



Munich Personal RePEc Archive

# **Labour Market Imperfection and HOS Model: Some Counterintuitive Results**

Chaudhuri, Sarbajit

University of Calcutta

20 May 2009

Online at <https://mpra.ub.uni-muenchen.de/18034/>  
MPRA Paper No. 18034, posted 22 Oct 2009 09:54 UTC

## **Labour Market Imperfection and HOS Model: Some Counterintuitive Results**

**Sarbajit Chaudhuri**  
**Professor**  
**Dept. of Economics**  
**University of Calcutta**  
**56A, B.T. Road**  
**Kolkata 700 050**  
**India.**

**Address for communication:** Dr. Sarbajit Chaudhuri, 23 Dr. P.N. Guha Road, Belgharia, Kolkata 700083, India. Tel: 91-33-541-0455 (R), 91-33-557-5082 (C.U.) Fax: 91-33-2844-1490  
E-mail: [sarbajitch@yahoo.com](mailto:sarbajitch@yahoo.com)

**(May 2009)**

**ABSTRACT:** This note introduces labour market imperfection in an otherwise Heckscher-Ohlin-Samuelson (HOS) model and provides a theory of unionized wage formation. It demonstrates that this framework satisfies the Stolper-Samuelson theorem and the magnification effect and that it is capable of producing certain results which are contrary to the standard HOS and the Corden and Findlay (1975) results.

**Keywords:** Labour market imperfection, Heckscher-Ohlin-Samuelson model, Corden and Findlay model, Foreign capital, Trade liberalization.

**JEL Classification:** F2, F21, O17.

## **Labour Market Imperfection and HOS Model: Some Counterintuitive Results**

### **1. Introduction**

The Heckscher-Ohlin-Samuelson (hereafter, HOS) model with Stolper-Samuelson (SS) theorem at its core is the foundation of the neoclassical theory of international trade. In the HOS model both commodity and factor markets are perfectly competitive that limits the application of such a model for the purpose of analyzing the problems of a developing economy where the existence of factor market imperfections is a salient feature. Attempts have been made to use derivatives of the HOS model with labour market imperfection to address the problems of such economies. One example of this is the two-sector mobile capital version of the Harris-Todaro (HT) model (known as the Corden and Findlay (1975) (CF) model). The CF is a dual economy model with exogenous labour market distortion in the urban sector and does not satisfy the Stolper-Samuelson theorem completely. For example, if the price of the labour-intensive commodity changes the return to capital does not change. Otherwise, this structure more or less behaves like the standard HOS model and certain important trade theoretic results remain undisturbed. For example, the welfare effect of foreign capital with full repatriation of foreign capital earnings is immiserizing if the import-competing sector is capital-intensive and is protected by an import tariff. This is the standard ‘Brecher-Alejandro’ (1977) (BA) proposition which is valid in the CF set-up despite the presence of labour market distortion. This is due to the ‘envelope property’ of the CF model where the average wage of the workers is equal to the rural sector wage. A reduction in import tariff and a reduction in the unionized wage in the absence of the tariff are welfare-improving.

The purpose of this paper is to introduce labour market imperfection in an otherwise HOS model and provide a theory of unionized wage formation. We will demonstrate that this modified framework satisfies the SS theorem and the magnification effect. Besides, we show that this structure is capable of producing certain counterintuitive results which the CF model cannot generate.

## 2. The Model

We consider the standard HOS model with labour market imperfection in sector 2. In sector 2 (formal sector) workers receive the unionized wage,  $W^*$ , while their counterparts in sector 1 (an informal sector) receive a low and competitive wage,  $W$ . All other standard assumptions of the HOS model are retained. Commodity prices are given by the small open economy assumption.

### *Determination of unionized wage*

We consider a competitive formal sector industry. Production requires two inputs: labour (L) and capital (K). The capital market is perfect while the labour market facing the industry is unionized. Each firm in the industry has a separate trade union. While determining unionized wage, labour is considered as the only variable input of production. The unionized wage is determined as a solution to the Nash bargaining game between the representative firm and the representative labour union.

The representative firm's profit function is given by:

$$\Pi = P_2 Q(L, K) - W^* L \quad (1)$$

where  $P_2$  is the exogenously given price of the formal sector's product.

The representative labour union maximizes the aggregate wage income of its members net of their opportunity wage income i.e.

$$\Omega = (W^* - W)L \quad (2)$$

The informal sector wage,  $W$ , is the opportunity wage to the workers in the industry. This is because any worker failing to get employment in the formal will surely be getting a job in the informal sector.

We consider a cooperative game between the firm and the labour union that leads to determination of the unionized wage,  $W^*$  and the employment level,  $L$ . If the two parties fail to

reach an agreement no production will take place and the workers have to accept jobs in the informal sector. So given the objective functions of the two parties, represented by equations (1) and (2), the disagreement pay-off is:  $[0, 0]$

The Nash bargaining solution is obtained from the following optimization exercise.

$$\text{Max } J = [P_2 Q(L, K) - W^* L]^{(1-U)} \times [(W^* - W)L]^U \quad (3)$$

$W^*, L$

where  $U$  is the bargaining strength of the labour unions.

The first-order conditions for maximization are

$$(1-U)[(W^* - W)L] = U[P_2 Q(\cdot) - W^* L] \quad (4)$$

and,

$$(1-U)(P_2 Q_L - W^*)L = -U[P_2 Q(\cdot) - W^* L] \quad (5)$$

Using (4) and (5) one obtains

$$P_2 Q_L = W \quad (6)$$

Differentiation of (6) leads to

$$\left(\frac{\partial L}{\partial W}\right) = \frac{1}{P_2 Q_{LL}} < 0; \left(\frac{\partial L}{\partial P_2}\right) = -\frac{Q_L}{P_2 Q_{LL}} > 0 \quad (7)$$

Simplification from (4) yields

$$W^* = U \frac{P_2 Q(L, K)}{L} + (1-U)W \quad (8)$$

Equation (7) is the unionized wage function. In general form it is written as

$$W^* = W^*(P_2, W, U) \quad (8.1)$$

Differentiating (8) and using (7) we find

$$\begin{aligned}
\left(\frac{\partial W^*}{\partial U}\right) &= \left(\frac{1}{L}\right)(P_2 Q(\cdot) - WL) > 0 \\
\left(\frac{\partial W^*}{\partial W}\right) &= (1-U) + \frac{U(WL - P_2 Q(\cdot))}{L^2 P_2 Q_{LL}} > 0; \\
\left(\frac{\partial W^*}{\partial P_2}\right) &= \left[\frac{UP_2 Q(\cdot)(Q_{LL}L + Q_L) - UWLQ_L}{P_2 Q_{LL}L^2}\right] > 0 \text{ if } (Q_{LL}L + Q_L) \leq 0 \text{ i.e. } \xi_L \geq 1, \\
\text{where } \xi_L &= -\left(\frac{LQ_{LL}}{Q_L}\right) \text{ is the elasticity of marginal product curve of labour.}^1
\end{aligned} \tag{9}$$

This establishes the following proposition.

***Proposition 1:*** *The unionised wage is a positive function of the informal wage and the bargaining strength of the labour union. It is also a positive function of the commodity price if  $\xi_L \geq 1$ .*

From (8) it can also be checked that

$$\begin{aligned}
E_W &= \left(\frac{\partial W^* W}{\partial WW^*}\right) = \left[\frac{\{(1-U)L^2 Q_{LL} + U(LQ_L - Q(\cdot))\}W}{uP_2 f(L^*, K) + WL^*(1-u)}\right] > 0; \\
E_{P_2} &= \left(\frac{\partial W^* P_2}{\partial P_2 W^*}\right) = \left(\frac{P_2 U}{LQ_{LL}}\right) \left[\frac{\{Q(\cdot)LQ_{LL} - Q_L(LQ_L - Q(\cdot))\}}{(UP_2 Q(L, K) + WL(1-U))}\right] > 0 \text{ if } \xi_L \geq 1 \text{ and,} \\
(E_W + E_{P_2}) &= 1
\end{aligned} \tag{10}$$

### ***The S-S theorem and magnification effect***

The general equilibrium setup is given by the following set of equations.

$$Wa_{L1} + ra_{K1} = P_1 \tag{11}$$

$$W^*(P_2, W, U)a_{L2} + ra_{K2} = P_2 \tag{12}$$

$$a_{K1}X_1 + a_{K2}X_2 = K \tag{13}$$

---

<sup>1</sup> This is only a sufficient condition. There can be another sufficient condition under which  $\left(\frac{\partial W^*}{\partial P_2}\right) > 0$ .

$$a_{L1}X_1 + a_{L2}X_2 = L \quad (14)$$

where  $a_{ji}$ s is the requirement of the  $j$ th factor required to produce one unit of output of sector  $i$  for  $j = L, K$ ; and,  $i = 1, 2$ .

(11) and (12) are the two zero-profit conditions for the two sectors while equations (14) and (15) are the full-employment conditions for capital and labour, respectively. We assume that sector 1 is more (less) labour-intensive (capital-intensive) than sector 2 in value sense i.e.

$$\frac{Wa_{L1}}{a_{K1}} > \frac{W^*a_{L2}}{a_{K2}}.$$

Differentiating equations (11) and (12) totally, applying the envelope conditions and arranging terms in a matrix notation we obtain

$$\begin{bmatrix} \theta_{L1} & \theta_{K1} \\ \theta_{L2}E_W & \theta_{K2} \end{bmatrix} \begin{bmatrix} \hat{W} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} \hat{P}_1 \\ \hat{P}_2(1 - E_{P2}\theta_{L2}) \end{bmatrix} \quad (15)$$

where the determinant to the coefficient-matrix is

$$\begin{aligned} |\theta| &= (\theta_{L1}\theta_{K2} - \theta_{K1}\theta_{L2}E_W) = (\theta_{L1} - \theta_{L2}E_W) - \theta_{L1}\theta_{L2}(1 - E_W) \\ &= (\theta_{K2} - \theta_{K1}E_W) - \theta_{K1}\theta_{K2}(1 - E_W) > 0 \end{aligned} \quad (16)$$

$\theta_{ji}$  is the distributive share of the  $j$ th factor in the  $i$ th sector; and, “ $\hat{\phantom{x}}$ ” means proportional change.

We now state and prove the following proposition.

**Proposition 2:** *If  $\hat{P}_1 > \hat{P}_2$  the following ranking must hold:  $\hat{W} > \hat{P}_1 > \hat{P}_2 > \hat{r}$ .*

Proof:

Solving (16) by Cramer's rule

$$\hat{W} = \frac{\theta_{K2}\hat{P}_1 - \theta_{K1}(1 - E_{P2}\theta_{L2})\hat{P}_2}{|\theta|} \quad (17)$$

Now if  $\hat{P}_1 > \hat{P}_2$  from (17) it follows that

$$\left(\frac{\hat{W}}{\hat{P}_1}\right) > \frac{(\theta_{K2} - \theta_{K1}) + \theta_{K1}\theta_{L2}E_P}{|\theta|} = B \text{ (say)}$$

Now  $\left(\frac{\hat{W}}{\hat{P}_1}\right) > B \geq 1$  iff  $(\theta_{K2} - \theta_{K1}) + \theta_{K1}\theta_{L2}E_P \geq |\theta|$ . Using (16) and simplifying one gets

$$\left(\frac{\hat{W}}{\hat{P}_1}\right) > B \geq 1 \text{ iff } (E_{P2} + E_W) \geq 1. \text{ Using (10) it follows that}$$

$$\hat{W} > \hat{P}_1 \tag{19}$$

Similarly, solving (16) it is easy to show that

$$\hat{r} < \hat{P}_2 \tag{20}$$

Combining (19) and (20) one can write

$$\hat{W} > \hat{P}_1 > \hat{P}_2 > \hat{r}.$$

$$\text{If } \hat{P}_1 > \hat{P}_2 = 0, \hat{W} > \hat{P}_1 > \hat{P}_2 = 0 > \hat{r}$$

This completes the proof.

It is important to note that the Rybczynski effect and the Rybczynski-type effect that occurs following an S-S effect if production technologies are of variable coefficient type also hold in this case.

### ***Some counterintuitive results***

Now suppose that there a tariff on sector 2. Equation (12) is, therefore, modified as

$$W^*(P_2, W, u)a_{L2} + ra_{K2} = P_2(1+t) \tag{12.1}$$

We also assume that the aggregate capital stock of the economy consists of both domestic capital ( $K_D$ ) and foreign capital ( $K_F$ ) and these are perfect substitutes. The capital endowment equation is now given by

$$a_{K1}X_1 + a_{K2}X_2 = K_D + K_F = K \tag{13.1}$$



The strictly quasi-concave social welfare function is given by

$$V = V(D_1, D_2) \quad (14)$$

where  $D_i$  denotes the demand for the  $i$ th commodity for  $i=1,2$ .

Given that international trade occurs, trade balance requires

$$(X_1 - D_1) = P_2(D_2 - X_2) + rK_F \quad (15)$$

where  $(X_1 - D_1)$  is the amount of  $X_1$  exported and  $(D_2 - X_2)$  denotes the amount of  $X_2$  that is imported.

Differentiating (14) yields

$$(dV / V_1) = dD_1 + P_2(1+t)dD_2 \quad (16)$$

National income at domestic prices is given by

$$Y = X_1 + P_2(1+t)X_2 + tP_2M - rK_F \quad (17)$$

where,  $M$  denotes the volume of import and is given by

$$M = D_2(P_2(1+t), Y) - X_2 \quad (18)$$

### ***Welfare consequences of foreign capital inflows, trade liberalization and labour market reform***

Differentiating (15), (17), (18), (13.1), (14) and the production functions and substituting into (16) the following expressions can be obtained.

$$(1 / V_1)(dV / dK_F) = v[(W^* - W)(dL_2 / dK_F) - tP_2(dX_2 / dK_F)] \quad (19)$$

$$(1 / V_1)(dV / dt) = v[(W^* - W)(dL_2 / dt) + tP_2(HP_2 - (dX_2 / dt))] \quad (20)$$

$$(1 / V_1)(dV / dU) = v[(W^* - W)(dL_2 / dU) - tP_2(dX_2 / dU)] \quad (21)$$

where:  $v = [(1+t) / \{1+t(1-m)\}] > 0$ ;  $m = P_2(1+t)(\partial D_2 / \partial Y)$  is the marginal propensity to consume commodity 2 ( $1 > m > 0$ ); and,  $H = [(\partial D_2 / \partial P_2(1+t)) + D_2(\partial D_2 / \partial Y)] < 0$  is the Slutsky's pure substitution term.

From (19) – (21) the following proposition can be established.

***Proposition 3:*** *While an inflow of foreign capital and/or labour market reform may improve social welfare a policy of trade liberalization may be welfare-worsening.*

It is worthwhile to mention that in the CF model we get exactly the opposite results due to the ‘envelope property’.

We explain proposition 3 as follows. An inflow of foreign capital leads to a Rybczynski effect. Sector 2 expands and sector 1 contracts as the former sector is capital-intensive. As the higher wage-paying sector expands, both in terms of output and employment, the aggregate wage income of the workers increase. This we call the *labour reallocation effect* that works positively on social welfare and is captured by the first term in the right-hand side of (20). On the other hand, an expansion of sector 2 leads to a further misallocation of economic resources, lowers volumes of trade thereby exerting a downward pressure on welfare. This is the cost of the tariff protection of the supply side which is captured by the second term in the right-hand side of (19). Welfare improves if the positive labour reallocation effect outweighs the increased cost of tariff protection.

A policy of trade liberalization, on the other hand, lessens,  $t$ . This lowers  $r$  and raises  $W$  and hence  $W^*$ . The higher wage-paying sector contracts both in terms of output and employment due to a Rybczynski type effect. This is the negative labour reallocation effect. Secondly, the consumers would be consuming more of commodity 2 leading to a decrease in cost of tariff protection of the demand side that works positively on welfare. As the protected sector contracts the efficiency of allocation of economic resources improves. The cost of protection of the supply side decreases which also works favourably on welfare. If the labour reallocation effect is stronger than the combined effect of the last two effects, welfare decreases due to trade liberalization. See (20).

Finally, a policy of labour market reform lowers the bargaining strength of the labour unions,  $U$ . This lowers the unionized wage,  $W^*$ . Sector 2 expands as the wage cost falls. As the higher

wage-paying sector expands there occurs a positive effect on welfare. But the supply side cost of protection rises which affects welfare adversely. Welfare may increase if the labour reallocation effect dominates over the increase in the protectionary cost of tariff. The change in welfare is given by (21).

In the absence of any labour market imperfection,  $W^* = W$ . The model boils down to the standard HOS model. There is no labour reallocation effect. The first terms in the right-hand side of equations (19) and (20) vanish and we get the following standard results: an inflow of foreign capital worsens welfare. On the contrary, a policy of trade liberalization improves social welfare.

### 3. Concluding remarks

In this note we have shown that an HOS model with labour market imperfection not only satisfies all the standard properties like S-S theorem, magnification and Rybczynski effects but also is capable of producing certain counterintuitive results. A two-sector mobile capital version of the HT model does not satisfy these properties. Hence, the present framework may be useful in explaining as to why the developing countries are yearning for foreign capital despite the standard immiserizing result and why these countries are not reducing the tariff rates beyond certain levels.

### References:

Brecher, R.A. and C.F. Diaz Alejandro (1977): 'Tariffs, foreign capital and immiserizing growth', *Journal of International Economics* 7, 317-22.

Corden, W. M. and Findlay, R. (1975): 'Urban unemployment, intersectoral capital mobility, and development policy in a dual economy', *Economica* 42, 59-78.

Harris, J. R. and Todaro, M. (1970): 'Migration, unemployment, and development: a two-sector analysis', *American Economic Review* 60, 126-142.