Risk, Leverage, and Regulation of Financial Intermediaries

Wang Tianxi

University of Essex

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Abstract

This paper presents a model on the leverage of financial intermediaries, where debt are held by risk averse agents and equity by the risk neutral. The paper shows that in an unregulated competitive market, financial intermediaries choose to be leveraged over the social best level. This is because the leverage of one intermediary imposes a negative externality upon others by reducing their profit margins. The paper thus founds capital adequacy regulation upon the market failure and suggests that this regulation should bind not only commercial banks, but *all* financial intermediaries, including private equities and hedge funds.

Key words: Risk Difference in Risk Preference Leverage Regulation Externality

JEL: G00, G01, D52, D62

*University of Essex. Correspondence: Department of Economics, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, Essex, UK. Email: wangt@essex.ac.uk. I am indebted to John Moore and Motty Perry for their enormous help and encouragement. I also thank Sanjay Banerji, Xuewen Liu, Francesco Squintani for their helpful comments.

I. Introduction

Leverage is an important ingredient of financial alchemy to boost the return of capital. For example, consider a project that requires investment of \$100 and returns \$101. If a private equity fund outlays the entire \$100 out of its own pocket, it earns a poor return rate of 1%; if it invests only \$1 of its own fund and borrows \$99 from a bank, then at the return date, it repays \$99 to the bank and earns \$2, with a return rate of 100%. Indeed, "(T)he reliance of private equity firms on high levels of debt to generate profits has been starkly revealed by an industry study... The aggregate investment return of the 14 biggest deals realized in 2005-7 was 3.3 times, or 330 percent of the return achieved by FTSE ALL Share in the same industry sector and timeframes. Of this 330 percent return, ...167 came from the use of extra debt over the amount at comparable companies at the same sector..."[1] Moreover, the problem of banks being overleveraged is at the core of current crisis.[2] All these raise the question: are financial intermediaries (FIs) naturally prone to being overleveraged? More fundamentally, how do FIs decide their leverage levels?

This paper presents a model on the leverage of FIs. Built on the model, it shows that the leverage of FIs is in itself subject to market failure; thus being overleveraged is a natural feature of financial intermediation. The paper, therefore, suggests to regulate indebtedness of the *whole* financial sector where leverage is pervasively used to boost performance.[3]

The building block model of this paper is based on difference in risk preference, illustrated as follows. There are many deep pockets and a lot more households. Deep pockets are risk neutral and each has \$4. The households are risk averse and each has \$1. Fund is invested either in one-to-one storage or in a venture with the gross return rate of 0.8 or 1.4, each with a half chance. Suppose a household cannot split his dollar between the two types of investment and is deterred by the risk of the venture. He thus chooses to store his dollar. A deep pocket, being risk

[1] The study is carried out by British Private Equity & Venture Capital Association and Ernest & Young; see "Buy-out Profits Tied to Debt", *Financial Times*, January 15, 2009.

[2] See Brunnermeier(2009) and many essays published in the media by practitioners and policy makers.

[3] This echos the point of view of Sir Andrew Large, a former deputy governor of the Bank of England, in "Central Banks Must Be the Debt Watchdogs", *Financial Times*, January 6, 2009.

neutral, will invest all his funds in the venture and will earn in expectation $\$4 \times \frac{0.8+1.4}{2} = \$4.4$. However, he earns more by borrowing from households and becoming a FI. Suppose he borrows \$1 from a household and invests the entire \$5, one borrowed dollar plus his own \$4, in a venture. The investment returns $\$5 \times 0.8 = \$4$ in the bad state and $\$5 \times 1.4 = \$7$ in the good state. The household is satisfied with getting back \$1 in each state. The deep pocket thus earns, with this minor leverage, $0.5 \times \$(4-1) + 0.5 \times \$(7-1) = \$4.5$, which is \$0.1 more than without any leverage. This extra \$0.1 is simply the difference between \$1.1, the earning of the borrowed dollar from the venture, and \$1, the repayment to the lender, whom it satisfies since it is risk free. Therefore, the deep pocket keeps earning a margin of \$0.1 from each borrowed dollar, so long as his borrowing bears no risk, which requires two conditions. One, he gives priority to the lenders' claims in the bad state, so his own funds compose the equity of the FI and act as the cushion to absorb the loss to the lenders in the bad state. The other, the amount borrowed is not over \$16: at this level, the bad state earning, $\$(4+16) \times 0.8 = \$16$, exactly suffices to service the risk free debt. It turns out, however, that he will borrow more and the debt will be risky. The optimal leverage is decided by the trade-off between the scale and the margin. Given the margin, the more he borrows, the more he gains; however, the more the borrowing, the further below 1 the bad state return rate to the lenders, hence as compensation, the higher the good state rate to them, and the smaller the profit margin. This model captures two realistic points: leverage serves to boost the return of the capital, and the capital serves to cushion the risk to debt holders.

The building block model illustrated above, where the venture's return rates are exogenously given, provides no scope for regulation. However, market failure arises if the return rates are endogenized, as follows. There are many perfectly correlated ventures, each run by a risk neutral entrepreneur, with decreasing return to investment scale. The equilibrium return rates of ventures clear the credit market between entrepreneurs and FIs. An individual FI, when deciding its leverage, takes these rates as given, but the rates decrease with aggregate credit supply. Then, the leverage of one FI imposes a negative externality upon others: its leverage expands the credit supply, which marginally lowers the credit return rate in the bad state, which in turn lowers the bad state return rate that other FIs offer to their lenders; consequently, they have to offer a higher good state return rate, and obtain a smaller profit margin. Because of this externality, the equilibrium leverage is over the social best level.[4]

The regulation that prohibits FIs from being leveraged over the social best level will restore the social best allocation in the market. This regulation shrinks credit supply, raises credit return rates, and increases the profit of the financial sector. However, individual FIs, facing the raised return rates, want to be leveraged over the regulation limit. They would like to set up off balance vehicles, if they can.

The market failure happens only when deep pockets are FIs. If entrepreneurs could issue papers to both deep pockets and households, they would issue equity to risk averse deep pockets and debt to risk neutral households, and would be leveraged at the social best level in equilibrium.[5] In this paper, deep pockets have to do financial intermediation because of the following friction. Households cannot, but deep pockets can, distinguish a genuine entrepreneur from a layman, who only wants to play with others' funds. Households only recognize deep pockets. Therefore, they accept only papers issued by deep pockets, but not papers issued by some self-claimed entrepreneurs.

Relation with the Literature

The capital adequacy requirement plays a key part in Macro and Micro prudential regulation of banking. The paper discovers the market failure in the leverage of FIs for the first time, and founds capital adequacy regulation upon it. Moreover, the paper suggests that such regulation should be imposed upon, not only commercial banks, but the *whole* financial sector. The most received reasoning for the regulation is based on the government's provision of insurance for demand deposit: because of the insurance, debt is cheap to banks, which are then inclined to be

[4] By a similar externality, competition reduces the total profit of the suppliers of a good, but increases consumers' surplus. The trick here, however, is that the externality presents itself even when the utility of households is fixed at the given level.

[5] Merton Miller (1991) noted in his Nobel Prize lecture, "(C)apital markets have built in controls against overleveraging...." (page 481). This paper then suggests that the assertion is only half true, true of the leverage of real sectors, not of the financial sector.

overleveraged to exploit the insurance and should thus be subject to capital adequacy regulation. The insurance for deposit is justified by Bryant (1980), Diamond and Dybvig (1983), and also Gorton and Pinnachhi (1990).[6] Notice that by this line of reasoning, only commercial banks, namely those FIs that take demand deposits, should be so regulated. Alternatively, Dewatripont and Tirole (1994) address banking regulation from the perspective that the regulator represents small depositors.[7] And Morrison and White (2005) consider banking regulation based on the assumption that the regulator has capacities that private agents do not have. Most papers on capital adequacy regulation take it as given and consider its effects. Among those papers, Gorton and Winton (2000), which considers other differences between social and private costs of bank capital also, uses a general equilibrium framework, while others use partial equilibrium, such as Merton (1977), Kim and Santomero (1988), Flannery (1989), Furlong and Keeley (1989), Gennotte and Pyle (1991), Rochet (1992), and Besanko (1996). See Bhattacharya et al. (1998), Gorton and Winton (2002), and Freixas and Rochet (2008) for a general reference on banking and its regulation.

The review of the following literature concerns the building block model of this paper on the leverage of a FI. This literature, however, presents no market failures, thus no role for regulation.

The building block model of this paper deviates from Modigliani and Miller (1958) (MM) by introducing a difference in risk preference on the asset demand side.[8] In contrast, the literature following MM on capital structure, as is surveyed and summarized by Harris and Raviv (1991) and Myers (2001), deviates by introducing either tax benefits of debt (trade-off theory) or asymmetric information, such as Jensen and Meckling (1976), Ross (1977), and Myers and

[6] However, this literature does not provide argument for why the insurance should be provided by the government rather than the market, and there is no research that strictly compares these two ways.

[7] By their argument, it is clear that the representation is necessary. But it is not clear that this representation service cannot be provided by some private institute, for example, "the assembly of the depositors", over which the depositors presumably have more and directer control than over the government.

[8] If there is such a difference, as Myers (2001) has noticed, the irrelevance proposition of MM may not hold. For example, suppose a firm yields \$80 or \$120, each with probability one half. Suppose only a part of investors are risk neutral, who afford exactly \$20. Then, the firm can be sold for \$100, only by issuing \$80 risk free debts to risk averse investors, and \$20 equities to those who are risk neutral.

Majluf (1984).

The building block model of this paper relates the asset size of a FI to the leverage rate, similar to Fostel and Geanakoplos (2008) (FG) and Adrian and Shin (2008) (AS).[9] And in the three papers, the equilibrium leverage rate decreases with the risk. Leverage serves to boost the return of the FI's equity in both this paper and AS, while it is driven by the difference in the belief as to the future in FG (the pessimist lend to the optimist). The optimal leverage is decided by the trade-off between the scale and the profit margin in this paper, while in AS, it is decided by incentive compatibility in the risk shifting problem of Jensen and Meckling (1976), namely that the bank, if overleveraged, will choose the riskier but lower NPV project for the interest of the equity holders, because they reap the benefit when the project's upper side is realized while the debt holders undertake the loss of the down side.

In the building block model, the capital of a FI works to cushion the risk to debt holders, who are thus willing to invest in the FI. In contrast, in Diamond and Rajan (2000), the bank capital helps prevent bank runs, but gives rent to the banker. And while bank capital is also used to reduce the risk to debt holders in Gorton and Pinnacchi (1990), that is not for risk sharing, as is in this paper, but for creating risk free saving instruments.

The rest of the paper is organized as follows. Section II presents the model. Section III considers competitive equilibrium, where the building block model is embedded when the credit supply is analyzed. Section IV figures out the social best allocation and the regulation that restores this allocation in the competitive market. Section V concludes. All technical proofs are relegated to Appendices.

II. The Model

There are two dates, today for contracting and investment and tomorrow for return and distribution. There is a continuum of N units of households, 1 unit of deep pockets, and 1 unit of entrepreneurs. Households are risk averse, with the Bernoulli utility function U satisfying

[9] By MM, the asset side and the liability side can be considered separately. The literature on capital structure takes the asset side as given and focuses on the liability side.

$U' > 0, U'' < 0$. Each household has a small amount of extra capital to invest, normalized to 1 unit. Deep pockets are risk neutral, each having K units of capital.[10] Entrepreneurs are risk neutral and penniless. Each of them runs a venture. Capital is invested either in one-to-one storage or in a venture. Anyone can access storage, whereas only the entrepreneurs know how to run ventures. If capital I is invested in a venture, it returns $Y = \widetilde{A} h^{1-\alpha} I^{\alpha}$, where h is the amount of the human capital of the entrepreneur and $\widetilde{A}$ is the macroeconomic shock and the same for all ventures. Without loss of generality, suppose $h = 1$. $\widetilde{A}$ is resolved tomorrow; today, it is publicly known that $\widetilde{A} = \overline{A}$ with probability q and $\widetilde{A} = \underline{A}$ with probability $1 - q$, and both contingencies are contractible. Let $A_e \equiv q\overline{A} + (1-q)\underline{A}$ denote the mean. All agents are protected by limited liability. Further assumptions are made below.

Assumption 1: Households cannot distinguish an entrepreneur from a layman (household), but can recognize deep pockets. Deep pockets recognize entrepreneurs.

The first part of the assumption is based on the fact that entrepreneurs have human capital and deep pockets have physical capital, and human capital is much harder to observe than physical capital. In this economy, a household, a layman, could have an idea of nonsense and would be happy to play it with others' funds. Households cannot distinguish this nonsense from a sensible idea, and thus will not accept papers issued by anyone who claims himself an entrepreneur. But they accept financial contracts issued by one whom they recognize is a deep pocket, since he plays with his own funds.

The assumption presents the sole friction of the model, because of which regulation plays a role. By this assumption, entrepreneurs have no way to signal their human capital (namely their type) to households, and have to be financed by deep pockets. This is clearly extreme, but simplifies the analysis considerably. The assumption necessitates deep pockets to provide intermediation service for households and become Financial Intermediaries (FIs; hereinafter FI and deep pocket are used interchangeably).[11]

[10] Assuming deep pockets to be risk neutral is more for the convenience of exposition than for substantial reason; what matters is that they are less risk averse than households.

[11] Financial intermediation is driven by a similar assumption of limited participation in He and Krishnamurthy (2008). However, even under this kind of assumptions, there is actually an alternative way of organizing financing.

Assumption 2: A household cannot split its unit of capital between investing in storage and in a FI.

This assumption simplifies the decision problem of households and does not affect the mechanics of the paper.

Assumption 3: $\underline{A} < \alpha A_e$ and $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$.

The former part ensures that ventures are risky enough, and the latter that the capital deep pockets is scarce.

Assumption 4: $N > \overline{A}^{\frac{1}{1-\alpha}}$.

It says that the overall funds of households are abundant enough; particularly, as will be shown, not all these funds are poured into FIs.

There are two markets to be cleared, corresponding to the two sides of the balance sheet of FIs. On the asset side, there is the credit market between FIs and entrepreneurs; let I be the credit demand of each entrepreneur and let $\overline{R}$ and $\underline{R}$ be the return rate of credit when $\widetilde{A} = \overline{A}$ and $\widetilde{A} = \underline{A}$ respectively. On the liability side, there is the market for absorbing households' funds. Since the return rates to households offered by a FI depend on the amount it borrows, which is under its decision, this market is cleared by certainty equivalent return rate, r, that is, the households investing in FIs obtain $U(r)$. Since the households can alternatively choose storage, $r \geq 1$, and if $r = 1$, they are indifferent between storage and investing in FIs. Let L denote the leverage level of a FI, namely the amount borrowed from households. As the population of deep pockets and entrepreneurs are both one, I, K and L also denote the according aggregate amount.

In this paper, for example, deep pockets can sell their knowledge of sensible projects directly to households, namely, they becomes rating agents (RAs). In the setting of this paper, the FI-way dominates the RA-way. The latter is subject to collusion in which a deep pocket recommends a fake entrepreneur and shares with him the funds absorbed. This problem is removed in the FI-way, where the deep pocket has to clear the liabilities to the creditor-households before allowed to consume anything; he would thus undergo the loss before the households if investing in fake entrepreneurs. For details see Wang (2009), which compares various ways of organizing financing.

Definition 1 $\{\overline{R}, \underline{R}, r; I, L\}$ *is a competitive equilibrium if and only if*

(i) Given $\{\overline{R}, \underline{R}\}$, $I = \arg\max q(\overline{A}I^\alpha - \overline{R}I) + (1-q)\max(\underline{A}I^\alpha - \underline{R}I, 0)$.

(ii) Given $\{\overline{R}, \underline{R}, r\}$, *each deep pocket chooses the optimal* L.

(iii) If $r > 1$, $L = N$, *and if* $L < N$, $r = 1$.

(iv) $I = K + L$.

(v) If $\underline{A}I^\alpha - \underline{R}I \leq 0$, *then* $\underline{A}I^\alpha - \underline{R}I = 0$.

Condition (i) says that given the return rates of credit, each entrepreneur chooses the optimal demand of the credit; the term $\max(\underline{A}I^\alpha - \underline{R}I, 0)$ reflects the fact that he may default in the bad state. Condition (ii) says that given the prices on the two sides of the balance sheet, each FI chooses the optimal leverage. Condition (iii) describes how the liability side market is cleared: if $r > 1$, all households strictly prefer investing in FIs to storage and thus the market is cleared at $L = N$; if only a part of households invest in FIs ($L < N$), households must be indifferent between this investment and storage, that is, $r = 1$. Condition (iv) clears the credit market. Lastly, condition (v) is the condition of rational expectation, which says that if default is going to happen in the bad state, deep pockets foresee it today and calculate the return rate in the state accordingly.

Lemma 1 $r = 1$ *in any competitive equilibrium.*

Proof. If otherwise, $r > 1$, then $L = N < I$. On the other hand, $\frac{\overline{A}I^\alpha}{I} \geq 1$ in equilibrium, as the output of a venture must cover the investment costs in the good state; it follows that $I \leq \overline{A}^{\frac{1}{1-\alpha}}$, which is strictly less than N by Assumption 4, contradictory to $N < I$. ■

By Assumption 3, $\underline{A} < A_e\alpha$, which implies that the return of the credit is always risky, as follows.

Lemma 2 $\overline{R} > 1 > \underline{R}$ *in any competitive equilibrium.*

Proof. Otherwise, suppose $\overline{R} = 1 = \underline{R}$. Consider the credit demand by entrepreneurs, which depends on whether default happens in the bad state or not. If not, then entrepreneurs choose I to maximize $q(\overline{A}I^\alpha - I) + (1-q)\underline{A}I^\alpha I$, which implies $I = (A_e\alpha)^{\frac{1}{1-\alpha}}$. But then the bad state output

$\underline{A}I^\alpha < A_e\alpha I^\alpha = A_e\alpha(A_e\alpha)^{\frac{\alpha}{1-\alpha}} = (A_e\alpha)^{\frac{1}{1-\alpha}} = I$. It follows that $\underline{R} \leq \frac{\underline{A}I^\alpha}{I} < 1$, contradictory to the supposition that $\underline{R} = 1$. If entrepreneurs default in the bad state, they choose I to maximize $q(\overline{A}I^\alpha - I)$, which implies $I = (\overline{A}\alpha)^{\frac{1}{1-\alpha}} > (A_e\alpha)^{\frac{1}{1-\alpha}}$. Then $\underline{R}$ is further lower than 1, contradictory to the supposition that $\underline{R} = 1$. ■

The next section solves the competitive equilibrium with the minimum difference between $\overline{R}$ and $\underline{R}$. This equilibrium features the least risk and is thus the most efficient one, and the equilibrium allocation is comparable to the social best allocation. As the liability side market is cleared at $r = 1$ by Lemma 1, we are concerned with only the credit market.

III. The Competitive Equilibrium

Let us start with the supply side of the credit market. The supply of credit equals $K + L$. L is also the leverage level of each deep pocket. How he decides the level is examined by the following model, which is the building block of the paper.

A. The Building Block: Risk and Leverage

The model is interesting in itself. So let us recap briefly the setting for individual deep pockets. A deep pocket, risk neutral, has K units of funds. He faces two kinds of investment opportunities, one-to-one storage and ventures with the gross return rate, $\widetilde{R}$, being $\overline{R}$ or $\underline{R}$ with probability q and $1 - q$ respectively. $\widetilde{R}$ is certainly decided by the credit market, but is taken as given by each deep pocket. There are also a lot of households, with Bernoulli utility function $U(\cdot)$, which satisfies $U' > 0, U'' < 0$. Each household has 1 unit of funds, which cannot be split between two investments. Let $R_e \equiv q\overline{R} + (1 - q)R$. Furthermore, assume

$$R_e > 1 \text{ and } qU(\overline{R}) + (1 - q)U(\underline{R}) < U(1) \tag{1}$$

It will be proved that (1) holds true in the general equilibrium, but for this subsection, let us simply take it as an assumption.

Note that as $qU(\overline{R}) + (1 - q)U(\underline{R}) < U(1)$, households will not invest in ventures, even if they know what a genuinely sensible venture is, that is, even without Assumption 1.

A deep pocket, being risk neutral, will invest all his funds in a venture. Without any leverage, he earns the expected return of R_e per unit of his capital. He can, however, boost the return rate of his capital by borrowing from households. To draw their interest, he promises to them a certainty equivalent return rate, r. We saw $r = 1$, given there are too many households.

Suppose that the deep pocket takes in L households' funds. He will invest all the $K + L$ units of funds in a venture; he gains nothing putting a borrowed dollar into storage: it returns one dollar, which is wholly repaid back to the lender. The security offered to the households are represented by the profile of the return rates, $\{\overline{h}, \underline{h}\}$, which satisfies the following individual rationality constraint (IR),

$$qU(\overline{h}) + (1-q)U(\underline{h}) = U(1) \tag{2}$$

Overall the investment of the deep pocket returns $(K + L)\underline{R}$ in the bad state and $(K + L)\overline{R}$ in the good state; he pays out $L\underline{h}$ and $L\overline{h}$ respectively. Thus, his expected profit is $(1-q)((K+L)\underline{R} - L\underline{h}) + q((K + L)\overline{R} - L\overline{h})$. His problem is then to choose $\{L, \overline{h}, \underline{h}\}$ to maximize this profit subject to (2) and $L\underline{h} \leq (K + L)\underline{R}$, the limited liability constraint. This problem is simplified a lot by considering how he benefits from leverage.

With each unit of households' funds, the deep pocket obtains R_e from the investment, repays $h_e = q\overline{h} + (1-q)\underline{h}$, and gains the margin of $R_e - h_e$. The margin is $R_e - 1 > 0$, so long as the security is risk free, namely $\overline{h} = \underline{h} = 1$. This is possible if and only if $L \leq (K+L)\underline{R}$, equivalently $L \leq \frac{\underline{R}}{1-\underline{R}}K$, by the limited liability constraint. Therefore, the deep pocket never stops borrowing if $L < \frac{\underline{R}}{1-\underline{R}}K$. When $L \geq \frac{\underline{R}}{1-\underline{R}}K$, however, the security becomes risky, namely $\overline{h} \geq 1 \geq \underline{h}$. In order to mostly reduce the risk to the households, he let them take all the earnings in the bad state. The limited liability constraint is thus binding,

$$\underline{h} = \frac{(K + L)\underline{R}}{L} \tag{3}$$

Therefore, the security to the households is debt, with the face value of $\overline{h}$, and the deep pocket's funds form the capital (the equity) of the FI. He gains net profit only in the good state. His problem becomes:

Problem 1 $\max_{L,\overline{h},\underline{h}} \Pi = q((K + L)\overline{R} - L\overline{h})$ *s.t. (2) and (3).*

It is equivalent to maximize $L(\overline{R}-\overline{h})$. The optimal leverage is decided by the trade-off between the scale (L) and the profit margin ($\overline{R}-\overline{h}$): the higher is L, the smaller is $\underline{h}$ by (3), then the larger is $\overline{h}$ by (2), and hence the lower is $\overline{R}-\overline{h}$. At the margin of $\overline{R}-\overline{h}$ and the scale of L, the gain from borrowing dL more is $(\overline{R}-\overline{h})\cdot dL$. However, for the already borrowed L units of funds, the profit margin decreases by $d\overline{h}$, leading to the total loss of $L\cdot d\overline{h}$. In the optimization, the two sides are equalized, $(\overline{R}-\overline{h})\cdot dL = L\cdot d\overline{h}$; equivalently, $\overline{R}-\overline{h}-L\frac{d\overline{h}}{dL}=0$. Note that $\frac{d\overline{h}}{dL}=\frac{d\overline{h}}{d\underline{h}}\cdot\frac{d\underline{h}}{dL}$, calculated from (2) and (3) respectively. Then we have the following first order condition.

$$\overline{R}-\overline{h}-\frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{K\underline{R}}{L}=0 \tag{4}$$

Then, the system of (2), (3), and (4) decides the optimal $\{L,\overline{h},\underline{h}\}$, for the given $\{\overline{R},\underline{R}\}$. All the three equations depend on K only through $l\equiv\frac{L}{K}$, the leverage rate. Therefore, the optimal l is a function of $\overline{R},\underline{R}$, independent of K. Let the function be $l(\overline{R},\underline{R})$, which could equal ∞ for some $\{\overline{R},\underline{R}\}$.

The leverage of each deep pocket is then $L=Kl(\overline{R},\underline{R})$.

We saw that the debt is risk free until $l=\frac{\underline{R}}{1-\underline{R}}$.[12] But the deep pocket will not stop borrowing at this level, by the lemma below.

Lemma 3 *$l(\overline{R},\underline{R})>\frac{\underline{R}}{1-\underline{R}}$ and hence the debt of FIs is risky.*

Proof. It suffices to show that $\frac{d\Pi}{dL}|_{L=\frac{\underline{R}}{1-\underline{R}}K}>0$, so that the deep pocket will keep borrowing at $l=\frac{\underline{R}}{1-\underline{R}}$. $\frac{d\Pi}{dL}=q(\overline{R}-\overline{h}-\frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{K\underline{R}}{L})$. When $L=\frac{\underline{R}}{1-\underline{R}}K$, $\overline{h}=\underline{h}=1$. Substitute these $L,\overline{h}$,and $\underline{h}$ into the equation, $\frac{d\Pi}{dL}|_{L=\frac{\underline{R}}{1-\underline{R}}K}=q(\overline{R}-1-\frac{(1-q)}{q}(1-\underline{R}))=R_e-1>0$. ■

The intuition for the lemma is simple. At $l=\frac{\underline{R}}{1-\underline{R}}$, the borrowing is still risk free and the overall margin is still $R_e-1>0$, as the proof shows, and hence the deep pocket keeps borrowing.

$\delta\equiv q(\overline{R}-R_e)=(1-q)(R_e-\underline{R})$ measures the risk of the projects. $l(\overline{R},\underline{R})$ can be transformed into a function of δ and R_e. The following lemma says that the leverage increases with the return rates of the projects and decreases with the risk.

[12] In Fostel and Geanakoplos (2008), borrowing is assumed to be risk free, and hence $l\leq\frac{\underline{R}}{1-\underline{R}}$.

Lemma 4 $\frac{\partial l}{\partial \overline{R}} > 0$ *and* $\frac{\partial l}{\partial \underline{R}} > 0$; $\frac{\partial l}{\partial R_e} > 0$ *and* $\frac{\partial l}{\partial \delta} < 0$.

Proof. See the appendices. Notice that $\frac{\partial l}{\partial R_e} > 0$ is simply implied by the former half of the lemma. ■

The lemma can be intuitively understood through the trade-off between the scale and the profit margin. For example, if $\overline{R}$ increases, then given L, the profit margin $\overline{R}-\overline{h}$ increases, which gives room to raise the scale. Similarly, given R_e fixed, suppose δ increases. Then $\overline{R}$ increases, and so does $\overline{h}$, since $\underline{h}$ decreases when $\underline{R}$ decreases. The net effect on the margin is negative – hence l goes down – since $\overline{h}$ increases by more than $\overline{R}$ due to two effects. One is that because of leverage, $\underline{h}$ decreases by more than $\underline{R}$; indeed, $d\underline{h} = \frac{1+l}{l} d\underline{R}$ by (3). The other is that because the households are risk averse, $\overline{h}$ has to increase by more than to compensate the decrement in $\underline{h}$ with h_e fixed (that is $d\overline{h} > \frac{1-q}{q}(-d\underline{h})$). On the other hand, $\overline{R}$ increases by exactly to compensate the decrement in $\underline{R}$, as R_e is fixed.

In the next subsection, we move on to the demand side.

B. The Demand Side

As $\overline{R} - \underline{R}$ is reduced to the most in this equilibrium, entrepreneurs let the creditors, namely the FIs, take all the yields in the bad state, $I\underline{R} = \underline{A}I^{\alpha}$. Therefore,

$$\underline{R} = \underline{A}I^{\alpha-1} \tag{5}$$

They then care only for the profit in the good state, and choose I to maximize $\overline{A}I^{\alpha} - I\overline{R}$, which implies

$$\overline{R} = \overline{A}\alpha I^{\alpha-1} \tag{6}$$

Basically, (6) decides the demand I, and then (5) decides the bad state return rate $\underline{R}$.

The next subsection fits the demand with the supply and pins down the competitive equilibrium.

C. The Market Clearing

The market is cleared if the demand equals the supply, that is, $I = K(1 + l(\overline{R}, \underline{R}))$. Substitute $\overline{R}$ and $\underline{R}$ with (6) and (5) respectively,

$$I = K(1 + l(\overline{A}\alpha I^{\alpha-1}, \underline{A}I^{\alpha-1})) \tag{7}$$

(7) has a unique solution, and hence the competitive equilibrium exists uniquely, by the following reason. The left hand side (LHS) of (7) increases with I, while the right hand side (RHS) decreases with it by Lemma 4. Moreover, if $I \to \underline{A}^{\frac{1}{1-\alpha}}$ and thus $\underline{R} = \underline{A}I^{\alpha-1} \to 1$, then $l \to \infty$ by Lemma 3; on the other hand, if $I \to \infty$ and thus both rates go to 0, then $l \to 0$. Therefore, the LHS and RHS intersect, and only once.

(1) is assumed in the building block model (subsection 3.1). Now it is time to prove it is an outcome of the equilibrium.

Lemma 5 $R_e > 1$, *and* $qU(\overline{R}) + (1-q)U(\underline{R}) < U(1)$.

Proof. See the Appendices. ■

$R_e > 1$ is driven by the scarcity of deep pockets' capital, namely the smallness of K as is set in Assumption 3. Nevertheless, for any $K > 0$, however small it is, leverage will expand the credit supply to the level at which the interest rates satisfy $qU(\overline{R}) + (1-q)U(\underline{R}) < U(1)$. The intuition is that so long as the interest rates are higher, in the sense that $qU(\overline{R}) + (1-q)U(\underline{R}) \geq U(1)$, it will be profitable for deep pockets to be leveraged more and to expand the credit supply, until the interest rates are pulled down to satisfy the inequality.

Notice that in the partial equilibrium model of Subsection 3.1, where $\{\overline{R}, \underline{R}\}$ are given, the total credit supply, $K + L = K(1 + l(\overline{R}, \underline{R}))$, is proportional to K. The model implies, therefore, that the credit supply goes down linearly with FIs' capital, especially it goes to 0 when the capital goes to 0. This seems to have well justified the worry of banks issuing too little credit when they suffers substantial loss. When $\{\overline{R}, \underline{R}\}$ is endogenized in general equilibrium, however, Lemma 5 above says that the credit does not decrease so fast as linearly, and later Proposition

1 will show that actually there is always too much credit. This illustrates the point that when addressing the problem of credit shortage, those "partial equilibrium" models that take the credit returns as given, such as Holmstrom and Tirole (1997) and He and Krishnamurthy (2008), may not provide robust conclusions.

Notice also that it was set in Assumption 1 that households do not directly invest in entrepreneurs because they fail to observe the entrepreneurial human capital ex ante. That $qU(\overline{R}) + (1-q)U(\underline{R}) < U(1)$ implies that even if they can recognize a genuine entrepreneur ex post (for example, by observing him financed by a deep pocket), they will not directly invest in him because of the risk involved.

Substitute (5) and (6) into (3) and (4),

$$\underline{h} = \frac{\underline{A}(K+L)^a}{L} \tag{8}$$

$$\alpha\overline{A}(K+L)^{\alpha-1} - r - \frac{(1-q)U'(\underline{h})}{qU'(r)} \frac{\underline{A}(k+L)^{\alpha-1}}{L} K = 0 \tag{9}$$

The system of these two equations plus (2), the IR to households, decides the equilibrium $\{L, \overline{h}, \underline{h}\}$, which we denote by $\{L^E, \overline{h}^E, \underline{h}^E\}$. And the equilibrium return rates are denoted by $\overline{R}^E$ and $\underline{R}^E$. The leverage rate is $l^E = \frac{L^E}{K}$ and the total credit is $I^E = K(1+l^E)$. $\Delta \equiv q(\overline{A} - A_e) = (1-q)(A_e - \underline{A})$ measures the macroeconomic risk. We have the following comparative statics results.

Lemma 6 *When the capital of the FIs (K) increases, the leverage rate (l^E) decreases, but the total credit (I^E) increases. Given K, the leverage rate and the total credit decrease with the macroeconomic risk (Δ).*

Proof. It suffices to prove $\frac{\partial l^E}{\partial K} < 0$, $\frac{\partial I^E}{\partial K} > 0$, and $\frac{\partial l^E}{\partial \Delta} < 0$ (keep A_e fixed). See the Appendices. ■

Notice that $\frac{\partial l^E}{\partial K} < 0$ is an effect of general equilibrium, as in the partial equilibrium of Subsection 3.1, the leverage rate is independent of K. On the other hand, the result that the leverage rate decreases with the risk holds true in both partial and general equilibrium.

The next section considers the social best allocation, which is restored in the marekt by a proper regulation.

IV. The Social Best Allocation and the Capital Adequacy Regulation

We show first that FIs are overleveraged and then that the social best allocation is implemented by the competitive market if FIs are regulated not to be leveraged over the social best level.

A. The Social Best Allocation

Because of the difference in risk preference between the agents, there is a continuum of Pareto optimal allocations. To be comparable with the equilibrium allocation, we consider the allocation that maximizes the total sum of the profits of deep pockets and entrepreneurs subject to giving households the equilibrium utility, $U(1)$. Therefore, the social planner's problem is

Problem 2 $Max_{L,\overline{h},\underline{h}} q[\overline{A}(K+L)^{\alpha} - L\overline{h}] + (1-q)[[\underline{A}(K+L)^{\alpha} - L\underline{h}]$, *s.t.*
(a): $qU(\overline{h}) + (1-q)U(\underline{h}) = U(1)$; *and (b):* $\underline{A}(K+L)^{\alpha} - L\underline{h} \geq 0$.

Here, $\{\overline{h}, \underline{h}\}$ is the consumption profile allocated to each of the L households that contribute their capital to ventures. Constraint (a) ensures that they obtain $U(1)$ and is the same as (2), the IR for households to invest in FIs; constraint (b) is the resource constraint, equivalent to the limited liability constraint.

As in the deep pocket's problem, the social planner, whenever possible, wants $\{\overline{h}, \underline{h}\}$ to be risk free: $\overline{h} = \underline{h} = 1$; this is possible if and only if $L \leq \underline{A}(K+L)^{\alpha}$. Given L, the marginal product of a unit of households' capital is $M(L) \equiv q\overline{A}\alpha(K+L)^{\alpha-1} + (1-q)\underline{A}a(K+L)^{\alpha-1}$. If the risk free consumption profile is available and $M(L) \geq 1$, the social planner keeps drawing households' capital into ventures. Indeed, $M(L) \geq 1$, when the consumption profile is risk free, according to the following lemma.

Lemma 7 $M(L) \geq 1$ *for* $L \leq \underline{A}(K+L)^{\alpha}$.

Proof. Parallel to the proof of Lemma 5. See the Appendices. ■

The lemma is driven by Assumption 3, which says that both K and $\underline{A}$ are small enough.

The social planner, therefore, takes in households' capital beyond the level of the risk free consumption profile being available. This implies, as in the deep pocket's problem, that all the outputs in the bad state are distributed to the households in Problem 2. That is, constraint (b) of Problem 2 is binding, which gives rise to (8). Then, the social planner's problem becomes:

Problem 3 $\max_{L,\overline{h},\underline{h}} \overline{A}(K+L)^{\alpha} - L\overline{h}$, *s.t. (2) and (8).*

The first order conditions of the problem are (2), (8), and

$$\alpha\overline{A}(K+L)^{\alpha-1} - \overline{h} - \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{\underline{A}(K+L)^{\alpha-1}}{L}(K+(1-\alpha)L) = 0 \tag{10}$$

The system of these three equations decides the social best $\{L, \overline{h}, \underline{h}\}$, which we denote by $\{L^B, \overline{h}^B, \underline{h}^B\}$. The system that decides $\{L^E, \overline{h}^E, \underline{h}^E\}$ is composed of (2), (8), and (9). The difference between (9) and (10) is that the term $(1-\alpha)L$ is present not in the former, but in the latter. This difference leads to the following result.

Proposition 1 $L^B < L^E$, *that is, in the competitive market FIs are over-leveraged.*

Proof. See the Appendices. ■

To obtain an intuition for this result, let us check where the term $(1-\alpha)L$ comes from. Compare Problem 1 to Problem 3. The LHS of (9) is the derivative of the deep pocket's objective, $(K+L)\overline{R} - L\overline{h}$, with respect to L, while the LHS of (10) is the corresponding derivative of the planner's objective, $\overline{A}(K+L)^{\alpha} - L\overline{h}$. As $\overline{R} = \overline{A}\alpha I^{\alpha-1}$ in the equilibrium, the difference is only in the term $L\frac{d\overline{h}}{dL}$. It equals $L\frac{d\overline{h}}{d\underline{h}}\frac{d\underline{h}}{dL} = L\frac{-(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{d\underline{h}}{dL}$ in both cases; however, $\underline{h} = \frac{(K+L)\underline{R}}{L}$ in the deep pocket's problem, whereas $\underline{h} = \frac{\underline{A}(K+L)^{\alpha}}{L}$ in the planner's problem. In equilibrium $\underline{R} = \underline{A}(K+L)^{\alpha-1}$ so that the two $\underline{h}$ are equal for any given L. However, when choosing L, each deep pocket takes $\underline{R}$ as given and ignores the effect of his choice of L on it, whereas the planner takes this effect into account, which gives rise to the term $(1-\alpha)L$ in (10).[13] Intuitively, there is a negative externality: the leverage of one FI expands the credit supply, which marginally lowers the return of the credit, and thus the bad state return rate which another FI offers to its creditors; consequently that FI has to offer to them a higher good state return rate, which reduces the FI's profit margin. Because of this negative externality, FIs are overleveraged.

Remark: this inefficiency is robust to the renegotiation between deep pockets and entrepreneurs. Consider the easy case where an entrepreneur is financed by only one deep pocket who, on the other hand, only finances the entrepreneur. The deep pocket may suggest that the entrepreneur invest less, only $I^B = K + L^B$, and obtain in the good state $(1-\alpha)\overline{A}(I^E)^\alpha$, exactly what he would obtain if turning to the credit market. Suppose the entrepreneur accepts the suggestion and signs the agreement, which the deep pocket shows to L^B households, in order to convince them that they will obtain $\underline{h}^B (> \underline{h}^E)$ in the bad state and thus should accept the return rate of $\overline{h}^B (< \overline{h}^E)$ in the good state. The deep pocket would achieve the social best leverage if the households were convinced that the agreement would be honored; in that case, the equilibrium considered above is not robust. However, they will not be convinced, for two reasons. First, the households do not know whether the entrepreneur who signs the agreement is a genuine entrepreneur and thus the agreement is a genuine agreement. Second, even if the agreement is genuine, it will not be honored. After taking in the households' capital, the deep pocket will invest all $K + L^B = I^B$ units of funds in the credit market instead of in the entrepreneur as is agreed, thus the households going to obtain $\frac{I^B \underline{R}^E}{L^B} < \underline{h}^E$:[14] the revenue in the good state is $I^B \overline{R}^E$ by the former investment and $\overline{A}(I^B)^\alpha - (1-\alpha)\overline{A}(I^E)^\alpha$ by the latter; $I^B\overline{R}^E > \overline{A}(I^B)^\alpha - (1-\alpha)\overline{A}(I^E)^\alpha$ always.[15]

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[13]Let "$\prime$" denote the derivative to L. Then $(\frac{\underline{A}(k+L)^a}{L})' = (\frac{(K+L)\underline{R}}{L})' = \frac{-K}{L^2}\underline{R} + \frac{K+L}{L}\underline{R}' = \frac{-\underline{R}}{L^2}(K + (1-\alpha)L)$, since $\underline{R}' = -(1-\alpha)\frac{\underline{R}}{K+L}$. Therefore, $\frac{K+L}{L}\underline{R}'$ contributes the term $(1-\alpha)L$.

[14] $\frac{I^B \underline{R}^E}{L^B} = \frac{(K+L^B)\underline{A}(K+L^E)^{\alpha-1}}{L^B} < \frac{(K+L^B)\underline{A}(K+L^B)^{\alpha-1}}{L^B} = \underline{h}^B$.

[15]the inequality is equivalent to $(1-\alpha)\overline{A}(I^E)^\alpha > \overline{A}(I^B)^\alpha - I^B\overline{R}^E$. As $I^B < I^E$, it follows from the fact that $f(I) \equiv \overline{A}I^\alpha - I\overline{R}^E < (1-\alpha)\overline{A}(I^E)^\alpha$ for any $I < I^E$. That is true, because $f(I^E) = (1-\alpha)\overline{A}(I^E)^\alpha$, and $f'(I) = \overline{R}(I) - \overline{R}^E > 0$ for $I < I^E$, which is because $\overline{R}(I) = \overline{A}\alpha I^{\alpha-1}$ decreases with I and $\overline{R}^E = \overline{R}(I^E)$.

This market failure stated by Proposition 1 exists only because the deep pockets become FIs standing between households and entrepreneurs. It would not arise if entrepreneurs could issue financial contracts directly to both deep-pockets and households. In that case, they would issue equity to former and the debt to the latter, according to the risk preference. The return rates of the debt offered by one entrepreneur would depend not upon the leverage of another project, but only upon that of his own, which cuts off the logical link leading to the externality. Indeed, the social best allocation would be achieved, as the following proposition states.

Proposition 2 *When entrepreneurs could issue financial contracts directly to households, the competitive equilibrium would implement the social best allocation,* $\{L^B, \overline{h}^B, \underline{h}^B\}$.

Proof. See the appendices. ■

We return to the model, where deep pockets become FIs which are overleveraged by Proposition 1. We move on to consider the regulation under which the market equilibrium implements the social best allocation.

B. The Capital Adequacy Regulation

The social best allocation can be restored by the capital adequacy regulation that prohibits the share of a FI's equity in the book value from being less than $\frac{K}{K+L^B}$, that is, prohibits the leverage rate of a FI from being more than $\frac{L^B}{K}$, the social best level. The regulation leads to the following results.

Proposition 3 *Under the regulation that* $l \leq \frac{L^B}{K}$, *the competitive equilibrium implements* $\{L^B, \overline{h}^B, \underline{h}^B\}$, *and the profit of entrepreneurs decreases and the profit of deep pockets increases.*

Proof. Since $I^E = K + L^E$ is the unique solution of (7) and is larger than $K + L^B$ by Proposition 1, $K + L^B < K(1 + l(\overline{R}(I^B), \underline{R}(I^B))$. This inequality implies $L^B < Kl(\overline{R}(I^B), \underline{R}(I^B))$. That is, L^B is less than the optimal leverage level of a FI when facing $\overline{R}(I^B), \underline{R}(I^B)$. Each FI would want to borrow more, which is, however, disapproved by the regulation. Therefore, all FIs are leveraged to the regulation limit, $\frac{L^B}{K}$, and $\{L^B, \overline{h}^B, \underline{h}^B\}$ is implemented. Compare the payoff for each group with the regulation to without. Households get the same, $U(1)$. The entrepreneurs get less: they are paid off only in the good state, with $\overline{A}I^{\alpha} - I\overline{R} = (1-\alpha)\overline{A}I^{\alpha}$, and the regulation shrinks I from $K + L^E$ to $K + L^B$. Thus, deep pockets get more, since the regulation increases the sum total of the payoffs of entrepreneurs and deep pockets. ■

The fact that the financial sector as a whole gains from the regulation does not mean that individual FIs are happy to abide by it. On the contrary, as the proof above shows, given the high return rates of credit brought by this regulation, which shrinks the credit supply, individual FIs want to be leveraged over the regulation limit. They would like to set up off-balance "Special Investment Vehicles", if possible.

V. Conclusion

The paper presents a general equilibrium model on the leverage of financial intermediaries (FIs), which is driven by the difference in risk preference. In the paper, the equity of FIs is held by risk neutral agents (deep pockets), and the debt is held by risk averse agents (households). Equity holders use their capital as the cushion to absorb the loss to debt holders in the bad state, so the debt holders are willing to invest in the FIs; the debt holders' capital is used to boost the return of the equity. With each unit of this capital, the equity holders gain the margin between the expected return rate on the asset side and that repaid to the debt holders. The margin goes down with the scale of the leverage: the more the borrowing, the less the average capital used to cushion each borrowed unit, the greater the risk to the debt holders, hence the higher the expected return they demand, and the lower the profit margin. The trade-off between the margin and the scale decides the optimal leverage rate of a FI, which decreases with the risk on its asset side.

In the paper, the asset side of FIs is determined by the credit market between FIs and entrepreneurs. With the asset side endogenized, the paper finds that in an unregulated market, FIs are leveraged over the social best level, because of the following externality. The leverage of one FI expands the credit supply, which marginally lowers the credit return rate in the bad state, which in turn lowers the bad state return rate that other FIs offer to their lenders; as compensation, they have to offer a higher good state return rate, and obtain a lower profit margin. This paper suggests, therefore, that the *whole* financial sector should be subject to this capital adequacy regulation. Moreover, the paper shows that the aggregate credit supply indeed goes down with the aggregate capital of FIs, but in a much slower pace than linearly (which is the prediction of the models that take the credit return rates as given). Indeed, however small the aggregate capital is, the aggregate credit is always abundant, in the sense that the resulting credit return rates are so low that households find it uninteresting to directly invest in entrepreneurs.

The model of the paper is static. It could be extended to a fully fledged RBC style model, which would be useful to investigate the dynamics of the leverage of FIs and whether debt accumulation by the financial sector necessarily leads to systematic risks.

Appendices

The Proof of Lemma 4:

To show comparative statics, we resort to the following principle: if some change leads margin $(\overline{R}-\overline{h})$ to increase for any given l, then it will increase l, the leverage rate. For the optimal leverage rate is decided by the trade-off between the margin and the scale; thus the increment in the former gives the room for increase in the latter.

Given l, fixed $\underline{R}$ implies fixed $\underline{h}$ and hence fixed $\overline{h}$. Thus, if $\overline{R}$ increase, then $\overline{R}-\overline{h}$ is increased. On the other hand, if $\underline{R}$ increase, then so does $\underline{h}$. By (2), $\overline{h}$ decreases. Thus, the margin increases also. Therefore, in both cases, l is increased, by the principle above.

Consider the effect of $d\delta > 0$. $d\overline{R} = \frac{d\delta}{q}$ and $d\underline{R} = \frac{-d\delta}{1-q}$. Given l, $d\underline{h} = \frac{1+l}{l}d\underline{R} = \frac{1+l}{l}\frac{-d\delta}{1-q}$. To keep $h_e = q\overline{h} + (1-q)\underline{h}$ unchanged, $d\overline{h} = \frac{1-q}{q}(-d\underline{h})$. Because the households are risk averse, to satisfy their IR, the required increment $d\overline{h} > \frac{1-q}{q}(-d\underline{h}) = \frac{1+l}{ql}d\delta$. Then change in the margin is $d(\overline{R}-\overline{h}) = \frac{d\delta}{q} - d\overline{h} < \frac{-1}{ql}d\delta < 0$. Thus $dl < 0$. Q.E.D.

The Proof of Lemma 5:

To prove $R_e > 1$, it suffices to show that $R_e = 1 \Rightarrow K > (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$, which violates

Assumption 3. With each unit of borrowed capital, the profit margin of a deep pocket is $R_e - h_e$. If $R_e = 1$, the margin is 0, as $h_e \geq 1$, and is strictly negative when $h_e > 1$, which is the case if the repayment $\{\underline{h}, \overline{h}\}$ bears any risk by the IR, (2), since households are risk averse. Therefore, if $R_e = 1$, leverage happens only if $\{\underline{h}, \overline{h}\}$ is risk free, that is, $\underline{h} = 1 = \overline{h}$. Then by the limited liability constraint, $L \leq (K+L)\underline{R} \Leftrightarrow L \leq \frac{\underline{R}}{1-\underline{R}}K$. In equilibrium, $I = K + L$, therefore, $I \leq \frac{1}{1-\underline{R}}K \Leftrightarrow K \geq (1-\underline{R})I$. By (5), $I = (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$. Therefore,

(A1): $K \geq (1-\underline{R})(\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$.

On the other hand, by (5) and (6), $\overline{R} = \frac{\overline{A}\alpha}{\underline{A}}\underline{R}$. So $q\overline{R} + (1-q)\underline{R} = 1 \Rightarrow \frac{q\overline{A}\alpha + (1-q)\underline{A}}{\underline{A}}\underline{R} = 1 \Rightarrow \underline{R} = \frac{\underline{A}}{q\overline{A}\alpha + (1-q)\underline{A}}$. By (A1), $K \geq f(\frac{\underline{A}}{q\overline{A}\alpha + (1-q)\underline{A}})$, where $f(x) = (1-x)(\frac{\underline{A}}{x})^{\frac{1}{1-\alpha}}$. Since f is decreasing and $\frac{\underline{A}}{\alpha A_e} > \frac{\underline{A}}{q\overline{A}\alpha + (1-q)\underline{A}}$, $K \geq f(\frac{\underline{A}}{q\overline{A}\alpha + (1-q)\underline{A}}) > f(\frac{\underline{A}}{\alpha A_e}) = (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$, violating Assumption 3. This ends the proof of the former part.

To prove the latter part of the lemma, it suffices to show that if $qU(\overline{R}) + (1-q)U(\underline{R}) \geq U(1)$, the optimal $L = \infty$, which cannot be true in equilibrium. In order to show that, it suffices to prove $L = \infty$ for the case where $qU(\overline{R}) + (1-q)U(\underline{R}) = U(1)$, because by Lemma 4, higher is $\widetilde{R}$, the larger is L.

To prove that, it suffices to show $\frac{d\Pi}{dL} > 0$ for any finite L. $\frac{1}{q}\frac{d\Pi}{dL} = \overline{R} - \overline{h} - \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{K\underline{R}}{L}$. By (2), $qU(\overline{h}) + (1-q)U(\underline{h}) = U(1) = qU(\overline{R}) + (1-q)U(\underline{R})$. It follows that $q(U(\overline{R}) - U(\overline{h})) = (1-q)(U(\underline{h}) - U(\underline{R})) \Leftrightarrow qU'(\xi)(\overline{R} - \overline{h}) = (1-q)U'(\zeta)(\underline{h} - \underline{R})$, for some ξ, ζ such that $\overline{R} > \xi > \overline{h} > \underline{h} > \zeta > \underline{R}$. Then $\overline{R} - \overline{h} = \frac{(1-q)U'(\zeta)}{qU'(\xi)}(\underline{h} - \underline{R}) = \frac{(1-q)U'(\zeta)}{qU'(\xi)}\frac{K\underline{R}}{L}$, where the last equation applies $\underline{h} = \frac{(K+L)\underline{R}}{L}$ by (3). Substitute this formula for $\overline{R} - \overline{h}$ into the equation of $\frac{d\Pi}{dL}$, $\frac{1}{q}\frac{d\Pi}{dL} = \frac{(1-q)K\underline{R}}{qL}(\frac{U'(\zeta)}{U'(\xi)} - \frac{U'(\underline{h})}{U'(\overline{h})})$. Because $\underline{h} > \zeta$ and U' is decreasing, $U'(\zeta) > U'(\underline{h})$; similarly $U'(\xi) < U'(\overline{h})$ since $\xi > \overline{h}$. Therefore, $\frac{U'(\zeta)}{U'(\xi)} > \frac{U'(\underline{h})}{U'(\overline{h})}$. Hence $\frac{d\Pi}{dL} > 0$ for any L. Q.E.D.

The Proof of Lemma 6:

First, let us establish that $\frac{\partial I^E}{\partial K} > 0$. Suppose otherwise $\frac{\partial I^E}{\partial K} \leq 0$. Then, $\frac{\partial \underline{R}^E}{\partial K} \geq 0$ and $\frac{\partial \overline{R}^E}{\partial K} \geq 0$. By Lemma 4, $l^E = l(\overline{R}^E, \underline{R}^E)$ increases with K, which implies $I^E = K(1 + l^E)$ strictly increases with K, contradictory to the supposition that $\frac{\partial I^E}{\partial K} \leq 0$. Therefore, $\frac{\partial I^E}{\partial K} > 0$. Second, $\frac{\partial I^E}{\partial K} > 0$ implies that both $\frac{\partial \underline{R}^E}{\partial K} < 0$ and $\frac{\partial \overline{R}^E}{\partial K} < 0$, which, by Lemma 4, implies $\frac{\partial l^E}{\partial K} < 0$.

As to the effects of Δ, let us establish $\frac{\partial I^E}{\partial \Delta} < 0$, so that $\frac{\partial l^E}{\partial \Delta} < 0$, as $I^E = K(1 + l^E)$ and K is fixed. Suppose otherwise $\frac{\partial I^E}{\partial \Delta} \geq 0 \Leftrightarrow \frac{\partial l^E}{\partial \Delta} \geq 0$. Then, $\frac{\partial R_e}{\partial \Delta} < 0$, since $R_e = ((\alpha q + 1 - q)A_e - (1 - \alpha)\Delta)(I^E)^{\alpha - 1}$. Meanwhile, δ, the variance of $\widetilde{R}$, increases with Δ. Both changes strictly decrease l^E by Lemma 4, contradictory to the supposition that $\frac{\partial l^E}{\partial \Delta} \geq 0$. Q.E.D.

The Proof of Lemma 7:

The key condition for this lemma is $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}}$, as is for the former part of Lemma 5. Since $M(L)$ decreases with L, it suffices to prove that $M(L) \geq 1$ for the L such that $L = \underline{A}(K+L)^\alpha$. To see the parallel to the proof of Lemma 3, here let $\underline{R} \equiv \underline{A}(K+L)^{\alpha-1}$. Then $M(L) = \frac{A_e \alpha}{\underline{A}} \underline{R}$, and $K + L = (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$. By the latter, the condition $L = \underline{A}(K+L)^\alpha \Leftrightarrow (\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}} - K = \underline{A}(\frac{\underline{A}}{\underline{R}})^{\frac{\alpha}{1-\alpha}}$, which is equivalent to

(D1): $K = f(\underline{R}) \equiv (1 - \underline{R})(\frac{\underline{A}}{\underline{R}})^{\frac{1}{1-\alpha}}$.

(D1) is parallel to (A1) of the proof of Lemma 3. By Assumption 3, $K \leq (\alpha A_e - \underline{A})(\alpha A_e)^{\frac{\alpha}{1-\alpha}} = f(\frac{\underline{A}}{\alpha A_e})$. Then $\underline{R} = f^{-1}(K) \geq \frac{\underline{A}}{A_e \alpha}$, as $f(\underline{R})$ is decreasing. It follows that $M(L) = \frac{A_e \alpha}{\underline{A}} \underline{R} \geq 1$. Q.E.D.

The Proof of Proposition 1:

The only difference is between (9) and (10). These two equations can be unified into

(F1): $\alpha \overline{A}(K+L)^{\alpha-1} - \overline{h} - \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})} \frac{\underline{A}(K+L)^{\alpha-1}}{L}(K + \theta L) = 0$.

(F1) is (9) for $\theta = 0$ (the competitive equilibrium case) and is (10) for $\theta = 1 - \alpha$ (the social best case). Let $L(\theta)$ be the solution of (F1), where $\underline{h}$ and $\overline{h}$ are implicit functions of L decided by (8) and (2), which are rewritten below.

(F2): $\underline{h} = \frac{\underline{A}(K+L)^\alpha}{L}$.

(F3): $qU(\overline{h}) + (1-q)U(\underline{h}) = U(1)$.

Because $L(0) = L^E$ and $L(1-\alpha) = L^B$, to prove $L^B < L^E$, it suffices to show that $\frac{dL}{d\theta} < 0$ for $\theta \leq 1 - \alpha$.

Let $F(L, \theta)$ be the LHS of (F1). Then, $\frac{dL(\theta)}{d\theta} = -\frac{\partial F/\partial \theta}{\partial F/\partial L}$. Simply $\frac{\partial F}{\partial \theta} = -S(L)\underline{A}(K+L)^{\alpha-1} < 0$, where $S(L) = \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})} > 0$, an implicit functions of L. To prove $\frac{dL(\theta)}{d\theta} < 0$, it suffices to show $\frac{\partial F}{\partial L} < 0$. $\frac{\partial F}{\partial L} = \alpha(\alpha-1)\overline{A}(K+L)^{\alpha-2} - \frac{d\overline{h}}{dL} - \frac{dS}{dL}\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L} - Sd\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}/dL$. The first term is negative; as to the third term, $\frac{dS}{dL} > 0$, since with L increasing, $\underline{h}$ decreases and accordingly $\overline{h}$ increases, and hence $S = \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}$ increases. To show $\frac{\partial F}{\partial L} < 0$, it thus suffices to prove that $-\frac{d\overline{h}}{dL} - Sd\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}/dL < 0$. By (F3), $\frac{d\overline{h}}{d\underline{h}} = -S$. Then $\frac{d\overline{h}}{dL} = \frac{d\overline{h}}{d\underline{h}}\frac{d\underline{h}}{dL} = -S\frac{d\underline{h}}{dL}$. Thus, $-\frac{d\overline{h}}{dL} - Sd\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}/dL = S(\frac{d\underline{h}}{dL} - d\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}/dL)$, which is negative if and only if $\frac{d\underline{h}}{dL} < d\frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}/dL$. Therefore, it suffices to show that $\frac{d\underline{h}}{dL} < \frac{\partial g}{\partial L}(L,\theta)$ for $\theta \leq 1-\alpha$, where $g(L,\theta) \equiv \frac{\underline{A}(K+L)^{\alpha-1}(K+\theta L)}{L}$.

$\frac{\partial g}{\partial \theta} = \underline{A}(K+L)^{\alpha-1}$. Then $\frac{\partial g_\theta}{\partial L} < 0$. So $\frac{\partial g_L}{\partial \theta} = \frac{\partial g_\theta}{\partial L} < 0$, that is $\frac{\partial g}{\partial L}(L,\theta)$ decreases with θ. Notice that $\underline{h} = g(L,1)$. Then, $\frac{d\underline{h}}{dL} = \frac{\partial g}{\partial L}(L,1) < \frac{\partial g}{\partial L}(L,\theta)$ for $\theta \leq 1-\alpha$. Q.E.D.

The Proof of Proposition 2:

An entrepreneur issues equity to deep pockets and debt to households. Similarly, to reduce the risk to households to the most, all the yields are distributed to debt holders in the bad state. The equity is thus represented by π, the return rate in the good state, which will clear the market for deep pockets' funds. The market for households' funds is cleared by certainty equivalent return rate, r. Let $\{\overline{h}, \underline{h}\}$ be the profile of the return rates of the debt, K_d denote the entrepreneur's demand for deep pockets' funds, and L denote the demand for households' funds. Since the debt holders obtain all the yields in the bad state,

(G1): $\underline{h} = \frac{\underline{A}(K_d+L)^\alpha}{L}$.

$\overline{h}$ is decided through the IR, that is,

(G2): $qU(\overline{h}) + (1-q)U(\underline{h}) = U(r)$.

Given $\{\pi, r\}$, the entrepreneur decides K_d and L, by solving the following problem:

$\max_{K_d,L,\overline{h},\underline{h}} \overline{A}(K_d+L)^\alpha - \pi K_d - \overline{h}L$, s.t. (G1) and (G2).

The first order conditions include G(1) and (G2) and the following.

(G3): $\alpha\overline{A}(K_d+L)^{\alpha-1} - \overline{h} - \frac{(1-q)U'(\underline{h})}{qU'(\overline{h})}\frac{\underline{A}(K_d+L)^{\alpha-1}}{L}(K_d+(1-\alpha)L) = 0$

In the equilibrium, $K_d = K$, and $r = 1$ since N is large. The equilibrium $\{L, \overline{h}, \underline{h}\}$ is to be found by substituting $K_d = K$ and $r = 1$ into equations (G1-3). The three equations are exactly the same as (8) and (2) and (10) respectively. Therefore, the same profile of $\{L^B, \overline{h}^B, \underline{h}^B\}$ is implemented in the equilibrium. Q.E.D.

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