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Strategic interactions and heterogeneity in a overlapping generations model with negative environmental externalities

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Abstract

We analyze an overlapping generations model where individuals' welfare depends on the stock of a free access environmental good E and on the consumption C of a private good.

We assume that the production process of the private good depletes the natural resource but that specific investments alleviate these damages. In such context, we show that strategic behaviour and heterogeneity in preferences may be a source of complex dynamics.

1 Introduction

Understanding the consequences of heterogeneity of economic agents for the management of common access environmental resources (commons) is one of the most appealing objectives of environmental economics. Does heterogeneity favor a sustainable management of commons or instead lead to over-exploitation? No clear cut answer to this issue emerges from economic literature; in some cases, the answer is “no”, in others it is “yes”; see e.g. Lekakis and Kousis (2001), Bardhan and Dayton-Johnson (2002), Cardenas et al. 2002, Dayton-Johnson and Bardhan (2002), Roca (2003), Potete and Ostrom (2004), Rutan (2006, 2008), Naidu (2008), Windrum et al. (2009).

By heterogeneity is meant both economic inequalities (e.g. in wealth and income) as well as socio-cultural differences (e.g. in preferences, class, religion). The main lesson that emerges from literature is that the effect of heterogeneity is highly context-dependent; it depends on the type of heterogeneity is considered, socio-cultural or economic, and on how the “success” in the management of

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commons is measured, in terms of collective action or in terms of the level of the collective good provided¹.

Socio-cultural heterogeneity seems to have a more clearly negative effect than does economic heterogeneity (Rutan 2006, 2008) in that it may inhibit the emergence of environment-preserving social norms in the economy. There are several causal mechanisms according to which socio-cultural heterogeneity may negatively affect natural resources management; for example, heterogeneity may inhibit the formation of trusting relationships that are a precondition of successful collective action (see e.g. Ostrom and Walker 2003) or may prevent agreement over the allocation of benefits deriving from the resources (see e.g. Singleton 2001).

The role of economic heterogeneity is less understood. There are a lot of studies that support a positive correlation between economic heterogeneity and environment preservation (see e.g. Ruttan and Borgerhoff Mulder 1999, Varughese and Ostrom 2002, Bardhan and Dayton-Johnson 2002, Dayton-Johnson and Bardhan 2002). In fact, the existence of “privileged” individuals in the community may favor the provision of the collective good via a better internalization of benefits and costs (the so called “Olson effect”, Olson 1965). However, some studies find U-shaped correlations between economic heterogeneity and various measures of success in resources management (see e.g. Bardhan and Dayton-Johnson 2002, Naidu 2008)² while others show that inequality may fuel distributional conflicts that inhibit collective action (see e.g. Jhonson and Libecap 1982, Singleton 2001). Exhaustive reviews of the mechanisms found in literature according to which heterogeneity may influence resources management can be found in Naidu (2008) and Rutan (2008).

The objective of our paper is to analyze, to our knowledge for the first time, the effects on environmental dynamics of socio-cultural heterogeneity by a “standard” overlapping generations model built on John and Pecchenino (1994)’s seminal work and the subsequent Zhang (1999)’s work, where a simplified version of John and Pecchenino’s model is studied. Differently that in Zhang’s model, where a social planner internalizes intra-generational externalities, we consider a strategic context in which each agent’s choices are taken on the basis of the (perfectly foresighted) expected behavior of the other agents. In this framework, where environmental externalities play a key role not only in determining future generations’ welfare but also in producing intra-generational undesirable over-exploitation results, we introduce socio-cultural heterogeneity by assuming that individuals have heterogeneous preferences over consumption goods and environmental quality.

¹It could be argued that successful collective action is a necessary precondition for the provision of the collective public good. However, this is not necessarily true (see e.g. Ruttan 1998, 2008) in that privileged individuals may be motivated to unilaterally provide the public good.

²When inequality is high, the rich unilaterally provides the public good; with low levels of inequality, all individuals face the same incentives and cooperation emerges. With intermediate levels of inequality, cooperation does not emerge and the rich are not sufficiently incentivated to unilaterally sustain the cost of environmental management, so heterogeneity negatively affects the resource management.

We start our analysis by comparing dynamics in Zhang's model with those in ours, in a context of a homogeneous population of agents. The main result of this part is that strategic behavior of economic agents favors the rise of complex dynamics in Zhang's model. In the remaining part of the paper, we investigate the role of heterogeneity in individuals' preferences; as in Bosi and Seegmuller (2008), we adopt a mean-preserving approach according to which an increase in heterogeneity is defined as an increase in individuals' dispersion around a constant mean value characterizing preferences. In such context, we find that an increase in heterogeneity lowers environmental quality and capital accumulation evaluated at the unique fixed point of dynamics and favors the rise of chaotic dynamics.³⁴

The paper is organized as follows. Section 2 introduces the overlapping generations model; sections 3-7 analyze it and section 8 concludes.

2 Set up of the model and equilibrium dynamics

We consider an overlapping generations economy where two generations coexist at each period of time t , the *young* and the *old*. The number of individuals belonging to each generation is constant and equal to N . Following the framework of John and Pecchenino (1994), we assume that individuals born at t work only when they are young and consume only when they are old. At time t , the young agent $i \in \{1, \dots, N\}$ supplies inelastically his time-endowment, normalized to one, to the productive sector receiving the real wage w_t , which is allocated between savings s_t^i for old-age consumption c_{t+1}^i and environmental defensive expenditures m_t^i aimed at improving the quality of environmental goods at time $t + 1$, measured by the quality index E_{t+1} . Individual i 's welfare depends on c_{t+1}^i and E_{t+1} ⁵ and is represented by a utility function $U^i(c_{t+1}^i, E_{t+1})$ assumed twice continuously differentiable, satisfying $U_1^i > 0$, $U_2^i > 0$, $U_{1,1}^i < 0$, $U_{2,2}^i < 0$, $U_{1,2}^i > 0$ and the *Inada* conditions.

The consumption good is produced by N identical firms which act competitively; each firm produces output y according to the following Cobb-Douglas technology:

³The analysis of the interplay between ecological and economic dynamics is a very interesting research field. It is well-known that starting from very simple economic and/or ecological dynamics the resulting dynamics may be very complex (see e.g. Hommes and Rosser 2001, Rosser 2001 and 2002, Lamantia 2006, Bischi and Lamantia 2007). In our model, the equation describing ecological dynamics is linear and augmenting this equation with the effects of economic activity we obtain dynamics that may be chaotic.

⁴Several works in economic literature build on the assumption of heterogeneity of economic agents, however no clear-cut result seems to emerge concerning the role played by heterogeneity in favoring complex dynamics. In some works (see e.g. Bischi et al. 1999, Bischi and Lamantia 2007, Naimzada and Ricchiuti 2008) heterogeneity fuels complex dynamics while in others the opposite is observed (see e.g. Bosi and Seegmuller 2008).

⁵This assumption is adopted in several overlapping generations models (see, among the others, Zhang 1999, Seegmuller and Verchère 2005). It simplifies our analysis by abstracting from the consumption-saving choices of agents.

$$y = Af(k_t) = Ak_t^\alpha$$

where k_t is physical capital per worker, A is a positive parameter representing technological progress, $\alpha \in (0, 1)$.

The evolution of the quality index E_t is described by the following linear difference equation:

$$E_{t+1} = (1 - b)E_t - \beta \sum_{j=1}^N c_t^j + \gamma \sum_{j=1}^N m_t^j \quad (1)$$

where $(1 - b)E_t$, $b \in (0, 1)$, describes the natural evolution of the environmental quality index in absence of the effects due to economic activity; $\sum_{j=1}^N c_t^j$ represents aggregate consumption of old individuals at time t and $\sum_{j=1}^N m_t^j$ represents aggregate environmental defensive expenditures of young individuals at time t ; the parameters $\beta > 0$ and $\gamma > 0$ measure, respectively, the negative impact of aggregate consumption and the efficiency of environmental defensive expenditures.

When old, individual i supplies his savings s_t^i to firms to be invested in physical capital and capital accumulation dynamics are given by the following equation:

$$N \cdot k_{t+1} = \sum_{i=1}^N s_t^i \quad (2)$$

Savings s_t^i gives rise to the gross return $(1 + r_{t+1} - \delta)$, where r_{t+1} is the real interest rate and $\delta \in (0, 1)$ is the depreciation rate of capital. So, individual i faces the following life-cycle budget constraints:

$$c_{t+1}^i = (1 + r_{t+1} - \delta)s_t^i \quad (3)$$

$$w_t = s_t^i + m_t^i \quad (4)$$

At each period t , firms maximize profits; from the usual optimality conditions, the following equilibrium equations for wage and interest rate are obtained:

$$w_t = A \cdot (1 - \alpha) \cdot k_t^\alpha \quad (5)$$

$$r_t = A \cdot \alpha \cdot k_t^{\alpha-1} \quad (6)$$

Differently from Zhang's model, where a social planner solution is studied, we consider a decentralized allocation problem where individuals act strategically. In particular, we assume that individual i behaves competitively taking as given the wage rate w_t and the interest rate r_{t+1} ; furthermore, in period t , he observes environmental quality E_t and aggregate consumption $\sum_{j=1}^N c_t^j$ of

the old generation and formulates *expectations* about other individuals' environmental defensive expenditures m_t^j , $j = 1, \dots, N$, $j \neq i$. In this context, individual i maximizes, with respect to the choice variables c_{t+1}^i , s_t^i and m_t^i , the objective function:

$$U^i(c_{t+1}^i, E_{t+1}^{e,i})$$

under the constraints (3) and (4); $E_{t+1}^{e,i}$ indicates his expectations about future environmental quality E_{t+1} :

$$E_{t+1}^{e,i} = (1-b)E_t - \beta \sum_{j=1}^N c_t^j + \gamma \left[m_t^i + \sum_{j=1, j \neq i}^N (m_t^j)^{e,i} \right]$$

where $(m_t^j)^{e,i}$ represents agent i 's expectations about agent j 's environment preserving expenditures m_t^j , $j = 1, \dots, N$, $j \neq i$.

We assume that individual i maximizes U^i , given his expectations about other individuals' environmental defensive expenditures; furthermore, we assume that his expectations are correct, that is $(m_t^j)^{e,i} = m_t^j$, $j = 1, \dots, N$, $j \neq i$ (ex post) hold. Under this perfect foresight assumption, the first order conditions become:

$$-U_1^i(c_{t+1}^i, E_{t+1}) \cdot (1 + r_{t+1} - \delta) + \gamma \cdot U_2^i(c_{t+1}^i, E_{t+1}) = 0 \quad i = 1, \dots, N \quad (7)$$

These implicitly define individuals' choices m_t^{i*} , s_t^{i*} , c_t^{i*} , $i = 1, \dots, N$, that can be plugged in equations (1) and (2) to get the values of E_{t+1} and k_{t+1} as functions of E_t and k_t only. To make the problem analytically more tractable, we introduce the same assumption used in Zhang:

Assumption Define

$$\eta^i := \frac{E_{t+1} U_2^i}{c_{t+1}^i U_1^i}$$

This elasticity parameter is assumed to be constant⁶.

Notice that η^i increases if (ceteris paribus) the marginal utility of the environmental good increases with respect to the marginal utility of the consumption good. So a "high" value of η^i implies a relatively high dependence of individual i 's welfare on environmental quality.

This assumption allows us to derive c_t^{i*} and m_t^{i*} in terms of E_t and E_{t+1} :

$$\begin{aligned} c_t^{i*} &= \alpha A \left(\frac{E_t}{\gamma \eta^i} \right)^\alpha + \frac{(1-\delta)E_t}{\gamma \eta^i} \\ m_t^{i*} &= (1-\alpha)A \left(\frac{E_t}{\gamma \eta^i} \right)^\alpha - \frac{E_{t+1}}{\gamma \eta^i} \end{aligned} \quad (8)$$

⁶The logarithmic, Cobb-Douglas, CES utility functions satisfy this property.

and to write economic dynamics by a single nonlinear difference equation in E_t :

$$E_{t+1} = G(E_t) := \frac{\eta}{N + \eta} \left[(1 - b)E_t + AN [\gamma(1 - \alpha) - \alpha\beta] \left(\frac{E_t}{\gamma\eta} \right)^\alpha - \beta N(1 - \delta) \frac{E_t}{\gamma\eta} \right] \quad (9)$$

where:

$$\eta := \frac{N}{\sum_{i=1}^N \frac{1}{\eta^i}} \quad (10)$$

is the *harmonic mean* of the elasticities η^i , $i = 1, \dots, N$.

Analogously to the model of Zhang, environmental and capital accumulation dynamics are linked by the equation:

$$k_{t+1} = \frac{E_{t+1}}{\gamma\eta}$$

Notice that, *ceteris paribus*, the value of k_{t+1} is inversely correlated with the parameter γ , measuring the productivity of the investment in environmental preservation; in fact, if γ rises, then the opportunity cost of savings for future consumption increases and, consequently, capital accumulation decreases. The opportunity cost of savings also increases if the harmonic mean η increases; that is, if individuals' preferences become more environment oriented.

3 Fixed points of dynamics

The following propositions deal with the existence and stability conditions of the fixed points of dynamics (9) and give some comparative statics results. The proofs are straightforward and are therefore omitted.

Proposition 1 *The map (9) admits the fixed point $E_{ss}^0 = 0$, for all feasible parameters' values, and the interior fixed point:*

$$E_{ss} = \gamma\eta \left[\frac{AN [\gamma(1 - \alpha) - \alpha\beta]}{\beta(1 - \delta)N + \gamma(N + b\eta)} \right]^{\frac{1}{1-\alpha}}$$

if $\gamma(1 - \alpha) - \alpha\beta > 0$.

From this proposition we have that the interior fixed point exists only if, given α , the negative impact of consumption on the environmental resource (measured by the parameter β) is low enough with respect to the positive impact of defensive expenditures (measured by γ). Notice that the population size N and the harmonic mean η do not play any role in the existence conditions for the interior fixed point; however, we shall see that they play a key role in determining its stability properties.

The following proposition shows how the interior fixed point E_{ss} varies in response to variations in the more relevant parameters of the model.

Proposition 2 *The value of E_{ss} is positively correlated with the values of the parameters γ , η and N and negatively correlated with the value of β .*

Remember that β , γ , η and N represent, respectively, the negative impact of consumption on environmental quality, the efficiency of environmental defensive expenditures, the harmonic mean of the elasticities η^i and the population size. It is worth to stress that the positive correlation between E_{ss} and N holds under the existence condition $\gamma(1 - \alpha) - \alpha\beta > 0$ of the interior fixed point, which requires (ceteris paribus) a high enough value of γ with respect to the value of β .

Let's now consider local stability properties of fixed points. It is simple to check that $\lim_{E_t \rightarrow 0} G'(E_t) = +\infty$, consequently the fixed point $E_{ss}^0 = 0$ is always repulsive. To study the stability properties of the interior fixed point E_{ss} , we consider the first order derivative evaluated at E_{ss} :

$$G'(E_{ss}) = \frac{\eta}{N + \eta} \left[1 - b(1 - \alpha) + \frac{N[\alpha\gamma - \beta(1 - \alpha)(1 - \delta)]}{\gamma\eta} \right]$$

By straightforward calculations, it is simple to verify that $G'(E_{ss}) < 1$ for all parameters' values. As a consequence, the interior fixed point is locally attracting if $G'(E_{ss}) > -1$, while it is repulsive if $G'(E_{ss}) < -1$. So, the following proposition can be stated:

Proposition 3 *The fixed point $E_{ss}^0 = 0$ is always repulsive while the interior fixed point is locally attractive if:*

$$\frac{N}{\eta} < \frac{\gamma[2 - b(1 - \alpha)]}{\beta(1 - \alpha)(1 - \delta) - \gamma(1 + \alpha)}$$

and repulsive if the opposite inequality holds.

By the above proposition, the interior fixed point is repulsive if, ceteris paribus, the size N of each generation is high enough or if the harmonic mean η is low enough. Remember that a "low" value of η^i implies a relatively low dependence of individual i 's welfare on environmental quality.

In the next sections we compare the dynamics generated by our model with those in Zhang (1999).

4 Dynamics in an economy with identical individuals

We first analyze economic dynamics in a population of identical individuals, that is we assume $\eta = \eta^i$, $\forall i$, and compare our results with those obtained by Zhang (1999). To this end, we extend Zhang's model to the case with N individuals; the representative agent's problem in such case becomes:

$$\begin{aligned}
& \max_{c_{t+1}, m_t, s_t} U(c_{t+1}, E_{t+1}) \\
& w_t = s_t + m_t \\
& c_{t+1} = (1 + r_{t+1} - \delta)s_t \\
& E_{t+1} = (1 - b)E_t - \beta N c_t + \gamma N m_t
\end{aligned}$$

and the corresponding dynamics are given by:

$$E_{t+1} = G_Z(E_t) = \frac{\eta}{1 + \eta} \left[\left((1 - b) - \frac{\beta(1 - \delta)}{\gamma\eta} \right) E_t + AN^{1-\alpha} [\gamma(1 - \alpha) - \beta\alpha] \left(\frac{E_t}{\gamma\eta} \right)^\alpha \right]$$

The following proposition holds.

Proposition 4 *The map (9) admits the fixed point $E_Z^0 = 0$, for all feasible parameters' values, and the interior fixed point:*

$$E_Z = \eta\gamma \left[\frac{A[\gamma(1 - \alpha) - \alpha\beta]}{\beta(1 - \delta) + \gamma(1 + b\eta)} \right]^{\frac{1}{1-\alpha}} N$$

if $\gamma(1 - \alpha) - \beta\alpha > 0$. Furthermore, $E_Z > E_{ss}$ always holds.

According to the above proposition, the existence conditions for E_{ss} and E_Z are the same, however the fixed point associated to our decentralized model is characterized by a greater depletion of natural resources due to underinvestment in environmental protection.

Notice that E_Z is an increasing linear function of the generations size N while E_{ss} , for $N \rightarrow \infty$, approaches the limit:

$$\bar{E} = \eta\gamma \left[\frac{A[\gamma(1 - \alpha) - \alpha\beta]}{\gamma + \beta(1 - \delta)} \right]^{\frac{1}{1-\alpha}}$$

Let's now consider the stability properties of fixed points in Zhang's model; it is easy to check that $E_Z^0 = 0$ is always repulsive and that:

$$G'_Z(E_Z) = \frac{\eta}{1 + \eta} \left[1 - b + \frac{\beta(\delta - 1) + \alpha[\beta(1 - \delta) + \gamma(b\eta + 1)]}{\gamma\eta} \right]$$

It is interesting to observe that in Zhang's model the local stability of the interior fixed point does not depend on the population size N . Furthermore, the following proposition can be easily checked.

Proposition 5 *The inequality $|G'_Z(E_Z)| < |G'(E_{ss})|$ always holds for $N > 1$.*

Therefore, if E_{ss} is locally attractive, then the same is true for E_Z ; however, parameters' values exist according to which E_Z is attractive while E_{ss} is repulsive. Consequently, we can say that strategic behavior has a destabilizing effect on the interior fixed point.

5 Some numerical exercises

In this section we show how the introduction of a strategic context in Zhang's model makes complex dynamics simpler to emerge. In Figures 1.a-1.d we show how the parameter N affects the map G and the long run behavior of environmental quality. In such exercise we have posed $\alpha = 0.12$, $\beta = 0.23$, $\gamma = 0.123$, $\delta = 0.16$, $\eta = 0.8$, $A = 5$, $b = 0.22$. In this context, when N increases, the fixed point E_{ss} undergoes a period doubling route to chaos⁷. For $N = 2$ the map G is represented by the dotted line in Figure 1.a and the fixed point is attracting (see Figure 1.b); for $N = 7$ the map G is represented by the semi-dotted line in Figure 1.a and the equilibrium orbits approach a 2-period cycle (see Figure 1.c); for $N = 28$ the map G is represented by the continuous line in Figure 1.a and the system becomes chaotic (see Figure 1.d).

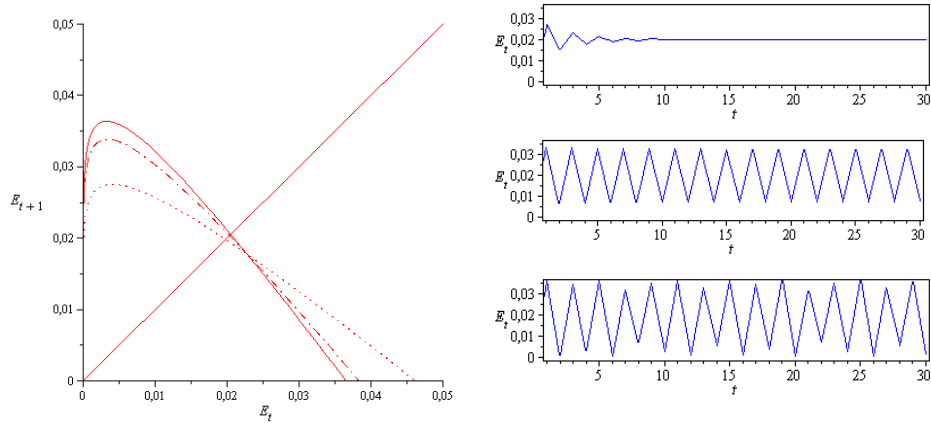


Figure 1: The effects of variations in N on the map G and on the long run behavior of environmental quality.

In the following numerical exercises we fix $\alpha = 0.12$, $\gamma = 0.123$, $\delta = 0.16$, $\eta = 0.8$, $A = 5$, $N = 4$, $b = 0.22$ and use β as bifurcation parameter. Figures 2.a and 2.b show the bifurcation windows corresponding to our model and to Zhang's one, respectively. In both contexts, both maps undergo a period doubling route to chaos; however, in our model the start up point of this phenomenon and the subsequent bifurcations occur *before* than in the other. That is, in the decentralized economy, complex dynamics occur for lower values of the parameter β , which measures the negative impact of consumption on environmental goods. We obtain similar results varying the parameter γ , which represents the efficiency of environmental defensive expenditures. Figures 3.a and 3.b show the bifurcation windows corresponding to our model and to Zhang's one,

⁷Notice, however, that N varies on the integer set.

respectively, obtained with the parameters' values: $\alpha = 0.1$, $\beta = 0.3$, $\delta = 0.16$, $\eta = 0.8$, $A = 1$, $N = 2$, $b = 0.2$.

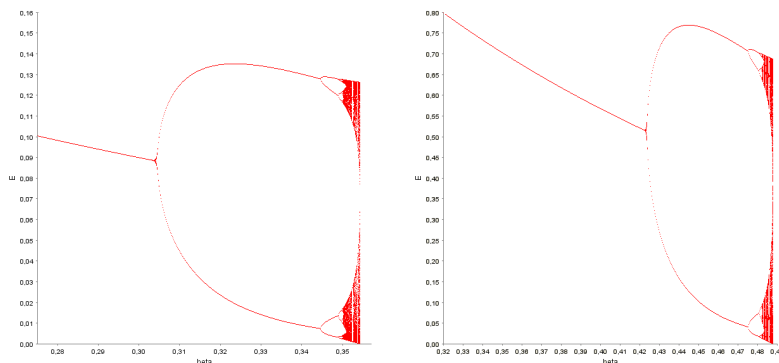


Figure 2: Bifurcation diagrams corresponding to our model (on the left) and to Zhang's model (on the right), obtained varying the parameter β .

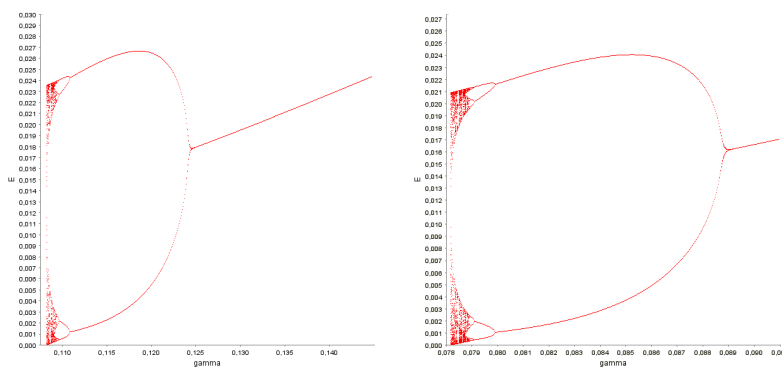


Figure 3: Bifurcation diagrams corresponding to our model (on the left) and to Zhang's model (on the right), obtained varying the parameter γ .

6 Dynamics in a population with heterogenous individuals

In this section we analyze the context where individuals have heterogeneous preferences, that is $\eta^i \neq \eta^j$ for $i \neq j$. In such case, according to (9), environmental dynamics depend on the harmonic mean of individuals' elasticities η^i . This section aims to study the role that an "increase" in heterogeneity plays on the dynamics of the model. To simplify our analysis, we set $N = 2$; let η^1 and η^2 represents environmental preferences of individuals 1 and 2, respectively. To introduce a measure of heterogeneity, we adopt a mean-preserving approach (see e.g. Bosi and Seegmuller 2008) by considering a variation of η^1 and η^2 such that

their arithmetic mean $\omega = \frac{\eta^1 + \eta^2}{2}$ remains constant. Under this assumption, we can express η^1 and η^2 as:

$$\begin{aligned}\eta^1 &= \omega - \frac{\varepsilon}{2} \\ \eta^2 &= \omega + \frac{\varepsilon}{2}\end{aligned}$$

with $\varepsilon \in [0, 2\omega]$ and substituting in (10) we have that η can be written as a function of ε and ω :

$$\eta(\varepsilon, \omega) = \frac{(\omega - \frac{\varepsilon}{2})(\omega + \frac{\varepsilon}{2})}{\omega} = \omega - \frac{\varepsilon^2}{4\omega}$$

and the dynamics (9) becomes:

$$E_{t+1} = \frac{\omega - \frac{\varepsilon^2}{4\omega}}{2 + \omega - \frac{\varepsilon^2}{4\omega}} \left[(1-b)E_t + 2[\gamma(1-\alpha)A - \alpha\beta A] \left[\frac{E_t}{\gamma(\omega - \frac{\varepsilon^2}{4\omega})} \right]^\alpha - 2\beta(1-\delta) \frac{E_t}{\gamma(\omega - \frac{\varepsilon^2}{4\omega})} \right] \quad (11)$$

In our analysis, we will take the value of the parameter ε as a measure of the heterogeneity between the two individuals; that is, heterogeneity increases if the value of ε increases. The following two propositions hold.

Proposition 6 *The interior fixed point of (11) is:*

$$E_{ss} = \gamma \left(\omega - \frac{\varepsilon^2}{4\omega} \right) \left[\frac{2A[\gamma(1-\alpha) - \alpha\beta]}{b\gamma \frac{\omega - \frac{\varepsilon^2}{4\omega}}{2 + \omega - \frac{\varepsilon^2}{4\omega}} + 2[\gamma + \beta(1-\delta)]} \right]^{\frac{1}{1-\alpha}}$$

Therefore, E_{ss} is a decreasing function of ε .

The above proposition says that if heterogeneity increases, then the fixed point value of environmental quality decreases. From the derivative:

$$G'(E_{ss}) = \frac{\omega - \frac{\varepsilon^2}{4\omega}}{2 + \omega - \frac{\varepsilon^2}{4\omega}} \left[1 - b(1-\alpha) + \frac{2[\alpha\gamma - \beta(1-\alpha)(1-\delta)]}{\gamma[\omega - \frac{\varepsilon^2}{4\omega}]} \right]$$

we can observe that an increase in heterogeneity makes $\eta = \omega - \frac{\varepsilon^2}{4\omega}$ lower and the value of $|G'(E_{ss})|$ consequently increases. In particular, the following proposition holds.

Proposition 7 *The interior fixed point E_{ss} is locally attractive if the following inequality holds:*

$$\varepsilon < 2\sqrt{\frac{\omega[\gamma\omega(2 + b\alpha - b) + 2\gamma(1 + \alpha) - 2\beta(1 - \alpha)(1 - \delta)]}{\gamma[2 - b(1 - \alpha)]}}$$

It is repulsive if the opposite inequality holds.

Therefore, an increase in heterogeneity destabilizes the fixed point. Figure 4 shows a bifurcation diagram obtained varying the parameter ε . The other parameters are fixed at the values $\alpha = 0.21$, $\beta = 0.35$, $\gamma = 0.123$, $\delta = 0.18$, $\eta = 0.8$, $\omega = 0.7$, $A = 5$, $b = 0.22$.

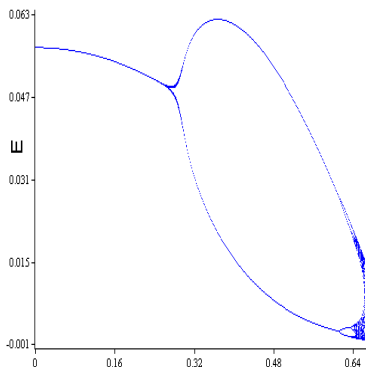


Figure 4: Bifurcation diagram obtained varying the parameter ε .

Notice that when heterogeneity increases, the dynamics of our model undergo a period doubling route to chaos. In order to understand the effect of homogeneity breaking in the preferences we examine the change in environmental defensive expenditures of individuals 1 and 2, m^1 and m^2 , when asymmetry is introduced. For $\varepsilon = 0$, the symmetry implies that the dynamics of environment preserving expenditures are located on the diagonal $\Delta = \{(m^1, m^2) | m^1 = m^2\}$ of the plane (m^1, m^2) . Figures 5.a-5.d are obtained posing $\alpha = 0.21$, $\beta = 0.35$, $\gamma = 0.123$, $\delta = 0.18$, $\eta = 0.8$, $\omega = 0.7$, $A = 5$, $b = 0.22$. The point $(m_1, m_2) = (2.62, 2.62)$ is the unique attractive fixed point of these variables for $\varepsilon = 0$ (see Figure 5.a). With a little increase in ε ($\varepsilon = 0.2$), the fixed point E_{ss} loses its stability through a flip bifurcation and the environment preserving expenditures follow a 2-period cycle below the diagonal Δ (see Figure 5.b). A further increase of the degree of heterogeneity ($\varepsilon = 1.04$) creates a chaotic attractor below the diagonal Δ (see Figure 5.c) and the behavior of the environmental defensive expenditures m^1 and m^2 follows very different paths as shown in Figure 5.d, where the time evolution of the difference $m_1 - m_2$ is represented.

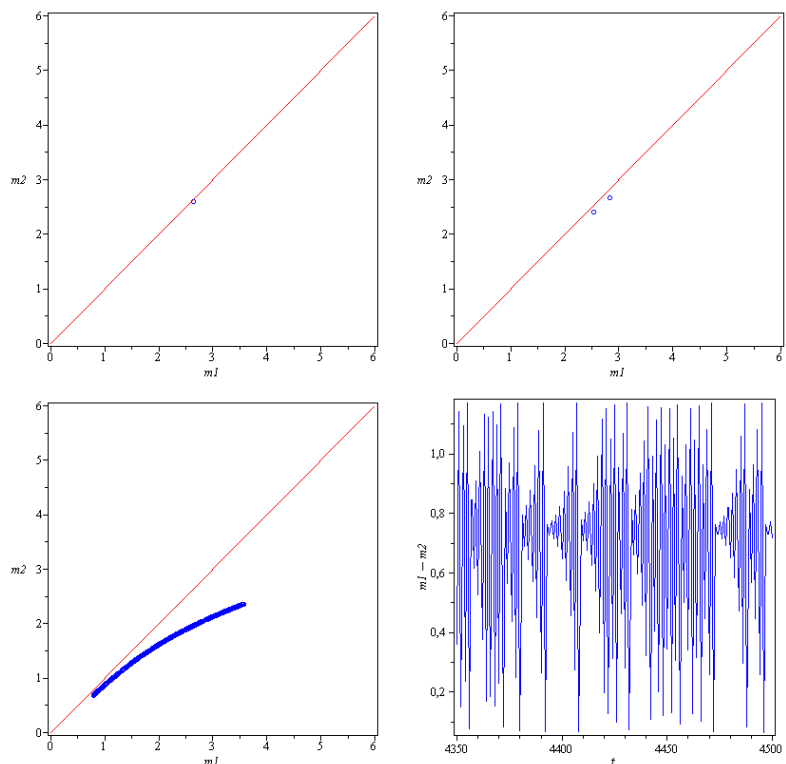


Figure 5: The effects of homogeneity breaking in the preferences on environmental defensive expenditures of individuals 1 and 2, m^1 and m^2 .

7 Conclusions

We have analyzed an overlapping generations economy in which individuals act strategically and are endowed with heterogeneous environmental preferences. Our model builds on the model of Zhang (1999), where individuals are identical and decisions are coordinated by a social planner. Comparing the evolution of environmental quality E generated by our model and by Zhang's one, we have showed that both models admit a unique interior fixed point, respectively E_{ss} and E_Z , and that $E_{ss} < E_Z$ always holds also in a context in which individuals are identical and the unique difference between the two models relies on the strategic behavior of individuals. Introducing heterogeneity, we have proved that the value of E_{ss} is negatively correlated with heterogeneity, measured by the parameter ε . This allows us to say that, in our specific context, (socio-cultural) heterogeneity leads to a higher depletion of environmental resources. Such result is in line with other findings in environmental economics literature according to which socio-cultural heterogeneity favors over-exploitation of common access environmental resources (see the introduction of this paper).

Passing to stability analysis of E_{ss} and E_Z , we have shown that, in a context

with identical individuals, strategic behavior has a destabilizing effect. That is, for the same parameters' values, local attractiveness of E_{ss} implies local attractiveness of E_Z , but the vice-versa does not hold. Heterogeneity generates a further destabilizing effect on E_{ss} ; more specifically, E_{ss} becomes repulsive for a high enough value of ε . This destabilizing effect may favor the rise of complex dynamics and chaotic behavior via period doubling bifurcations, as numerical simulations suggest.

References

- [1] Bardhan P. and Dayton-Johnson J., 2002, Unequal irrigators: Heterogeneity and commons management in large-scale multi-variate research. In: Ostrom E., Dietz T., Dolsak N., Stern P.C., Stonich S. and Weber E.U. (Eds), *The drama of the commons*, National Academy Press, Washington, DC, pp. 87-112.
- [2] Bischi G.-I. and Lamantia F., 2007, Harvesting dynamics in protected and unprotected areas, *Journal of Economic Behavior & Organization* 62 (3), pp. 348-370.
- [3] Bischi G.-I., Gallegati M. and Naimzada A., 1999, Symmetry-breaking bifurcations and representative firm in dynamic duopoly games, *Annals of Operation Research* 89, pp. 253-272.
- [4] Bosi S. and Seegmuller T., 2008, Can heterogeneous preferences stabilize endogenous fluctuations?, *Journal of Economic Dynamics & Control* 32, pp. 624-647.
- [5] Cardenas J. C., Stranlund J. and Willis C., 2002, Economic inequality and burden-sharing in the provision of local environmental quality, *Ecological Economics* 40, pp. 379-395.
- [6] Dayton-Johnson J. and Bardhan P., 2002, Inequality and conservation on the local commons: A theoretical exercise, *The Economic Journal* 112, pp. 577-602.
- [7] Hommes C. H. and Rosser J. B., 2001, Consistent expectations equilibria and complex dynamics in renewable resource markets, *Macroeconomic Dynamics* 5, pp.180-203.
- [8] John A. and Pecchenino R., 1994, An overlapping generations model of growth and the environment, *The Economic Journal* 104, pp. 1393-1410.
- [9] Johnson R.N. and Libecap G.D., 1982, Contracting problems and regulation: The case of the fishery, *American Economic Review* 72 (5), pp. 1005-1023.
- [10] Lamantia F., 2006, Variable effort management of renewable natural resources, *Chaos, Solitons & Fractals* 29 (3), pp. 771-782.

- [11] Leevcs G.D. and Herbert R. D., 2002, Economic and environmental impacts of pollution control in a system of environment and economic interdependence, *Chaos, Solitons & Fractals* 13 (4), pp. 693-700.
- [12] Naidu S. C., 2008, Heterogeneity and collective management: evidence from common forests in Himachal Pradesh, India, *World Development* 37 (3), pp. 676-686.
- [13] Naimzada A. and Ricchiuti G., 2008, Heterogeneous fundamentalists and imitative processes, *Applied Mathematics and Computation* 199, pp. 171-180.
- [14] Olson M., 1965, *The logic of collective action*, Harvard University Press, Cambridge, MA.
- [15] Ostrom E. and Walker J., 2003 (Eds), *Trust and reciprocity: Interdisciplinary lessons from experimental research*, Russell Sage Foundation, New York.
- [16] Potete A. and Ostrom E., 2004, Heterogeneity, group size and collective action: The role of institutions in forest management, *Development and Change* 35, pp. 435-461.
- [17] Rosser J. B. Jr, 2001, Complex ecologic-economic dynamics and environmental policy, *Ecological Economics* 37, pp. 23-37.
- [18] Rosser J. B. Jr, 2002, Complex coupled system dynamics and the global warming policy problem, *Discrete Dynamics in Nature and Society* 7 (2), pp. 93-100.
- [19] Ruttan L.M., 1998, Closing the commons: Cooperation for gain or restraint?, *Human Ecology* 26, pp. 43-66.
- [20] Ruttan L.M., 2006, Sociocultural heterogeneity and the commons, *Current Anthropology* 47, pp. 843-853.
- [21] Ruttan L.M. and Borgerhoff Mulder M., 1999, Are East African pastoralists truly conservationists?, *Current Anthropology* 40, pp. 621-652.
- [22] Seegmuller T., Verchère A., 2005, Environment in an overlapping generations economy with endogenous labour supply, Document de travail n. 2005-05, Bureau d'économie théorique et appliquée (BETA), France.
- [23] Singleton, S., 2001, How distributional conflicts shape comanagement institutions and collaborative environmental policy in the United States. Paper prepared for the workshop Economic Inequality, Collective Action and Environmental Sustainability, Santa Fe Institute, September 2001.
- [24] Varughese G. and Ostrom E., 2001, The contested role of heterogeneity in collective action: Some evidence from community forestry in Nepal, *World Development* 29, pp. 747-765.

- [25] Windrum P., Ciarli T. and Birchenhall C., 2009, Consumer heterogeneity and the development of environmental friendly technologies, *Technological Forecasting & Social Change* 76, pp. 533-551.
- [26] Zhang J., 1999, Environmental sustainability, nonlinear dynamics and chaos, *Economic Theory* 14, pp. 489-500.