Reputation, social identity and social conflict

Smith, John

Rutgers University-Camden

22 October 2009

Reputation, Social Identity and Social Conflict*

John Smith†
Rutgers University-Camden

October 22, 2009

Abstract

We interpret the social identity literature and examine its economic implications. We model a population of agents from two exogenous and well defined social groups. Agents are randomly matched to play a reduced form bargaining game. We show that this struggle for resources drives a conflict through the rational destruction of surplus. We assume that the population contains both unbiased and biased players. Biased players aggressively discriminate against members of the other social group. The existence and specification of the biased player is motivated by the social identity literature. For unbiased players, group membership has no payoff relevant consequences. We show that the unbiased players can contribute to the conflict by aggressively discriminating and that this behavior is consistent with existing empirical evidence.

*The author would like to acknowledge helpful comments from Roland Benabou, Armin Falk, Faruk Gul, Jo Hertel, Wolfgang Pesendorfer, Jack Worrall and the participants of the Social Identity Theory Seminar in the Princeton Psychology Department organized by Debbie Prentice.
†Email: smithj@camden.rutgers.edu; Phone: (856) 225-6319.

1 Introduction

Experimental research has found that placing people into social groups can cause some to have a preference for discrimination: favoring members of their own group at the expense of members of other groups.[1] Indeed, this is the primary insight of the vast literature on social identity, which we describe in more detail below. In this paper, we model a heterogeneous population, partially composed of agents who behave as described by this literature. The interesting questions are then, what can we say about agents with no such preference for discrimination and what can we say about outcomes in such a society.

We present a model in which each player lives for two periods and in each is matched to play a reduced form bargaining stage game. In each stage game, both players have a better material outcome by agreeing to a distribution than by not agreeing. Also, in the stage game, each player has a better material outcome by securing the larger share of the surplus. We assume that every agent is a member of one of two social groups and that this status is observable.

Players are assumed to be either unbiased or biased. Unbiased players are motivated entirely by material payoffs. In other words, group membership contains no payoff relevant consequences for unbiased players. By contrast, a biased player has payoffs which are affected by group membership. Consistent with the social identity literature, we make the following assumptions regarding biased players. When matched with a member of their own group (an ingroup match), biased players are cooperative. When matched with a member of the other group (an outgroup match) biased players intransigently destroy surplus rather than accept a payoff lower than the outgroup opponent.

We find that when preferences are unobservable, a social conflict can emerge. In particular, we show that the conflict does not require an entire population of biased agents. Rather, unbiased players can contribute to the conflict through the destruction of surplus in outgroup matches by mimicking biased agents. Unbiased agents might find it beneficial to behave as such in order to obtain a reputation for being biased and hence secure more favorable outcomes in the future.[2]

Our first main result (Proposition 1) shows that the inefficiency in a society tends to be increasing in the heterogeneity of that society. Our second main result (Proposition 2) shows that inefficiency is increasing in the inequitability of the environment. These results relate to the following two strands of literature.

[1] See Tajfel et. al. (1971) for a classic reference and see Miller et. al. (1998) for a particularly interesting application.

[2] Modeling reputation is standard in game theory and was pioneered by Kreps and Wilson (1982). The novelty in our approach lies in merging this technique with our interpretation of the social identity literature. Like Silverman (2004), this paper models matching in a two-sided reputation setting in order to explore outcomes not generated a perfect information model.

Researchers have examined the relationship between social heterogeneity and economic conditions. For instance, Easterly and Levine (1997), Mauro (1995), Posner (2004) and Montalvo and Reynal-Querol (2005) show that measures of heterogenous populations are negatively related to economic development. We contend that our model contributes to the understanding of this stylized fact. As individuals of different social groups compete for material benefits, disagreement and inefficiency can result. We demonstrate the positive relationship between our measure of social heterogeneity and social conflict as measured by such inefficiency.[3]

Additionally, researchers have noted the relationship between the level of social conflict and the inequitability of the environment. Falk and Zweimuller (2005) show a relationship between local economic conditions and aggressive behavior. Specifically, the authors show that higher local unemployment rates (and hence, larger probabilities of inequitable outcomes) lead to higher incidences of right-wing extremist crimes. It is important to note that the authors find that it is the threat of a worse economic position, and not the economic position per se, which induces this conflict. Therefore, we interpret these findings as evidence of a positive relationship between the inequitability of the environment[4] and social conflict. There is also a large sociological literature relating various forms of social conflict to the inequitability of the environment. For instance, Olzak (1992) finds a positive relationship between the inequitability of the environment and ethnic conflict, as measured by violent events.[5] Our model also provides an explanation for these findings. Specifically we show that the amount of social conflict is increasing in the inequitability of the environment.

Our specification of the biased player is motivated by the social identity literature. A very large literature has found that placing people into groups is a sufficient condition for discriminating behavior.[6] Of particular interest is the finding that people tend to prefer better material outcomes for ingroup members than outgroup members and that they are also prepared to create inefficiencies (destroy surplus) to secure this outcome. For instance, the discriminating person would prefer to allocate \$6 to an ingroup member and \$2 to an outgroup member rather than \$5 to each. Tajfel et. al. (1971) find that these preferences imply the maximization of the payoff difference between the groups.[7] In other words, the discriminating person will accept some inefficiency in allocating resources in order to secure a better material outcome for the ingroup.

[3] Also see Vigdor (2002) for a paper with a similar goal.

[4] What we refer to as "inequitability of the environment" sociologists refer to as "competition." Sociologists define competition to be the threat of a worse economic position. Here, we believe this term to be inappropriate as "competition" has a different meaning to economists.

[5] Lubbers and Scheepers (2001), Scheepers et. al. (2002), Quillian (1995, 1996) also find a positive relationship between the inequitability of the environment and social conflict, as measured by prejudiced beliefs. Olzak, Shanahan, and West (1994) find the relationship in the context of school busing in U.S. cities.

[6] A very small sample of this enormous literature would include Sumner 1906, Murdock (1949), Sherif et. al. (1961), Tajfel (1970), Tajfel et. al. (1971), Tajfel (1978), Tajfel and Turner (1979), Kramer and Brewer (1984), Tajfel and Turner (1986), Dawes, Van De Kragt, and Orbell (1988).

[7] There is, however, no consensus on this statement. Messick and Mackie (1984 pg. 64) point out that some authors find that discrimination can come in the form that the joint allocation is maximized "as long as the ingroup gets more than the outgroup." This perspective also suffices to justify our specification of behavioral players.

We view the social identity literature as providing specific justification for our model. First we assume the formation of social groups based on some shared characteristic and that membership in these groups might affect the preferences of some, but not all. Secondly, we assume that all players are *nice* in an ingroup match and in an outgroup match, some players are not nice in that they pick the action which maximizes the difference between the groups. The condition that some people prefer ingroup members to have better outcomes than outgroup members does not have bite in our ingroup matches. Therefore, we assume that biased players are nice in ingroup matches.

1.1 Related Literature

Recently economists have devoted attention to modeling identity.[8] For instance, Akerlof and Kranton (2000) present a general model of identity and economics. The authors assume that an agent's identity related preferences are affected by the actions of others, therefore their notion of a social group is fluid. By contrast, we model a social conflict between well defined social groups which are not fluid and not defined by behavior. Similar to Akerlof and Kranton, the behavior in our model is optimal from the perspective of the agent. However, the behavior in both models can be suboptimal in other ways: in our model discrimination leads to inefficiencies and in Akerlof and Kranton agents can engage in destructive activities.[9]

Insights on identity have been recently appearing in the experimental economics literature.[10] For instance, Ferraro and Cummings (2007) describe the results of an experiment where subjects play an anonymous version of the ultimatum game, although subjects know the distribution of the ethnicity of potential opponents. The authors find that the lowest offer which a subject would accept as a responder is decreasing in the fraction of players of the same ethnicity. We the work on identity within the experimental economics literature as supporting our assumptions of the model.

There exists a literature which formally models social conflict, however each strand focuses on different issues than we do here.[11] For instance, Fearon and Laitin (1996) and Nakao (2009) focus on the role in which ingroup policing helps to maintain social order by avoiding social conflict between groups. Specifically, it is assumed that information is differentially better for the histories of ingroup members than outgroup members and that no agents have a preference for discrimination. By contrast, we examine the implications of the preference for discrimination. Benhabib and Rustichini (1996), Bridgman (2008) and Strulik (2008) also model the relationship between social heterogeneity and conflict. These papers are able to make nuanced statements regarding outcomes in such a society, however groups are modeled as cohesive units. By contrast we assume a rather general stage game and model each unbiased

[8] See Phelps (1972) and Arrow (1973) for early theoretical work on identity and discrimination.

[9] For more on identity in economics, see Sobel (2004), Kirman and Teschl (2004) and Davis (2006) See Lindqvist and Ostling (2009) and Shayo (2009) for the application of identity to redistribution.

[10] See Ahmed (2007), Charness et. al. (2007), Goette et. al. (2006) and Guth et. al. (2008). Also see, Chen and Li (2008) who use econometric techniques to estimate the form of social preferences involving identity.

[11] Also see Caselli and Coleman (2006), Dion (1997), Esteban and Ray (2008, 2009) and Robinson (2001).

player as maximizing individual material payoffs. Finally, Orbell, Zeng and Mulford (1996) use computer simulation techniques to model social conflict as driven by individual incentives.

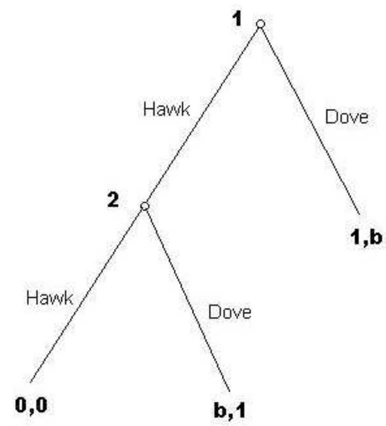
Like Basu (2005), we model social conflict in a heterogenous society[12] containing some members with a preference for discrimination. Additionally, we both show how the presence of these types can induce those without such a preference to discriminate. Basu models a one-shot game with multiple equilibria in material payoffs which can be Pareto ranked. The presence of types with a preference for discrimination can cause those without such a preference to select the action associated with the Pareto dominated equilibrium. By contrast our stage game has a single equilibrium in material payoffs. Actions other than the equilibrium actions are played only for the purpose of improving future outcomes. Therefore, in Basu the presence of special types of agents induces a more defensive posture in other agents, in our paper the resulting behavior is a more aggressive posture. In other words, the inefficiencies in Basu are driven by fear of aggressive behavior of the opponent and in our model the inefficiencies are driven by the aggressive behavior of unbiased agents induced by material gains.

Rohner (2008) also introduces a game theoretic model which seeks to link the social composition of a heterogenous population with economic outcomes in that population. Like we do here, Rohner presents a reputation model where types are unobservable. However, in Rohner's model no agent has a preference for discrimination but rather differential access to information. While agents in our model wish to obtain a reputation for biased preferences, agents in Rohner's model wish to avoid obtaining a reputation for toughness. The differences also include that Rohner uses contest functions, we use a reduced bargaining game; Rohner's stage game is infinitely repeated whereas ours is only repeated twice; and in our paper information regarding histories is very precise and it is very coarse in Rohner. Despite these differences, our main results are relatively congruent. Our Proposition 1 shows that the economic inefficiency in a society tends to be increasing in the heterogeneity of that society. Similarly, Proposition 4 of Rohner shows that social tension is increasing in (what we refer to as) the heterogeneity of the population. Given the large differences between Rohner and the present paper, it is somewhat surprising that, roughly, we come to the same conclusion regarding social heterogeneity and economic outcomes.

2 The Model

We study a sequential chicken stage game repeated for $T = 2$ periods. The stage game payoffs are described by the following game tree $\mathcal{T}$:

[12] Esteban and Ray (1994, 1999) provide an axiomatization relating the amount of polarization (and hence potential for conflict) in a society to the distribution of characteristics of individuals in that society. Although the authors accommodate a more rich profile of characteristics than considered here, we focus on the individual behavior which might yield such a conflict.



where b is strictly larger than one.[13] In each repetition of the stage game, player 1 chooses an action of either Hawk (H) or Dove (D). In the event that player 1 selects H, player 2 chooses between H and D. We do not allow transfers between agents.

There is a continuum of players $i \in [0,1]$. Each player is a member of exactly one of two social groups. This group identity is described by the social identity parameter $\theta \in (0.5, 1)$. All agents such that $i \in [0, \theta] = M$ are in the majority group and all agents such that $j \in (\theta, 1] = m$ are in the minority group. In each period, agents are matched to play the stage game where the matching probability is uniform on the population. In each match, the probability of being a player 1 is identical to that of being a player 2. If two players i, j such that $i \in M$ and $j \in m$ are matched, we refer to this as an outgroup match, otherwise it is an ingroup match.

In each group, there are two types of players: unbiased and biased. The unbiased players have their payoffs described by $\mathcal{T}$. Biased players always play H in an outgroup match and have payoffs as described by $\mathcal{T}$ in an ingroup match. Group membership is observable however the preferences of the opponent are unobservable. The ex-ante fraction of biased players, in each group, is γ. The entire game Γ is therefore described by $\Gamma = (\mathcal{T},\, b,\, \theta,\, \gamma)$.

To simplify the subsequent analysis, note that in every ingroup match the subgame perfect equilibrium of the stage game is played: player 1 plays H and the player 2 plays D. No player has an incentive to deviate. Player 2 gains no future benefit by playing H. Player 1, knowing this, plays H. Therefore, we take the ingroup matches as given and focus exclusively on the behavior in outgroup matches.

Player i's action is denoted $a \in \{H, D\} = A$. We define the condition of the match as $c \in \{1, H\} = C$. Here $c = 1$ indicates that i is a player 1. Likewise, $c = H$ indicates

[13] All of the following would hold if instead we exchanged b and 1 with x and $1-x$ respectively where $x = \frac{b}{b+1} > \frac{1}{2}$.

that i is a player 2 whose opponent played H. The history of the matched opponent is perfectly observed. We can write the relevant set of histories for player i in the first period as $h^i \in \mathcal{H}^i = \{I, H1, D1, HH, DH, E\}$. The first element refers to an ingroup match. The following two elements refer to playing H and D as a player 1. Likewise the next two refer to playing H and D as a player 2 against a player 1 who played H. The last element refers to a player 2 matched against a player 1 who played D. We define the set of player histories $\mathcal{H}_D$ in which the action of D has been observed in an outgroup match

$$\mathcal{H}_D = \{D1, DH\}$$

A first period strategy for player i is a mapping $\sigma_1^i : C \to \Delta A$ and the second period strategy as a player 1 is a mapping $\sigma_2^i : C \times \mathcal{H}^j \to \Delta A$. We define $\sigma^i = \sigma_1^i \times \sigma_2^i$. We also define $\sigma = \times_{i \in [0,1]} \sigma^i$. We denote $\sigma^i(\cdot)$ as the probability that H is played. After a history of h^i the posterior belief that player i is biased is denoted $p^i(h^i)$. Players maximize the sum of expected utility payoffs. We assume no discounting. In period 2, for a given history h_1^j and condition c, player i's expected payoff from the profile of strategies is defined to be $U_2^i(\sigma|c, h^j)$. In period 1, for a given c, player i's expected payoff from the profile of strategies in periods 1 and 2 is defined to be $U_1^i(\sigma|c)$.

Recall that our goal is to model a general conflict situation with as few asymmetries as possible. Specifically, we designed the model in such a way that the groups are as meaningless as possible. As such, we have assumed that each group has an identical fraction of biased players (γ). We have also assumed that the probabilities that an agent is designated as a player 1 and player 2 are equal for agents in each group. Despite these symmetry assumptions, we still observe the inefficiencies associated with a social conflict. Indeed our assumptions regarding γ are weaker than warranted by the experimental evidence. For instance, Cho and Connelley (2002) find that the competitiveness of an outgroup setting is associated with a higher degree of identification of subjects. We interpret this finding as evidence of a positive relationship between γ and b. Although we do not assume such a relationship, our results would be stronger if we did.

In our solution concept, we use the following definition:

Definition 1 *Beliefs $p^j(h^j)$ satisfy condition $(*)$ if $h^j \in \mathcal{H}_D$ then $p^j(h^j) = 0$.*

Condition $(*)$ requires beliefs to be updated in an intuitive manner. On or off-the-equilibrium path, it requires that if player j ever played D in an outgroup match, opponents ascribe probability 0 to j being biased.

Now we define the notion of equilibrium which we will use throughout the paper.

Definition 2 *A strategy profile σ is a Symmetric Perfect Bayesian Equilibrium (SPBE) if:*

$$(i)\;\; U_1^i(\sigma|c) \geq U_1^i(\widetilde{\sigma}^i, \sigma^{-i}|c) \text{ for every } i,\ \widetilde{\sigma}^i \neq \sigma^i \text{ and } c \in \{1, H\}$$
$$(ii)\;\; U_2^i(\sigma|c, h^j) \geq U_2^i(\widetilde{\sigma}^i, \sigma^{-i}|c, h^j) \text{ for every } i,\ \widetilde{\sigma}^i \neq \sigma^i,\ c \in \{1, H\} \text{ and } h^j \in \mathcal{H}^j$$
$$(iii)\;\; \text{for any } i, k \in M \text{ and any } j, l \in m,\ \sigma^i = \sigma^k \text{ and } \sigma^j = \sigma^l$$

Furthermore, beliefs $p^j(h^j)$ must satisfy condition $(*)$ and are updated using Bayes Rule wherever possible, for all j and $h \in \mathcal{H}$.

Definition 2 is a slightly more restrictive version of a Perfect Bayesian Equilibrium (PBE). Condition (i) requires that period 1 actions are optimal, as both a player 1 and 2, given any set of initial beliefs. Condition (ii) is the analogous requirement for period 2. Condition (iii) requires that every member of a group use the same strategy. Note that in equilibrium, this requirement only bites when players are indifferent between actions. In such a case, condition (iii) allows us to break ties in a manner consistent with a social identity interpretation. Condition (iii) also allows us to refer to strategies for the group rather than for the individual. For instance, $\sigma_1^M(1)$ refers to the strategy of the majority group as a first period player 1. Finally, we require that beliefs are updated using Bayes Rule wherever possible and that a player who selected D in the first period is known with certainty to be unbiased.

Finally note that we speak of *aggressive discrimination* whenever the actions (H, H) are observed. This terminology is appropriate as the outcome (H, H) never occurs in equilibrium in an ingroup match. More generally we refer to a play of H (in any period) as aggressive play. Note that all unbiased players always play D as a second player in period 2 ($\sigma_2^i(H, h^j) = 0$ for all $h^j \in \mathcal{H}$ and $i \in \{m, M\}$). As there is no confusion, we write $\sigma_2^i(1, h^j)$ as $\sigma_2^i(h^j)$ in order to conserve notation.

Again, note that in a game without biased players ($\gamma = 0$), the unique subgame perfect equilibrium is to play H as a player 1 and play D against H as a player 2. When $\gamma > 0$, there are conditions under which an unbiased player will optimally destroy surplus in order to secure a reputation for being a biased player. This destruction of surplus can take one of the following two forms.

Definition 3 *Agent i exhibits Reputation as a Player 2 ($P2$) if the SPBE is such that:*

$$\sigma_1^i(H) > 0$$

If player i exhibits $P2$, he will play H with positive probability in response to a player 1 selecting H, even though playing H means forgoing a certain payoff of 1 in order to have more favorable future matches. However, another type of reputation can be observed when the agent is a player 1.

Definition 4 *Agent i exhibits Reputation as a Player 1 ($P1$) if the $SPBE$ is such that:*

$$\sigma_1^i(1) > 0$$
$$(1-\gamma)(1-\sigma_1^j(H))b < 1$$

If player i exhibits $P1$, he will play H with positive probability as a player 1, even though playing D would yield a larger expected payoff in the first period. In order to compare the two definitions, note that if an agent displays $P2$ then the player exchanges a first period stage game payoff of 1 for a payoff of 0. However, a player 1 selecting H could be myopically optimal if the matched opponent is sufficiently likely to play D. In this case, we could not claim that the player is motivated by reputation concerns. Therefore, we require the second condition so that the first period action does not maximize first period payoffs.

The following lemma states that $P1$ and $P2$ will never both occur in any $SPBE$.

Lemma 1 *There are no parameter values such that if one player exhibits $P1$ ($P2$) then any player exhibits $P2$ ($P1$).*

Proof: See Appendix.

To see that parameter values cannot be such that $P1$ and $P2$ are both present, note that if a player exhibits $P1$ then the fraction of biased players is sufficiently high, $\gamma \geq \gamma'$, otherwise the definition of $P1$ cannot be satisfied. The smallest such fraction of biased players γ' makes the exhibition of $P2$ by any player unprofitable. Similarly, if a player exhibits $P2$ then it is sufficiently unlikely that a future opponent is a biased player, $\gamma \leq \gamma''$, otherwise $P2$ would not be profitable. However the largest such fraction of biased players γ'' renders playing H as a player 1 myopically optimal, thus the agent cannot exhibit $P1$.

3 Comparative Statics: Social Fragmentation and Inequitable Environments

In this section, we present our main results. We examine the relationship between social conflict, as measured by inefficiency, and social heterogeneity. We also examine the relationship between social conflict, as measured by inefficiency, and inequitable environments. These results provide an individually rational explanation for the relevant empirical results.

Many authors use the fragmentation index, defined as the probability that two randomly selected people are from different social groups, as a measure of social heterogeneity. In the present context, this would imply that the fragmentation index is $2\theta(1-\theta)$. By contrast we use $1-\theta$ as a measure of social heterogeneity. Both measures are maximized on $[0, 0.5]$ at $\theta = 0.5$ and are strictly decreasing in θ. Furthermore, nothing is gained by considering the more complicated measure of heterogeneity.

To formally state our results, we first define the total efficiency loss in the $SPBE$ as $\mathcal{I}(b,\theta)$. This quantity is the probability of aggressive discrimination ((H,H) outcomes) in either period multiplied by the total material surplus which could have been achieved, $b+1$. We state $\mathcal{I}(b,\theta)$ as explicitly depending on θ and b but not on γ (fraction of biased players), as we will shortly explore the implications of varying the first two but not the last parameter. Furthermore, γ is hard to measure and to our knowledge, no empirical papers have studied the matter.

Definition 5 *$\mathcal{I}(\theta,b)$ is the total efficiency loss in the $SPBE$:*

$$\mathcal{I}(b,\theta) = (b+1)\left[P((H,H)\ in\ t=1) + P((H,H)\ in\ t=2)\right],$$

Note that $\mathcal{I}$ is not a measure of social welfare. Specifically, $\mathcal{I}$ is not the average of the utilities of the agents in the game. The value of $\mathcal{I}$ is intended to provide a measure of the material payoffs not captured in the bargaining procedure. While it is often assumed that a social planner seeks to maximize the utility of every agent, with standard assumptions regarding utility, this condition is equivalent to maximizing the material surplus of each agent. However, in our case, these two notions are not identical. Indeed, to be consistent with the spirit of the social planner, we would seek to maximize the volume of trade rather than accommodate the discriminatory preferences of the biased players. The value of $\mathcal{I}$ provides a measure of the material outcomes in the population and we therefore consider it to be the most appropriate objective function.

The next result shows that there exists a level of heterogeneity such that for every smaller value of heterogeneity, $\mathcal{I}$ is strictly increasing in heterogeneity. Although the statement of Proposition 1 is rather intricate, it roughly states that inefficiency tends to be increasing in heterogeneity. We now state this formally.

Proposition 1 *For all (b,γ), there exists a $1-\theta^* > 0$ such that for all $1-\theta < 1-\theta^*$ inefficiency $\mathcal{I}$ is strictly increasing in $1-\theta$.*

Proof: *See Appendix.*

The intuition behind the proposition is as follows: when heterogeneity increases, the occurrence of outgroup matches also increases. Within these outgroup matches are matches involving only biased players and matches involving at least one unbiased player. Obviously, in the biased-only matches, an increase in heterogeneity will, by assumption, imply a greater inefficiency. Also, matches involving exactly one unbiased player will imply a greater inefficiency unless every unbiased agent always plays D. However, unbiased-only matches will also exhibit inefficiency if either player exhibits $P1$ or $P2$ and this inefficiency is increasing in heterogeneity.

To better understand the nuanced statement of the proposition we consider the four possibilities of the relationship between inefficiency and $1-\theta$ for a given b and γ. In each of these four cases, there is no inefficiency at $1-\theta=0$. A particularly simple case is illustrated by Figure 1. Here b and γ are such that for every $1-\theta$, inefficiency is strictly and continuously increasing.

FIGURE 1 HERE

As illustrated in Figure 1, there exist values of b and γ for which a single qualitative $SPBE$ describes the behavior for all values of heterogeneity. However, it could also be the case that, as heterogeneity increases, a qualitatively different $SPBE$ can occur. As $1-\theta$ gets larger, the minority reputation becomes less valuable and the majority reputation becomes more valuable. Therefore, only two types of such "jumps" can occur as $1-\theta$ becomes larger. Either the majority does not exhibit reputation for any heterogeneity whereas the minority exhibits reputation for small $1-\theta$ and for large values does not exhibit reputation (Figure 2). Or it can be that the minority always exhibits reputation and for small $1-\theta$ the majority does not display reputation and for large values, the majority does (Figure 3).

FIGURE 2 HERE
FIGURE 3 HERE

As illustrated in Figures 1, 2 and 3, there exist parameter values such that inefficiency strictly increases almost everywhere from $1-\theta \in (0, 0.5)$ with at most one point of discontinuity. In other words, for these values there does not exist an interior extrema. However, there also exists parameter values where such an interior extrema exists. Figure 4 illustrates a possible relationship.

FIGURE 4 HERE

Here in Figure 4, for $1-\theta$ less than 0.49 the minority displays $P2$ and the majority does not. However, for $1-\theta$ greater than 0.49 neither the majority nor the minority displays $P2$. There is an interior maximum of inefficiency at 0.485. Therefore, for such a case to hold we need the interior maximum on the inefficiency function where only m displays $P2$ to occur at a smaller degree of heterogeneity than the point of discontinuity. Although the extremum is always "close" to 0.5, it still remains that there is a small region for which inefficiency is decreasing in heterogeneity.[14]

In order to relate the figures to the proposition, note that in the cases of Figures 1 and 3 inefficiency is everywhere strictly increasing in heterogeneity, therefore $1-\theta^*=0.5$. In the case of Figure 2, $1-\theta^*$ is at the point of downward continuity. And in Figure 4, $1-\theta^*$ is at the interior maximum.[15]

[14] Note that this interior maximum only ranges from $1-\theta^*=0.4833$ to 0.5.

[15] Here only m displays $P2$. The mixing probability of m is decreasing in heterogeneity and this effect dominates when inefficiency otherwise becomes nearly constant. When both m and M display $P2$, the probability mix of M increases in heterogeneity, and the changes in the mixing of m are offset by the mixing of M.

This completes our discussion of the relationship between social conflict and social heterogeneity. We now turn to the relationship between social conflict and the inequitability of the environment. We show that increasing the inequitability of the environment leads to an increase in social conflict as measured by inefficiency.[16]

Proposition 2 *$\mathcal{I}$ is strictly increasing in b.*

The map $\frac{\mathcal{I}}{b+1}$ is a function in b with five points of upward discontinuity. The intuition behind the result is as follows: as b increases, playing H becomes more attractive. This leads to an increase in the probability which unbiased agents play H and this increases inefficiency. Figure 5 illustrates a typical relationship between $\frac{\mathcal{I}}{b+1}$ and b.[17]

FIGURE 5 HERE

Our model provides an explicit account of the individual behavior which drives the social conflict. Specifically, the presence of biased players means that inefficiency is increasing in the inequitability of the environment. Furthermore, Proposition 2 is free of the built-in inefficiency present in Proposition 1. Any increases beyond the smallest value of $\frac{\mathcal{I}}{b+1}$ in Figure 5 are driven exclusively by the behavior of the unbiased agents.

4 Characterization of SPBE

We now characterize the $SPBE$. We start by characterizing the $SPBE$ where b is small and therefore neither group displays $P2$ (Proposition 3). We then characterize the $SPBE$ where b is intermediate and therefore the minority group displays $P2$ but the majority does not (Proposition 4). Subsequetntly we characterize the $SPBE$ where b is large and therefore both groups displays $P2$ (Proposition 5).[18]

Proposition 3 *If $\frac{2}{(1-\theta)(1-\gamma)}+1 > \frac{2}{\theta(1-\gamma)}+1 > b$ then the unique $SPBE$ is such that $\sigma_1^i(H) = 0$, $\sigma_2^i(h_1^j) = 0$ if $h_1^j = HH$ and $\sigma_2^i(h_1^j) = 1$ if $h_1^j \in \mathcal{H}_D$ for all $i \in \{m, M\}$. Furthermore, if in addition to $\frac{2}{\theta(1-\gamma)}+1 > b$ it is also the case that*

(i) $\gamma < \frac{b-1}{b}$ then the unique $SPBE$ is such that $\sigma_1^i(1) = 1$, $\sigma_2^i(h_1^j) = 0$ if $h_1^j \neq HH$ for all $i \in \{m, M\}$.

(ii) $\gamma \in (\frac{b-1}{b}, \overline{\gamma}_M)$ where $\overline{\gamma}_M = \frac{b-1+\left(\frac{1-\theta}{2}\right)(b-1)}{b+\left(\frac{1-\theta}{2}\right)(b-1)} > \frac{b-1}{b}$ then the unique $SPBE$ is such that for $i \in N$, $\sigma_1^i(1) = 1$, $\sigma_2^i(h_1^j) = 0$ if $h_1^j = \{I, H1, E\}$.

(iii) $\gamma \in (\overline{\gamma}_M, \overline{\gamma}_m)$ where $\frac{b-1+\left(\frac{\theta}{2}\right)(b-1)}{b+\left(\frac{\theta}{2}\right)(b-1)} = \overline{\gamma}_m > \frac{b-1+\left(\frac{1-\theta}{2}\right)(b-1)}{b+\left(\frac{1-\theta}{2}\right)(b-1)} = \overline{\gamma}_M > \frac{b-1}{b}$ then the unique $SPBE$ is such that $\sigma_1^m(1) = 1$, $\sigma_1^M(1) = 0$, $\sigma_2^i(h_1^j) = 0$ if $h_1^j \in \{I, H1, E\}$.

[16] The proof is available from the author upon request.

[17] To better understand the values for which the $SPBE$ is not unique, see Proposition 8.

[18] In each of these propositions the $SPBE$ is unique. In the appendix, we characterize the $SPBE$ where it is not unique.

(iv) $\gamma > \overline{\gamma}_m$ *then the unique SPBE is such that for all* $i \in \{m, M\}$, $\sigma_1^i(1) = 0$, $\sigma_2^i(h_1^j) = 0$ *for* $h_1^j \notin \mathcal{H}_D$

Proof: See Appendix.

Proposition 3 states that for small b, neither group will display $P2$ because it will not be profitable to play H as a player 2 in order to enter the second period with a posterior even as high as 1. Descriptively, for small γ, (case (i)) both groups play aggressively as a player 1. The only situation where the strategy of playing H as a player 1 is not $SPBE$ is when the second period opponent played H in the first period. This is because a history of HH is the only history leading to a posterior greater than $\frac{b-1}{b}$. In both periods, the optimal strategy turns out to be the one which myopically maximizes payoffs. For case (ii), both groups display $P1$. In the first period, both groups play H as a player 1 rather than D, despite the fact that the latter yields a higher stage game payoff. Here, D is myopically superior to H despite the fact that first period player 2 does not play H. The myopic action is not selected because the first period player 1 selecting D forfeits reputation in the second period and it is sufficiently valuable. For case (iii), only m displays $P1$. This asymmetry arises because M does not find it profitable to maintain its reputation. For case (iv), neither player selects H in the first period as a player 1 because of the high likelihood of being matched with a biased player. No unbiased agent plays H as a second period player 1 unless the opponent has played D in the first period.

Proposition 4 *If* $\frac{2}{(1-\theta)(1-\gamma)} + 1 > b > \frac{2}{\theta(1-\gamma)} + 1$ *then the unique SPBE must be that* $\sigma_1^M(H) = 0$, $\sigma_2^m(h^M) = 0$ *for* $h^M = HH$, $\sigma_2^M(h^m) = 1 - \frac{1}{\left(\frac{\theta}{2}\right)(b-1)(1-\gamma)}$ *for* $h^m = HH$, $\sigma_1^m(H) = \beta^* \in (0,1)$ *such that* $p^m(HH) = \frac{b-1}{b}$ *where* $\beta^* = \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1}{b-1}\right)$ *and for* $i \in \{m, M\}$, $\sigma_1^i(1) = 1$, $\sigma_2^i(h^j) = 1$ *for* $h^j \notin HH$.

Proof: See Appendix.

For intermediate b, the minority finds it profitable to play H as a player 2 with probability strictly between 0 and 1 in order to enter the next period with a posterior of $\frac{b-1}{b}$. Unlike m, M never finds it profitable to play H as a first period player 2 even if it secures a posterior of 1 in the second period. Therefore, m displays $P2$ and M does not. Note that by Lemma 2, we can restrict attention to $\gamma < \left(\frac{b-1}{b}\right)^2$ and therefore every agent plays H as a first period player 1. By being able to restrict attention to $\gamma < \left(\frac{b-1}{b}\right)^2$ we do not have the number of cases that we had in the Proposition 3. Every first period player demands H and in the second period demands H in response to a history of $H1$ as the probability of a biased player is sufficiently low.

Proposition 5 *If* $b > \frac{2}{(1-\theta)(1-\gamma)} + 1 > \frac{2}{\theta(1-\gamma)} + 1$ *then the SPBE must be that* $\sigma_2^m(HH) = 1 - \frac{1}{\left(\frac{1-\theta}{2}\right)(b-1)(1-\gamma)}$, $\sigma_2^M(HH) = 1 - \frac{1}{\left(\frac{\theta}{2}\right)(b-1)(1-\gamma)}$, $\sigma_1^i(H) = \beta^* \in (0,1)$ *such that* $p^i(HH) = \frac{b-1}{b}$ *and* $\sigma_2^i(h^j) = 1$ *where* $h^j \in \{I, E, H1\}$ *and* $\sigma_1^i(1) = 1$.

Proof: See Appendix.

For large b, both groups exhibit $P2$. Much of the reasoning above involving m now holds for both groups. Again, by Lemma 2, we restrict attention to $\gamma < \left(\frac{b-1}{b}\right)^2$. Both groups play H as a player 1 in the first period and play H as a second period player 1 against a player with a history of $H1$.

Propositions 3, 4 and 5 characterize the $SPBE$. Figure 6 demonstrates, given a value of θ, the regions of b and γ which are consistent with a $SPBE$.

FIGURE 6 HERE

The northwest portion of the graph corresponds to the values of b and γ which yield the $SPBE$ as described in Proposition 5. In other words, for high b and low γ, both groups exhibit $P2$. The band to the right of this corresponds to the parameters which yield the $SPBE$ as described in Proposition 4. To the right of this band, there are three small bands which correspond to the parameters which yield the $SPBE$ as described in Proposition 3 (i), (ii) and (iii). Finally, the southeast portion of the graph corresponds to the values of b and γ which yield the $SPBE$ as described in Proposition 3 (iv).

We now provide the following example in order to facilitate a more intuitive understanding of the model. While we vary b, we assume specific values for θ and γ. In the first case ($b = 3$) neither group displays $P2$, in the second case ($b = 5$) only the minority displays $P2$ and in the final case ($b = 7$) both groups display $P2$.

Example 1 *Consider an SPBE where the majority group composes* 60% *of the population* ($\theta = 0.6$)*, each group contains a* 10% *fraction of biased players* ($\gamma = 0.1$) *and the prize b is either* 3, 5, *or* 7*:*

(i) *In the case that $b = 3$, the SPBE strategies look similar to that of the unperturbed game.*[19] *The only difference being that those matched with a player who played H as a player* 2 *in the first period will play D as a player* 1. *The SPBE strategies are:*

$$\begin{aligned}\sigma_1^i(1) &= 1 \textit{ and } \sigma_1^i(H) = 0 \textit{ for } i \in \{m, M\}\\ \sigma_2^i(1, h^j) &= 0 \textit{ if } h^j = HH \textit{ for } i \in \{m, M\}\\ \sigma_2^i(1, h^j) &= 1 \textit{ if } h^j \neq HH \textit{ for } i \in \{m, M\}\end{aligned}$$

When $b < \frac{2}{\theta(1-\gamma)} + 1 \approx 4.7$ *(and thus* $b < \frac{2}{(1-\theta)(1-\gamma)} + 1 \approx 6.6$*) the minority (majority) has no incentive to deviate from* $\sigma_1^i(H) = 0$. *Here, in both majority and minority groups, only biased players destroy surplus.*

(ii) *In the case that $b = 5$ the incentives (and therefore first period strategies) are identical to the $b = 3$ case for M, but not for m. Here $\sigma_1^m(H) = 0$ cannot be part of an SPBE.*

[19] The interested reader is referred to the appendix for Propositions 3 (i), 4 and 5 respectively for the proofs of the strategies given in parts (i), (ii) and (iii) of the example.

However it also cannot be that $\sigma_1^m(H) = 1$ *because this would imply* $p^m(HH) = \gamma$ *and thus* $\sigma_2^M(HH) = 0$ *for* M *as* $\gamma < \frac{b-1}{b}$. *Therefore* $\sigma_1^m(H)$ *must be such that* $p^m(HH) = \frac{b-1}{b} = \frac{4}{5}$. *This is the posterior which makes the agent as a player* 1 *indifferent between* H *and* D. *This mixing probability occurs at* $\sigma_1^m(H) = \frac{\gamma}{(1-\gamma)(b-1)} = 0.028$.

(iii) In the case that $b = 7$, *both* m *and* M *will mix such that* $p^i(h) = \frac{6}{7}$. *This mixing probability occurs at* $\sigma_1^i(H) = 0.0185$. *Similarly both groups must mix as a second period player* 2 *in order to keep the first period player* 2 *indifferent between playing* H *and* D *against an* H.■

5 SPBE Results

We now characterize some basic properties of the $SPBE$. We illustrate the underlying asymmetry in payoffs by showing that the majority always does strictly better for parameter values such that both groups have identical equilibrium strategies. We also show that reputation is always more valuable for the minority players. Hence, we find that minority players will always exhibit weakly more aggressive behavior in the first period, than do majority players.[20]

Although the $SPBE$ is generically unique, depending on the particular parameters of the game, the equilibrium can have significantly different properties. For some parameter values, $SPBE$ strategies and therefore equilibrium payoffs can exhibit some asymmetry. However, there is also a basic asymmetry inherent in our model, which is best illustrated when attention is restricted to strongly symmetric strategies - that is, first period strategy profiles which are identical across groups. This motivates the following definition:

Definition 6 *Let* σ^Γ *be the* $SPBE$ *of* Γ. *Then* Γ *is strongly symmetric if the first period strategies in* σ^Γ *can be written without reference to group membership.*

We say that a game is strongly symmetric if its parameters are such that all players have identical first period equilibrium strategies. However, even in such a markedly symmetric environment, the majority does strictly better than the minority, as the next result shows.[21]

Proposition 6 *If* Γ *is strongly symmetric, the majority has a strictly higher ex-ante payoff than the minority.*

This result follows from the fact that majority group members are more likely to be in an ingroup match than minority group members. If Γ is strongly symmetric, an ingroup match is more profitable than an outgroup match. Additionally, the posteriors for a given history are identical across groups which implies that second period strategies are also identical. These facts combine to produce the result.

[20] As this paper proposes a general model of social conflict, the only assumed asymmetry involves the probability of an outgroup match. The following results crucially depend on this symmetry. In modeling a particular situation, where the symmetry assumptions are not justified, a modified version of our model will suffice.

[21] The proof is available from the author upon request.

Note that this result crucially depends on the existence of the biased players ($\gamma > 0$). In the unperturbed game, members of both groups have an expected payoff of $b + 1$. Therefore if there are biased players then we observe no payoff differences based on group membership.

Although Proposition 6 demonstrates that for strongly symmetric Γ, the majority always does better than the minority, the majority can do worse if the equilibrium strategies across groups are sufficiently asymmetric. We now present an example of such an $SPBE$ where the minority has a larger expected payoff than the majority.

Example 2 *Suppose that $\theta = 0.6$, $b = 2$, and $\gamma = 0.55$. The $SPBE$ which corresponds to these parameter values is described by Proposition 3 (iii). In this $SPBE$ the minority displays $P1$ and the majority does not. Therefore, the $SPBE$ is not strongly symmetric. If we let E^i represent the ex-ante payoff of player i, then it follows that:*

$$E^m = 2.825 > E^M = 2.687$$

■

The above example demonstrates the necessity of the strong symmetry assumption in Proposition 6. The intuition behind Example 2 is that the majority does not obtain a reputation while the minority does. Hence, the minority does sufficiently better than the majority in outgroup matches and so the minority does better overall.

In Example 2, the minority exhibits more aggressive behavior in the first period than does the majority. This is a general feature of the $SPBE$, as we show in the next proposition. Specifically, we show that the minority is always at least as likely as the majority to play H as a first period player 1 and player 2.

Proposition 7 *In every generic $SPBE$, a minority member m plays at least as aggressively as a majority member M :*

$$\sigma_1^M(1) \leq \sigma_1^m(1) \text{ and } \sigma_1^M(H) \leq \sigma_1^m(H).$$

Proof: *See Appendix.*

The intuition behind Proposition 7 is that reputation is more valuable to the minority than the majority, as the former is more likely to be in a second period outgroup match. Note that we assume very little asymmetry between the groups; we assume uniform matching, an equal probability of being a player 1 and 2 in each period for both groups, and an equal fraction of biased players in each group. The only assumed asymmetry relates to the composition of society. One could imagine a situation where these symmetry assumptions are not appropriate. However, the purpose of this paper is to investigate social outcomes when assuming as little between group asymmetry as possible. Therefore, we do not explore these issues.

We interpret Proposition 7 to be consistent with psychology literature related to the group identity of majorities and minorities. Psychologists find that minorities have a stronger group identity than do majorities.[22] As a result of this stronger identity, we expect stronger behavior; and in the context of our model, stronger behavior means more aggressive play.

6 Concluding Remarks

We have modeled a social setting containing some agents as described by our interpretation of the social identity literature. We have demonstrated that the struggle for resources, in the presence of agents with a taste for discrimination, can induce agents without such a taste to aggressively discriminate. The paper showed that for games which induce a sufficiently symmetric equilibrium, the majority has a greater ex-ante payoff than the minority. Additionally, we showed that the minority always plays the game at least as aggressively as the majority. We interpret this result as consistent with the experimental findings that minorities have stronger group identities than do majorities.

We showed that our model is consistent with empirical papers which find a relationship between social conflict and a measure of the social heterogeneity. Our results are also consistent with the literature identifying a relationship between social conflict and the inequitability of the environment. Indeed our model provides an individually rational explanation for these results. One possible alternative explanation for the empirical results is that *every* member of the society has a preference for better material outcomes for ingroup members, however the fraction of agents intransigently playing H in outgroup matches is increasing in b or $1-\theta$. We regard our explanation as superior to this alternate explanation, as the latter effectively assumes the result.

It should be noted that there remain interesting, unanswered questions. For instance, it could be fruitful to investigate a model in which information is less than perfect. Obviously some information is required for these results to hold, however it might prove productive to investigate weaker assumptions. It would also be interesting to model the presence of three of more groups. It could be the case that there is be an interaction among the groups which is not present with only two groups.

In light of the recent interest in fairness, it is useful to note that there exist aspects of every society which could be described as unfair. In every society, economic inequalities persist on the basis of race, religion and gender. We argue that, in economic situations, *unfairness* is at least as important than *fairness*. It is also our opinion that the social identity literature is useful in providing direction for the study of unfairness.

[22] See Gurin et. al. (1999).

7 Appendix

The appendix is arranged as follows. First we prove some technical results which we use subsequently. Then we prove our characterization of the $SPBE$ where it is unique (Propositions 3, 4 and 5). Next we prove Proposition 7 then Proposition 1. Finally we characterize the $SPBE$ where it is not unique (Proposition 8).

Before we begin, note that characterizing the $SPBE$ boils down to characterizing $\sigma_1^i(1)$, $\sigma_1^i(H)$ and $\sigma_2^i(h^j)$ for all $i \in M(m)$, $j \in m(M)$ and all $h^j \in \mathcal{H}$. Also, we define $v_i(h^i)$ as the expected payoff of i entering period 2 with a history of h^i. The difference in continuation payoffs can be summarized by the difference in expected payoffs as a second period player 2 as strategy for an ingroup and outgroup as a player 1 are independent of the player's own history. The following two lemmas provide useful technical results and together prove Lemma 1.

Lemma 2 *If $\gamma \geq \left(\frac{b-1}{b}\right)^2$ then $b < \frac{2}{\theta(1-\gamma)} + 1$*

Proof: Note that $b < \frac{2}{\theta(1-\gamma)} + 1$ is equivalent to

$$\gamma > \frac{\theta(b-1)-2}{\theta(b-1)}. \tag{1}$$

With a domain of $\theta \in [0.5, 1]$, the right hand side of (1) attains a maximum at $\theta = 1$. Therefore,

$$\frac{b-3}{b-1} \geq \frac{\theta(b-1)-2}{\theta(b-1)}.$$

Notice that for all $b > \frac{1}{3}$

$$\left(\frac{b-1}{b}\right)^2 > \frac{b-3}{b-1} \tag{2}$$

and so (2) implies that if $\gamma \geq \left(\frac{b-1}{b}\right)^2$ then it must be that $\gamma > \frac{\theta(b-1)-2}{\theta(b-1)}$. Therefore, the lemma is proved.■

Lemma 3 *$b < \frac{2}{(1-\theta)(1-\gamma)} + 1$ ($b < \frac{2}{\theta(1-\gamma)} + 1$) if and only if M (m) does not exhibit $P2$.*

Proof: It must be that $\sigma_1^M(H) > 0$ if and only if

$$1 + \left(\frac{1-\theta}{2}\right) \geq 0 + \left(\frac{1-\theta}{2}\right)(b(1-\gamma)+\gamma).$$

The left side represents the expected utility heading into the second period with a posterior of 1 and the right side represents the expected utility entering the second period known to be unbiased. The analogous reasoning holds for m.■

Corollary 1 *$P2$ cannot occur in any $SPBE$ if $\gamma \geq \left(\frac{b-1}{b}\right)^2$*

This corollary follows from Lemmas 2 and 3 since $b \geq \frac{2}{\theta(1-\gamma)} + 1$ ($b \geq \frac{2}{(1-\theta)(1-\gamma)} + 1$) is a necessary condition for m (M) to display $P2$. This is the lower bound of b for which a player would sacrifice an immediate payoff of 1 in order to find entering the second period with a posterior of 1. This allows us to restrict attention to the $SPBE$ which contains $P2$ to $\gamma < \left(\frac{b-1}{b}\right)^2$. Furthermore, note that the second condition for $P1$ requires that $(1-\gamma)(1-\sigma_1^j(H))b < 1$. This implies that $P1$ only occurs when $\gamma \geq \frac{b-1}{b}$ as $\frac{b-1}{b} > \left(\frac{b-1}{b}\right)^2$. In other words, there are no parameter values for which the $SPBE$ exhibits both $P1$ and $P2$, which proves Lemma 1.

Proof of Proposition 3: In any $SPBE$ with

$$\frac{2}{(1-\theta)(1-\gamma)} + 1 > \frac{2}{\theta(1-\gamma)} + 1 > b$$

it must be that $\sigma_1^i(H) = 0$, by Lemma 3. This implies posteriors of $p^i(h^i) = 1$ for $h^i = HH$ and $p^i(h^i) = 0$ for $h^i = DH$ and strategies $\sigma^j(h^i) = 0$ for $h^i = HH$. If $\sigma_1^i(H) = 0$ then $p(HH) = 1$ and therefore $\sigma_2^i(HH) = 0$. It also must be that $\sigma_2^i(h_1^j) = 1$ if $h_1^j \in \mathcal{H}_D$. Furthermore, there can be no other $SPBE$ strategies.

(i) It will be that $\sigma_2^i(h^j) = 1$ if $h^j \in \{I, E\}$ because $p^j(h^j) = \gamma < \frac{b-1}{b}$. It remains to determine $\sigma_1^i(1)$ and $\sigma_2^i(H1)$. It cannot be that $\sigma_1^i(1) = 0$ as this would imply that $p^i(H1) = 1$ and $\sigma_2^j(H1) = 0$. However, a deviation is easy to find as both the first period stage game payoffs are higher for H:

$$b(1-\gamma) > 1 \tag{3}$$

and

$$v_i(H1) > v_i(D1) \tag{4}$$

because $p^i(H1) = 1 > \frac{b-1}{b} > p^i(D1) = 0$. Therefore, $\sigma_1^i(1) \neq 0$. It cannot be that $\sigma_1^i(1) = \alpha^* \in (0,1)$ because the first period player 1 cannot be indifferent between playing H and D as a player 1. Therefore, $\sigma_1^i(1) = 1$ and $p^i(H1) = \gamma$ so that $\sigma_2^i(h^j) = 1$. Furthermore, there can be no other $SPBE$ strategies.

(ii) Here it cannot be that $\sigma_1^i(1) = 0$ as this would imply that $p^i(H1) = 1$, $\sigma_2^i(h^j) = 0$ for $h^j = H1$. However, a deviation exists for M:

$$\begin{gathered} b(1-\gamma) + v_M(H1) > 1 + v_M(D1) \\ b(1-\gamma) + \left(\frac{1-\theta}{2}\right)(b-1)(1-\gamma) > 1 \\ \frac{b-1+\left(\frac{1-\theta}{2}\right)(b-1)}{b+\left(\frac{1-\theta}{2}\right)(b-1)} = \overline{\gamma}_M > \gamma. \end{gathered} \tag{5}$$

And similarly for m:

$$\frac{b-1+\left(\frac{\theta}{2}\right)(b-1)}{b+\left(\frac{\theta}{2}\right)(b-1)} = \overline{\gamma}_m$$

where $\overline{\gamma}_m > \overline{\gamma}_M > \frac{b-1}{b}$. Therefore, $\sigma_1^i(1) > 0$ despite the fact that the first period stage game payoff for D is greater than that of H for a player 1 of both groups. Hence, both m and M display $P1$. It also cannot be that $\sigma_1^i(1) \in (0,1)$. In order for the first period player 1 to mix, it would require:

$$b(1-\gamma) + v_i(H1) = 1 + v_i(D1). \tag{6}$$

Since $\gamma > \frac{b-1}{b}$, (or $b(1-\gamma) < 1$), (6) will only hold if $v_i(H1) > v_i(D1)$. Expression (6) only holds when $\sigma_1^i(1)$ is such that $p^i(H1) \geq \frac{b-1}{b}$. Since $\overline{\gamma}_M > \gamma$

$$b(1-\gamma) + v_M(H1) > 1 + v_M(D1)$$

if $p^i(H1) > \frac{b-1}{b}$. Therefore, the only way to satisfy (6) is to select $\sigma_2^j(h^i)$ for $h^i = H1$ such that $p^i(H1) = \frac{b-1}{b}$ and this is impossible given that the prior γ is strictly greater than $\frac{b-1}{b}$. If $\sigma_1^i(1)$ is such that $p^i(H1) > \frac{b-1}{b}$ then $\sigma_2^j(h^i) = 0$ for $h^i = H1$. Therefore, the optimal choice is $\sigma_1^i(1) = 1$ and as a consequence $\sigma_2^j(h^i) = 0$ for $h^i = H1$. It also follows that since $\gamma > \frac{b-1}{b}$ that $\sigma_2^i(h^j) = 0$ for $h^j \in \{I, E\}$. Indeed, this last fact holds for the final three sections of the proof. Furthermore, there can be no other $SPBE$ strategies.

(iii) Since $\gamma \in (\overline{\gamma}_M, \overline{\gamma}_m)$ we can make identical arguments as those given in part (ii) only for m and not M. Therefore $\sigma_1^m(1) = 1$ and $\sigma^M(h^m) = 0$ such that $h^m = H1$. In the case of M, it cannot be that $\sigma_1^M(1) = 1$ because (5) no longer holds. It cannot be that $\sigma_1^M(1) \in (0,1)$ because (6) cannot be satisfied by any value in this range. Therefore, $\sigma_1^M(1) = 0$ and $\sigma_2^m(h^M) = 0$ for $h^M = H1$ as $p^M(H1) = 1$ as it is no longer for worthwhile for M to display $P1$. Furthermore, there can be no other $SPBE$ strategies.

(iv) Now the arguments supporting $\sigma_1^i(1) \in (0,1]$ in cases (ii) and (iii) do not hold for either group. Therefore, $\sigma_1^i(1) = 0$ and $\sigma_2^i(h^j) = 0$ for $h^j = H1$ as $p^i(H1) = 1$. It is no longer for either group to display $P1$. Furthermore, there can be no other $SPBE$ strategies.■

Proof of Proposition 4: In any $SPBE$ with $\frac{2}{(1-\theta)(1-\gamma)} + 1 > b > \frac{2}{\theta(1-\gamma)} + 1$, it must be that $\sigma_1^m(H) = \beta^* \in (0,1)$ such that $p^m(HH) = \frac{b-1}{b}$ and $\sigma_1^M(H) = 0$. By Lemma 3, it cannot be that $\sigma_1^M(H) > 0$. Therefore, $\sigma_1^M(H) = 0$ and $\sigma_2^m(h^M) = 0$ when $h^M = HH$. In the case of m, it cannot be that $\sigma_1^m(H) = 0$. It also cannot be that $\sigma_1^m(H) = 1$ as this implies that $p^m(HH) = \gamma < \frac{b-1}{b}$ and so $v_m(HH) = v_m(DH)$. Therefore, $\sigma_1^m(H) = 0$ is a profitable deviation. It must be that $\sigma_1^m(H) = \beta^*$ such that

$$\begin{aligned} p^m(HH) &= \frac{b-1}{b} = \frac{\gamma}{\gamma + (1-\gamma)\beta^*} \\ \beta^* &= \left(\frac{\gamma}{1-\gamma}\right)\left(\frac{1}{b-1}\right) \end{aligned}$$

If $\sigma_1^m(H) > \beta^*$ then $p^m(HH) < \frac{b-1}{b}$ which would imply $\sigma_2^M(h^m) = 1$ where $h^m = HH$. There would be no benefit for $\sigma_1^m(H) > 0$, and so it must be that $\sigma_1^m(H) \leq \beta^*$. If $\sigma_1^m(H) < \beta^*$ then $p^m(HH) > \frac{b-1}{b}$ which would imply that $\sigma_2^M(h^m) = 0$ where $h^m = HH$. However, if

$\sigma_2^M(h^m) = 0$ where $h^m = HH$ then $\sigma_1^m(H) = 1$ is optimal. By the above argument this cannot be the case, therefore $\sigma_1^m(H) = \beta^*$. The $SPBE$ requires

$$0 + v_m(HH) = 1 + v_m(DH)$$
$$0 + \left(\frac{\theta}{2}\right)[b(1-\gamma)(1-\sigma_2^M(HH)) + (1-\gamma)\sigma_2^M(HH) + \gamma] = 1 + \left(\frac{\theta}{2}\right)$$

so that

$$\sigma_2^M(HH) = \frac{\left(\frac{\theta}{2}\right)(b-1)(1-\gamma) - 1}{\left(\frac{\theta}{2}\right)(b-1)(1-\gamma)}.$$

Therefore, $\sigma_1^m(H) = \beta^*$ such that $p^m(HH) = \frac{b-1}{b}$. Additionally, since $\gamma < \left(\frac{b-1}{b}\right)^2 < \frac{b-1}{b}$, it must be that $\sigma_2^i(h^j) = 1$ for $h^j \in \{I, E, D1, DH\}$. Since $\gamma \leq \left(\frac{b-1}{b}\right)^2$ the $SPBE$ must be that $\sigma_1^M(1) = 1$ because

$$b(1-\gamma)(1-\beta^*) + v_M(H1) \geq 1 + v_M(D1) \tag{7}$$

$v_M(H1) = v_M(D1)$ as $p^M(H1) < \frac{b-1}{b}$. Therefore, (7) holds when $\gamma \leq \left(\frac{b-1}{b}\right)^2$. Furthermore, $\sigma_1^m(1) = 1$ and $\sigma_2^M(h^m) = 1$ for $h^m = H1$. This is true as $v_m(H1) = v_m(D1)$ and $b(1-\gamma) > 1$. Furthermore, there can be no other $SPBE$ strategies.■

Proof of Proposition 5: In any $SPBE$ with $b \geq \frac{2}{(1-\theta)(1-\gamma)} + 1 > \frac{2}{\theta(1-\gamma)} + 1$, it must be that $\sigma_1^i(H) = \beta^* \in (0,1)$ such that $p^i(HH) = \frac{b-1}{b}$. Here, the argument presented in the proof of Proposition 4 goes through for both M and m. It also must be that $\sigma_2^i(h^j) \in (0,1)$ where $h^j = HH$. Just as in Proposition 4, in order to determine $\sigma_2^M(HH)$ it must be that

$$0 + \left(\frac{\theta}{2}\right)(b(1-\gamma)(1-\sigma_2^M(HH)) + (1-\gamma)\sigma_2^M(HH) + \gamma) = 1 + \left(\frac{\theta}{2}\right)$$

and similarly for $\sigma_2^m(HH)$. Additionally, Lemma 2 allows us to restrict attention to $\gamma < \left(\frac{b-1}{b}\right)^2 < \frac{b-1}{b}$. This allows us to determine that $\sigma_2^i(h^j) = 1$ for $h^j \in \{I, E\}$. Since $\gamma \leq \left(\frac{b-1}{b}\right)^2$ arguments in the proof of Proposition 4 apply to both M and m therefore $\sigma_1^i(1) = 1$ and $\sigma_2^i(h^j) = 1$ for $h^j = H1$. Furthermore, there can be no other $SPBE$ strategies.■

Proof of Proposition 7: We begin by showing that $\sigma_1^M(H) \leq \sigma_1^m(H)$. Suppose there was an $SPBE$ such that

$$\sigma_1^M(H) > \sigma_1^m(H).$$

First note that by Lemma 2, if $\sigma_1^i(H) > 0$ then $\gamma < \left(\frac{b-1}{b}\right)^2$. If $\sigma_1^i(H) = 1$ and $\gamma < \frac{b-1}{b}$ then there is no benefit to foregoing payment in the first period because $p^i(HH) = \gamma < \frac{b-1}{b}$. Furthermore, arguments advanced in the Proof of Proposition 4 show that if $\sigma_1^i(H) \in (0,1)$ then it must be that $\sigma_1^i(H) = \beta^*$ such that $p^i(HH) = \frac{b-1}{b}$. Therefore, $\sigma_1^i(H) \in \{0, \beta^*\}$. To satisfy the inequality it must be that $\sigma_1^M(H) = \beta^* > \sigma_1^m(H) = 0$. In order to support this $SPBE$ it must be that

$$1 = v_M(HH) - v_M(DH)$$

and therefore

$$1 = \left(\frac{1-\theta}{2}\right)(b-1)(1-\gamma)(1-\sigma_2^m(HH)).$$

It must also be that

$$\begin{aligned} 1 &> v_m(HH) - v_m(DH) \\ 1 &> \left(\frac{\theta}{2}\right)(b-1)(1-\gamma). \end{aligned}$$

This is a contradiction as

$$\left(\frac{1-\theta}{2}\right)(b-1)(1-\gamma)(1-\sigma_2^m(HH)) < \frac{\theta}{2}(b-1)(1-\gamma)$$

and so it is proved that $\sigma_1^M(H) \leq \sigma_1^m(H)$.

Now we show that $\sigma_1^M(1) \leq \sigma_1^m(1)$. By way of contradiction, suppose that:

$$\sigma_1^M(1) > \sigma_1^m(1).$$

In the case that $\gamma > \frac{b-1}{b}$, for all $\sigma_1^i(H) \in [0,1]$, it must be that $p^i(HH) > \frac{b-1}{b}$ and so $\sigma_2^j(HH) = 1$. Therefore, in order for $\sigma_1^i(H) \in (0,1)$, it must be that[23]

$$b(1-\gamma) + v_i(H1) = 1 + v_i(D1).$$

This condition only holds for $\overline{\gamma}_M$ in the case of the majority and $\overline{\gamma}_m$ in the case of the minority. Since we are restricting attention to generic parameters, we can exclude $\sigma_1^i(H) \in (0,1)$. Therefore, the only remaining case for $\gamma > \frac{b-1}{b}$ is: $1 = \sigma_1^M(1) > \sigma_1^m(1) = 0$. This implies that

$$\begin{aligned} b(1-\gamma) + v_M(H1) &> 1 + v_M(D1) \\ b(1-\gamma) + v_m(H1) &< 1 + v_m(D1) \end{aligned}$$

and so

$$\left(\frac{\theta}{2}\right)(b-1)(1-\gamma) < 1 - b(1-\gamma) < \left(\frac{1-\theta}{2}\right)(b-1)(1-\gamma).$$

This is a contradiction and so for $\gamma > \frac{b-1}{b}$, it must be that $\sigma_1^M(1) \leq \sigma_1^m(1)$.

In the case that $\gamma < \frac{b-1}{b}$, $\sigma_1^i(H)$ will affect $\sigma_2^j(HH)$. We investigate $\sigma_1^i(H) \in (0,\alpha^*) \cup \{\alpha^*\} \cup (\alpha^*,1)$ where $\alpha^* = \left(\frac{\gamma}{(1-\gamma)(b-1)}\right)$ which implies $p^i(HH) = \frac{b-1}{b}$. In order for i to mix, it must be that:

$$b(1-\gamma)(1-\sigma_1^j(H)) + v_i(H1) = 1 + v_i(D1). \tag{8}$$

It must be that $v_i(H1) \geq v_i(D1)$. Since $\gamma < \frac{b-1}{b}$, (8) only holds when $\sigma_1^j(H) > 0$. However, since $\sigma_1^j(H)$ only takes one nonzero value: $\frac{\gamma}{(1-\gamma)(b-1)}$. Since a player is displaying $P2$, by

[23] Note that since $\gamma > \frac{b-1}{b}$ no player displays $P2$.

Lemma 2 it must be that $\gamma < \left(\frac{b-1}{b}\right)^2$. However, $b\left(1-\frac{b\gamma}{b-1}\right) = 1$ is not satisfied by any $\gamma < \left(\frac{b-1}{b}\right)^2$ therefore $b\left(1-\frac{b\gamma}{b-1}\right)+v_i(H1) = 1+v_i(D1)$ cannot be satisfied by any $\gamma < \left(\frac{b-1}{b}\right)^2$. Therefore, the only remaining case for $\gamma < \frac{b-1}{b}$ is: $1 = \sigma_1^M(1) > \sigma_1^m(1) = 0$. In this case, $v_M(H1) = v_M(D1)$ as $p^M(H1) = \gamma < \frac{b-1}{b}$. A deviation of m would imply $p^m(H1) = 1$ and therefore, $v_m(H1) > v_m(D1)$. This leads to a contradiction as it cannot be that

$$b(1-\gamma) > 1$$

and

$$b(1-\gamma) + v_m(H1) < 1 + v_m(D1).$$

Therefore, $\sigma_1^M(1) \leq \sigma_1^m(1)$ for generic parameter values.■

Proof of Proposition 1: For every set of parameter values (b, θ, γ), the statement of Propositions 3, 4 and 5 map to the corresponding values of $\mathcal{I}$. Therefore in the proof of Proposition 1, we note the trajectory of $\mathcal{I}$, given b and γ, as θ varies. As $1-\theta$ changes, the incentives for each group changes. Specifically, as $1-\theta$ gets larger, the minority reputation becomes less valuable and the majority reputation becomes more valuable. As $1-\theta$ becomes large one of the following three possibilities occur. In the first case, no qualitative change occurs in the $SPBE$. In the second case, the majority does not exhibit reputation whereas the minority exhibits reputation for small $1-\theta$ and for large values does not exhibit reputation. In the third case, the minority always exhibits reputation and for small $1-\theta$ the majority does not display reputation and for large values, the majority does display reputation.

Now we characterize the relationship between $\mathcal{I}$ and $1-\theta$ for every pair of (b,γ). If $b \leq \frac{2+(1-\gamma)}{3(1-\gamma)}$, then for all values of $1-\theta$, it will be that $\mathcal{I} = (b+1)\theta(1-\theta)[4\gamma^2]$. This implies that for values of (b,γ) in this region $\mathcal{I}$ is strictly increasing and continuous in $1-\theta$. Therefore $1-\theta^* = 0.5$.

If $b \in (\frac{2+(1-\gamma)}{3(1-\gamma)}, \frac{4+(1-\gamma)}{5(1-\gamma)})$ then for small values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)[\gamma(1+3\gamma)]$ and for large values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)[4\gamma^2]$. Intuitively, for small $1-\theta$ the minority exhibits $P1$. However, for large $1-\theta$, it is no longer profitable for the minority to exhibit $P1$. This downward discontinuity occurs at $1-\theta$ such that $b = \frac{2+\theta(1-\gamma)}{(2+\theta)(1-\gamma)}$. Note that at this downward discontinuity the minority is indifferent between displaying $P1$ or not. Therefore, $\mathcal{I} \in [(b+1)\theta(1-\theta)[4\gamma^2], (b+1)\theta(1-\theta)[\gamma(1+3\gamma)]]$ at $1-\theta$ where $b = \frac{2+\theta(1-\gamma)}{(2+\theta)(1-\gamma)}$. Hence, $1-\theta^*$ is where $b = \frac{2+\theta(1-\gamma)}{(2+\theta)(1-\gamma)}$ and this is strictly larger than zero.

If $b = \frac{4+(1-\gamma)}{5(1-\gamma)}$ then $\mathcal{I} = (b+1)\theta(1-\theta)[\gamma(1+3\gamma)]$ for all values of $1-\theta$. This implies that for values of (b,γ) such that $b = \frac{4+(1-\gamma)}{5(1-\gamma)}$ then $\mathcal{I}$ is strictly increasing and continuous in $1-\theta$. Therefore, $1-\theta^* = 0.5$.

If $b \in (\frac{4+(1-\gamma)}{5(1-\gamma)}, \frac{1}{1-\gamma})$ then for small values $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)[\gamma(1+3\gamma)]$ and for large values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)2\gamma(1+\gamma)$. Intuitively, for small $1-\theta$ the majority does not exhibit $P1$ however for large $1-\theta$ the reputation of the majority becomes sufficiently profitable to display $P1$. This upward discontinuity occurs at $1-\theta$ such that $b = \frac{2+(1-\theta)(1-\gamma)}{(3-\theta)(1-\gamma)}$. Note that at this discontinuity, the majority is indifferent between displaying $P1$ or not. Thus, $\mathcal{I} \in [(b+1)\theta(1-\theta)[\gamma(1+3\gamma)], (b+1)\theta(1-\theta)2\gamma(1+\gamma)]$ at $1-\theta$ where $b = \frac{2+(1-\theta)(1-\gamma)}{(3-\theta)(1-\gamma)}$. As there is a single upward discontinuity and is increasing at every point of continuity therefore $1-\theta^* = 0.5$.

If $b = \frac{1}{1-\gamma}$ then for all values of $1-\theta$ it will be that $\mathcal{I} \in [(b+1)\theta(1-\theta)[\gamma(1+3\gamma)], (b+1)\theta(1-\theta)\gamma(3.5+0.5\gamma)]$. Note that for these particular values of b and γ any value of $\mathcal{I}$ in the above specified region will suffice. However, given any second period strategies for the histories I, $H1$ or E, inefficiency is increasing and continuous in $1-\theta$. Therefore, $1-\theta^* = 0.5$.

If $b \in (\frac{1}{1-\gamma}, \frac{2}{1-\gamma}+1]$ then for all values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)\gamma(3.5+0.5\gamma)$. This implies that for values of (b,γ) in this region $\mathcal{I}$ is strictly increasing and continuous in $1-\theta$. Therefore, $1-\theta^* = 0.5$.

If $b \in (\frac{2}{1-\gamma}+1, \frac{4}{1-\gamma}+1)$ then for small values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}+\left(\frac{1-\theta}{2}\right)\gamma)$ and for large values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)\gamma(3.5+0.5\gamma)$. Intuitively, for small $1-\theta$ the minority exhibits $P2$ and for large $1-\theta$ the minority does not exhibit $P2$. This boundary occurs at $1-\theta \in (0, 0.5)$ such that $b = \frac{2}{\theta(1-\gamma)}+1$. Although the minority is indifferent between exhibiting $P2$ or not, it is not the case that any combination will suffice. Therefore, at $1-\theta''$ where $b = \frac{2}{\theta(1-\gamma)}+1$, the minority either exhibits $P2$ or not: $\mathcal{I} \in \{(b+1)\theta(1-\theta)\gamma(3.5+0.5\gamma), (b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}+\left(\frac{1-\theta}{2}\right)\gamma)\}$. Due to the particular behavior of $(b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}+\left(\frac{1-\theta}{2}\right)\gamma)$ we denote its interior maximum as $1-\theta' = \frac{9-\gamma-\sqrt{\gamma^2+6\gamma+57}}{3(1-\gamma)}$. The quantity $1-\theta'$ is increasing from 0.4833 when $\gamma = 0$ to 0.5 when $\gamma = 1$. Therefore, $1-\theta^* = \min\{1-\theta', 1-\theta''\}$ and this is bounded away from zero.

If $b = \frac{4}{1-\gamma}+1$ then for all values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}+\left(\frac{1-\theta}{2}\right)\gamma)$. This implies that for values of b and γ in this region $\mathcal{I}$ is strictly increasing and continuous in $1-\theta$. Therefore, $1-\theta^* = 1-\theta'$

If $b \in (\frac{4}{1-\gamma}+1, \infty)$ then for small values of $1-\theta$ it will be that $\mathcal{I} = (b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}+\left(\frac{1-\theta}{2}\right)\gamma)$ and for large values of $1-\theta$ it will be that $(b+1)\theta(1-\theta)4\gamma$. Intuitively, for small $1-\theta$ the majority does not find it profitable to exhibit $P2$ however for large $1-\theta$ the reputation of the majority becomes sufficiently profitable. This upward discontinuity occurs at $1-\theta$ such that $b = \frac{2}{(1-\theta)(1-\gamma)}+1$. Although the majority is indifferent between exhibiting $P2$ or not, it is not the case that any combination will suffice. Therefore, the majority either exhibits $P2$ or not: $\mathcal{I} \in \{(b+1)\theta(1-\theta)\gamma(3.5+\frac{\theta}{2}), (b+1)\theta(1-\theta)4\gamma\}$ at $1-\theta$ where $b = \frac{2}{(1-\theta)(1-\gamma)}+1$. Therefore, $1-\theta^* = 1-\theta'$.

Therefore, for every value of (b,γ) there exists $1-\theta^* > 0$ such that for all $1-\theta < 1-\theta^*$, inefficiency $\mathcal{I}$ is increasing in $1-\theta$.■

7.1 Non-generic parameter values

The $SPBE$ is generically unique, as the following corollary shows. Following the corollary, is a result which describes the $SPBE$ for non-generic parameter values. There exists a set Ψ, of measure zero, in the parameter space for which the $SPBE$ is not unique. For parameter values not contained in Ψ, the $SPBE$ is unique. We explicitly define Ψ as

$$\begin{aligned}\Psi &= \{(b,\theta,\gamma) : b \in \{\frac{2}{\theta(1-\gamma)}+1, \frac{2}{(1-\theta)(1-\gamma)}+1\} \\ \text{or } \gamma &\in \{\frac{b-1}{b}, \frac{b-1+\left(\frac{1-\theta}{2}\right)(b-1)}{b+\left(\frac{1-\theta}{2}\right)(b-1)}, \frac{b-1+\left(\frac{\theta}{2}\right)(b-1)}{b+\left(\frac{\theta}{2}\right)(b-1)}\}\}\end{aligned}$$

The following corollary follows from Propositions 3, 4 and 5.

Corollary 2 *If parameters (b, θ, γ) are not contained in the set Ψ then the σ satisfying the conditions for $SPBE$ will be unique.*

Lemma 2 demonstrates that either a condition for b can be satisfied or a condition for γ can be satisfied, but not both. The values of b given above are the values for which the minority (respectively majority) will be indifferent between displaying $P2$ or not. The first value of γ represents the value for which a second period player 2 will be indifferent between playing H and D against an opponent with a history h such that $p^i(h) = \gamma$. The second (and third) value(s) of γ denotes the parameter for which the majority (minority) is indifferent between displaying $P1$ and not.

Now, we characterize the $SPBE$ for each element of Ψ.

Proposition 8 *(a) If $b < \frac{2}{\theta(1-\gamma)} + 1$ and $\gamma = \frac{b-1}{b}$ then the $SPBE$ is not unique as the strategies specified in Proposition 3 (i) or (ii) or any mixture will suffice.*

(b) If $b < \frac{2}{\theta(1-\gamma)} + 1$ and $\gamma = \frac{b-1+\left(\frac{1-\theta}{2}\right)(b-1)}{b+\left(\frac{1-\theta}{2}\right)(b-1)}$ then the $SPBE$ is not unique as the strategies specified for M in Proposition 3 (ii) or (iii) or any mixture will suffice.

(c) If $b < \frac{2}{\theta(1-\gamma)} + 1$ and $\gamma = \frac{b-1+\left(\frac{\theta}{2}\right)(b-1)}{b+\left(\frac{\theta}{2}\right)(b-1)}$ then the $SPBE$ is not unique as the strategies specified for m in Proposition 3 (iii) or (iv) or any mixture will suffice.

(d) If $b = \frac{2}{\theta(1-\gamma)} + 1$ then the $SPBE$ is not unique as the strategies specified for m in Proposition 4 and those specified in Proposition 3 (i), however no mixture between them will suffice.

(e) If $b = \frac{2}{(1-\theta)(1-\gamma)} + 1$ then the $SPBE$ is not unique as the strategies specified for M in Proposition 5 and those specified in Proposition 4, however no mixture between them will suffice.

Proof: In the case of (a), any $\sigma_2^i(h) \in [0,1]$ where h such that $p^j(h) = \gamma$ is an $SPBE$. For such histories, the second period player 2 is indifferent between actions. For histories I, $H1$ and E any second period strategies will suffice. In the case of (b), the majority is indifferent between displaying $P1$ or not. Any $\sigma_1^M(1) \in [0,1]$ will constitute an $SPBE$. These first period player 1 strategies will induce posteriors strictly between γ and 1. Therefore, the second period strategies are unchanged. In the case of (c), the minority is indifferent between displaying $P1$ or not. Reasoning similar to case (b) applies to m. In the case of (d), the minority is indifferent between displaying $P2$ or not. However, unlike the previous cases, the $SPBE$ cannot contain any mixture between the equilibria will not form a $SPBE$. Given condition (iii) of the definition of $SPBE$ it must be either $\sigma_1^m(H) \in \{0, \left(\frac{\gamma}{(1-\gamma)(b-1)}\right)\}$. Any other value would imply $p^m(HH) \neq \frac{b-1}{b}$. Unlike the cases of (a), (b), and (c), the first period strategy nontrivially affects the second period posteriors, as $\gamma < \frac{b-1}{b}$. For the parameter values given, there is no deviation from the m strategy given in Proposition 4. Likewise, there is no deviation from the strategy given in Proposition 3(i). In the case of (e), the majority is indifferent between displaying $P2$ or not. Reasoning similar to case (d) applies to M.■

The statement of Proposition 8 elucidates Figure 5 in the body of the paper. In this figure, the relationship between $\mathcal{I}$ and b is connected at 3 points of discontinuity ((a), (b) and (c)) and not connected at two points of discontinuity ((d) and (e)).

8 References

Ahmed, A. (2007) Group identity, social distance and intergroup bias, *Journal of Economic Psychology*, 28, 324-337.

Akerlof, G., and R. Kranton (2000) Economics and identity, *Quarterly Journal of Economics*, 115(3), 715-753.

Alesina, A., A. Devleeschauwer and W. Easterly (2003) Fractionalization, *Journal of Economic Growth*, 8, 155-194.

Arrow, K. (1973) The theory of discrimination, in *Discrimination in Labor Markets,* O. Ashenfelter and A. Rees, eds., Princeton, NJ, Princeton University Press, pp. 3-33.

Basu, K. (2005) Racial conflict and the malignancy of identity, *Journal of Economic Inequality*, 3, 221-241.

Benhabib, J. and A. Rustichini (1996) Social conflict and growth, *Journal of Economic Growth*, 1, 125-142.

Bridgman, B. (2008) Why are ethnically divided countries poor? *Journal of Macroeconomics*, 30, 1-18.

Caselli, F. and W. Coleman (2006) On the theory of ethnic conflict , unpublished CEPR Discussion Paper No. 5622

Charness, G., L. Rigotti and A. Rustichini (2007) Individual behavior and group membership, *American Economic Review*, 97(4), 1340-1352.

Chen, Y. and X.S. Li (2009) Group identity and social preferences, *American Economic Review*, 99(1), 431-457.

Cho, B. and D. Connelley (2002) The effect of conflict and power differentials on social identity and intergroup discrimination, AoM Conflict Management Division 2002 Mtgs. No. 12522. Available at SSRN: http://ssrn.com/abstract=320286.

Davis, J. (2006) Social identity strategies in recent economics, *Journal of Economic Methodology*, 13(3), 371-390.

Dawes, R., A. Van De Kragt and J. Orbell (1988) Not me or thee but we: The importance of group identity in eliciting cooperation in dilemma situations: Experimental manipulations, *Acta Psychologica*, 68, 83-97.

Dion, D. (1997) Competition and ethnic conflict, *Journal of Conflict Resolution*, 41(5), 638-648.

Easterly, W. and R. Levine (1997) Africa's growth tragedy: Policies and ethnic divisions, *Quarterly Journal of Economics*, 112(4), 1203-1250.

Esteban, J. and D. Ray (1994) On the measurement of polarization, *Econometrica*, 62(4), 819-851.

Esteban, J. and D. Ray (1999) Conflict and distribution, *Journal of Economic Theory*, 87, 379-415.

Esteban, J. and D. Ray (2008) On the salience of ethnic conflict, *American Economic Review*, 98(5), 2185-2201.

Esteban, J. and D. Ray (2009) A model of ethnic conflict, *Journal of the European Economic Association*, forthcoming.

Falk, A. and J. Zweimuller (2005) Unemployment and right-wing extremist crime, unpublished IZA and Bonn.

Fearon, J. and D. Laitin (1996) Explaining interethnic cooperation, *American Political Science Review*, 90(4), 715-735.

Ferraro, P. and R. Cummings (2007) Cultural diversity, discrimination, and economic outcomes: An experimental analysis, *Economic Inquiry*, 45(2), 217-232.

Goette, L., D. Huffman and S. Meier (2006) The impact of group membership on cooperation and norm enforcement: Evidence using random assignment to real social groups, *American Economic Review*, 96(2), 212-216.

Gurin, P., T. Peng, G. Lopez and B. Nagda (1999) Context, identity and intergroup relations, in *Cultural Divides: Understanding and Overcoming Group Conflict*, D. Prentice and D. Miller, eds., New York: Russel Sage Foundation, pp. 133-170.

Guth, W., M.V. Levati and M. Ploner (2008) Social identity and trust: An experimental investigation, *Journal of Socio-Economics*, 37, 1293-1308.

Kirman, A. and M. Teschl (2004) On the emergence of economic identity, *Revue De Philosophie Economique*, 9, 59-86.

Kramer, R. and M. Brewer (1984) Effects of group identity on resource utilization in a simulated commons dilemma, *Journal of Personality and Social Psychology*, 46, 1044-1057.

Kreps, D. and R. Wilson (1982) Reputation and imperfect information, *Journal of Economic Theory*, 27, 253-279.

Lindqvist, E. and R. Ostling (2009) Identity and redistribution, working paper Stockholm University.

Lubbers, M. and P. Scheepers (2001) Explaining the trend in extreme right-wing voting: Germany 1989-1998, *European Sociological Review*, 17(4), 431-449.

Mauro, P. (1995) Corruption and growth, *Quarterly Journal of Economics*, 110(3), 681-712.

Messick, D. and D. Mackie (1989) Intergroup relations, *Annual Review of Psychology*, 40, 45-81.

Miller, D., J. Downs and D. Prentice (1998). Minimal conditions for the creation of a unit relationship: The social bond between birthdaymates, *European Journal of Social Psychology*, 28, 475-481.

Montalvo, J. and M. Reynal-Querol (2005) Ethnic diversity and economic development, *Journal of Development Economics*, 76, 293-323.

Murdock, G. (1949) *Social Structure*, New York: Macmillan.

Nakao, K. (2009) Creation of social order in ethnic conflict, *Journal of Theoretical Politics*, 21(3), 365-394.

Olzak, S. (1992) *Dynamics of Ethnic Competition and Conflict*, Palo Alto, CA: Stanford University Press.

Olzak, S., S. Shanahan and E. West (1994) School desegregation, interracial exposure, and antibusing activity in contemporary urban America, *American Journal of Sociology*, 100(1), 196-241.

Orbell, J., L. Zeng and M. Mulford (1996) Individual experience and the fragmentation of societies, *American Sociological Review*, 61, 1018-1032.

Phelps, E. (1972) The statistical discrimination of racism and sexism, *American Economic Review,* 62, 659-661.

Posner, D. (2004) Measuring ethnic fractionalization in Africa, *American Journal of Political Science*, 48(4), 849-863.

Quillian, L. (1995) Prejudice as a response to perceived group threat: Population composition and anti-immigrant and racial prejudice in Europe, *American Sociological Review*, 60(4), 586-611.

Quillian, L. (1996) Group threat and regional change in attitudes toward African-Americans, *American Journal of Sociology*, 102(3), 816-860.

Robinson, J. (2001) Social identity, inequality and conflict, *Economics of Governance*, 2, 85–99.

Rohner, D. (2008) Reputation, group structure and social tensions, unpublished, HiCN Working paper 40.

Scheepers, P., M. Gijberts and M. Coenders (2002) Ethnic exclusion in European countries: Public opposition to civil rights for legal migrants as a response to perceived ethnic threat, *European Sociological Review*, 18(1), 17-34.

Shayo, M. (2009) A model of social identity with an application to political economy: Nation, class and redistribution, *American Political Science Review*, 103(2), 147-174.

Sherif, M., O.J. Harvey, B.J. White, W. Hood and C. Sherif (1961) *Intergroup Conflict and Cooperation: The Robbers Cave Experiment*, Norman: University of Oklahoma Book Exchange.

Silverman, D. (2004) Street crime and street culture, *International Economic Review*, 45(3), 761-786.

Sobel, J. (2005) Interdependent preferences and reciprocity, *Journal of Economic Literature*, 43(2), 392-436.

Strulik, H. (2008) Social composition, social conflict and economic development, *Economic Journal*, 118, 1145-1170.

Sumner, W. (1906) *Folkways: A Study of the Sociological Importance of usages, manners, customs mores and morals*, Boston: Ginn.

Tajfel, H. (1970) Experiments in intergroup discrimination, *Scientific American*, 223, 96-102.

Tajfel, H. (1978) *Differentiation Between Social Groups: Studies in the Social Psychology of Intergroup Relations*, London: Academic Press.

Tajfel, H., C. Flament, M. Billig and R. Bundy (1971) Social categorization and intergroup behavior, *European Journal of Social Psychology*, 1, 149-178.

Tajfel, H. and J. Turner (1979) An integrative theory of intergroup conflict, in *The Social Psychology of Intergroup Relations*, W.G.Austin and S. Worchel, eds., Monterey, CA: Brooks/Cole, pp. 33-48. Reprinted (2001) in *Intergroup Relations*, M. Hogg and D. Abrams, eds., Ann Arbor, MI, Psychology Press, pp. 94-109.

Tajfel, H. and J. Turner (1986) The social identity theory of intergroup behavior, in *Psychology of Intergroup Relations*, S. Worchel and W. Austin, eds., Chicago: Nelson-Hall, pp. 7-25.

Vigdor, J. (2002) Interpreting ethnic fragmentation effects, *Economics Letters*, 75, 271-276.

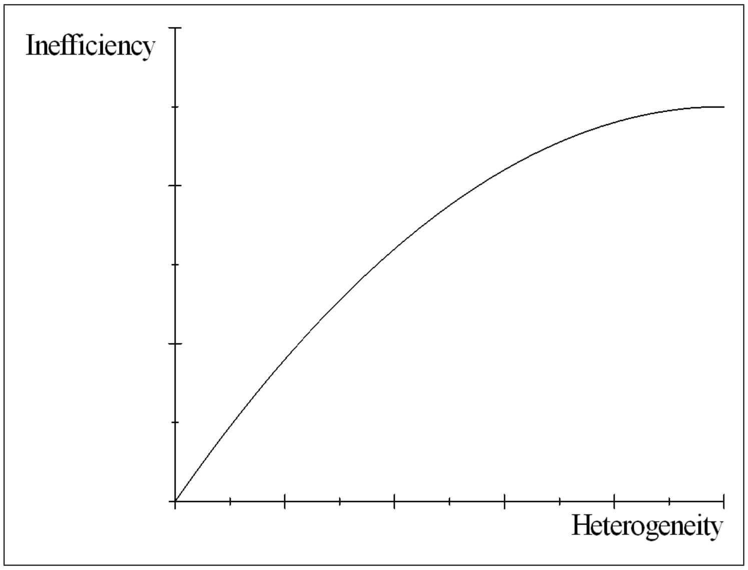


Figure 1-Inefficiency strictly increasing in heterogeneity.

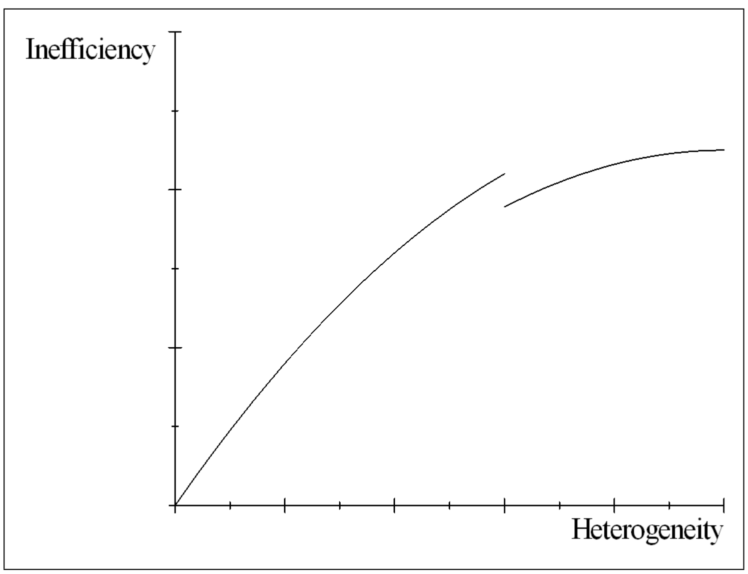


Figure 2-Inefficiency almost everywhere increasing in heterogeneity, with a single downward discontinuity.

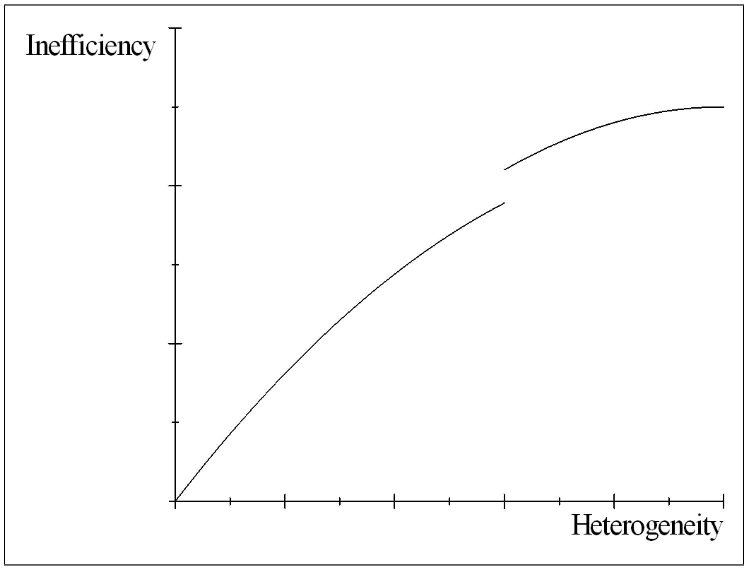


Figure 3-Inefficiency everywhere increasing in heterogeneity, with a single upward discontinuity.

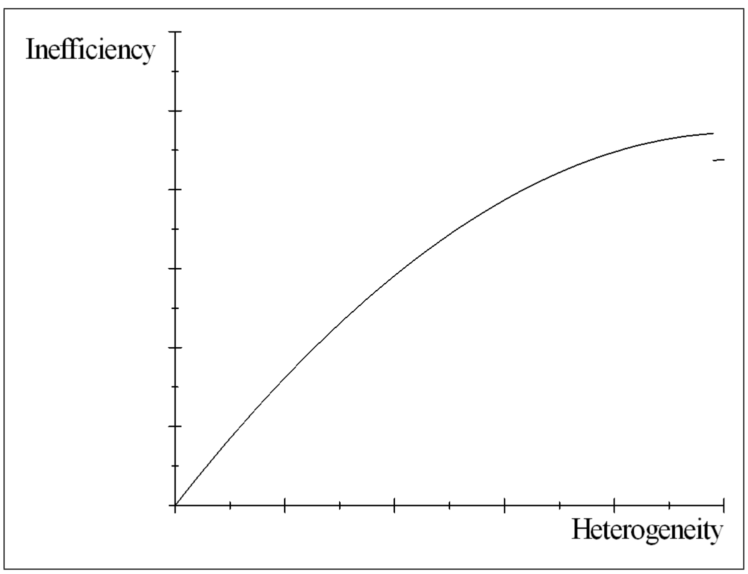


Figure 4-Inefficiency increasing in heterogeneity, with a maximum at 0.485 and a downward discontinuity at 0.49.

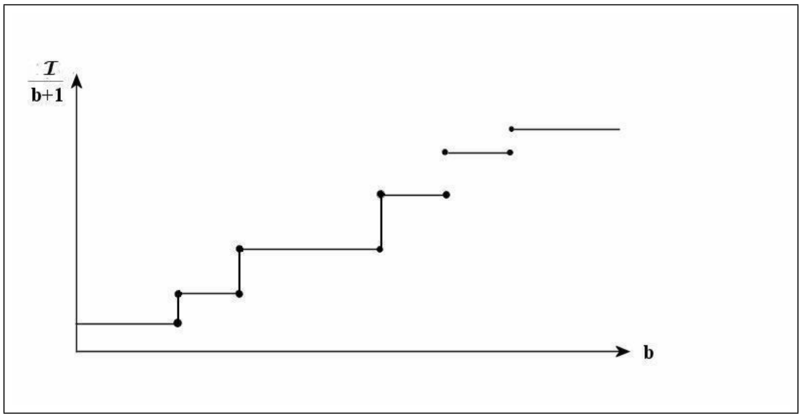


Figure 5-Probability of inefficient outcome and inequitability of the environment.

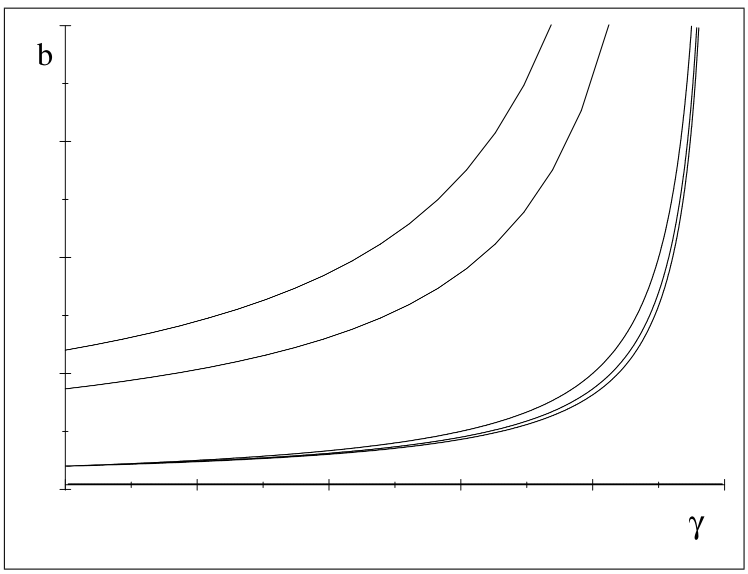


Figure 6-SPBE regions of b and γ given θ