterion

Similarly to the criterion on the CPI aggregate inflation we interpret the inequality (49):

$$E(\hat{R}_t) \leq E(\hat{R}_t^*) + C_R - kSD(\hat{R}_t) \quad (194)$$

where  $k$  is large enough to prevent from frequent violating the criterion (49) and  $C_R = \sqrt[4]{1,02} - 1$ . As in the case of the foreign inflation we assume that  $\hat{R}_t^* = 0 \forall t$ . So the restriction (194) becomes:

$$kSD(C_R - \hat{R}_t) \leq E(C_R - \hat{R}_t). \quad (195)$$

This inequality can be represented as a set of two inequalities:

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t) \geq 0, \quad (196)$$

$$(1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t)^2 \leq K \left( (1 - \beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \hat{R}_t) \right)^2. \quad (197)$$

## 9.6 The constrained loss function

We provide the proof of the Proposition 1 stated in the main text. Since all the sets of the constraints have a similar structure the proof concerns the optimal monetary policy with only one constraint on the CPI inflation rate. The proof is based on the proof of Proposition 6.9 in Woodford (2003).

**Proposition 2** *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1 - \beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\} \quad (198)$$

*subject to (51) - (52). Let  $m_{1,\pi}$ ,  $m_{2,\pi}$  be the discounted average values of  $(B_\pi - \hat{\pi}_t)$  and  $(B_\pi - \hat{\pi}_t)^2$  associated with the optimal policy. Then the optimal policy also minimizes a modified discounted loss*

criterion of the form (198) with  $L_t$  replaced by:

$$\tilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \hat{\pi}_t)^2 \quad (199)$$

under constraints represented by the structural equations. Importantly  $\Phi_\pi \geq 0$  and takes strictly positive value if and only if the constraint (52) binds. Moreover if the constraint (52) binds the corresponding target value  $\pi^T$  is negative and given by the following relation:

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \quad (200)$$

**Proof.** Let  $m_{1,\pi}$  and  $m_{2,\pi}$  be the discounted average values of  $(B_\pi - \pi_t)$  and  $(B_\pi - \pi_t)^2$  associated with the policy that solves the constrained optimization problem stated in the corollary. Let  $m_{1,\pi}^*$  and  $m_{2,\pi}^*$  be the values of these moments for the policy that minimizes (198) without additional constraints. Notice that since  $m_{1,\pi}^* = B_\pi$  the constraint (51) does not bind.<sup>40</sup> We identify the deterministic component of policy, i.e.  $m_{1,\pi}$  and also the stabilization component of policy which is:  $m_{2,\pi} - (m_{1,\pi})^2$ . Moreover we also conclude that  $m_{1,\pi} \geq m_{1,\pi}^*$  since there is no advantage from choosing  $m_{1,\pi}$  such that:  $m_{1,\pi} < m_{1,\pi}^*$  - both constraints set only the lower bound on the value of  $m_{1,\pi}$  for any value of the stabilization component of policy. If one chooses  $m_{1,\pi}$  such that:  $m_{1,\pi} > m_{1,\pi}^*$  then one can relax the constraint (52). So  $m_{1,\pi} \geq m_{1,\pi}^*$ . Based on the above discussion we formulate two alternative constraints to the constraints (51, 52):

$$(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t) \geq m_{1,\pi}, \quad (201)$$

$$(1 - \beta)E_0 \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t)^2 \leq m_{2,\pi}. \quad (202)$$

Observe that any policy that satisfies the above constraints satisfies also the weaker constraints: (51, 52). Now we take advantage of the Kuhn – Tucker theorem: the policy that minimizes (198) subject to (201, 202) also minimizes the following loss criterion:

$$E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t L_t \right\} - \mu_{1,\pi} E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t) \right\} + \mu_{2,\pi} E_0 \left\{ (1 - \beta) \sum_{t=0}^{\infty} \beta^t (B_\pi - \hat{\pi}_t)^2 \right\} \quad (203)$$

---

<sup>40</sup>Means of all the variables under the unconstrained optimal policy are zero.

where  $\mu_{1,\pi}$  and  $\mu_{2,\pi}$  are the Lagrange multipliers which are nonnegative. If (52) binds then we obtain the following relation between the multipliers:

$$\mu_{1,\pi} = 2Km_{1,\pi}\mu_{2,\pi} \quad (204)$$

since  $m_{2,\pi} = Km_{1,\pi}^2$ .

Rearranging the terms in (203) we can define the new loss function as:

$$\tilde{L}_t \equiv L_t + \mu_{2,\pi} \left( (B_\pi - \hat{\pi}_t) - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}} \right)^2 \quad (205)$$

where the final term appears only when  $\mu_{2,\pi} > 0$ . Therefore  $\Phi_\pi = \mu_{2,\pi} \geq 0$  and takes a strictly positive value only if (52) binds. Moreover for  $\Phi_\pi > 0$  we have that:

$$\pi^T = B_\pi - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}} = B_\pi - Km_{1,\pi}. \quad (206)$$

Notice that the target value for the CPI inflation is negative (since  $K > 1$  and  $m_{1,\pi} \geq B_\pi$ ):

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \quad (207)$$

■

## 9.7 Constrained optimal monetary policy

We derive the first order conditions for the optimal monetary policy that satisfy the additional criteria on the nominal interest and the CPI aggregate inflation.

$$\begin{aligned} \min L_{t_0} = U_C \bar{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} & \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_{T^d} (\hat{T}_t^d - \hat{T}_t^{dT})^2 + \right. \\ & \frac{1}{2} \Phi_T (\hat{T}_t - \hat{T}_t^T)^2 + \Phi_{T^d} \hat{T}_t^d \hat{T}_t + \Phi_{Y T^d} \hat{Y}_t \hat{T}_t^d + \Phi_{YT} \hat{Y}_t \hat{T}_t + \\ & \frac{1}{2} \Phi_{\pi_H} \hat{\pi}_{H,t}^2 + \frac{1}{2} \Phi_{\pi_N} \hat{\pi}_{N,t}^2 + \frac{1}{2} \Phi_R (\hat{R}_t - R^T)^2 \\ & \left. + \frac{1}{2} \Phi_\pi (\hat{\pi}_t - \pi^T)^2 \right] + tip + O(3) \quad (208) \end{aligned}$$

*subject to :*

$$\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}_t^d + m_{N,T}\widehat{T}_t + m_{N,A_N}\widehat{A}_{N,t} + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t) + \beta\widehat{\pi}_{N,t+1}, \quad (209)$$

$$\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}_t^d + m_{H,T}\widehat{T}_t + m_{H,A_N}\widehat{A}_{N,t} + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t) + \beta\widehat{\pi}_{H,t+1}, \quad (210)$$

$$\widehat{C}_t^* = \widehat{Y}_t + n_{T^d}\widehat{T}_t^d + n_T\widehat{T}_t + n_B\widehat{B}_t, \quad (211)$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}), \quad (212)$$

$$\begin{aligned} \widehat{R}_t &= b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) \\ &\quad + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho * ct d(\widehat{T}_{t+1}^d - \widehat{T}_t^d) + \\ &\quad (\rho * ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), \end{aligned} \quad (213)$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}). \quad (214)$$

First order conditions of the minimization problem:

- wrt  $\widehat{\pi}_{N,t}$ :

$$0 = \Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - b\beta^{-1}\gamma_{5,t-1} - b\gamma_{6,t}, \quad (215)$$

- wrt  $\widehat{\pi}_{H,t}$ :

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - (1-b)\beta^{-1}\gamma_{5,t-1} - (1-b)\gamma_{6,t}, \quad (216)$$

- wrt  $\widehat{Y}_t$ :

$$\begin{aligned} 0 &= \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} \\ &\quad - k_H m_{H,Y}\gamma_{2,t} - \gamma_{3,t} + \rho\gamma_{5,t} - \rho\beta^{-1}\gamma_{5,t-1}, \end{aligned} \quad (217)$$



- wrt  $\widehat{T}_t^d$  :

$$0 = \Phi_{Td}(\widehat{T}_t^d - \widehat{T}_t^{dT}) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d} \gamma_{1,t} \\ - k_H m_{H,T^d} \gamma_{2,t} - n_{T^d} \gamma_{3,t} + \gamma_{4,t} \\ - \beta \gamma_{4,t+1} + \rho ctd \gamma_{5,t} - \rho ctd \beta^{-1} \gamma_{5,t-1}, \quad (218)$$

- wrt  $\widehat{T}_t$  :

$$0 = \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T} \gamma_{1,t} \\ - k_H m_{H,T} \gamma_{2,t} - n_T \gamma_{3,t} + a \gamma_{4,t} - \beta a \gamma_{4,t+1} \\ + (\rho ct + a(1-b)) \gamma_{5,t} - (\rho ct + a(1-b)) \beta^{-1} \gamma_{5,t-1} \\ - a(1-b) \gamma_{6,t} + a(1-b) \beta \gamma_{6,t+1}, \quad (219)$$

- wrt  $\widehat{R}_t$  :

$$0 = \Phi_R(\widehat{R}_t - R^T) + \gamma_{5,t}, \quad (220)$$

- wrt  $\widehat{\pi}_t$  :

$$0 = \Phi_\pi(\widehat{\pi}_t - \pi^T) + \gamma_{6,t}. \quad (221)$$

## 9.8 The constrained loss function - general case

We provide the proposition<sup>41</sup> that summarizes the discussion in Section 6 regarding the foreign economy. Since sets of the constraints for the CPI inflation rate and the nominal interest rate have the same structure the proposition concerns the optimal monetary policy with only one constraint on the CPI inflation rate.

**Proposition 3** *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1-\beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\} \quad (222)$$

subject to

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*) \geq 0, \quad (223)$$

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)^2 \leq K \left( (1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*) \right)^2. \quad (224)$$

---

<sup>41</sup>We do not present the proof for this proposition since it resembles to a great extent the proof of Proposition 2.

Let  $n_{1,\pi}$ ,  $n_{2,\pi}$  be the discounted average values of  $(B_\pi - \hat{\pi}_t + \hat{\pi}_t^*)$  and  $(B_\pi - \hat{\pi}_t + \hat{\pi}_t^*)^2$  associated with the optimal policy. Moreover assume that the average of the foreign CPI inflation rate ( $m_{1,\pi^*}$ ) satisfies the following inequality:  $m_{1,\pi^*} \geq -B_\pi$ . Then the optimal policy also minimizes a modified discounted loss criterion of the form (222) with  $L_t$  replaced by:

$$\tilde{L}_t \equiv L_t + \Phi_\pi (\pi_t^T - \hat{\pi}_t)^2 \quad (225)$$

under constraints represented by the structural equations. Importantly  $\Phi_\pi \geq 0$  and takes strictly positive value if and only if the constraint (224) binds. Moreover if the constraint (224) binds the corresponding target value  $\pi_t^T$  is given by the following relation:

$$\pi_t^T = B_\pi + \hat{\pi}_t^* - K n_{1,\pi} < \hat{\pi}_t^*. \quad (226)$$

## 9.9 Robustness analysis

We provide the results of our robustness analysis.

Table 4: Robustness analysis - structural parameters										
The unconstrained optimal policy and the Maastricht criteria										
$\mu \backslash \lambda$	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC
0.1	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC
0.2	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC
0.3	CPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC
0.4	CPI	CPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC
0.5	CPI	CPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.6	CPI	CPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.7	CPI	CPI	CPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.8	CPI	CPI	CPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.9	CPI	CPI	CPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC

note: CPI - CPI criterion satisfied; nCPI - CPI criterion not satisfied;  
nSOC - second order conditions not satisfied

Table 5: Robustness analysis - structural parameters										
The constrained optimal policy and the Maastricht criteria										
$\mu \backslash \lambda$	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	nNER	nNER	nNER	nNER	nNER	NER	NER	NER	NER	nSOC
0.1	nNER	nNER	nNER	nNER	NER	NER	NER	nSOC	nSOC	nSOC
0.2	nNER	nNER	nNER	nNER	NER	NER	nSOC	nSOC	nSOC	nSOC
0.3	nNER	nNER	nNER	nNER	NER	nSOC	nSOC	nSOC	nSOC	nSOC
0.4	nNER	nNER	nNER	nNER	NER	nSOC	nSOC	nSOC	nSOC	nSOC
0.5	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.6	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.7	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.8	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.9	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC

note: NER - nominal exchange rate (NER) criterion satisfied;  
nNER - NER criterion not satisfied; nSOC - second order conditions not satisfied

Table 6: Robustness analysis - stochastic environment			
variances of the Maastricht variables under the unconstrained policy (in $(\%)^2$ )			
shocks	CPI inflation	nominal interest rate	nominal exchange rate
$A_N, A_H, B, C^*$	0.2638	0.3525	16.6195
$A_N, B, C^*$	0.1252	0.1028	8.5792
$A_H, B, C^*$	0.0649	0.1063	5.7338
$A_N, A_H$	0.2372	0.3424	13.2265
$A_N^r, A_H^r, B, C^*$	0.0356	0.0228	3.9607
bound	0.0356	0.0651	58.57

note:  $SD(A_N^r) = 0.2SD(A_N)$ ;  $SD(A_H^r) = 0.2SD(A_H)$   
where  $SD$  - standard deviation