Measuring productivity increase by long-run prices: The early analyses of G.R. Porter and R. Giffen

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Abstract. The 19th century economic commentators did not possess a formal measure of the rate at which productivity was increasing during the industrial take-off. Yet they did develop an intuitive method based on the comparative change in prices and wages. This paper reviews the contributions of G.R. Porter and R. Giffen and, in the light of some modern contributions, presents an assessment of their rationality and improvability under current standards. It is argued, in particular, that a proper measure of industrial productivity increase based on the change in real earnings rates is the mathematical dual of a Solovian measure of the industrial Total Factor Productivity growth.

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1. Introduction

The 19th century economic commentators were certainly aware of the unprecedented increase in productivity that was taking place during the industrial take-off, but they did not possess a fully rational method for measuring it. The assessments of 'how great' the improvement was largely relied on intuitive, common sense measures. The most immediate and crudest was, of course, the increase in output per worker in an individual firm, in accordance, for example, with Smith's pin-maker illustration. These commentators also used a more elaborate measure, however. This was in terms of the degree of 'cheapening' in the production of commodities, in accordance, for example, with Ricardo's illustration regarding machinery. G.R. Porter, in particular, has been the first to analyse long time series of average prices with the purpose 'to form some judgement as to the economy which has been introduced into the process of manufacture' (Porter, 1851, p. 181. A more complete quotation can be found below). This second kind of assessment was more promising, and in fact it was refined in some detail during the 19th century for a series of reasons. First, price data were available on a larger and more systematic scale than output and input data, as exemplified by Tooke's monumental *History of Prices*. Secondly, the assessment of productivity increase by prices was more suitable than direct productivity data to shed some light on the closely related assessment of the degree of progress in the condition of the working classes, which became an increasingly important issue from the 1840s (e.g., Ludlow and Jones, 1876; see also Opocher, 2010). Finally, the wide use of price indexes allowed for some kind of aggregation, which was beyond any possibility of measurement based on occasional local data on output per worker. A proper 'price accounting' method aimed at extracting productivity data from price series was introduced, in the second half of the century, by R. Giffen in a series of articles for the *Journal of* *the Statistical Society of London* (later *Journal of the Royal Statistical Society*) from 1879 to 1888. They presented, among other things, a rather refined '*measure* of the increase of the return to the industry of the community' (Giffen, 1888, p. 716). As he proudly remarked, with this measure 'all facts [were] in harmony' (*Giffen, 1888*, p. 746).

The evolution of the early 'price accounting' measures of productivity increase, which is outlined in sections 2 and 3 of this paper, poses a number of interesting questions concerning their degree of rationality, their improvability over current standards, and their relation to the familiar Solovian method. Answering these questions will lead us to reconsider some relatively neglected modern contributions which happen to formally develop the basic ideas of Porter and Giffen. In particular, we shall see in section 4 that Lydall (1969) proposed, like Giffen, some price indexes of output, wages and profits aimed at formalizing 'the intuitively plausible statement that, when the average earnings of the factors increase more rapidly than the average price of the products, there is technical progress' (Lydall, 1969, p. 4). Steedman (1983) worked out a complete theoretical method for the definition and aggregation of industrial rates of 'cost reduction', for the case of Leontief technologies, and adopted a coherent conception of long-run prices, thus formalizing (in an extended context) Porter's analysis of the technological information embedded in prices which are 'at or near their natural level' (Porter, 1851 [1834], p. 427). In section 5 we shall extend Steedman's argument somewhat further, in order to deal with the case of any (constant returns) production function (or, more exactly, unit cost function), and reach a simple but previously unnoticed result: in principle, at the level of the industry, a measure based on price accounting is numerically identical to the Solovian measure based on growth accounting. They are two sides of the same coin (and, in mathematical terms, they are duals), the only difference being that they use different data: output and input *prices* the former, and output and input *quantities* the latter. With reference to the industry, therefore, we may indifferently say that Solow's 'primal' implicitly contains a rational version of the Porter-Giffen measures of real cheapening, or that the early analyses of the 'dual' to some extent anticipated Solow's Total Factor Productivity. Section 6 concerns the aggregation of the industrial rates of productivity increase. We shall argue that the method proposed by Steedman (1983), based on the shift in the real wages – rate of interest (profit) frontier provides an important missing link with the tradition initiated by Porter. Section 7 concludes.

2. Long-run price series, money and productivity: Porter and Jevons

During the 1830s in England there was a flourishing of statistical studies: the foundation of the Statistical Society of London (LSS) in 1834, the publication of Porter's *Progress of the Nation* (in 1836, followed by further editions in 1846 and 1851), and of Tooke's *History of Prices* (a monumental publication started in 1838 and thereafter widened and updated until 1857, with the collaboration of W. Newmarch), have been at the basis of a subsequent flow of statistical analyses of the many aspects of the late 18th century and 19th century industrial revolutions.

The analysis of the sources of the variations of (money) prices was one of the most intriguing and controversial topics. According to Jevons, 'a true understanding of the course of prices can alone explain many facts in the statistical and commercial history of the country' (Jevons, 1865, p. 294). The tables of prices frequently embraced very long periods of time: Tooke's series ranged from 1782 to the 1850s, and Porter's series from the start to the middle of the century; likewise, the proceedings of the LSS frequently carried studies with series of fifty years or more. The price series reflected at once a variety of factors such as bad harvests, commercial crises, taxation, speculation and wars, together with more lasting causes, such as the relative abundance or scarcity of gold and the relation of gold to paper currency, as well as a permanent paramount cause, which was 'the continuous progress of invention and production' (Jevons, 1865, p. 308). Hence the need to separate, first of all, the local peaks and troughs from more lasting variations; this was typically done by averaging across prices and times, thus obtaining something similar to what we now call a trend variation in prices. Secondly, a more difficult problem was to distinguish between two sources of the 'trend' in money prices: currency appreciation or depreciation, and productivity growth.

Even though Tooke's work soon became the main source for further studies, his mode of presentation of data has been criticized on the ground that his narrative presented a succession of heterogeneous facts and the tables failed to distinguish amongst them. According to Jevons,

> …large tables of figures are but a mass of confused information for those causally looking into them. They will probably be the source of error to those who pick out a few figures only; a systematic (…) course of *calculation and reduction* is necessary to their safe and complete use. (Jevons, 1865, p. 294; emphasis added)

Some elementary tools for 'eliciting the general facts contained in them [the tables of prices]' (Jevons, *Ibid.*) and in particular the role of currency appreciation or depreciation, have been proposed by Porter and consisted of the calculation of an aggregate of prices by means of index numbers:

> There is perhaps no single circumstance more pregnant with instruction on this subject [whether the currency be at any time redundant or otherwise] than a *general* rise or fall of prices when viewed and adjusted in combination with local or temporary causes of disturbances. (Porter, 1851 [1834], p. 431)

The basis of the index numbers should be a period

> …in which prices were considered to be at or near their natural level, and in which the mercantile community in this kingdom were believed to be principally engaged in their regular and legitimate business; a period, in fact, which should be free from any undue depression on the one hand, and without the excitement of speculation on the other. (Porter, 1851 [1834], p. 427)

This double distinction between individual and general changes, and between temporary and permanent changes, marked a considerable progress in the interpretation of the statistical evidence of prices, and all the local and temporary factors which were at the heart of Tooke's narrative could be considered as disturbances of more general and lasting factors.

Along these lines, Jevons distinguished between three long alternating periods of elevation and fall of general prices: some thirty years of elevation around the turn of the 18th century, followed by a period of about the same length of a general fall, until the middle of the new century, and then again a prolonged elevation until 1865 (which was to last until the mid-1870s).

Abstracting from the local peaks and troughs within each period due to temporary factors, these secular movements were, according to Jevons, the result of two fundamental forces which may act in accordance or in opposition with each another: 'the production of gold' and 'the progress of invention' (Jevons, 1865, p. 303).

He argued that the 'great fall, proceeding from 1818 to 1830, and reaching its lowest point as yet in 1849' was not difficult to understand:

> The production of almost all articles has been improved, extended, and cheapened during this period, and all the imported articles must, too, have been affected by improvements in navigation, while there was no corresponding improvement in the production of precious metals, from the derangement of the American mines in 1810 to the Californian discoveries in 1849. (Jevons, 1865, p. 303)

Jevons's analysis here closely follows in the steps of Porter's book. In the chapters which provide minutely detailed evidence of the improvements in the cotton, linen, silk, wool, iron and steel industries, and in mining during the first half of the 19th century, Porter invariably provided a scalar, synthetic measure of such improvements by the individual price reduction that they allowed. His comment on the spectacular fall in the price of cotton wool in the thirty years from 1820 to 1849 is worth quoting at some length, because it shows that Porter was quite close to conceiving of the measure of productivity growth as the outcome of a price accounting exercise:

> …the average price [of cotton wool] per yard, which in 1820 was $12_{2/3d}$, [fell] in 1849 to $3_{2/5d}$. The average price of twist in 1820 was $2s.5_{1/2d}$, and in 1849 was little more than $10_{3/4d}$ per pound. If, in addition to these values, we take account of the reduction that has occurred in the price of raw cotton, we may be enabled to *form some judgment* as to the economy which has been introduced into the process of manufacture during the last 30 years, and be besides able to *apportion* the degrees of that economy which appertain to the spinning and to the weaving branches of the manufacture respectively. (Porter, 1851 [1834], pp. 179-81; emphasis added)

Interpreting 'judgement' in the sense of 'measure', we have here a clear statement of the idea that the industrial rate of productivity growth can be measured by comparing the evolution of output and input prices. In the above case, explicit attention has been limited to the comparative evolution of produced inputs, whose prices, too, were reduced by technological improvements in related industries. Porter does not mention wages, as one might have expected, and this requires some explanation. His tables, in fact, never fail to also record *wage* changes for specific kinds of labour in specific places, and they show that during the same period (money) wages have been roughly *constant* (see, e.g., p. 184 for weavers, p. 194 for spinners), so that in the illustration above, the recorded fall in the output price had to be 'corrected' only for produced inputs. Moreover, wage records were less general and complete than price records, as Porter himself remarked elsewhere (Porter, 1850), so that small changes were not very significant. We may conclude, then, that in the period referred to by Porter the price of manufactured goods fell decidedly *relative to wages*, and this was the 'real' change detecting technological improvement. With constant wages, the nominal fall in prices (as averaged over long periods of time) was a fairly good measure of productivity growth.

The other two periods considered by Jevons, however, have been characterized by a general *increase* in prices. This raised a clear problem of interpretation: 'The progress of our industry (…) has been *continuous*, and its only change that of acceleration in recent years. There is nothing in such constant progress that can account for a great rise in price' (Jevons, 1865, p. 303). Evidently, Jevons argued, 'if the progress of invention causes a fall of price, then we need even more potent causes to raise prices in opposition to it' (Jevons, 1865, p. 303). These causes were concerned with the availability of gold. The 'current of gold' was 'considerable' at the turn of the 18^{th} century, thus making for rising prices. The proof of this relationship, according to Jevons, was a sort of price-specie flow mechanism involving England and India, in which prices were comparatively higher in the country with comparatively more precious metals (England), thus determining a compensating flow of gold towards India (Cf. Jevons, 1865, p. 304). The abundance of gold determined an elevation of prices of greater force than the downward pressure due to technical improvements. As the current of gold 'greatly fell off' in correspondence to the Mexican War of Independence (1810-21) and remained low for the next thirty years or so, such a pressure could operate undisturbed. In the third period, the price fall was interrupted and reverted into a general rise, by 'the Californian and Australian discoveries of gold, which were followed almost immediately by the great drain, unremitted to the present time' (Jevons, 1865, p. 305). Similarly to what had happened in the years around the turn of the 18th century, the abundance of gold more than compensated for the effect of technical improvements on prices.

The merit of Jevons's analysis is to draw attention to general and secular movements in nominal prices, and in so doing he could make it clear that from a long-run perspective the importance of gold production could hardly be overstated. It also has a fundamental limit, however, which consists of the failure to *separate* the effect of 'gold' from the effect of technical improvements. In fact, according to Jevons's argument, only in the absence of productivity change would the trend variation in general nominal prices reflect 'gold' alone. And, conversely, only if there were no comparative change in 'gold' would the change in nominal prices reflect only productivity change. But in general, he presents no criteria for assessing the individual contributions to price change. Even worse, he appears to think that such criteria did not exist: he maintained, in fact, that the measure of 'the fall in prices which *might have been expected* from the continuous progress of invention and production (…) is *necessarily* unknown' (Jevons, 1865, p. 308; emphasis added).

Yet, as we have seen, one can find in Porter's comments on prices and wages in the first half of the 19th century some distinct elements for a method of analysis of nominal changes which can provide a measure of productivity change, and this method is based on a *relative* change in input and output prices. Porter did not develop his own argument any further because during that period the nominal price of the main input, labour, did not show a systematic direction of change, so that he was content to say that (nominal) 'cheapening', in the sense of the observed systematic fall in price, was a fair approximation of the 'real' rate of cost reduction in any industry.

3. Towards a coherent measure of 'real cheapening': Giffen

It was not until Giffen's series of articles from 1879 to 1888 that a method for separating technical change from 'gold' components in the long-run variations of prices was proposed. Giffen's argument is based on two main premises.

The first premise is that the question of appreciation or depreciation of money should be deprived of the abstract prejudices of the time, and dealt with in practical and conventional terms. It was all a matter of relative change, according to Giffen. An appreciation of money *in terms of commodities* is *by definition* a general and lasting fall in commodity prices as expressed in monetary terms, and the question of appreciation or depreciation can only be settled, not in general terms, but with reference to a *specific* set of 'things' in terms of which money is evaluated:

> It is of the utmost importance (…) that the question of the appreciation of money at the present time [1888] should be discussed for its own sake as a question of fact merely, and as a purely statistical rather than an economic question (…). It is convenient to employ the phrases appreciation of money and depreciation of money, (…) when the expressions are used scientifically, as the mere equivalents of the fall or rise of the prices of those articles or groups of articles with which money is compared. (Giffen, 1888, p. 714)

The second and complementary premise consists of the observation that the proportions among prices and between prices and wages/income per capita normally changed over time. Only in a 'stationary community, which goes on from year to year with the same population, producing and consuming the same things' would 'the fall or rise of prices (…) extend to all commodities equally, and to wages and incomes also' (*Giffen, 1888*, p. 715). In such a case, 'nothing would be easier apparently than to ascertain appreciation or depreciation' (*Giffen, 1888*, p. 716). But this is contrary to historical experience. Most 19th century communities were all but stationary. Moreover, Giffen dismisses the case of 'retrograding communities' as 'a very rare one'. Therefore he concentrated on an 'advancing community' in which the 'average prices of commodities' (*Giffen, 1888*, p. 740) have a systematic tendency to *fall relative to* average 'wages and incomes per head' (*Giffen, 1888*, p.716): through the 'advance in the return to the industry of the community', there were in fact more 'real things to divide' (*Ibid.*). This had nothing to do with money and was an entirely 'real' change. As a consequence, however, appreciation or depreciation of money was *automatically different* according to whether it was measured by commodities or by incomes, and the analyst can only (and indeed must) make this very explicit and transparent.

On the basis of these two premises, Giffen was able to perform a price accounting analysis which separated money appreciation/depreciation from productivity growth. He distinguished between three cases of appreciation, three cases of depreciation and a further 'mixed' case. For brevity of exposition, let us denote by $\hat{P}$ the proportional average change of prices, and by $\hat{I}$ the proportional average change of wages and incomes per head. The obvious cases of appreciation are

i) $\hat{P} < 0 = \hat{I}$

ii) $\hat{P} < \hat{I} < 0$

In the first case, $-\hat{P}$ is the rate of productivity growth, as in Porter's analysis: 'the fall of [commodity] prices might be the measure of the increase of the return to the industry of the community, assuming that the labour employed in services improves generally as does the labour employed in the production of commodities' (Giffen, 1888, p. 716). At the same time, we also have *one form* of appreciation, still equal to $-\hat{P}$, if measured by commodities.

The second case depicts a different and stronger form of appreciation which extends to wages. The rate of productivity growth is now measured by $-\left(\hat{P}-\hat{I}\right)$: 'the *difference* between it [the fall of prices] and the fall in wages and incomes might represent the advance in the return to the industry of the community' (*Giffen, 1888*, p. 716; emphasis in original).

A more extreme form of appreciation is distinguished by Giffen as a third case, and this occurs when not only the wage *rates* and incomes *per capita* fall, but also the aggregate nominal income does, notwithstanding the increase in working population. Denoting by $\hat{N}$, the rate of working population growth, we have

iii) $\hat{P} < \left(\hat{I} + \hat{N}\right) < 0$

Symmetrically, there are two obvious forms of depreciation:

iv) $0 = \hat{P} < \hat{I}$

v) $0 < \hat{P} < \hat{I}$

In Case iv), depreciation is mild and is detected only if measured by incomes. Moreover, the rate of productivity growth would be measured by $\hat{I}$ if productivity grew uniformly in manufacturing and non-manufacturing sectors: 'the increase in [wages and incomes per head] might correspond with the increase of the return to the industry of the community' (Giffen, 1888, p. 717); likewise, this rate also measures one special form of appreciation. In Case v), depreciation is so strong that the 'natural' tendency of prices to fall reverts into a general rise. The rate of productivity growth is still measured by $-\left(\hat{P} - \hat{I}\right)$: 'the improvement in the [return to the industry] might be measured by the *difference* between the rise in the prices of commodities and the rise in wages and incomes' (*Giffen, 1888, p.717*; emphasis in original).

Giffen's sense of symmetries generated a third, even stronger case of depreciation, in which there was 'absolute inflation in all prices along with a continued cheapening of production' (*Ibid.*), but it is unclear how this case

vi) $0 << \hat{P} < \hat{I}$

should be different from case v).

A seventh distinct and more interesting case is singled out by Giffen:

vii) $\hat{P} < 0 < \hat{I}$

This case 'may be described as intermediate between the mildest types of appreciation and depreciation above specified' (*Ibid.*) (that is, Cases i) and iv)). Notwithstanding a general fall in commodity prices, it would be inappropriate to speak of monetary appreciation (or depreciation, either) in this case.

For our purposes, it may be interesting to note that, algebraically, the rate of productivity growth is measured by $-\left(\hat{P}-\hat{I}\right)$ *in all cases*. An even more explicit statement of this general rule is made by Giffen where he compared the recent evolutions (1876-1888) of prices and wages/incomes in different countries:

> Thus the phenomenon of falling prices of commodities and stationary or, at least, not greatly declining incomes and wages, appears to be very general in gold-using countries. It does not follow that the result should be the same in every country. We cannot assume that the rate of advance in material progress to be the same in each, or that the margin between the average prices of commodities and the average income should widen in the same way. But although the same result precisely is not to be looked for, if we could measure with the necessary degree of fineness, we cannot but assume that the communities of all the countries named [Germany, Belgium, France, Italy] are progressing to some extent. (Giffen, 1888, pp. 139-40)

The phrase 'average income', like 'wages and incomes per head' and similar phrases reported so far may seem ambiguous, and the reader may wonder what assumptions allowed Giffen to treat them as a single magnitude.

Giffen was a leading expert on wages; his 1883 inaugural address to the LSS was on 'The Progress of the Working Classes in the Last Half Century', and a few years later he published a long paper with some 'Further Notes' on the same subject. For the purpose of illustrating the economic basis of the marked progress in living standards made by the working classes in the fifty years around the middle of the century (which he, like Jevons, judged to be much more 'decisive' to what had taken place in the period covered by Porter's data: see Giffen, 1886, pp. 30-31), a series of concordant evolutions in wage rates in some relevant districts and for some 'typical' kinds of labour in leading sectors of the British economy was certainly enough:

> While no precise answer [on the degree of the improvement] is possible, I wish to point out that the reasons for believing in a very considerable degree of improvement, almost if not quite to the extent of enabling us to say that the working classes are twice as well off as they were fifty years ago, are so strong as to be beyond reasonable doubt. The data may be incomplete, but read with a little care they show us that the minimum limit of the improvement must be a very high one. (Giffen, 1886, p. 32)

He evaluated that, by the general rise in *monetary* wages and the average constancy or decline in prices over the same period (with the exception of house rents and 'meat': see Giffen, 1886, p. 47; Giffen, 1883, pp. 601-605; Giffen 1879, p. 39), the improvement was 'at least between 50 and 100 per cent., and with an allowance for the shortening of the hours of labour, may be placed nearer the 100 than the 50, if not over the 100' (Giffen, 1886, p. 33).

When confronted, however, with the problem of assessing the degree of productivity growth and of monetary appreciation/depreciation by comparing the change in prices and incomes, Giffen found an obstacle in the 'want of records of wages'. In principle, in fact, a weighted average of the change in a very wide variety of wages was needed. 'But no such records are in existence. Instead there are only records of isolated rates of wages, not weighted in any way' (Giffen, 1888, p.728). His strategy, at least for the twenty years from 1867 to 1887, was to consider a proxy of the overall income per capita, and he identified it in the 'income tax incomes', for which the record was 'tolerably complete' (*Ibid.*). Of course, they admittedly represented mainly 'the earnings of profit on capital' (*Ibid.*), but he maintained that 'what we do know of wages points in the same direction' (Giffen, 1888, p. 729). Thus, for instance, he evaluated that the average index number of commodity prices of imports and exports in the ten years 1878-1887 fell by 16.5%, as compared with the average in the ten years 1868-77 (see table on p. 722; the table refers to the estimate of 'Economist', which was Giffen's own pseudonym). At the same time, income tax incomes per capita rose by 13.3% (see table on p. 728). Under the assumption that wages also rose by about the same proportion (which would be realistic on the basis of the table on p. 731), the ten-year rate of productivity growth was about 30%, which amounts to a yearly average rate of 2.66%.

For earlier periods, Giffen did not venture to present data in aggregate index numbers, and therefore we cannot make similar calculations. He did, however, present a series of qualitative results (in the main consistent, of course, with Porter's and Jevons's) which allow us to fit each period into his classification:

a) 'Towards the closing of last century, and the early part of the present century [that is, about 1775-1810], there was a remarkable rise in prices, and an equally remarkable, if not more remarkable, rise in incomes, indicating that, on the whole, the community was then advancing' (Giffen, 1888, p. 747): This was case v) of his classification
b) By contrast, in the following period, from the early 19th century through about 1850, there was a 'steady fall of general prices (…) [while] average money incomes increased very little' (Giffen, 1888, p. 746), in accordance to Porter's evidence: This was case i) or vii)
c) Then, in 1850-1873, there was a 'great rise in money incomes accompanied by a much less rise in commodities' (Giffen, 1888, p. 746): Case v) once again
d) The next period, from the middle 1870s to the late 1880, was finally characterized by 'stationary and almost slightly declining incomes, accompanied by a great fall in the prices of commodities' (*Ibid.*): Case i) or ii)

Despite these oscillations, during the whole period there was constantly some productivity growth, and Giffen thought that in the two later periods it was 'much the same' (*Ibid.*), while he was 'inclined to think that (…) before 1845 [it] was not so great as it has since been' (*Giffen, 1888*, p.747).

Some broad technological facts hiding behind monetary prices and wages/incomes are unveiled by Giffen's price accounting analysis and, at the same time, the much debated issues concerning alternating periods of monetary appreciation or depreciation are settled on a technically sound ground. What he calls 'real cheapness' (e.g. Giffen, 1888, p. 748), that is, low prices in relation to incomes, coexisted 'with any (…) range of money prices or any (…) change in that range' (*Ibid.*). As he remarked with pride, 'much confusion has arisen from the neglect of this distinction' (*Ibid.*), while, by his analysis, 'the facts are all in harmony' (*Giffen, 1888*, p.746).

4. Formalizing Giffen's aggregate measure

We can say that Giffen measured the rate of productivity increase by the ratio of an index of 'wages and incomes per head', *I*, and an index of output prices, *P*, which is equivalent to considering, as he did, the difference of their proportional rates of change. Because of the absence of systematic records on wages and the unavailability of an empirically-based system of weights, he proxied *I* with an index of income tax incomes per capita, under the assumption that the index of wages did not differ significantly from the index of other incomes (mainly profits). Denoting by *G* the overall index of productivity relative to the base period, we can express his idea with the equation

$$G = \frac{I}{P}$$

Had Giffen the possibility to consider complete records of wages and profits he would, it may be presumed, have liked to build separate indexes for each kind of income, as shown by the quotations above. This is precisely what Lydall (1969) suggested to do some 80 years later (without reference to Giffen). Lydall made some assumptions which ensure that profits on all capital items (old and new) can be defined as a uniform proportion of the replacement cost of each of them (see Lydall, 1969, p. 5). Expressing by *C* an index of the aggregate replacement cost of the capital stock and assuming the rate of profit constant over time, C will also be an index of profits. Denoting by *W* an index of wages, and by α, β the shares of profits and wages on net income in the base period, Lydall defined the index of productivity increase (see Lydall, 1969, p. 6) as

$$L = \alpha\frac{C}{P} + \beta\frac{W}{P}$$

Since Giffen's index of 'wages and incomes per head' is ideally a weighted average of indexes of wages and profits ($I = aC + bW$), Lydall's measure can be considered as a rationalization of Giffen's early measure.

To be sure, a simple accounting identity is hidden behind Giffen's and Lydall's measures. Let in fact Q denote an index of output. Clearly, $PQ = I = \alpha C + \beta W$, and $Q = G = L$. Since I, C, and W are price indexes, based on the same capital and labour physical amounts as in the base period, Q may be thought to represent the hypothetical index of output obtained with 'constant' physical inputs (as in the case of Solow's aggregate measure). At the same time, G and L may be thought to represent the hypothetical *common* rate of increase of all earnings *rates* in terms of the product.

The simple elaboration of an accounting identity on Giffen-Lydall lines can rightly be considered as an interesting empirical approximation. On theoretical grounds, however, this is not enough. One should explicitly prove that a specific weighted average of the proportional changes in 'real' earnings rates theoretically tends, for small variations, to the hypothetical *common* proportional change (which is, no doubt, an agreeable definition of productivity increase). This requires us to consider some microeconomic conditions at the industry level.

5. Defining productivity increase at the industry level: Porter reconsidered

G.R. Porter thought that the 'degrees of the economy' brought about by technological improvements in the various industries can be 'judged' on the basis of the change in each 'natural price', and that such a price reflected the cost of production under normal business conditions. As we have seen, things were complicated by the fact that an improving industry uses some inputs which are the outputs of some other improving industries, so that the change in one price reflects the improvements in many industries, and one should develop a method apt to 'apportion' it.

A proper elaboration of Steedman's (1983) method of measurement of the industrial rates of productivity increase can clarify and rationalize Porter's intuition.

Let us define the industrial rate of productivity increase as the proportional rate of reduction of its cost of production *at constant input prices* (Steedman, 1983, p. 225). Steedman refers this definition to Leontief technologies, but we can safely refer it to industries characterized by generic (constant returns) production functions.[1]

Let us denote by $\mathbf{w}$ the vector of wages of different kinds of labour and by $\boldsymbol{\pi}$ the vector of rental rates for the services of all other inputs. These rental rates may include an interest payment at rate r. For instance, in the case of a raw material or intermediate good bought from another industry at price p_j, we have $\pi_j = (1+r)p_j$. We do not need in the present context to enter into details, and just assume the correct π to be calculated for each item. By duality theory, the *unit* cost function, *c*, will suffice to fully describe technical conditions:

$$c = c(\mathbf{w}, \boldsymbol{\pi}, t) \qquad (1)$$

where *t* denotes (logical) time.

The rate of productivity increase, $\boldsymbol{\gamma}$, is

$$\gamma \equiv -\frac{\dot{c}}{c} \qquad (3)$$

Porter's 'natural price' require that

$$p = c(\mathbf{w}, \boldsymbol{\pi}, t) \qquad (2)$$

Let us consider, for illustrative purposes, the case with two inputs (say, one kind of labour and one intermediate input). Dividing both sides of equation (2) by *p*, we obtain, by homogeneity of degree 1 of the cost function,

[1] Steedman also allows for general joint production and fixed capital, but we shall stick to the simpler case of single output industries and circulating capital.

$$1 = c\left(\frac{w}{p}, \frac{\pi}{p}, t\right) \qquad (2')$$

Equation (2') implicitly traces a curve in real input price space – the real input price frontier — whose position depends on the shift parameter t, as in Figure 1. Let us draw a ray through P_1, the pair of real input prices at time 1, until it crosses the frontier at time 2. We have here a *hypothetical* vector on the new frontier, characterized by constant proportions. The distance between the two frontiers along the ray *as a proportion* of the length of the vector at P_1 is clearly our theoretical measure γ.

[Figure 1 about here]

As it happens, however, relative input prices normally change along with productivity increase, if only because the change in π/p depends on the exogenous productivity change in another industry, so that the new real input prices may be at a point like P_2. Put another way, re-distribution of the industrial output normally accompanies productivity increase. Do we have, then, a method for approximating γ on the basis of *any* observed pair on the new frontier? Fortunately we do, and this method turns out to be the mathematical dual of Solow's method, as referred to the same kind of industry.

It will be clear that, for sufficiently small variations, point P_{21} approximates the hypothetical 'constant-proportions' point on the new frontier. It follows that the approximated $(1+\gamma)$ is the ratio of the distance between 0 and P_{21} to the distance between 0 and P_1. By similar triangles, we also have

$$(1+\gamma) = \frac{(w/p)_{21}}{(w/p)_1} = \frac{(\pi/p)_{21}}{(\pi/p)_1} \qquad (3)$$

Now the tangent to the new frontier at P_2 has slope $-a/l$, where a and l are, respectively, the cost minimising use of the commodity input and that of labour, per unit of output, at P_2. It follows that

$$\frac{(w/p)_{21}-(w/p)_2}{(\pi/p)_2-(\pi/p)_{21}}=-\frac{a}{l}$$

Making use of equation (3) we obtain

$$(1+\gamma)=\frac{wl}{p}\frac{(w/p)_2}{(w/p)_1}+\frac{\pi a}{p}\frac{(\pi/p)_2}{(\pi/p)_1} \qquad (4)$$

where wl/p and $\pi a/p$ are clearly the input shares at time 2. The index of industrial productivity increase, therefore, can always be calculated as a weighted average of the index of the real wage and the index of the real rental rate of the intermediate input.

The interested reader may wish to check our Figure 1 against Figure 1 in Solow (1957) setting $a/l=k$, and find that the two methods are duals. An equivalence proof is given in Opocher (2009), together with the proof that, for infinitely small variations, the exact rate γ can be obtained on the basis of any pair of instantaneous changes in real input rentals.

We can at this point reconsider Porter's 'apportionment' argument in terms of equation (4). Let us assume that the rate of interest is constant, so that the index of (π/p) reflects only the change in the relative commodity price (or, in Lydall's terminology, the change in the replacement cost of the commodity input, in terms of the output). We have three possible cases. The first is when

$$(1+\gamma)=\frac{(w/p)_2}{(w/p)_1}=\frac{(\pi/p)_2}{(\pi/p)_1}\ .$$

Taking Porter's point of view, we may say that the output price has fallen at the same rate relative to both input rentals. There is therefore no apportionment to make, and the relative fall in the output price *is* the measure of productivity increase. If, however

$$\frac{(w/p)_2}{(w/p)_1}>(1+\gamma)>\frac{(\pi/p)_2}{(\pi/p)_1}$$

then the fall in the output price relative to the wage would overestimate the increase in industrial productivity, and part of this fall (the difference with $(1+\gamma)$ as defined in equation 4) depends on the fact that the price of the commodity input, too, has fallen. This 'portion' of the price fall, according to Porter, is due to the improvements taking place in the input-producing industry. The third obvious case (not to be found in Porter's illustration) is defined by inverting the inequalities: for concreteness, one may think of an imported commodity input whose price has increased relative to both the industrial output and the wage. We have here a sort of 'reverse apportionment', since the fall in the industrial price relative to the wage would *underestimate* productivity increase.

More generally, equation (4) allows us to distinguish between two components of the change in real earnings rates: productivity increase and redistribution (both within and between industries). This again provides a counterpart to Solow's method, which distinguishes between productivity increase and change in factor intensities. It is beyond the scope of the present paper to analyse this aspect any further, and the interested reader is again referred to Opocher (2009).

6. Aggregating the industrial rates of productivity increase

There is a clear analogy between our industrial productivity index $(1+\gamma)$ and Lydall's aggregate index L. Unfortunately, however, we have no theoretical ground for interpreting equation (4) as an aggregate relation. In the practical case with produced inputs and a positive interest (profit) rate, the aggregate counterpart of the industrial real input price frontier is the so-called 'real wages – rate of interest (profit) frontier'. Under the conditions of validity of Samuelson's 'surrogate production function', it would indeed be possible to interpret the frontier as the dual representation of an aggregate production function. In this case, a linear approximation of the shift in the frontier may work as in the industrial case. But it is well known that, if industrial heterogeneity is taken seriously, a surrogate production function does not exist. Besides, the 'growth accounting' approach itself has developed a method which first defines productivity increase at the industry level and then explicitly aggregates the industrial rates. The classical references are Domar (1961) and Hulten (1978). Likewise, even the ‘price accounting’ view has its own theoretical method of aggregation, developed in Steedman’s mentioned article.

The main premise of Steedman’s contribution, apart from constant returns to scale, is that the wage rates (in a common numéraire) and the rate of interest tend to be uniform across industries. Moreover, commodities are paid for by the input-receiving industries their long-run competitive price. Under these premises, which are fully shared also by the Domar-Hulten methods (see Jorgenson, 1990, p. 67), prices, wages and interest rates for the economy as a whole are tied together by a system of equations à la Sraffa-Leontief. The aggregate rate of productivity growth can at this point be expressed by a measure of the shift in the real wages-rate of profit (interest) frontier. Steedman argued that each potential measure can be expressed as a weighted sum of sectoral rates of productivity increase. In this respect, the main qualitative result is that the aggregate rate of productivity growth ‘can be far greater than the average process-level improvement rate, due to the role of produced means of production’ (Steedman, 1983, p. 232), and even greater than that expected on the basis of Domar’s weights.

We have seen in section 2 that Porter had the remarkable intuition that long-run prices and wages at the level of the individual industry contained valuable information on the pace of industrial technological improvements, but it was beyond his possibilities to extract from these data any clear evaluation of the overall increase. Nonetheless the idea that, in the economy as a whole, technological improvements in one or more industries allow for increasing real wages at constant rates of profit was in his general economic background. Kurz (this issue) has argued, in fact, that Smith’s and Ricardo’s views of technological improvements can be rationalized in terms of outward shifts in the real wages – rate of profit frontier. If this is true, then Steedman’s contribution can rightly be considered as a further development in the long tradition initiated by Porter.

9. Concluding remarks

Productivity increase is currently measured using the Solovian ‘growth accounting’ method, and its theoretical foundations have been continuously refined over the past fifty years. Yet we have argued in this paper that another, less known method that can be called ‘price accounting’ has been developed over the course of a long time. A first distinct contribution, mainly focused on individual industries, dates back to Porter, around the middle of the 19th century; a more refined method for measuring aggregate productivity increase on the basis of indexes of nominal prices and nominal wages and incomes per capita was developed by Giffen at the end of the century. These contributions were based more on intuitive ideas than on formal logic. Further steps have been made much later, as a consequence of the 1960s criticism of capital theory and the renewed interest in the classical perspective. Lydall discussed the general theoretical foundations of the price view on productivity increase, and in so doing he happened to refine Giffen’s aggregate approach. Steedman later implemented a formal definition of the rate of productivity increase at the level of the individual process and a method for aggregating over processes. His method is based on long-run prices and can be considered as a rationalization of Porter’s method.

The less known tradition reviewed in this paper was initiated much earlier than the Solovian tradition, and has been partially developed in opposition to it. It certainly has its own potentialities for empirical research, because it connects productivity increase with output redistribution – a subject which is attracting increasing interest. Nonetheless it would be wrong to consider it as entirely independent of the Solovian method. Quite the contrary, we have shown that, at the industry level, the price accounting measure is mathematically dual to a corresponding Solovian measure: while the latter is able to distinguish between a shift in the production function and a movement on it, the former is able to distinguish between a shift in the real input price frontier and a movement on it.

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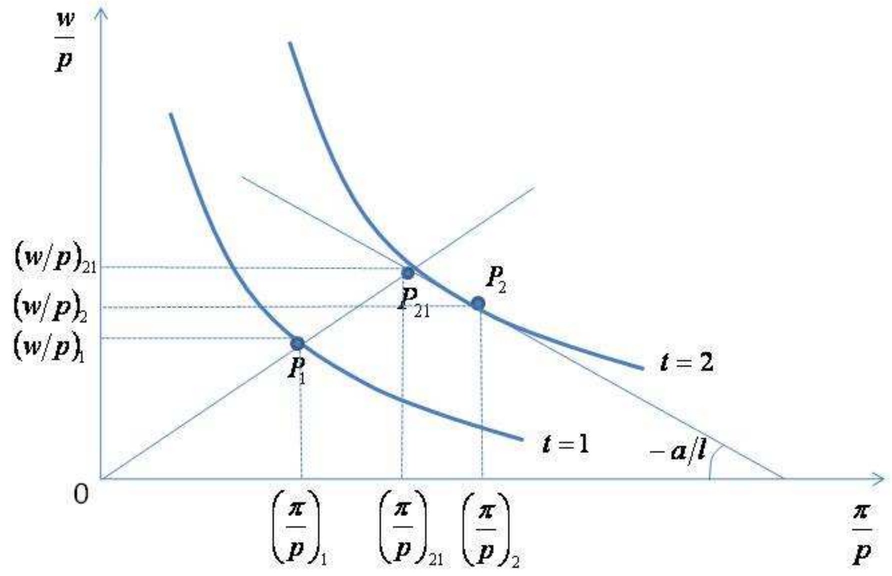

$\frac{w}{p}$
$(w/p)_{21}$
$(w/p)_2$
$(w/p)_1$
P_2
P_{21}
P_1
$t = 2$
$t = 1$
$-a/l$
0
$\left(\frac{\pi}{p}\right)_1$
$\left(\frac{\pi}{p}\right)_{21}$
$\left(\frac{\pi}{p}\right)_2$
$\frac{\pi}{p}$

Figure 1