Connecting labour values and relative prices

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October 2009

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10/2009

Abstract

A rate of profit, expressed in labour magnitudes, that allows matching total plus value with total profits is determined. The relative prices, expressed in wage units, associated with this rate ensure that total profits are equal to the difference between the sum of relative prices and the sum of labour values as well.

Summary of the procedure

Vector of live labour, L, Leontief matrix, A, and basket of wage goods, B, are given with which labour values, Λ, and plus value are obtained (see definitions and assumptions in Annex 1). Concerning plus value, two quantities can be differentiated, one limited to the current period plus value and the other including, in addition, to the plus value obtained in the production of the circulating capital:

$$Pl = L' - \Lambda' BL' = L'(1 - \Lambda' B)$$

$$Total\_plusv = \Lambda' - \Lambda' \Lambda' B = \Lambda'(1 - \Lambda' B)$$

Similarly, profits and total profits can be distinguished. In order to have plus value, prices and labour values expressed in the same way, total plus value and total profits concepts are used.

Relative prices when expressed in wage units, are always higher than labour values, coincide with them when profit is zero and increase with the rise in the rate of profit. Thus, subtracting the sum of prices and the sum of values gives the total profits and makes it possible, as well, to avoid the presence of the prices vector in any formula (see expression (2) below). With the units employed, wage does not appear either (see (1) below). This total profit magnitude is then equated to total plus value by means of the unique remaining unknown: the profit rate (see (3) below).

From this point, a rate of profit expressed only in labour magnitudes is calculated, which equates total plus value and profits. A set of relative production prices is associated to this rate. The sum of the new relative production prices in the selected wage unit is equal to the sum of total plus value and labour values, as can be checked in the numerical example (Annex 2).

Development

Let the prices – taken in wage units[1] - and labour values be represented by these two expressions:

$$P'_w = (1+r)L'(I + (1+r)A + (1+r)^2 A^2 + ... + (1+r)^n A^n + ...) \quad (1)$$

[1] I must be noted that the use of the wage unit removes its presence in the formula (1)

$$\Lambda' = L'(I + A + A^2 + ... + A^n + ...)$$

or similarly:

$$P'_w = (1+r)L'(I-(1+r)A)^{-1}$$
$$\Lambda' = L'(I-A)^{-1}$$

in which, as long as there is a positive profit rate, it always holds, as it is already known[2], that:

$$P'_w \rangle\rangle \Lambda'$$

Subtracting both expressions, that is to say, the sum of prices and the sum of values - and knowing also that prices are equal to values when there is no profit -, we obtain the expression of profit:

$$P'_w I_v - \Lambda' I_v = profits^{3}$$

There should be a precise relation between profits in the price system and the plus value in the labour values system in order for profits to have full meaning.

Let $\Lambda' B$ be the expression of the time-unit wage and $1 - \Lambda' B$ the plus value rate,

the total system plus value is:

$$\Lambda' - \Lambda' \Lambda' B = \Lambda_{Total\_plusv}'$$

which is also equal to the following expression:

$$\Lambda_{Total\_plusv}' = (1-\Lambda' B)L' I + (1-\Lambda' B)L' A + (1-\Lambda' B)L' A^2 + ... + (1-\Lambda' B)L' A^n + ...)$$
$$= (1-\Lambda' B)L'(I-A)^{-1}$$

representing the unpaid or retained part of the total labour force employed in the present and in the past.

Connecting values and prices means that the total profit should be based on an existing total plus value as follows:

$$P'_w - \Lambda' = (1+r)L'(I-(1+r)A)^{-1} - L'(I-A)^{-1} = (1-\Lambda' B)L'(I-A)^{-1} (2)$$

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[2] Josep Maria Vegara i Carrió, **Economía Política y Modelos Multisectoriales**, Biblioteca Tecnos de Ciencias Económicas, Madrid: Editorial Tecnos, 1979, 190 pp.84-309-0795-5.

[3] I_v is a n column vector with all its components equal to one

$$L' = (2-\Lambda' B)L'(I-A)^{-1}I - (2-\Lambda' B)L'(I-A)^{-1}A(1+r) - L'r$$

$$L' - (2-\Lambda' B)L'(I-A)^{-1}I + (2-\Lambda' B)L'(I-A)^{-1}A$$
$$= -(2-\Lambda' B)L'(I-A)^{-1}Ar - L'r$$

$$\frac{L'}{(2-\Lambda' B)} - L'(I-A)^{-1} + L'(I-A)^{-1}A = -L'(I-A)^{-1}Ar - \frac{L'}{(2-\Lambda' B)}r$$

$$r = \frac{(L'(I-A)^{-1} - L'(I-A)^{-1}A - \frac{L'}{2-\Lambda' B})I_v}{(L'*(I-A)^{-1}A + \frac{L'}{2-\Lambda' B})I_v} = \frac{(\Lambda' - \Lambda' A - \frac{L'}{2-\Lambda' B})I_v}{(\Lambda' A + \frac{L'}{2-\Lambda' B})I_v} \quad (3)$$

At this profit rate (3) total plus value is equal to total profits and, in addition, the sum of new prices are equal to the sum of labour values and total plus value, or, similarly, to the sum of values and profits. The apparent contradiction of the former equality where plus value – or profits - is added to values is due to the unit taken for relative prices and would disappear if another unit for prices were used in which case the counteracting role of the wage would make prices come closer to values.

Conclusions

A set of relative prices exists that is determined by means of a rate of profit calculated in terms of labour magnitudes in the corresponding labour value system.

This rate of profit equates total plus value and total profits. This quantity coincides simultaneously with the difference between the sum of new relative prices, expressed in wage units, associated with this rate, and the sum of labour values. Thus, starting from a labour value system, a precise relation between labour values and prices and between total plus value and total profits is found. If labour is a good measure of economic costs and profits, the above relation between prices and values should not be disregarded.

Annex 1. Definitions and assumptions

A[4] Leontief matrix is used and the wage is expressed as a basket of goods. The resulting general price system is as follows:

$$P' = (1+r)P'\left(\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{21} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix} + \begin{bmatrix} b_1 l_1 & b_1 l_2 & \ldots & b_1 \mathrm{l_n} \\ b_2 l_1 & b_2 l_2 & \ldots & b_2 \mathrm{l_n} \\ \ldots & \ldots & \ldots & \ldots \\ b_n l_1 & b_n l_2 & \ldots & b_n \mathrm{l_n} \end{bmatrix}\right)$$

Unknowns

P', the row vector of prices
r, the common rate of profit

Data

a_{1i},…,a_{ni} (column of the *A* matrix), are the necessary inputs to produce a unit of product i
B is the column vector that corresponds to the basket of goods where components: b_1,... b_n, are the quantities of good, 1 to n, that need to be consumed to reproduce a unit of work
L' is a row vector where components l_1,….,.l_n, are the quantities of work needed to produce a unit of product of branches, 1 to n

A more abbreviate price system expression is as follows:

$$P' = (1+r)(P'A + P'BL')$$

If prices are measured taking wages as a unit we have:

$$P'_w = (1+r)(P'A + L')$$

Other definitions

I is a nxn identity matrix
I_v is a n column vector with all its components equal to one
Λ is a column vector of the total, direct and indirect, labour time incorporated in each unit of the commodities produced in the system

[4] Terminology of Josep Maria Vegara i Carrió

Assumptions are: A is a non-decomposable and productive matrix, a case of simple production is considered where only circulating capital exists. Every good or service considered is a basic commodity; there is only one quality of labour.

Annex 2. Numerical example

- Data

Input/Output matrix, A

	Product 1	Product 2	Product 3
Product 1	0,30	0,20	0,20
Product 2	0,10	0,10	0,00
Product 3	0,10	0,20	0,05

Live labour vector, L

1,00	0,10	2,00

Wage goods basket, B, per unit of wage

0,10	0,10	0,25

- Calculations

labour values per unit of good

1,9452	1,1023	2,5149

Wage per unit of time (value of the wage goods basket): $\Lambda' B$ *:*
0,9335

Labour value system	
Sum of values	5,5624
Variable capital	2,8938
Constant Capital	2,4624
Plus value	0,2062
Total plus value	0,3700
Rate of profit that equates Total profits and Total plus value – formula (3) above -	0,0360
Formula (3) check	
Sum of values + Total plus value	5,9324
Sum of new prices vector (2,08+1,20+2,64)	5,9284
Relative production price system calculated independently	
Sum of production prices vector - in wage units - (2,11+1,23+2,66)	6,00
Rate of profit associated with above vector	0,043