Tariff and Equilibrium Indeterminacy - A Global Analysis

Zhang, Yan and Chen, Yan

1 November 2009

Tariff and Equilibrium Indeterminacy—A Global Analysis

Yan Chen*

Shandong University

Yan Zhang†

Shanghai Jiao-Tong University

November 1, 2009

Abstract

Zhang (2009) shows that endogenous tariffs and endogenous labor income taxes (Schmitt-Grohe and Uribe, 1997) are equivalent in generating local indeterminacy. Using the method developed by Stockman (2009), we extend Zhang's analysis to prove that they are also equivalent in generating global indeterminacy (chaotic equilibria) under a balanced-budget rule. We show that the existence of Euler equation branching in an arbitrarily small neighborhood of a steady state can imply topological chaos in the sense of Devaney. In addition, the Euler equation branching occurs regardless of the local uniqueness of the equilibrium around the steady state(s).

Key words: Endogenous Tariff Rate, Regime Switching, Chaos.

JEL: E32, E62.

1. Introduction

Zhang (2009) shows that endogenous tariffs and endogenous labor income taxes (Schmitt-Grohe and Uribe, 1997) are equivalent in generating local indeterminacy. To be accurate, local indeterminacy

*Center for Economic Research, Shandong University, 27 Shanda Nanlu, Jinan, Shandong, China, 250100.

†We wish to thank David Stockman for his kind help. *Correspondence*: Yan Zhang, Department of Economics, School of Economics, Antai College of Economics & Management, Shanghai Jiao-Tong University, 535 FaHuaZhen Road, Shanghai, P.R. China, 200052; Tel and Fax: 86-21-52302560. Email: laurencezhang@yahoo.com.

can emerge when tariff rates levied on imported energy are endogenously determined by a balanced-budget rule with a constant level of government expenditures (or lump-sum transfers). In this paper, we extend Zhang's analysis to prove that they are also equivalent in generating global indeterminacy (chaotic equilibria) under this balanced-budget rule. A global analysis shows that as in Stockman (2009), the existence of Euler equation branching in an arbitrarily small neighborhood of a steady state can imply topological chaos in the sense of Devaney.[1] In addition, multiple equilibria and chaos through regime switching near a steady state can arise, regardless of the local uniqueness of the equilibrium around the steady state(s). These results show that (1) global indeterminacy always exists in the model of Zhang (2009), no matter whether the (low tariff) steady state is locally indeterminate or not, and (2) tariffs and labor income taxes are equivalent in generating global indeterminacy because Stockman (2009) shows that (endogenous) labor income taxes have the same effect on the model dynamics in a one-sector closed economy.

This type of regime switching sunspot equilibria are deterministic and once explored by Gardini et al. (2009), Christiano and Harrison (1999), and Stockman (2009) among others.[2] One important characteristic of this type of indeterminacy is that the dynamics going backward are single-valued, but multi-valued going forward (see, for example, Michener and Ravikumar, 1998).

In what follows, we describe our model in Section 2. In Section 3, we make a global analysis and explore the implications of Euler equation branching. In Section 4, we conclude the paper.

[1] Here Euler equation branching means that the dynamics going forward can be expressed by a differential inclusion of the form $\dot{x} \in \{f(x), g(x)\}$, i.e., a multi-valued dynamical system.

[2] For example, Christiano and Harrison (1999) analyzed this kind of regime switching sunspot equilibria in a one-sector economy with productive externalities. And Stockman (2009) explores it in a one-sector economy with fiscal increasing returns.

2. The One-Sector Economy With Tariffs

This is the one-sector oil-in the production RBC model studied by Zhang (2009). A representative agent maximizes the intertemporal utility function

$$\int_0^\infty e^{-\rho t}(\log c_t - bn_t)dt,\ b > 0, \tag{1}$$

where c_t and n_t are the individual household's consumption and hours worked, and $\rho \in (0, 1)$ is the subjective discount rate. We assume that there are no intrinsic uncertainties present in the model.

The budget constraint of the representative agent is given by

$$\dot{k}_t = (r_t - \delta)k_t + w_t n_t - c_t,\ k_0 > 0 \text{ given}, \tag{2}$$

where $\dot{k}_t$ denotes net investment and the other variables are k_t (capital), r_t (rental rate), w_t (real wage) and δ (depreciation rate).

On the production side, a single good is produced by the representative firm with a Cobb-Douglas production technology:

$$y_t = k_t^{a_k} n_t^{a_n} o_t^{a_0} \tag{3}$$

where y_t is total output, $a_k + a_n + a_0 = 1$ (constant returns to scale), and the third factor in the production, non-reproducible natural resources, say oil (o_t), is imported. Perfect competition in factor and product markets implies that factor demands are given by:

$$w_t = a_n \frac{y_t}{n_t}$$

$$r_t = a_k \frac{y_t}{k_t}$$

and

$$p^0(1+\tau_t) = a_0 \frac{y_t}{o_t},$$

where p^0 denotes the real price of oil (the imported goods) and τ_t is the tariff rate levied on the imported oil and uniform to all firms. Here we should emphasize that (1) in this standard neoclassical growth model, p^0 is the relative price of the foreign input in terms of the single good, which is the numeraire and tradable; and (2) the variable τ_t represents the endogenous tariff rate levied on the foreign input and we require that $\tau_t \geq 0$ to rule out the existence of import subsidies.[3]

Since we assume that the foreign input is perfectly elastically supplied, the factor price (p^0) is independent of the factor demand for o_t, we can substitute out o_t in the production function using

$$o_t = a_0 \frac{y_t}{p^0(1+\tau_t)}$$

to obtain the following production function:

$$y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}} \tag{4}$$

where $A_t = (\frac{a_0}{p^0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$ acts as the "Solow residual" in a neoclassical growth model, which is inversely related to the foreign factor price and τ_t.

[3]The model is based on the standard DSGE models that incorporate foreign energy as a third production factor. This class of models (such as those of Rotemberg and Woodford (1994), and ACW (2005, 2007, and 2008)) have been used widely to study the business-cycle effects of oil price shocks.

The government must select $\{\tau_t\}$ to balance its budget each period:

$$p^0 \tau_t o_t = G \tag{5}$$

with $G > 0$ given.

As in Stockman (2009), we consider a kind of global indeterminacy called "Euler equation branching". As we stated before, the model dynamics going forward can be expressed by a differential inclusion of the form $\dot{x} \in \{f(x), g(x)\}$. The Euler equation branching occurs in our model because multiple equilibria arise in the oil market. To be accurate, we consider paths for prices $\{w_t, r_t\}$ and tariffs $\{\tau_t\}$ that are piecewise continuous with the following property: for any finite time interval, there are at most a finite number of discontinuities. That is to say, the control variables should be piecewise continuous and the state variable should be continuous with piecewise continuous derivative with possible discontinuities which occur as the control variables and prices/tariffs are discontinuous. Moreover, at these discontinuous points, left and right limits should exist and be finite (the first kind of discontinuity).

A competitive equilibrium (**CE**) is defined as follows: A set of prices $\{w_t, r_t\}$, resource allocations $\{c_t, k_t, n_t\}$ and a fiscal policy $\{G, \tau_t\}$ can be a CE if $\{c_t, k_t, n_t\}$ is a solution of the household maximization problem, $\{k_t, n_t\}$ is solution of the firm profit–maximization problem and $\{G, \tau_t\}$ satisfies the government budget constraint.

The current value Hamiltonian for our problem is,

$$V(k_t, c_t, n_t, \Lambda_t, t) = (\log c_t - bn_t) + \Lambda_t[(r_t - \delta)k_t + w_t n_t - c_t], \tag{6}$$

where Λ_t is the costate variable. Using the same definitions of admissible trajectories and weak maximality as in Stockman (2009), we have sufficient conditions for the weakly optimal solution of our problem.[4]

Proposition 1. *Assume that prices $\{w_t, r_t\}$, tariffs $\{\tau_t\}$ and initial capital stock k_0 are given. The current-value Hamiltonian $V(k_t, c_t, n_t, \Lambda_t, t)$ is concave in $\{c_t, k_t, n_t\}$ for any given Λ_t and t. Suppose there exists a continuous and piecewise continuously differentiable function $\Lambda_t^* : R_+ \longrightarrow R$ and an admissible interior plan $\{c_t^*, k_t^*, n_t^*\}$ that satisfies the following conditions:*

$$\frac{1}{c_t^*} = \Lambda_t^*, \tag{7}$$

$$b = \Lambda_t^* w_t^*, \tag{8}$$

$$\dot{\Lambda_t^*} = (\rho + \delta - r_t)\Lambda_t^*, \text{ for almost } t \in R_+ \tag{9}$$

$$\dot{k_t^*} = (r_t - \delta)k_t^* + w_t n_t^* - c_t^*, \text{ for almost } t \in R_+ \tag{10}$$

$$\lim_{t \bar{\longrightarrow} \infty} e^{-\rho t}\Lambda_t^*(k_t - k_t^*) \geq 0, \text{ for all admissible paths.} \tag{11}$$

Then $\{c_t^*, k_t^*, n_t^*\}$ is weakly optimal.

Proof. The proof is similar to that of Proposition 1 in Stockman (2009). ■

3. Euler Equation Branching and Global Indeterminacy

We use the sufficient conditions given in the section above and government budget constraint to show the existence of global indeterminacy. As in Zhang (2009), equilibrium conditions can be expressed as follows:

[4] A trajectory $P := (c, n, k)$ is admissible if (a) $c(t)$, $n(t)$, $k(t) \geq 0$ and $k(0) = k_0 > 0$ is given; (b) c and n are piecewise continuous with at most a countable number of discontinuities and they satisfy the property that at most a finite number of discontinuities occur during any finite time interval $[a, b]$; and (c) k is continuous and piecewise continuously differentiable and $\dot{k_t} = (r_t - \delta)k_t + w_t n_t - c_t$ holds for almost t. Two admissible paths P^* and P are comparable if we define the following function: $D(P^*, P, Time) = \int_0^{Time} e^{-\rho t}(\log c_t^* - bn_t^*)dt - \int_0^{Time} e^{-\rho t}(\log c_t - bn_t)dt$. The path P^* is weakly optimal if for every admissible path P, $\underset{Time \longrightarrow \infty}{lim} D(P^*, P, Time) \geq 0$.

$$\dot{\Lambda}_t = \Lambda_t[\rho + \delta - a_k A_t k_t^{\frac{a_k}{1-a_o}-1} n_t^{\frac{a_n}{1-a_o}}], \tag{12}$$

$$\dot{k}_t = (1-a_o)\, A_t k_t^{\frac{a_k}{1-a_o}} n_t^{\frac{a_n}{1-a_o}} - \delta k_t - 1/\Lambda_t, \tag{13}$$

$$b/\Lambda_t = a_n A_t k_t^{\frac{a_k}{1-a_o}} n_t^{\frac{a_n}{1-a_o}-1}, \tag{14}$$

$$\frac{\tau_t a_o y_t}{1+\tau_t} = G. \tag{15}$$

In addition, any equilibrium path $\{\Lambda_t, k_t, n_t\}$ should also satisfy the conditions below

(i) k_t and Λ_t are continuous and piecewise continuously differentiable;

(ii) n_t is piecewise continuous with those restrictions that we stated in the section above; and

(iii) Λ_t, k_t, n_t are bounded from above and not zero for any t.

Any path $\{\Lambda_t, k_t, n_t\}$ that satisfies those conditions above can be a CE. Equation (15) will show that multiple equilibria in the oil market are the key of the Euler equation branching. To see this, first, from equation (14), we express n_t as a function of k_t, Λ_t and τ_t: $n_t = [a_n A_t k_t^{\frac{a_k}{1-a_o}} \Lambda_t/b]^{\frac{1}{1-\frac{a_n}{1-a_o}}}$. Second, using $y_t = A_t k_t^{\frac{a_k}{1-a_0}} n_t^{\frac{a_n}{1-a_0}}$, $n_t = [a_n A_t k_t^{\frac{a_k}{1-a_o}} \Lambda_t/b]^{\frac{1}{1-\frac{a_n}{1-a_o}}}$ and $A_t = (\frac{a_0}{p^0(1+\tau_t)})^{\frac{a_0}{1-a_0}}$, equation (15) can be rewritten as follows:

$$G = \tau_t \left(\frac{a_o}{1+\tau_t}\right)^{1+\frac{a_o}{a_k}} \left(p^0\right)^{-\frac{a_o}{a_k}} k_t (a_n \Lambda_t/b)^{\frac{a_n}{a_k}} \equiv M(\tau_t, k_t, \Lambda_t) \tag{16}$$

From the right-hand side of (16), one sees that the equilibrium oil demand curve is not monotonic because $\tau_t \left(\frac{a_o}{1+\tau_t}\right)^{1+\frac{a_o}{a_k}}$ is single peaked. Therefore, these two curves (the demand and supply curves) intersect twice. As in Stockman (2009), we find that (1) the equilibrium oil demand curve is initially beneath and ultimately below the oil supply curve, and (2) this branching is global and it exists in an arbitrarily small open neighborhood of a steady state (k^*, Λ^*).

Proposition 2. *A steady state exists for small G, and in a small open neighborhood of the steady state, Euler equation branching occurs. Moreover, we have the following results:*

(1) In the steady state, $\rho + \delta = a_k A^* k^{*\frac{a_k}{1-a_o}-1} n^{*\frac{a_n}{1-a_o}}$, $(1-a_o) A^* k^{*\frac{a_k}{1-a_o}} n^{*\frac{a_n}{1-a_o}} = \delta k^* + 1/\Lambda^*$, $n^* = [a_n A^* k^{*\frac{a_k}{1-a_o}} \Lambda^*/b]^{\frac{1}{1-\frac{a_n}{1-a_o}}}$ *and* $G = \tau^* \left(\frac{a_o}{1+\tau^*}\right)^{1+\frac{a_o}{a_k}} (p^0)^{-\frac{a_o}{a_k}} k^* (a_n \Lambda^*/b)^{\frac{a_n}{a_k}}$ *hold, where* $A^* = (\frac{a_0}{p^0(1+\tau^*)})^{\frac{a_0}{1-a_0}}$. *Suppose that* $(1-a_o) A^* k^{*\frac{a_k}{1-a_o}} n^{*\frac{a_n}{1-a_o}} > \delta k^*$. *Then there exists a steady state* (k^*, Λ^*) *which is a solution to the first three equations (given* τ^**), and* τ^* *is a solution to the last equation (given that* k^* *and* Λ^* *are functions of* τ^**).*

(2) In a small open neighborhood B *of* (Λ^*, k^*)*, there can be two solutions to equation (16), which are denoted by* $\tau_t = g^1(\Lambda_t, k_t)$ *and* $\tau_t = g^2(\Lambda_t, k_t)$. *Moreover,* $\tau^* = g^1(\Lambda^*, k^*)$ *and* $\tilde{\tau} = g^2(\Lambda^*, k^*) \neq \tau^*$. *Therefore, equations (12), (13), (14) and (16) define a multi-valued dynamical system , which form can be written as* $(\dot{\Lambda}_t, \dot{k}_t) \in \{\Phi(\Lambda_t, k_t), \Psi(\Lambda_t, k_t)\}$ *with* $0 = \Phi(\Lambda^*, k^*) \neq \Psi(\Lambda^*, k^*)$ *and* $\Phi(\Lambda_t, k_t) \neq \Psi(\Lambda_t, k_t)$ *for* $(\Lambda_t, k_t) \in B$. $\Phi(\Lambda_t, k_t)$ *and* $\Psi(\Lambda_t, k_t)$ *can be obtained from (12) and (13) by replacing* τ_t *with* $g^1(\Lambda_t, k_t)$ *and* $g^2(\Lambda_t, k_t)$. *In this case, Euler equation branching occurs on the set* B.

Proof. The proof is left as an exercise for the reader. Hint: The proof is similar to that of Prop. 3 in Stockman (2009). ∎

The key theorem in this paper is Theorem 1 in Section 4 of Stockman (2009).

Theorem 1. *Let* $X \subseteq R^2$ *be an open set containing* x^* *and consider the multi-valued dynamical system (MVDS) defined by* $\dot{x} \in \{\Phi(x), \Psi(x)\}$ *for all* $x \in X$ *where* $\Phi, \Psi : X \to R^2$ *are* C^r *functions as in Definition 5 of Stockman (2009). Suppose* x^* *is a steady state of the single-valued differential equation* $\dot{x} = \Phi(x)$*, i.e.,* $\Phi(x^*) = 0$*, and assume that* $\Psi(x^*) = \kappa \neq 0$ *is not collinear with any of the eigenvectors of the Jacobian matrix* $E = D\Phi(x^*)$ *evaluated at the steady state* x^*. *Then the MVDS is Devaney chaotic on an invariant compact set with a non-empty interior in each of the following three cases:*

1. (Saddle) The steady state x^* is a saddle under Φ, i.e., $E = D\Phi(x^*)$ has real eigenvalues λ_1, λ_2 with $\lambda_1 < 0 < \lambda_2$.

2. (Sink or source with distinct real roots) The steady state x^* is a sink or source under Φ with distinct real roots, i.e., $E = D\Phi(x^*)$ has distinct real eigenvalues with $0 < \lambda_1 < \lambda_2$ or $\lambda_2 < \lambda_1 < 0$.

3. (Sink or source with complex roots) The steady state x^* is a sink or source under Φ with complex roots, i.e., $E = D\Phi(x^*)$ has complex eigenvalues $u \pm vi$ with $u \neq 0$.

In a short sentence, this theorem says that a steady state associated with Euler equation branching implies chaos. To see this in numerical cases, we consider two examples for the low tariff steady state and find that no matter whether it is locally indeterminate or not, there always exist numerous Devaney chaotic invariant sets with nonempty interiors. Let us continue to consider these two equilibria in the oil market. Notice that rearranging terms in equation (16) gives:

$$G(a_n \Lambda_t / b)^{-\frac{a_n}{a_k}} = \tau_t \left(\frac{a_o}{1+\tau_t} \right)^{1+\frac{a_o}{a_k}} \left(p^0\right)^{-\frac{a_o}{a_k}} k_t = \Omega(\tau_t). \tag{17}$$

One can see that $\Omega(\tau_t)$ is single caved with $\Omega'(\tau_t) > 0$ for $\tau_t < \frac{a_k}{a_o}$ and $\Omega'(\tau_t) < 0$ for $\tau_t > \frac{a_k}{a_o}$. Therefore, we have the following results:

(1) $\Omega\left(\frac{a_k}{a_o}\right) = G(a_n \Lambda_t / b)^{-\frac{a_n}{a_k}}$. A unique equilibrium exists in the oil market with $\tau_t = \frac{a_k}{a_o}$.

(2) $\Omega\left(\frac{a_k}{a_o}\right) > G(a_n \Lambda_t / b)^{-\frac{a_n}{a_k}}$. Two equilibria exist in the oil market, which we call τ_{1t} and τ_{2t} with

$$0 < \tau_{1t} < \frac{a_k}{a_o} < \tau_{2t} < \infty.$$

Example 1. *(Local determinacy). We set those parameter values at the following baseline values: $\rho = 0.04$, $a_o = 0.21$, $a_n = 0.64$, $p^0 = 0.01$, $b = 0.5$, $\delta = 0.1$ and $G = 0.25$. We calculate the two steady states and eigenvalues around them and we have:*

1. Low tariff steady state values: $\tau^* = 0.3392$, $k^* = 5.0362$, $\Lambda^* = 0.31155$, $n^* = 1.8745$,

$c^* = 3.2097$, $y^* = 3.7719$; *eigenvalues* $\mu_1 = -0.7237$, $\mu_2 = 0.8903$.

2. High tariff steady state values: $\tau = 83.8794$, $k = 1.2907$, $\Lambda = 1.2156$, $n = 1.8745$, $c = 0.8226$, $y = 0.9667$; *eigenvalues* $\mu_1 = 0.2482$, $\mu_2 = -0.3494$.

It is obvious that these two steady states are locally determinate. Then we draw the trajectories from both branches near the low tariff steady state and we find that numerous Devaney chaotic invariants sets with non-empty interiors appear.

The caption of Figure 1: The low-tariff steady state is locally a saddle. The plotted trajectories from the high-tariff branch are flowing from the bottom-right to the top-left. But the plotted trajectories from the low-tariff branch are flowing down and to the right.

Example 2. *(Local indeterminacy). We set those parameter values at the following baseline values:* $\rho = 0.04$, $a_o = 0.21$, $a_n = 0.64$, $p^0 = 0.01$, $b = 0.5$, $\delta = 0.1$ *and* $G = 0.4$. *We calculate the two steady states and eigenvalues around them and we have:*

1. Low tariff steady state values: $\tau^* = 0.8092$, $k^* = 4.5628$, $\Lambda^* = 0.3439$, $n^* = 1.8745$, $c^* = 2.9080$, $y^* = 4.2586$; *eigenvalues* $\mu_1 = -0.5767 + 1.3309i$, $\mu_2 = -0.5767 - 1.3309i$.

2. High tariff steady state values: $\tau = 16.5738$, $k = 2.1640$, $\Lambda = 0.7251$, $n = 1.8745$, $c = 1.3792$, $y = 2.0197$; *eigenvalues* $\mu_1 = 0.2278$, $\mu_2 = -0.3341$.

It is obvious that the low-tariff steady state is locally indeterminate and the high-tariff one is locally determinate. Then we draw the trajectories from both branches near the low tariff steady state and we find that numerous Devaney chaotic invariants sets with non-empty interiors appear.

The caption of Figure 2: The low-tariff steady state is locally a sink. The plotted trajectories from the high-tariff branch are flowing from the top-left to the bottom-right. The plotted trajectories for the low-tariff branch are flowing counter clockwise.

4. Concluding Remark

We show that under a balanced–budget rule, endogenous tariffs and endogenous labor income taxes are equivalent in generating global indeterminacy in the form of Euler equation branching. The methodology in our paper comes from Stockman (2009). Similar to Stockman (2009), the existence of Euler equation branching depends crucially on an endogenous tariff rate. These findings show that those multiple equilibria due to a balanced-budget rule studied by Zhang can always exist and extend beyond local indeterminacy.

References

[1] Aguiar-Conraria L., Wen Y. Foreign trade and equilibrium indeterminacy. Working paper 2005-041a, Federal Reserve bank of St. Louis 2005.

[2] Aguiar-Conraria L., Wen Y. Understanding the large negative impact of oil shocks. Journal of Money, Credit, and Banking 2007; 39; 925–944.

[3] Aguiar-Conraria L., Wen Y. A note on oil dependence and economic instability. Macroeconomic Dynamics 2008; 12; 717–723.

[4] Christiano L.J., Harrison S.G. Chaos, sunspots, and automatic stabilizers in a business cycle model. Journal of Monetary Economics 1999; 44; 3–31.

[5] Gardini L., Hommes C., Tramontana F., de Vilder R. Forward and backward dynamics in implicitly defined overlapping generations models. Journal of Economic Behavior & Organization 2009; 71; 110–129.

[6] Michener R., Ravikumar B. Chaotic dynamics in a cash-in-advance economy. Journal of Economic Dynamics and Control 1998; 22; 1117–1137.

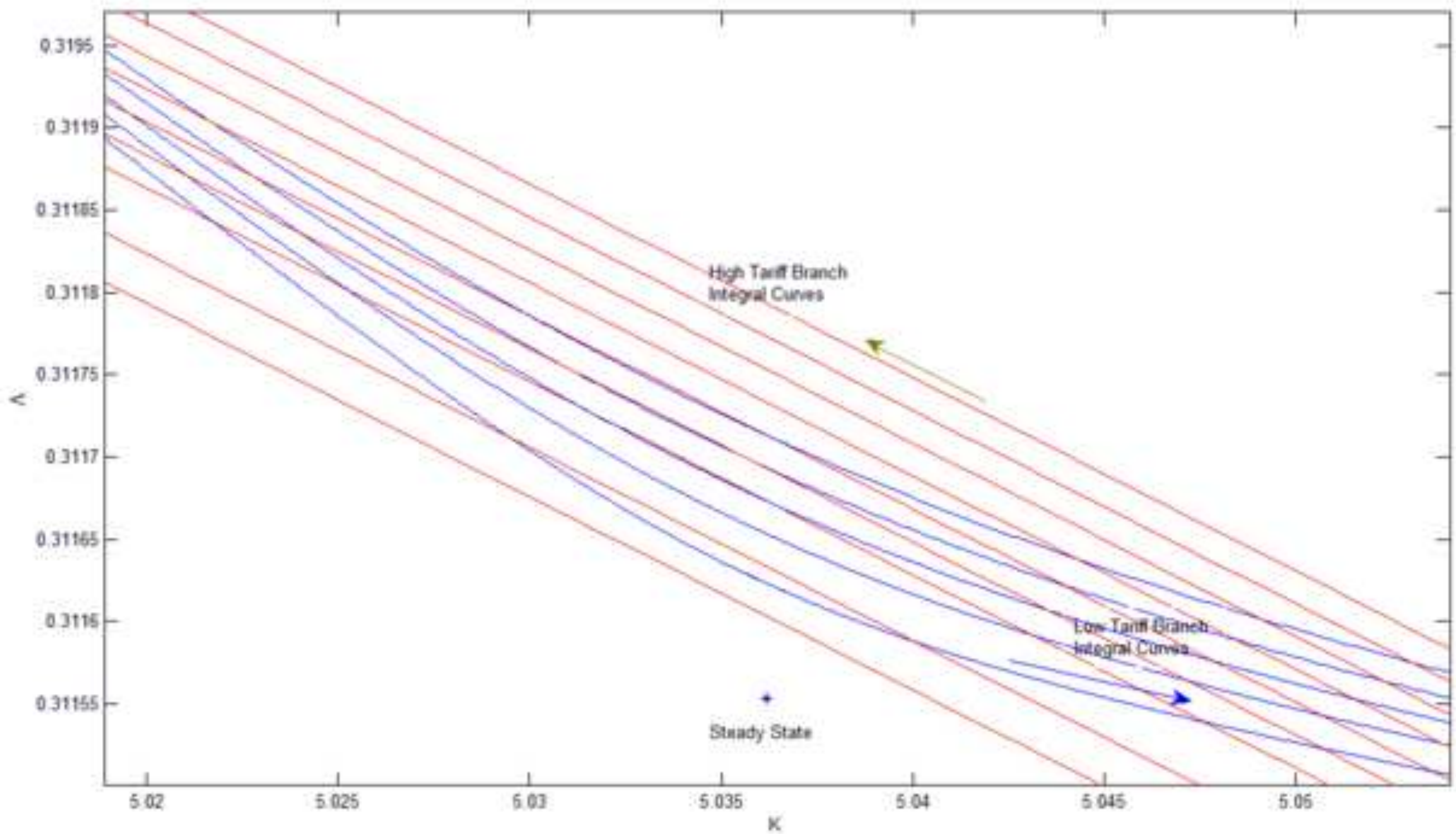
[7] Rotemberg J., Woodford M. 1994. Energy taxes and aggregate economic activity. In: Poterba J. (Ed), Tax Policy and the Economy, vol. 8. MIT press: Cambridge; 1994. p. 159–195.

[8] Schmitt-Grohe S., Uribe M. Balanced-budget rules, distortionary taxes and aggregate instability. Journal of Political Economy 1997; 105; 976–1000.

[9] Stockman D. Balanced-budget rules: chaos and deterministic sunspots. Journal of Economic Theory 2009 forthcoming.

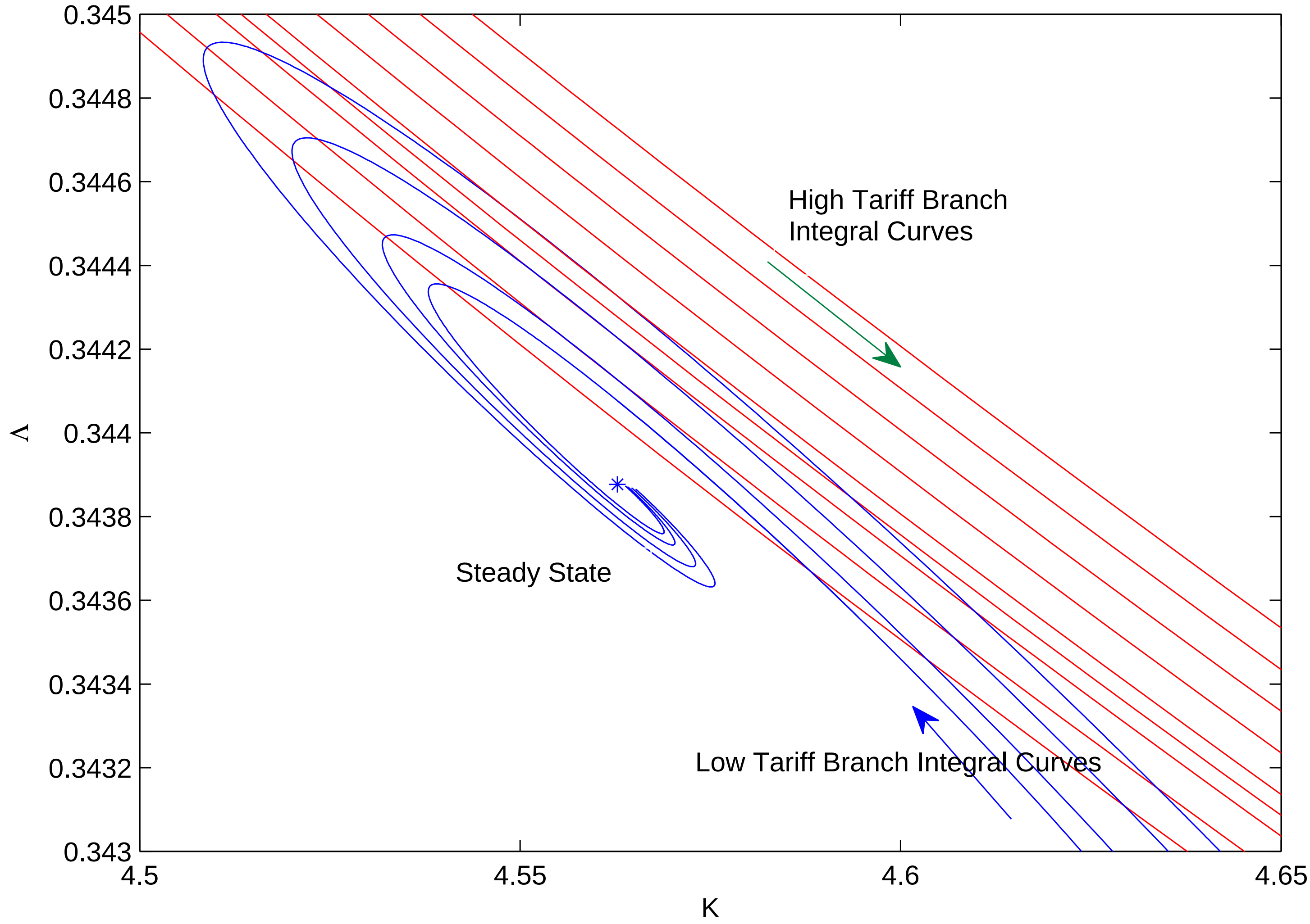
[10] Zhang Y. Tariff and Equilibrium Indeterminacy. Mimeo, New York University 2009. http://mpra.ub.uni-muenchen.de/13099/

Figure(s)



0.3195
0.3119
0.31185
0.3118
0.31175
0.3117
0.31165
0.3116
0.31155
λ
5.02
5.025
5.03
5.035
5.04
5.045
5.05
K
High Tariff Branch
Integral Curves
Low Tariff Branch
Integral Curves
Steady State

Figure(s)


0.345
0.3448
0.3446
0.3444
0.3442
0.344
0.3438
0.3436
0.3434
0.3432
0.343
Λ
4.5
4.55
4.6
4.65
K
High Tariff Branch
Integral Curves
Steady State
Low Tariff Branch Integral Curves