Will growth and technology destroy social interaction? The inverted U-shape hypothesis

Antoci, Angelo and Sabatini, Fabio and Sodini, Mauro

Università of Sassari, University of Siena, University of Pisa

2009

Online at https://mpra.ub.uni-muenchen.de/18229/
MPRA Paper No. 18229, posted 01 Nov 2009 14:20 UTC
Will growth and technology destroy social interaction? The inverted U-shape hypothesis*

Angelo Antoci†, Fabio Sabatini‡, Mauro Sodini§

July, 2009

Abstract

This paper addresses two hot topics of the contemporary debate, social capital and economic growth. Our theoretical analysis sheds light on decisive but so far neglected issues: how does social capital accumulate over time? Which is the relationship between social capital, technical progress and economic growth in the long run? The analysis shows that the economy may be attracted by alternative steady states, depending on the initial social capital endowments and cultural exogenous parameters representing the relevance of social interaction and trust in well-being and production.

When material consumption and relational goods are substitutable, the choice to devote more and more time to private activities may lead the economy to a “social poverty trap”, where the cooling of human relations causes a progressive destruction of the entire stock of social capital. In this case, the relationship of social capital with technical progress is described by an inverted U-shaped curve. However, the possibility exists for the economy to follow a virtuous trajectory where the stock of social capital endogenously and unboundedly grows. Such result may follow from a range of particular conditions, under which the economy behaves as if there was no substitutability between relational activities and material consumption.

---

*We are grateful to Pier Luigi Sacco and Paolo Vanin for the insightful conversations on the topics dealt with in this work. The paper benefited also from comments by participants at conferences in Paris (“The formation and the evolution of social networks”, 25-29 June 2009) and Forli (“Social economy: young economists’ contributions”, 19-20 June 2009). Needless to say, usual caveats apply.

†Dipartimento di Economia Impresa e Regolamentazione, Università di Sassari, via Torre Tonda 34, 07100 Sassari, Italy; e-mail: antoci@uniss.it;

‡Dipartimento di Economia Politica, Università di Siena, piazza San Francesco 5, 53100 Siena, Italy; e-mail: f.sabatini@unisi.it;

§Dipartimento di Statistica e Matematica Applicata all’Economia, Università di Pisa, via Cosimo Ridolfi 10, 56124 Pisa, Italy; e-mail: m.sodini@ec.unipi.it.

▲ Corresponding Author.
1 Introduction

The positive role of social capital in growth and development is one of the most popular and controversial theses standing in the contemporary economic debate. Even if theoretical and empirical research have produced a huge amount of papers on the topic, this field of research still suffers from some endemic problems. Firstly, economic studies generally focus on the possible effects of social capital on a range of supposed outcomes, with a strong attention to innovation, technology adoption and growth (Knack and Keefer, 1997, Rodrik, 1999, Zak and Knack, 2001, Isham, 2002, Amenn, 2003, Akçomakak and ter Weel, 2009). However, we lack a micro-founded theory explaining the causal mechanisms, and the reverse effect of growth and technical progress on human interactions is neglected in the literature.

Secondly, there is a surprising lack of studies addressing the sources of social capital. The dominant view in sociology is that social capital is a public good incidentally accumulated as a by-product of diverse relational activities (Coleman, 1988). This form of capital exists only when it is shared, i.e. it needs some kind of interaction to be exploited to the pursuit of common interests (Bourdieu, 1980, Coleman, 1990). By contrast, theoretical studies in economics are basically tied down to Becker’s (1974) theory of social interaction, where social capital is an individual resource created through rational self-interested investment decisions.

This paper aims to contribute both to the growth and the social capital literature by shedding light on the following questions: which are the dynamics of social capital’s interaction with economic growth? How is social capital created and accumulated over time? Does technical progress influence the accumulation dynamics? Do we have to trust the folk wisdom according to which technology is one of the major responsible for the widespread social isolation of our time?

To reach this goal, we develop a dynamic model where social capital enters as an argument in the agents’ utility functions and as an input in private and relational goods’ production functions. Social capital is here defined as the sum of networks of trust-intensive relations that the agents incidentally develop through the simultaneous production and consumption of relational goods. Relational goods are a distinctive type of good that can only be enjoyed if shared with others.

We integrate hints from sociology and political science into an economic framework by assuming that the stock of social capital accumulated as a by-product of social participation functions as a public good, which benefits all individuals equally. In this way, we explicitly model also one of the most popular claims raised in the social capital debate, i.e. its role in private production. Specifically, the assumption we draw from the empirical literature is that the diffusion of trust boosted by shared values and social participation reduces the average cost of transactions, thereby fostering the economic activity and growth (Knack and Keefer, 1997, Rodrik, 1999, Zak and Knack, 2001, Guiso, Sapienza and Zingales, 2004, Beugelsdijk and van Shaik, 2005, Antoci, Sacco and Vanin, 2007).
Our analysis starts from the observation that, even if private and relational goods satisfy different needs, the private good can be consumed as a substitute for the relational one. For example, when the social environment is poor, people may be constrained to replace human interactions, e.g. joining the meetings of a cultural circle or playing football with friends, with private consumption, e.g. staying at home and watching a TV show or playing a virtual match against the computer.

The analysis of agents’ time allocation choices accounts for the alternative cases of zero or positive Edgeworth substitutability between the two types of good. In our analytical framework, social participation is constant at a given level if the two goods are not Edgeworth substitutes. However, it seems to be more realistic to assume a positive degree of Edgeworth substitutability between the two types of goods. In such a case, an increase in the stock of social capital can enhance the productivity of time spent on social interaction, thereby encouraging the consumption of relational goods. This process results in a positive level of participation, which in turn creates and strengthens durable ties increasing the stock of social capital.

The analysis of dynamics shows that the economy may be attracted by alternative steady states, depending on the initial wealth of social capital and cultural exogenous parameters representing the importance of social interaction and trust in well-being and production.

The possibility exists for the economy to fall down in a “social poverty trap”, where agents devote all their time to private activities. In this case, the cooling of relations may cause a progressive destruction of the entire stock of social capital. According to common wisdom, the link leading from economic growth to social isolation is technology. In an interview to the New York Times, Robert Putnam stated that “The distinctive effect of technology has been to enable us to get entertainment and information while remaining entirely alone. It’s fundamentally bad because the lack of social contact, the social isolation means that we don’t share information and values and outlook that we should”.

Introducing exogenous technical progress in the production function of private goods indeed leads to interesting modifications in social capital’s accumulation dynamics. If there is not Edgeworth substitutability between private and relational goods, the stock of social capital can grow indefinitely along any trajectories or alternatively tend to zero according to the value of the model’s parameters. Its trend relative to technical progress can be monotonic (always increasing or decreasing) or can experience an initial decline followed by a growth, but not vice versa (a growth followed by a decline is impossible).

Under the more realistic assumption of positive Edgeworth substitutability, social capital may experience a growth followed by a decline, so that its relationship with technical progress is described by an inverted-u shaped curve. Since technical progress is in turn positively correlated with GNP, our results support Putnam’s (2000) intuition of the inverted-u shaped relationship between social capital and income. Along the inverted-u shaped curve, private activities infinitely expand at the expenses of social interaction, thereby trapping the economy in a situation of social poverty.
However, also in the context of substitutability, if social capital’s initial endowments are high enough, the economy can follow a growth path along which both technical progress and social capital grow indefinitely.

The outline of the paper is as follows. Section two briefly reviews the literature on social capital and growth with a particular attention for the hypotheses we integrate into the model. Sections three and four present the model and analyze its dynamics. Section five studies the effect of exogenous technical progress on social capital’s dynamics. The paper is closed by a discussion of results and policy implications.

## 2 Social capital and growth

The public good features of social capital are now commonly acknowledged in the recent debate. It is possible to argue that the functioning of the economy itself relies on those institutions (whether formal or informal) that the literature often groups together under the common label of social capital (e.g. norms of trust and reciprocity, moral sanctions, networks of relationships, and organizations). If that is the case, then the economy’s possibility of “reproducing” itself, thereby experiencing sustainable growth, depends also on its ability to foster - or, at least, to preserve - its endowments of social capital. On this basis, the idea is spreading that a better understanding of the role of norms and relationships would be a crucial step for the advancement of modern political economy. In the last two decades, such idea has informed the development of a huge number of empirical studies, exploring the effect of social capital on an immense range of phenomena, from political participation to the institutional performance, from health to the economic success of countries. The seminal study in the field is the so-called “Italian work” by Putnam, Leonardi and Nanetti (1993), which explains the different institutional and economic performance of the Italian regions as the result of the influence exerted by some aspects of the social structure, summarized into the multidimensional concept of “social capital”. Empirical research in economics has then been prompted by a series of notable tests of Putnam’s hypotheses (Knack and Keefer, 1997, La Porta et al., 1997, Temple and Johnson, 1998, Whiteley, 2000, Knack, 2003, Buigelsdijck and van Schaik, 2005).

We do not want to discuss the strengths and weaknesses of the empirical research here (see Durlauf and Fafchamps, 2005, and Sabatini, 2007, for exhaustive reviews). To our purposes, it is important to point out that this strand of the literature unanimously converges on the claim that one of the mechanisms through which social capital impacts economic efficiency is by enhancing the level of trust and reducing uncertainty. The basic idea is that a social environment rich of participation opportunities is a fertile ground for nurturing trust and shared values. Repeated interactions foster the diffusion of information and raise reputations’ relevance. The higher opportunity cost of free-riding in prisoners’ dilemma kind of situations makes the agents’ behavior more foreseeable causing an overall reduction of uncertainty. Therefore, an increase in trust-
based relations reduces monitoring costs and, more in general, the average cost of transactions. Our model accounts for these claims through the assumption that social capital enters as an input in the production of private goods.

The direction of the nexus connecting social capital to the economic performance and growth is another subject of contention in the empirical literature. Even if most studies agree on the positive role of networks and trust, we have few evidence on the reverse effect possibly played by growth on the accumulation of social capital. On this regard, two conflicting hypotheses emerge from the debate.

a) Growth has positive “social” externalities. The “workplace hypothesis”. 

Development may reinforce social participation by providing the material and cultural bases for an increase in relational goods’ production and consumption. Empirical studies have shown that higher levels of education and wealth are indeed associated with higher social participation (Temple and Johnson, 1998, Sabatini, 2008). In addition, it is noteworthy that more developed countries are generally governed by democratic regimes. The respect of civil rights associated with democracy is a necessary precondition for the development of associational activities and certainly encourages the consumption of relational goods (Fox, 1996, Offe, 1999, Rahn et al, 1999, Stolle, 2001, Paxton, 2002). Bilson (1982) shows that civil liberties are strongly associated with per capita income. The author’s interpretation is that economic performance determines freedoms, rather than the other way around.

On the contrary, in situations of underdevelopment and poverty, a non cooperative behavior may be regarded as the most effective survival strategy. For example, the mutual assistance mechanisms developed within the family unit, which Banfield (1958) referred to as a result of the “amoral familism”, could be looked on as a defence reaction against situations of economic and social poverty, where both the state’s and market’s institutions are weak (Sabatini, 2009).

The causal mechanism positively connecting economic growth to social participation may work through two main channels:

1) wealthier and more educated people generally have stronger cultural needs and civic awareness. Thus, they devote a higher amount of time to cultural consumption (e.g. going to the cinema) and associational activities (e.g. joining the meetings of cultural or political circles) which boost the production and consumption of relational goods.

2) On the job interactions may stimulate the creation of durable ties among workers. Friendships often start on the workplace, both spontaneously and as a result of precise human resources management strategies. As the business literature shows, nurturing a good relational climate inside the firm is generally a key task for management. In political science, several schools of thought claim that citizens can develop their relational and political attitudes at the workplace. Non-hierarchical work structures allowing workers to participate in decision making processes are a generator of face-to-face interactions that stimulate the sharing of social norms and the creation of interpersonal ties (Karasek 1976, Smith 1985, Peterson 1992, Goul Andersen and Hoff 2001, Schur, 2003).
Moreover, the workplace is a training ground where people improve those communication and organizational abilities which are crucial for the production and consumption of relational goods. In other words, such skills raise the productivity of time spent on social participation. Those who possess well-developed relational skills are likely to find social and political participation less daunting and costly (Elden 1981, Greenberg 1986, Burn and Konrad 1987, Brady et al. 1995, Verba et al. 1995). Remarkably, many authors claim that on the job interactions foster the development of democratic attitudes (Paterman, 1970, Verba et al., 1995), and active political participation (Sobel, 1993, Greenberg et al. 1996, Mutz and Mondak, 2006, Adman, 2008).

In this paper, we take hints from political science seriously and explicitly model the “workplace hypothesis”, assuming that private production fosters relational goods’ consumption and the accumulation of social capital as a spillover effect.

b) Growth has negative “social” externalities. The “Machine hypothesis”.

On the other side, it is possible to argue that the pressure exerted on time by economic growth acts as a factor hampering the consolidation of social ties, thereby leading to an erosion of the stock of social capital. As everyday life experience suggests, time constraints are ever more pressing in modern societies.

At the theoretical level, the negative externalities of growth have been quite neglected by the literature. Routledge and von Amsberg (2003) show that the technical change and innovation generally associated to growth influence social capital by rising labour mobility: higher levels of turnover may hamper the consolidation of social ties, both inside and outside the workplace. Moreover, the uncertainty of future incomes related to increased mobility affects any form of long-term planning of life activities such as marriage and procreation. Autoc, Sacco and Vanin (2007) show that the expansion of market activities implies a growing pressure on time, which compresses the social sphere of individuals. As a consequence, the process of economic growth may be accompanied by a progressive “social impoverishment” of the economy. As the authors state, an early account of this process is given by Hirsch (1976): “As the subjective cost of time rises, pressure for specific balancing of personal advantage in social relationships will increase. ... Perception of the time spent in social relationships as a cost is itself a product of privatized affluence. The effect is to whittle down the amount of friendship and social contact ... . The huge increase in personal mobility in modern economies adds to the problem by making sociability more of a public and less of a private good. The more people move, the lower are the chances of social contacts being reciprocated directly on a bilateral basis” (p.80).

According to common wisdom, growth’s perverse effects on social cohesion are bound to be exacerbated by technical progress. Exactly 100 years ago, E.M. Forster (1909) described one of the most intriguing accounts of technology and society in the future. In his short story “The Machine Stops”, the novelist describes a prophetic vision of the internet simply called “the Machine”: people around the globe have given up direct interaction with each other, opting instead to communicate and obtain new “ideas” through the Machine virtually.
Each person occupies their own room that supplies all their material needs and makes “the terrors of direct experience” unnecessary and undesirable. The idea that technology is one of the major responsible of the widespread social isolation of our time has get stronger and stronger in the last century, walking at the same pace of technological advance. Besides folk wisdom and literary fascinations, recent empirical studies suggest that communication technologies like TV (Corneo, 2005, Bruni and Stanca, 2009), cell phones (McPherson and Smith-Lovin, 2006), and internet (Gershuny 2003, Nie et al., 2002, Boase et al. 2006, Wellman et al. 2006) may interfere with face-to-face interactions. However, we lack theoretical analyses addressing the micro foundations of the possible linkage between technical progress and social isolation. In section five we test the “Machine hypothesis” by studying the effect of exogenous technical progress on social capital’s accumulation dynamics.

The model in section three integrates the hypotheses suggested above and in the introduction: agents chose how to allocate their time between labour, aimed at the production of private goods, and social participation activities, that generate social capital as a by-product. Private production exerts positive spillover effects on relational goods’ production and consumption (“workplace hypothesis”). In section four we analyze the long-run dynamics of the interaction between growth and the accumulation of social capital, showing that the economy may fall in a social poverty trap depending on the initial social capital endowments and cultural exogenous parameters representing the relevance of social interaction and trust in well-being and production.

3 The model

The notion of social capital taken into account within our framework is defined as the sum of networks of trust-intensive relations that the agents develop through the simultaneous production and consumption of relational goods.

We consider a population of size 1 constituted by a continuum of individuals. We assume that, in each instant of time $t$, the well-being of the individual $i \in [0, 1]$ depends on the consumption of two goods: a private good, $C_i(t)$, and a socially provided good, $B_i(t)$. We assume that $B_i(t)$ is produced through the joint action of the time devoted by agent $i$ to social activities, $s_i(t)$, the average social participation $s(t) = \int_0^1 s_i(t) \, dt$, and the stock of social capital $K_s(t)$:

$$B_i(t) = F(s_i(t), s(t), K_s(t))$$  \hspace{1cm} (1)

Since relational goods can be enjoyed only if shared with others, their production process depends on the others’ social participation and on the stock of networks existing in the surrounding environment.

The time agent $i$ does not spend for social participation, $1 - s_i(t)$, is used as input in the production of the output $Y_i(t)$ of the private good. As suggested by Antoci, Sacco and Vainin (2005, 2008), we assume that social capital plays also a role in the production process of the private good. In this way, we explicitly
model one of the most popular claims raised by the empirical literature. In addition, for simplicity, we assume that \( C_i(t) = Y_i(t) \), that is \( Y_i(t) \) cannot be accumulated, and that the production process of \( Y_i(t) \) requires only the input \( 1 - s_i(t) \) and \( K_s(t) \):

\[
C_i(t) = Y_i(t) = G(1 - s_i(t), K_s(t))
\]

The functions \( F \) and \( G \) in (1) and (2) are assumed to be strictly increasing in each argument. Note that, in such context, \( 1 - s_i(t) \) can be interpreted as the time spent both to produce and to consume \( C_i(t) \). A peculiarity of relational goods is that it is virtually impossible to separate their production from consumption, since they easily coincide.

The accumulation of social capital is highly path-dependent: on the one side, it improves the technology of production of relational goods; on the other side greater social participation taking the form of higher levels of relational goods’ production and consumption fosters the consolidation of ties and trust among people, thereby increasing the stock of social capital as a by-product. Of course we cannot exclude the possibility that agents engage in social activities for instrumental purposes (for example, to achieve a better job). However, following hints from rational choice sociology (Coleman, 1990), we assume that most of the times the creation of interpersonal ties does not depend on rational investment decisions. Thus, as in Autocci, Sacco and Vanin (2005, 2007, 2008), social capital is accumulated as a by-product of social participation. Following hints from political science, we model the “workplace hypothesis” described in the previous section by assuming that the production of private goods exerts a positive spillover on social capital’s accumulation. Furthermore, since human relations need care to be preserved, we introduce a positive social capital’s depreciation rate to account for their possible cooling over time:

\[
\dot{K}_s(t) = H \left[ \bar{B}(t), \bar{Y}(t) \right] - \eta K_s(t)
\]

where \( \dot{K}_s(t) \) indicates the time derivative of \( K_s(t) \), the parameter \( \eta > 0 \) is the depreciation rate of \( K_s(t) \), \( \bar{B}(t) = \int_0^1 B_i(t) \partial i \) and \( \bar{Y}(t) = \bar{Y}(t) = \int_0^1 Y_i(t) \partial i \) are the average production/consumption of the socially provided good and the average production/consumption of the private good, respectively. The resulting stock is a public resource, which enters as an argument in every agent’s utility function due to its ability to contribute to the production of both private and relational goods.

For simplicity, we consider the following specifications for (1),(2),(3):

\[
Y_i(t) = [1 - s_i(t)] \cdot K_s^\alpha(t)
\]

\[
B_i(t) = s^\beta(t) \cdot s^{1-\beta}(t) \cdot K_s^\gamma(t)
\]

\[
\dot{K}_s(t) = \left[ \bar{Y}(t) \right]^{\beta} \cdot \left[ \bar{B}(t) \right]^{\delta} - \eta K_s(t)
\]
where $\varepsilon \in (0,1)$ is the productivity of time spent on social interaction in the individuals’ production process of relational goods, and $\alpha, \beta, \gamma, \delta > 0$.

Note that a positive average social participation $\pi(t) > 0$ is essential for the production/consumption of $B_i(t)$, that is $B_i(t) = 0$ if $\pi(t) = 0$ whatever the values of $s_i(t)$ and $K_s(t)$ are. If no one participates, single agents have no possibility to enjoy relational goods, even in presence of a positive stock of social capital. If $\gamma > \alpha$, then the role of social capital is more relevant in the production/consumption of relational goods than in the production/consumption of private goods.

According to (4), $\pi(t)$ and $\Upsilon(t)$ are both essential factors for the accumulation of social capital, that is the stock of social capital $K_s(t)$ decreases (that is $K_s(t) < 0$) if $\pi(t) = 0$ or $\Upsilon(t) = 0$.

Finally, we assume that the instantaneous utility function of individual $i$ is:

$$U_i [C_i(t), P_i(t)] = \ln C_i(t) + b \ln P_i(t)$$

(5)

where $P_i(t)$ represents the whole of social needs, $C_i(t)$ are the agents’ private needs, and $b > 0$ measures the relative importance of social needs in respect to private ones. We assume that private goods can satisfy both private and social needs. As pointed out in the introduction, private goods can be used as an imperfect surrogate of human interactions to meet certain social needs, or to compensate the deprivation of socially enjoyed pleasures. If the surrounding environment is socially poor, agents may choose to replace human interactions with private consumption (e.g. they may play a virtual match against the computer instead of meeting friends on a sport field, or chat with unknown and distant people through the web instead of talking with neighbours). On the contrary, relational goods cannot satisfy primary needs such as food, security, clothing, and shelter. A useful way to describe this issue is to consider the following linear specification:

$$P_i(t) = B_i(t) + d \cdot C_i(t)$$

(6)

where the parameter $d$ measures the degree of Edgeworth substitutability between $B_i(t)$ and $C_i(t)$ with respect to the production of $P_i(t)$. If $d = 0$, then there is no Edgeworth substitutability between the two goods. Note that, if $d > 0$, the mixed partial derivative of $U_i$ with respect to $C_i(t)$ and $B_i(t)$ is strictly negative:

$$\frac{\partial^2 U_i}{\partial C_i \partial B_i} = -\frac{bd}{(dC_i + B_i)^2} < 0$$

This means that the lower the value of $B_i(t)$ is, the greater the marginal utility of private consumption $C_i(t)$ will be. As argued by Antoci, Sacco and Vanin (2007), it is more rewarding to interact with people in a context that offers many options for socially enjoyed leisure. On the contrary, if the social environment is poor, and people have few chances to meet and enjoy relational goods, private consumption is more rewarding. For the brevity’s sake, from now the term “substitutability” will mean “Edgeworth substitutability”.

9
Letting \( r \) be the discounting rate of future utility, the \( i \)-agent's maximization problem is:

\[
\max_{s_i(t)} \int_0^{+\infty} \left\{ \ln C_i(t) + b \ln P_i(t) \right\} e^{-rt} dt
\]  

(7)

subject to the dynamic constraint (4). The agent \( i \) solves problem (7) taking as exogenously given the value of \( K_s(t) \) and average values \( \bar{\pi}(t) \), \( \bar{B}(t) \) and \( \bar{Y}(t) \); this is due to the fact that the choice of \( s_i(t) \) by agent \( i \) does not modify the average values, being economic agents a continuum. As a consequence, by applying the Maximum Principle to problem (7) we obtain that the choices of individual \( i \) do not depend on the co-state variable associated to \( K_s(t) \) (that is the "price" of \( K_s(t) \)) in the maximization problem (7). Consequently, to solve problem (7), agent \( i \), in each instant \( t \), chooses the value of \( s_i(t) \) maximizing the value of the instantaneous utility function (5). This implies that the dynamics of \( K_s(t) \) we study do not represent the social optimum. However, since agent \( i \) plays the best response \( s_i(t) \), given the others' choices, the trajectories followed by \( K_s(t) \) represent Nash equilibria. In fact, along these trajectories, no agent has incentive to modify his choices if the other agents do not revise theirs as well.

To simplify our analysis, in this paper we focus on symmetric Nash equilibria. In particular, we assume that individuals are identical and make the same choices. This assumption allows us to study the choices of a representative agent. Thus we can omit the subscript \( i \) in the variables \( s_i(t) \), \( B_i(t) \), \( Y_i(t) \) and \( C_i(t) \) writing simply \( s(t) \), \( B(t) \), \( Y(t) \) and \( C(t) \). In this symmetric Nash equilibrium context, we have that ex ante average values \( \bar{\pi}(t) \), \( \bar{B}(t) \) and \( \bar{Y}(t) \) are considered as exogenously given by the representative agent; however, once \( s(t) \) is chosen, ex post it holds:

\[
\bar{\pi}(t) = s(t)
\]

\[
\bar{B}(t) = \bar{\pi}(t) \cdot 3^{1-\varepsilon}(t) \cdot K_s^\alpha(t) = s(t) \cdot K_s^\alpha(t)
\]

\[
\bar{Y}(t) = [1 - \bar{\pi}(t)] \cdot K_s^\alpha(t) = [1 - s(t)] \cdot K_s^\alpha(t)
\]

In this context, the representative agent, in each instant of time \( t \), chooses \( s(t) \) solving the following static optimization problem:

\[
\max_s \left\{ \ln [(1 - s) \cdot K_s^\alpha] + b \ln \left[ s^3 3^{1-\varepsilon} K_s^\alpha + d \cdot (1 - s) \cdot K_s^\alpha \right] \right\}
\]  

(8)

taking as exogenously given the values of \( \bar{\pi} \) and \( K_s \). The solution \( s(t) \) of the problem (8) has to be substituted to \( \bar{\pi}(t) \) in the equation (4) which, under our symmetric Nash equilibria assumption, can be written as follows:
\[ K_s(t) = \left[ Y(t)^\beta \cdot B(t)^\delta \right] - \eta K_s(t) = \]
\[ = \left[ (1 - \pi(t))^\beta \cdot K_s^\alpha(t) \right]^\beta \cdot \left[ \pi^t(t) \cdot \pi^1(t) \cdot K_s^\gamma(t) \right]^\delta - \eta K_s(t) = \]
\[ = \left[ 1 - \pi(t)^\beta \right] \cdot K_s^\alpha \cdot K_s^\beta + \gamma \delta(t) - \eta K_s(t) \quad (9) \]

Where \( \beta \) and \( \delta \) are strictly positive parameters. Note that under dynamics (9), social capital accumulation is negative if \( \pi(t) = 0 \) (no social participation) or if \( \pi(t) = 1 \) (the production/consumption of the private good is equal to zero).

As noted above, \( B(t) \) and \( Y(t) \) are both essential factors for the accumulation of social capital. Thus, even if people devote all their time to social participation, the stock of social capital is doomed to erosion if there is no private production so that primary needs cannot be satisfied.

According to (9), the value of \( \pi(t) \) that, given \( K_s(t) \), maximizes the rate of growth of \( K_s(t) \) is:

\[ \pi(t) = s^* := \frac{\delta}{\beta + \delta} \]

The latest expression can be interpreted as the “golden rule” for the accumulation of social capital. Note that \( s^* \to 0 \) if \( \delta \to 0 \), and \( s^* \to 1 \) when the value of \( \beta \) is negligible relative to that of \( \delta \).

### 4 Analysis of the model

#### 4.1 The time allocation choice

For simplicity, we limit our analysis to “robust” cases only, that is those not corresponding to equality conditions on parameters’ values. The following result concerns the choice of \( s(t) \) by the representative agent (due to space constraints, propositions’ proofs are omitted if straightforward).

**Lemma 1** Problem (8) admits solution and the time allocation choice \( s^*(t) \) of the representative agent is:

1. if \( \gamma - \alpha > 0 \)
   \[ s^*(t) = \begin{cases} 
   0, & \text{if } K_s(t) \leq \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma - \alpha}} \\
   \frac{bcK_s^{\gamma - \alpha}(t) - b(b+1)}{(1+bc)K_s^{\gamma - \alpha}(t) - b(b+1)}, & \text{if } K_s(t) > \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma - \alpha}} \end{cases} \quad (10) \]

2. if \( \gamma - \alpha < 0 \)
   \[ s^*(t) = \begin{cases} 
   \frac{bcK_s^{\gamma - \alpha}(t) - b(b+1)}{(1+bc)K_s^{\gamma - \alpha}(t) - b(b+1)}, & \text{if } K_s(t) \leq \left( \frac{d(b+1)}{bc+1} \right)^{\frac{1}{\gamma - \alpha}} \\
   0, & \text{if } K_s(t) > \left( \frac{d(b+1)}{bc+1} \right)^{\frac{1}{\gamma - \alpha}} \end{cases} \quad (11) \]
Social capital produces contrasting pressures on the representative agent’s time allocation choices. Remember that $\gamma$ and $\alpha$ are the exponents of $K_s$ in the production functions of $B(t)$ and $Y(t)$, respectively, $b > 0$ represents the weight of social needs, and $d$ measures the degree of substitutability between $B_i(t)$ and $C_i(t)$ in the satisfaction of social needs $P_i(t)$.

If $\gamma - \alpha > 0$ then (ceteris paribus) an increase in the stock of social capital $K_s$ has the effect to raise the productivity of time spent on social participation $s(t)$ relative to that of time spent on the production and consumption of the private good. In this case, if the stock of social capital is “high” enough, then social participation will be positive ($s(t) > 0$). If social capital’s initial endowments are “low”, then agents will devote all their time to private production: despite the improvement in the productivity of time devoted to social participation, the agents’ allocation choice will be guided by the necessity to meet the private needs $C_i(t)$ first.

If $\gamma - \alpha < 0$, an increase in the stock of social capital $K_s$ raises the productivity of time spent on private production more than the productivity of time devoted to social participation. Then, if the initial stock of social capital is “high”, agents will prefer to “exploit” its eventual increases to raise the production/consumption of private goods, and we will have $s(t) = 0$. Otherwise, if the stock of social capital is low, individuals will devote time to social interaction leading to a positive level of participation.

An implication of such result is that, according to equation (4), the stock of social capital cannot grow indefinitely when $\gamma - \alpha < 0$. In this case, an increase in the stock of social capital raises the productivity of time spent on private production therefore leading to a restriction of social participation, which in turn hampers the accumulation of social capital in the long run. By contrast, when $\gamma - \alpha > 0$, an increase in the stock of social capital can spark off a self-feeding process resulting in an increase in the production/consumption of relational goods and the formation and of new ties.

If there is no substitutability between private consumption $C$ and the socially provided good $B$, that is $d = 0$ in (5), it holds $d(b+1) = 0$; therefore $(\frac{d(b+1)}{bc})^{\frac{1}{\gamma-\alpha}} = 0$ if $\gamma - \alpha > 0$. In such context that, by (10)-(11), the following proposition holds.

**Proposition 2** Under the assumption $d = 0$, problem (8) gives the following time allocation choice $s^*(t)$ of the representative agent:

$$s^*_{d=0}(t) = \frac{be}{1 + be}, \text{ whatever the value of } K_s(t) \text{ is;}$$

When the two goods are not Edgeworth substitutes in the satisfaction of social needs, the reduction in the production/consumption of relational goods is not accompanied by an increase in the marginal utility of private goods. For example, the lack of amateur tennis players will not raise the marginal utility of playing virtual matches against a playstation, and the reduction in the number of friends available for going together to the cinema will not raise the
marginal utility of devices to watch movies alone. In this case, private goods will not replace socially provided goods for the satisfaction of social needs, and participation will be constant and such that $1 > s^*(t) > 0$, whatever the value of $K_s(t)$ is.

Notice that social participation $s^*(t)$ in (10)-(11) is (ceteris paribus) a strictly decreasing function of the parameter $d$, which measures the degree of substitutability between $B$ and $C$. Therefore, it always holds $s^*(t) < s^*_{b=0}(t)$. If agents admit the possibility to satisfy social needs (or to compensate for their deprivation) through the production/consumption of private goods (e.g. a playstation is considered as a substitute of a match on the tennis field), then the level of social participation will be lower, whatever the value of $K_s(t)$ is.

5 Dynamics of social capital accumulation and well-being analysis

Even if we have considered very simple specifications of functions $F$, $G$ and $H$, there may exist multiple steady states and poverty traps.

The following result concerns the evolution of representative agent’s well-being along the trajectories under dynamics (9).

**Proposition 3** Along the trajectories of (9), the values of the utility function $U$ and of $K_s$ are positively correlated. This implies that if there exist two steady states $K^1$ and $K^2$ such that $K^2 > K^1$, then $K^2$ Pareto-dominates $K^1$; that is $K^1$ is a poverty trap.

The following proposition defines social capital dynamics resulting from the time allocation choices of the representative agent described in Proposition (1).

**Proposition 4** If $\gamma - \alpha > 0$, social capital dynamics are given by:

$$
\dot{K}_s = \left\{ \begin{array}{ll}
\frac{-\eta K_s}{(1+bc)K_s^{\gamma-\alpha} - d(b+1)} & \text{for, respectively, } K_s \leq \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma-\alpha}} \text{ and } K_s > \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma-\alpha}}
\end{array} \right.
$$

$$
\dot{K}_s = \left\{ \begin{array}{ll}
\frac{-\eta K_s}{(1+bc)K_s^{\gamma-\alpha} - d(b+1)} & \text{for, respectively, } K_s \leq \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma-\alpha}} \text{ and } K_s > \left( \frac{d(b+1)}{bc} \right)^{\frac{1}{\gamma-\alpha}}
\end{array} \right.
$$

Notice that, if $d = 0$ (that is relational and private goods are not substitutes), the dynamics of social capital accumulation become:
\[ K_s = \frac{(be)^\delta}{(1 + be)^{\beta + \delta}} \cdot K_s^{\alpha \beta + \gamma \delta} - \eta K_s \]  

(14)

and the corresponding dynamic regimes are described by the following proposition.

**Proposition 5** Under the assumption of no Edgeworth substitutability \( d = 0 \), the basic features of dynamics are the following (whatever the sign of the expression \( \gamma - \alpha \) is):

a) If \( \alpha \beta + \gamma \delta < 1 \), there exist two steady states:

\[ K_s = \left[ \frac{\eta(1 + be)^{\beta + \delta}}{(be)^\delta} \right]^{\frac{1}{\alpha \beta + \gamma \delta}} \text{ and } K_s = 0 \]

then the economy approaches \( K_s \) (the steady state \( K_s = 0 \) is repulsive) whatever the initial value of \( K_s > 0 \) is.

b) If \( \alpha \beta + \gamma \delta = 1 \), then the steady state \( K_s = 0 \) is the unique steady state. In particular,

b.1) if \( \frac{(be)^\delta}{(1 + be)^{\beta + \delta}} < \eta \), the economy reaches the steady state \( K_s = 0 \); 

b.2) if \( \frac{(be)^\delta}{(1 + be)^{\beta + \delta}} > \eta \), the economy follows a trajectory along which the value of \( K_s \) grows indefinitely (that is \( K_s \to +\infty \)) at the constant rate \( \frac{(be)^\delta}{(1 + be)^{\beta + \delta}} - \eta \).

If private goods cannot substitute relational ones in the satisfaction of social needs, then the economy can follow a virtuous trajectory where the stock of social capital unboundedly grows, raising social participation and consolidating interpersonal ties. When there is some degree of substitutability between \( C \) and \( B \), that is \( d > 0 \), dynamics become more complicated and are characterized by the following propositions.

**Proposition 6** Under the assumption of substitutability \( d > 0 \) and if \( \gamma - \alpha > 0 \), the dynamics are characterized by the following properties:

1. The steady state \( K_s = 0 \) is always locally attractive (whatever the value of the expression \( \alpha \beta + \gamma \delta \) is).

2. If \( \alpha \beta + \gamma \delta < 1 \) then the number of steady states with \( K_s > 0 \) is (generically) zero (see Figure 1) or two (see Figure 2); if two steady states \( K_s^1 \) and \( K_s^2 \) \((K_s^1 < K_s^2)\) exist, then \( K_s^2 \) is attractive while \( K_s^1 \) is repulsive.

3. If \( \alpha \beta + \gamma \delta = 1 \) then there exists at least a steady state with \( K_s > 0 \); furthermore, the number of steady states with \( K_s > 0 \) is one (see Figure 3) or three; if an unique steady state exists, then it is repulsive; if three steady states \( K_s^1, K_s^2 \) and \( K_s^3 \) \((K_s^1 < K_s^2 < K_s^3)\) exist, then \( K_s^2 \) is attractive while \( K_s^1 \) and \( K_s^3 \) are repulsive. Finally, if \( \frac{(be)^\delta}{(1 + be)^{\beta + \delta}} > \eta \) and the initial value of \( K_s \) is greater than \( K_s^* \), where \( K_s^* \) is the steady state with the highest value of \( K_s \), then there exists an unbounded endogenous growth path with increasing well-being along which \( s \to \frac{be}{1 + be} \).
If agents tend to replace relational goods with private ones for the satisfaction of social needs or to compensate for the deprivation of human interactions, and if an increase in the stock of social capital $K_a$ has the effect to raise the productivity of time spent on social participation $s(t)$ relative to that of time spent on the production and consumption of the private good, then a steady state where $K_a = 0$ is always locally attractive: the risk exists of an erosion of the entire stock of social capital. Such risk can be warded off only if certain conditions hold, as it will be pointed out later.

Under the assumption of substitutability ($d > 0$) and if $\gamma - \alpha < 0$, the dynamics are characterized by the following properties:

**Proposition 7**  
1. The value of $K_a$ always approaches a steady state value lower than the upper bound $\left(\frac{d(b+1)}{bc+1}\right)^{\frac{1}{1+\gamma}}$ (see proposition 4), whatever the value of the expression $\alpha \beta + \gamma \delta$ is. Consequently, the stock of social capital $K_a$ cannot grow indefinitely.

2. If $\alpha \beta + \gamma \delta < 1$, then the steady state $K_a = 0$ is always repulsive; there exists at least a steady state with with $K_a > 0$; the number of steady states with $K_a > 0$ is (generically) one (see Figure 4) or three; steady states with an odd index are attractive and those with an even index are repulsive. Whatever the initial value of $K_a$ is, the economy approaches a steady state with $K_a > 0$.

3. If $\alpha \beta + \gamma \delta = 1$, then the steady state $K_a = 0$ is always locally attractive; the number of steady states with with $K_a > 0$ is zero (see Figure 5) or two (see Figure 6); the steady states with an odd index are repulsive and those with an even index are attractive\(^1\).

High initial endowments are a necessary but not a sufficient condition for an endogenous and unbounded growth of the stock of social capital. Necessary and sufficient conditions are as follows:

\[
\gamma - \alpha > 0 \quad \frac{(bc)^\beta}{(1 + bc)^{\beta + \delta}} > \eta \quad \alpha \beta + \gamma \delta = 1
\]

When these conditions hold, the poverty trap $K_a = 0$ is always locally attractive; however, when the initial value of $K_a$ is high enough, the economy follows

\(^1\)Values of parameters in **Figure 4**: $\alpha = 0.9, \beta = 0.5, \gamma = 0.4, \delta = 1, \epsilon = 0.7, \eta = 0.04, b = 4, d = 0.05$; **Figure 5**: $\alpha = 0.9, \beta = 0.8, \gamma = 0.4, \delta = 1, \epsilon = 0.7, \eta = 0.24, b = 4, d = 0.05$; **Figure 6**: $\alpha = 0.9, \beta = 0.8, \gamma = 0.38, \delta = 1, \epsilon = 0.5, \eta = 0.12, b = 4, d = 0.4$.

For $\alpha = 0.9, \beta = 0.5, \delta = 0.55, \gamma = 1, \epsilon = 0.5, \eta = 0.11, b = 4, d = 0.05$ we obtain three positive fixed points of coordinates $K_a^* = 1.27$ (attractive), $K_a^{**} = 1.41$ (repulsive), $K_a^{***} = 1.52$ (attractive).
a trajectory along which $K_s \to +\infty$. As stated above, along such trajectory we have that:

$$s \to \frac{b\varepsilon}{1 + b\varepsilon}$$

and the equation (12) “tends” (for $K_s \to +\infty$) to the equation:

$$\dot{K}_s = \frac{(b\varepsilon)^{\delta}}{(1 + b\varepsilon)^{\gamma+s}} \cdot K_s - \eta K_s$$

which coincides with the equation (14) describing social dynamics under the assumption $d = 0$ (no substitutability between $C$ and $B$). This allows to say that when the stock of social capital becomes high enough, then the dynamics under the assumption $d = 0$ and those under the assumption $d > 0$ become “very similar”. This result is more likely if $b$ and $\varepsilon \in (0, 1)$ exhibit high values and if the social capital’s depreciation rate $\eta$ is low. As pointed out above, $b$ measures the weight of social needs in determining people’s satisfaction. A high value of $b$ indicates that agents are not so prone to sacrifice their relational sphere for private needs. Such parameter may be determined by cultural factors acknowledging the importance of non market relations in respect to material consumption. For example, a culture exalting the prominence of cooperation and solidarity in social life is a good starting point. $\varepsilon$ represents the productivity of social participation in the individual production of relational goods. A high value of this parameter indicates that agents are more capable to influence their relational sphere with their own effort, independently of the current levels of the others’ social participation and of the stock of social capital. This ability may be due to “human” and “social” factors. As regards human factors, it is remarkable that people have diverse attitudes towards social interactions. Charismatic agents may be able to carry away other people in interpersonal relationships even if the surrounding environment is not particularly rich of social participation opportunities. Sociological studies claim that charismatic agents behaving as leaders in social networks may work as catalysts for the creation of social capital (Burt, 1999, Renshon, 2000, Roch, 2005). However, aggregative behaviors need a certain degree of generalized trust to take place. In a socially poor environment, agents may use their relational skills to produce relational goods, and to carry away other people in social participation only if there is a reasonable likelihood that their effort will not be wasted and will be repaid. The diffusion of social norms of reciprocity is a crucial precondition for such aggregative and pro-social behaviors. Individuals are more likely to resist the temptation to satisfy social needs through the substitution of relational goods with private ones if they perceive that their efforts will not be “betrayed”. It is noteworthy that, starting from Coleman (1987, 1988), part of the literature considers norms of trust and reciprocity as an integral part of the definition of social capital. Following such approach, it is possible to argue that the parameter $\varepsilon$ incorporates human and social factors whose development in the long run is influenced by the stock of social capital.

16
The following 6 figures show solow-like graphs where the intersection points between $H \left[ \frac{B(t)}{Y(t)} \right]$ and $\eta K_s(t)$ are fixed points of the differential equation. The black dot is used for attractive stationary points, the grey dot for repulsive stationary points.

Figure 1

Figure 2

Figure 3

Figure 4

\(^2\) Values of simulations in Figure 1: $\alpha = 0.01, \beta = 0.4, \gamma = 0.82, \delta = 1, \varepsilon = 0.12, \eta = 0.15, b = 4, d = 0.03$; Figure 2: $\alpha = 0.01, \beta = 0.4, \gamma = 0.82, \delta = 1, \varepsilon = 0.12, \eta = 0.04, b = 4, d = 0.03$; Figure 3: $\alpha = 0.4, \beta = 0.9, \gamma = 0.64, \delta = 1, \varepsilon = 0.7, \eta = 0.3, b = 4, d = 0.3$.

For $\alpha = 0.9, \beta = 0.5, \delta = 0.55, \gamma = 1, \varepsilon = 0.5, \eta = 0.11, b = 4, d = 0.05$ we obtain three positive fixed point of coordinates $K_s^* = 0.005$ (repulsive), $K_s^{**} = 0.02$ (attractive), $K_s^{***} = 93$ (repulsive).
6 The effects of (exogenous) technical progress

In previous sections, we argued that one of the main transmission mechanisms leading from economic growth to social poverty may be the pressure on the time allocation choices of the individuals. According to the folk wisdom, there is at least another, crucial, link connecting growth to the growing social isolation of our time: technology.

Technical progress may cause a decline in human interactions through two main channels. Firstly, it creates ever more sophisticated material goods which make private consumption seem more and more capable to replace relational goods. As stated by Robert Putnam in the New York Times interview, people can now get entertainment and information while remaining entirely alone. If the surrounding social environment is not intriguing from a cultural and relational point of view, isolated people could be forced to satisfy their social needs at home, by means of private consumption. Think for example of a teenager living in an isolated mountain community: technology allows him to satisfy a range of social needs simply connecting to the web for chatting, sharing photos, playing chess at distance, and so on. When it is impossible to satisfy social needs at distance, technical progress provides agents with a variety of opportunities to compensate the deprivation of social interaction. For example, a virtual match against the playstation can compensate for the lack of the time necessary to play an actual match on a sport field. The latest home theatre is a (cold) comfort for the lack of friends with whom watching movies at the cinema.

Secondly, technical progress and innovation adoption cause continuous changes in production processes, raising the mobility of workers (Routledge and von Amsberg, 2003). The rise in mobility may result in an increase in the “precariousness” of labour, which in turn can be seen as a barrier to social integration and as a factor of social capital’s destruction: a high level of flexibility on employment hampers the consolidation of ties, both inside and outside the workplace. While a stable and satisfactory work provides not only income, but also
an identity and a “sense of belonging”, precariousness generates discouragement and distrust towards labour market institutions that, at the macro level, may result in a more distrustful society. The high exposure to the risks of job loss and intermittent unemployment raises the uncertainty on future incomes, making people unhappy and less self-confident, more suspicious towards the others and less able to plan crucial relational activities such as marriage and procreation (Clarke and Oswald, 1994, Dockery, 2005, Di Tella et al., 1997). In other words, technical progress can “crowd out” the workplace’s ability to nurture social participation described in section 2.

In this section we study the dynamic effects generated by the introduction of exogenous technical progress $T(t)$ in the production function of the private good:

$$Y(t) = T(t) \cdot [1 - s(t)] \cdot K_s^\alpha(t)$$

where the growth rate of $T$ is assumed to be given by the equation:

$$\dot{T}(t) = \sigma T(t)$$

where $\sigma$ is a strictly positive parameter representing the growth rate of $T$. In such context, the time allocation choice $s^*(t)$ by the representative agent is described by the following proposition.

**Lemma 8** If the production function of the private good is given by (15), then problem (8) admits solution and the time allocation choice $s^*(t)$ of the representative agent is:

If $\gamma - \alpha > 0$

$$s^*(t) = \begin{cases} 0, & \text{if } K_s(t) \leq \left(\frac{d(b+1)T(t)}{bc}\right)^{\frac{1}{\gamma-\alpha}} \\ \frac{beK_s^{-\alpha}(t)-d(b+1)T(t)}{(1+bc)K_s^{-\alpha}(t)-d(b+1)T(t)}, & \text{if } K_s(t) > \left(\frac{d(b+1)T(t)}{be}\right)^{\frac{1}{\gamma-\alpha}} \end{cases}$$

(17)

If $\gamma - \alpha < 0$

$$s^*(t) = \begin{cases} 0, & \text{if } K_s(t) \leq \left(\frac{d(b+1)T(t)}{bc+\gamma+1}\right)^{\frac{1}{\gamma-\alpha}} \\ \frac{beK_s^{-\alpha}(t)-d(b+1)T(t)}{(1+bc)K_s^{-\alpha}(t)-d(b+1)T(t)}, & \text{if } K_s(t) > \left(\frac{d(b+1)T(t)}{bc+\gamma+1}\right)^{\frac{1}{\gamma-\alpha}} \end{cases}$$

(18)

### 6.1 The case without substitutability between C and B

If there is not substitutability between private consumption $C$ and relational goods $B$ (i.e. if $d = 0$), social participation is constant and strictly positive $s^*(t) = bc/(1+bc)$; in such context, the dynamic of social capital’s accumulation is given by the equation:

$$\dot{K}_s = \frac{(be)^{d}}{(1+bc)^{d+\gamma}} \cdot T^\beta K_s^{\alpha\beta+\gamma d} - \eta K_s$$

(19)
where the evolution of $T$ is described by the differential equation (16). Note that $K_s = 0$ for $K_s = 0$ and, if $\alpha\beta + \gamma\delta < 1$, along the graph of the function:

$$K_s = \left[\frac{(be)^\delta}{\eta(1 + b\varepsilon)^{2+\delta}}\right]^{1-\alpha-\gamma}. \frac{T^{\beta}}{T^{\alpha+\gamma}} \quad (20)$$

which is increasing in $T$. Note that it holds $\dot{K}_s < 0$ above the curve (20) and $\dot{K}_s > 0$ below it.

The basic features of dynamics under the assumption of no substitutability are described by the following Proposition.

**Proposition 9** Dynamics (19) have the following properties:

1) If $\alpha\beta + \gamma\delta < 1$, then both $T$ and $K_s$ grow without bound (i.e., $\lim_{t \to +\infty} T(t) = +\infty$ and $\lim_{t \to +\infty} K_s(t) = +\infty$) along any trajectory starting from a strictly positive initial value of $K_s$. Among these trajectories, there exists a trajectory represented by the equation:

$$K_s = \left[\frac{(1 - \alpha\beta - \gamma\delta)(be)^\delta}{[\beta\sigma + \eta(1 - \alpha\beta - \gamma\delta)](1 + b\varepsilon)^{2+\delta}}\right]^{1-\alpha-\gamma}. \frac{T^{\beta}}{T^{\alpha+\gamma}} \quad (21)$$

along which the growth rates of $T$ and $K_s$ are given by:

$$\frac{\dot{K}_s}{K_s} = \frac{\beta}{(1 - \alpha\beta - \gamma\delta)} \frac{T}{T} \quad (22)$$

where $T/T = \sigma$ by assumption and $\frac{\dot{K}_s}{K_s} > \frac{T}{T}$ if and only if $\frac{\beta}{1 - \alpha\beta - \gamma\delta} > 1$.

Along the remaining trajectories, the growth rate of $K_s$ approaches the value given in (22) as $t \to +\infty$ (see Figure 7).

2) If $\alpha\beta + \gamma\delta = 1$, then an explicit solution of (19) can be calculated:

$$K_s(t) = K_s(0)e^{A \frac{T(0)^{\beta}}{\sigma} - \frac{T(t)^{\beta}}{\sigma}} e^{A \frac{T(0)^{2\delta}}{\eta}}$$

where $A = \frac{(be)^\delta}{(1 + b\varepsilon)^{2+\delta}}$ and $K_s(0)$ and $T(0)$ are the initial conditions on social capital and technology (see Figure 8).

**Proof.** The first part of Proposition can be proved by defining a new variable:

$$x := \frac{T^{\beta}}{K_s^{1-(\alpha\beta + \gamma\delta)}} \quad (23)$$
and by calculating its time derivative:

\[
\begin{align*}
\dot{x} &= \frac{T^\beta}{K_s^{1-(\alpha \beta + \gamma \delta)}} \left( \frac{\dot{T}}{T} - (1 - (\alpha \beta + \gamma \delta)) \frac{\dot{K}_s}{K_s} \right) \\
&= x \left[ \beta \sigma + \eta (1 - \alpha \beta - \gamma \delta) \right] - \frac{(bc)^\delta}{(1 + bc)^{\beta + \delta}} (1 - \alpha \beta - \gamma \delta) x 
\end{align*}
\] (24)

Equation (25) has two stationary states:

\[
x^* = 0 \quad \text{and} \quad x^{**} = \frac{[\beta \sigma + \eta (1 - \alpha \beta - \gamma \delta)] (1 + bc)^{\beta + \delta}}{(1 - \alpha \beta - \gamma \delta) (bc)^\delta}
\] (25)

Notice that, since \(\alpha \beta + \gamma \delta < 1\), then \(x^{**} > 0\) is globally attractive in the positive \(x\)-axis; since, by (24), \(\dot{x} = 0\) if and only if (22), point 1) of Proposition is proved. The second part follows by a direct calculation. ■

It is interesting to note that:

1. If there is no substitutability between \(C\) and \(B\), then \(K_s\) can grow without bond, whatever the sign of the expression \(\gamma - \alpha\) is (we will show that this result no more holds when there is substitutability).

2. Along the trajectories under dynamics (19), the evolution of \(K_s\) is monotonic (always increasing or decreasing) or follows a U-shaped path, according to which \(K_s\) is initially decreasing and then becomes definitively increasing. It is worth to stress such result in that, as we will see, if there is substitutability between \(C\) and \(B\), then the evolution of \(K_s\) can take the shape of an inverted U curve.

![Figure 7](image_url)

![Figure 8](image_url)
6.2 The case with substitutability between $C$ and $B$

If $d > 0$, then social participation $s^\ast$ depends on the values of $T$ and $K_s$ and can assume the value 0. In particular, the graph of the function:

$$K_s = \left( \frac{d(b+1)T}{be} \right)^{\frac{1}{\gamma - \alpha}}$$

(26)

separates, in the plane $(T, K_s)$, the region where $s^\ast = 0$ (below the curve) from that where $s^\ast > 0$ (above it). Note that the function (26) is increasing (decreasing) in $T$ if $\gamma - \alpha > 0$ (respectively, if $\gamma - \alpha < 0$).

In the region where $s^\ast = 0$, social capital dynamics are given by $\dot{K_s} = -\eta K_s < 0$ and the slope of trajectories $\frac{dK_s}{dT} = -\frac{dK_s}{\eta} < 0$ is negative; this implies that along the trajectories below the curve (26), $T$ increases and $K_s$ decreases.

Above the curve (26), social capital dynamics are given by:

$$\dot{K_s} = \left[ \frac{K_s^{\gamma - \alpha}}{(1 + be)K_s^{\gamma - \alpha} - d(b+1)T} \right]^{\beta} \left[ \frac{beK_s^{\gamma - \alpha} - d(b+1)T}{(1 + be)K_s^{\gamma - \alpha} - d(b+1)T} \right]^{\delta} \cdot T^{\beta} K_s^{\gamma \alpha + \gamma - \alpha - 1} - \eta K_s$$

(27)

where $T$ evolves according to the differential equation (16).

The basic features of dynamics (27) are described in the following Proposition.

**Proposition 10** If $\gamma - \alpha > 0$, then the region under the curve (26) is positively invariant under dynamics: every trajectory entering such region cannot leave it. Along the trajectories under the curve (26), the value of $K_s$ approaches 0 for $t \to +\infty$ (see Figure 9).

If $\gamma - \alpha < 0$, then the region above (under) the curve (26) is positively invariant if $\sigma/\eta(\alpha - \gamma) \geq 1$ (respectively, if $\sigma/\eta(\alpha - \gamma) < 1$); in any case, along every trajectory the value of $K_s$ approaches 0 for $t \to +\infty$ (see Figure 10).

**Proof.** To check the results on positive invariance of the sets under or above the curve (26), we have simply to compare the slope of the trajectories $\frac{dK_s}{dT} = -\frac{dK_s}{\eta}$, evaluated along the curve (26), and the slope of (26). To prove the results about the evolution of $K_s$, notice that in the set where $s^\ast = 0$ it holds $\frac{dK_s}{dT} = -\eta K_s$ and consequently $K_s(t) = K_s(0) \cdot e^{-\eta t}$, where $K_s(0)$ is the initial value of $K_s$. Finally, note that, in case $\gamma - \alpha < 0$, every trajectory lies definitively in the set under the curve (26) or in the set above it; in any case, since $T \to +\infty$, the value of $K_s$ approaches 0.

Whatever the sign of the expression $\gamma - \alpha$ is, along the trajectories crossing the curve (26) we will show that the evolution of $K_s$ can take an inverted U-shape, differently from the case without substitutability.

---

Values of parameters in **Figure 9**: $\alpha = 0.7$, $\beta = 0.2$, $\gamma = 0.2$, $\delta = 0.3$, $\varepsilon = 0.4$, $\eta = 0.06$, $\sigma = 0.04$, $b = 3$, $d = 0.3$.

**Figure 10**: $\alpha = 0.3$, $\beta = 0.2$, $\gamma = 0.71$, $\delta = 0.3$, $\varepsilon = 0.4$, $\eta = 0.06$, $\sigma = 0.04$, $b = 3$, $d = 0.3$.  

---

22
According to the above Proposition, a necessary condition to have unbounded growth of $K_s$ is $\gamma - \alpha > 0$; that is, the importance (measured by $\alpha$) of $K_s$ as an input in the production process of the private good must be lower than its importance (measured by $\gamma$) in the production process of the relational good.

To analyze the behaviour of $K_s$ in case $\gamma - \alpha > 0$ we introduce the following definition.

**Definition 11** A Regular Growth Curve (RGC) is a curve in the plane $(T, K_s)$ along which the rate of growth of $T$ is equal to the exogenously given value $\sigma$ while the rate of growth of $K_s$ is equal to a constant strictly positive value $g$, possibly different from $\sigma$.

Notice that along a RGC, being $\frac{T}{K} = \sigma$ and $\frac{K_s}{K_s} = g$, it holds $\frac{dK_s}{dT} = \frac{K_s}{T} = \sigma$; consequently a RGC is the graph of a function $K_s(T) = CT^\frac{\sigma}{g}$, where $C$ is a positive arbitrary constant. RGCs are not trajectories under our dynamics. However, we aim to show that, for $T$ high enough, there exist values of $g$ and $C$, that we will denote by $\overline{g}$ and $\overline{C}$ respectively, such that the corresponding RGC $K_s(T) = \overline{C}T^\overline{\frac{\sigma}{g}}$ is asymptotically approached, for $T \to +\infty$, by the trajectories starting “near” to it. This implies that along such trajectories $\frac{K_s}{K_s} \to \overline{g}$ as $t \to +\infty$.

Notice that, for $T$ high enough, a RGC can be approached by the trajectories only if it lies above the curve (26); this requires that $g$ must satisfy the necessary condition: $\overline{g} > \frac{1}{\overline{g} \sigma}$, where $\overline{g} \sigma > 1$. That is, it must hold $g > \frac{1}{\sigma - \alpha}$ (where $\frac{\sigma}{\gamma - \alpha} > \sigma$). So we introduce a further definition.

**Definition 12** A Reachable Regular Growth Curve (RRGC) is a RGC satisfying the condition $g > \frac{\sigma}{\gamma - \alpha}$ (see Figure 6 11).

Looking at (17), it is easy to check that, along a RRGC, the social participation choice $s^*$ approaches the value $\frac{b\varepsilon}{(1 + b\varepsilon)}$ as $t \to +\infty$. Remember that $\frac{b\varepsilon}{(1 + b\varepsilon)}$ is the value of social participation in the context without substitutability. In other words, if $T$ and $K_s$ grow following a RRGC, then for $T$ and $K_s$ high enough, social participation is “almost” equal to the level achieved under the assumption of no substitutability. Now the problem is: are there values of $g$ and $C$ such that the associated RRGC may be approached by some trajectories of our dynamics? To solve this problem, we analyze the behavior of the variable:

$$x = \frac{T^\gamma}{K_s^{1-\alpha \beta - \gamma \delta}}$$

previously defined (see (23)) and limit our analysis to the case $1-\alpha \beta - \gamma \delta > 0$. Remember that in the context in which $s^* = \frac{b\varepsilon}{(1 + b\varepsilon)}$ always (i.e. in the context without substitutability), it holds $x = 0$ (see (24)) along the curve (see (21)):

---

6 Value of parameters: $\alpha = 0.82$, $\beta = 0.71$, $\gamma = 0.7$, $\delta = 0.92$, $\varepsilon = 0.6$, $\eta = 0.02$, $b = 3$, $d = 0.3$. 

23
\[ K_s = \overline{C} \cdot T^\overline{g} \]

where \( \overline{C} \equiv \frac{(1-\alpha_2-\gamma)(1-\beta)}{(1+\alpha_1+\gamma)(1-\alpha_1-\gamma_1)} \) and \( \overline{g} \equiv \frac{\beta}{1-\alpha_2-\gamma}. \) So, if \( \frac{\beta}{1-\alpha_2-\gamma} > \frac{\alpha}{\gamma} \) (i.e. \( \frac{\beta}{1-\alpha_2-\gamma} > \gamma > 1 \)), we have that, as \( T \to +\infty \), along \( K_s = \overline{C} \cdot T^\overline{g} \) the value of \( \dot{x} \) approaches 0 while \( \dot{x} \) becomes (see (24)) strictly positive (respectively, strictly negative) along the RRGCs corresponding to values of \( g < \overline{g} \) (respectively \( g > \overline{g} \)), with \( g \) and \( C \) near enough to \( \overline{g} \) and \( \overline{C} \), respectively. This implies that all trajectories starting (for \( T \) high enough) sufficiently near to \( K_s = \overline{C} \cdot T^\overline{g} \), approach \( K_s = \overline{C} \cdot T^\overline{g} \) as \( T \to +\infty \).

Notice that, by Proposition 3, all trajectories in the plane \((T, K_s)\) can be Pareto-ranked; in particular, we have that, given two trajectories \( \bar{K}_s(T) \) and \( \bar{K}_s(T) \), with \( \bar{K}_s(T) < \bar{K}_s(T) \), then \( \bar{K}_s(T) \) Pareto-dominates \( \bar{K}_s(T) \). Furthermore, well-being may be decreasing when the economy follows a trajectory along which \( K_s \to 0 \) (see Figures 12, 13, 14).

---

7 Values of parameters in Figures 12, 13, 14: \( \alpha = 0.7, \beta = 0.2, \gamma = 0.2, \delta = 0.3, \varepsilon = 0.4, \eta = 0.01, \sigma = 0.04, b = 3, d = 0.3. \)
7 Concluding remarks

Our framework addresses a range of hypotheses that have never been jointly taken into account within a theoretical model. Agents allocate their time between labour, aimed at the production of private goods, and social participation activities. Private consumption and relational goods are substitutable: if the environment is poor of participation opportunities and social participation are perceived as costly and frustrating, people can disengage from relational activities and devote more time and resources to private consumption. Following hints from political science, we account for the possibility of positive spillovers from private to relational production, due the ability of job interactions to stimulate the creation of durable ties. Following hints from the sociological literature, we assume that most of the times the creation of interpersonal ties does not depend on rational investment decisions. Rather, it is an incidental, not necessary, by-product of social participation. The resulting stock is a public resource, which enters as an argument in agent’s utility function and as an input in both “material” and “relational” goods’ production functions. Since human relations need a continuous care to be preserved over time, we account for a positive depreciation rate of the stock of social capital. The main results of our study can be summarized as follows.

In the framework without technical progress, an unbounded growth of social capital can be only observed in the following to cases:

a) The case in which private goods and relational ones are not Edgeworth substitutes.

b) The case in which the two types of goods are Edgeworth substitutes but relevance of the stock of social capital impact on the production/consumption process of relational goods is greater than in the production process of private ones (i.e. $\gamma - \alpha > 0$).

In both cases (a) and (b), social capital dynamics is path-dependent; in a “favourable” configuration of the model’s parameters, starting from an high
enough initial endowment of social capital, the economy follows a virtuous trajectory where the stock of social capital endogenously and unboundedly grows, raising social participation and consolidating interpersonal ties. On the other side, the reverse process may be self-feeding as well: if the initial endowment of social capital is low, then the economy experiences a simultaneous decline in social participation and social capital leading to “social poverty traps” where time spent on relational activities becomes more expensive (in terms of opportunity cost) and less productive (in terms of relational goods).

If private and relational goods are substitutes but (b) doesn’t hold (i.e. \( \gamma - \alpha < 0 \)), then no trajectory exists along which social capital grows without bound.

In the context without technical progress, the evolution of social capital is always monotonic, always increasing or decreasing; introducing exogenous technical progress in the production function of private goods leads to interesting modifications in social capital’s accumulation dynamics. In such context, social capital’s trend relative to technical progress can be non monotonic; in particular, it may experience an initial decline followed by a growth, but not vice versa (a growth followed by a decline is impossible), if relational and private goods are not substitutes, while under the assumption of substitutability the stock of social capital may exhibit a growth followed by a decline, so that its relationship with technical progress is described by an inverted U-shaped curve. Since technical progress is in turn positively correlated with GNP, our result supports Putnam’s (2000) intuition of the inverted U-shaped relationship between social capital and development.

Also under the assumption of exogenous technical progress, there exist trajectories along which social capital can grow without bound only in the contexts (a) and (b) described above. The possibility to find the path to sustainable growth of social capital crucially depends on the initial endowment of social capital: an environment rich of participation opportunities, a culture acknowledging the importance of non market relations, the diffusion of moral norms of reciprocity and cooperation certainly constitute a good, desirable “starting point”.

8 References


Econ Discussion Papers, University of Bonn.


