Why should central banks be independent?

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Abstract
Most explanations for the necessity of an independent central bank rely on the time-inconsistency model and therefore assume that governments are weak, foolish, or untruthful and tend to cheat people. The model in this paper indicates, however, that an independent central bank is not necessary because governments are weak or foolish. Central banks must be independent because governments are economic Leviathans. Only by severing the link between the political will of a Leviathan government and economic activities is inflation perfectly guaranteed not to accelerate. A truly independent central bank is necessary because it severs this link.

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I. INTRODUCTION

A great deal of emphasis has been placed on the necessity of an independent central bank. The reason given for this necessity is that there is a fundamental difference between governments and central banks – governments pursue both political and economic objectives while central banks generally pursue only economic objectives. For example, a government is responsible for the national defense while a central bank is not. If a central bank is not independent of a politically motivated government and government intervention is allowed, the central bank cannot optimize economic social welfare. This heterogeneity in objectives between the government and the central bank makes an independent central bank necessary. Hence, the essential reason for the necessity of independent central bank lies in the political objectives of government.

What kind of political objectives do governments pursue and how are these political objectives related to the development of inflation? Most studies on independent central banks have relied on the time-inconsistency model by Kydland and Prescott (1977) and Barro and Gordon (1983) to answer these questions (e.g., Berger, de Haan, and Eijffinger, 2000; Rogoff, 1985). Their explanation invokes two fundamental concepts: (1) if a government is pressured by interest groups to take an inflationary policy stance and intervene in a central bank’s decision-making, the central bank is unable to fully commit to its policies; and (2) if a central bank is unable to fully commit to its policies, it finds itself in a sub-optimal equilibrium.

These explanations do not seem to sufficiently explain a government’s action. What pressures would cause a government to take an inflationary policy stance? Even though the government’s political objectives are essential in explaining the necessity of independent central banks, no detailed mechanism of the inflationary political pressures is usually given. Also, is a government always so foolish that it obeys interest groups that represent only a part of its constituency? Why is a government so weak even though it wields great authority at will? Does
a government dare to take inflationary actions even if the majority of its constituency prefers low inflation and the government itself also desires low inflation? Is such behavior rational? In addition, why would people choose apparently untruthful and weak governments that are under control of small interest groups? Are people foolish? These questions appear quite reasonable and imply that the aforementioned explanations are not sufficient as the explanation for the necessity of an independent central bank. That is, these explanations require us to assume that governments are intrinsically so weak, foolish, or untruthful that they always tend to cheat people and that people are so foolish that they are always meekly cheated by governments. These assumptions are necessary because the time-inconsistency model needs them to generate high inflation. However, explanations that require such consistently weak, foolish, or untruthful governments and people clearly do not seem persuasive.

Many of these explanations also do not consider the interrelation between inflation and the constraints on government deficit financing. The importance of this relationship is stressed in the literature on the fiscal theory of the price level (FTPL), the basic idea of which goes back to Sargent and Wallace (1981). If a government’s deficit-financing behavior has an important impact on inflation, a government can affect the development of inflation not only through

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1 This view has also been criticized for other reasons, including that it needs a series of negative and persistent supply-side shocks that works to increase the natural rate of unemployment. Hence, as Ireland (1999) and Taylor (2002) argue, it is hard to explain the Great Inflation in Europe and the United States by the same mechanism because the movement of the natural rate of unemployment differed between them. In addition, it is difficult to explain the sharp decline of inflation in the 1980s by a demographic change that usually proceeds gradually. Another difficulty with this view is that it predicts that unemployment leads inflation but unemployment usually lags inflation. Thus, this view is clearly at odds with the data.

2 A related view on this issue is the union contract view. However, the basic reasoning in the union contract view is similar to the time inconsistency view in the sense that inflationary political pressures are the problem (see, e.g., Berger, de Haan, and Eijffinger, 2000).

intervention with the central bank but more directly through its own decisions on deficit financing. In this case, the government’s deficit-financing behavior and its interactions with the central bank must be studied to determine whether an independent central bank is necessary. Without modeling the government’s deficit-financing process, any results may not necessarily be regarded as sufficiently persuasive.

The purpose of this paper is to examine the aforementioned problems and present an alternative explanation for the necessity of an independent central bank. In this paper, I construct a model that analyzes the necessity of an independent central bank by explicitly incorporating (1) the political motive of a government that is not weak, foolish, or untruthful, and (2) the deficit-financing process of the government. This model indicates that, even though a government is not weak, foolish, or untruthful, the possibility of high inflation remains. This result implies that, without an independent central bank, there is no guarantee that inflation will not accelerate.

The paper is organized as follows. In section II, I examine the nature of government and construct a model that assumes an economically Leviathan government. The model shows that inflation is an inevitable consequence of heterogeneity in time preference rates between the Leviathan government and the representative household. The model in section II is combined with a conventional inflation model in section III. This combined model indicates that the rate of inflation is determined not only by the target rate of inflation but by the time preference of a Leviathan government. In section IV, I argue that it is only by severing the link between the political will of a Leviathan government and economic activities that inflation is guaranteed not to accelerate, and a truly independent central bank clearly severs the link. I offer some concluding remarks in section V.

II. A LEVIATHAN GOVERNMENT AND INFLATION
1. The government budget constraint

The budget constraint of a government in the model in this paper is

\[ \dot{B}_t = B_t R_t + G_t - X_t - S_t, \]

where \( B_t \) is the accumulated nominal government bonds, \( R_t \) is the nominal interest rate for government bonds, \( G_t \) is nominal government expenditure, \( X_t \) is nominal tax revenue, and \( S_t \) is the nominal amount of seigniorage at time \( t \). The tax is assumed to be lump sum. All variables are expressed in per capita terms. The government bonds are long-term, and the returns on the bonds, \( R_t \), are realized only after the bonds are held during a unit period, say a year. Government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time. \( R_t \) is composed of the real interest rate \( r_t \) and the expected change of the bonds’ price by inflation \( \pi_{by} \), such that \( R_t = r_t + \pi_{by} \). Let \( b_t = \frac{B_t}{p_t}, \ g_t = \frac{G_t}{p_t}, \ x_t = \frac{X_t}{p_t}, \) and \( s_t = \frac{S_t}{p_t} \), where \( p_t \) is the price level at time \( t \). Let also \( \pi_t = \frac{\dot{p}_t}{p_t} \) be the inflation rate at time \( t \). By dividing by \( p_t \), the budget constraint is transformed to

\[ \frac{\dot{B}_t}{p_t} = b_t R_t + g_t - x_t - s_t, \]

which is equivalent to

\[ \dot{b}_t = b_t R_t + g_t - x_t - s_t - b_t \pi_t = b_t (R_t - \pi_t) + g_t - x_t - s_t. \]

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if \( R_t \geq E_t \int_t^{t+1} (\pi_s + r_s) \) at time \( t \) where \( \overline{R}_t \) is the nominal interest rate for bonds bought at \( t \). Hence, by arbitrage, \( \overline{R}_t = E_t \int_t^{t+1} (\pi_s + r_s) \) and \( \overline{R}_t = E_t \int_t^{t+1} \pi_s ds + r_t \) if \( r_t \) is constant (i.e., if it is at steady state). This equation means that, during a sufficiently small period between \( t \) and \( t + dt \), the government’s obligation to pay for
the bonds’ return in the future increases not by $dt \pi$, but by $dt E_i \int_{s}^{s+1} \pi_x ds$. Because $B_{t+1} - B_{t} = \frac{\pi}{\int_{s}^{s+1} \pi_x ds + r} B_{t}$, where $B_{t+1}$ is the value of bonds at time $t+1$, which was issued at time $t$, then $B_{t} = \left( E_i \int_{s}^{s+1} \pi_x ds + r \right) B_{t}$. If $\pi_t$ is constant, then $\tilde{B}_{t} = \left( E_i \int_{s}^{s+1} \pi_x ds + r \right) \tilde{B}_{t}$, but if $\pi_t$ is not constant, they are not necessarily equivalent.

Because bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at $t$ are composed of bonds issued between $t-1$ and $t$. Hence, under perfect foresight, the average nominal interest rate for the total government bonds at time $t$ is the weighted sum of $\tilde{R}_t$ such that $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t$.

If the weights $\frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv}$ between $t-1$ and $t$ are not so different from each other, then approximately $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t$. The average nominal interest rate for the total government bonds, therefore, develops by $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t$. Here, if approximately

\[\tilde{R}_{t} = \left( \pi_t + r_t \right) \tilde{B}_{t}^{t+1} \]

has been used for many analyses because $\pi_t$ is usually assumed to be constant.

More precisely, if $\pi_t$ is constant, then $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t = \pi_t + r_t$ for any set of weights. If $\pi_t$ is increasing, then $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t > \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t$, in general because if new bonds are issued at $t$ only for refinancing the redeemed bonds, then $\tilde{B}_{t+1} = \left( 1 + \frac{\rho}{\tilde{R}} \right) \tilde{B}_{t-1,t-1}$. In addition, if $\pi_t$ is increasing, $\int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds > \pi_t$; thus, $R_t > \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t > \pi_t + r_t$. Nevertheless, if weights are nearly equal, then approximately $R_t = \int_{s}^{s+1} \tilde{R} \left( \frac{\tilde{B}_{r,t}}{\int_{s}^{s+1} \tilde{B}_{r,t} dv} \right) ds + r_t$. 

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\[\tilde{R}_{t} = \left( \pi_t + r_t \right) \tilde{B}_{t}^{t+1} \]
\[ \int_{t}^{t+1} \pi_s ds = \pi_{t+1} \] for some constant \( w \) (i.e., if \( \int_{t}^{t+1} \pi_s ds \) is represented by \( \pi_{t+1} \) for any \( t \)), then \( R_t = \int_{t-1}^{t} \pi_v dv ds + r_t = \int_{t-1}^{t+1} \pi_s ds + r_t \); thus, approximately \( \pi^{e}_{t} \) indicates a total price change by inflation during a unit period. If \( \pi_t \) is constant, then \( \pi^{e}_{b,t} = \int_{t-1}^{t+1} \pi_s dv ds = \pi_t \), but if \( \pi_t \) is not constant, \( \pi^{e}_{b,t} = \pi_t \) does not necessarily hold. The equation \( \pi^{e}_{b,t} = \pi_t \) is merely a special case of \( \pi^{e}_{b,t} \).

2. An economically Leviathan government

A Leviathan government is assumed in the model in this paper. As is known well, there are two extremely different views regarding government’s behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs, 1957; Brennan and Buchanan, 1980; Alesina and Cukierman, 1990). In a Leviathan government, politicians have their own preferences in responding to policy issues. In a benevolent government, politicians desire to behave in accordance with the will of voters, which also ensures that they will be reelected. In the Leviathan view, a government prioritizes pursuing its political objectives whereas, in the benevolent view, a government maximizes the same economic utility as the representative household. Because the political motivation of a government is essential to the explanation of the necessity of an independent central bank, it is appropriate to assume a Leviathan rather than a benevolent government for the analysis of the necessity of an independent central bank.

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6 The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

7 The time-inconsistency model implicitly assumes a benevolent government. Hence, unless a government is assumed to be weak, foolish, or untruthful, inflation cannot be accelerated by monetary policies. That is, if a benevolent government is not weak, foolish, or untruthful, the rate of inflation is perfectly kept at the target rate of inflation.
From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household but a Leviathan government does not. Unlike a benevolent government, a Leviathan government is therefore not managed by politically neutral bureaucrats who are obligated to mechanically maximize the expected economic utility of the representative household at any time and under any political party. It is instead managed by politicians who have strong political wills to achieve their own political objectives by all means.\(^8\) Hence, while the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government’s policy objectives. For instance, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly compared with the case in which a government sees defense as a low priority. If improvement of social welfare is the top priority, however, spending on social welfare will increase dramatically compared with the case in which a government sees social welfare as a low priority.

Is it possible, however, for a Leviathan government to hold office for a long period? It is possible if both economic and political points of view are considered. The majority of people will support a Leviathan government even though they know that the government does not necessarily pursue only the economic objectives of the representative household because people choose a government for both economic and political reasons. A government is generally chosen by the median of households under a proportional representation system, but the representative household usually presumed in the economics literature is basically the mean

\(^8\) The government behavior assumed in the FTPL reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.
household. Therefore, the economically representative household is not usually identical to the politically representative household. In other words, the Leviathan government argued here is an economically Leviathan government that maximizes the political utility of people whereas the conventional economically benevolent government maximizes the economic utility of people.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the political utility function of government (e.g., Edwards and Keen, 1996). A Leviathan government derives political utility from expenditure for its political purposes. Hence, the larger the expenditure is, the happier the Leviathan government will be. On the other hand, the Leviathan government knows that raising tax rates will provoke people’s antipathy and reduce the probability of being reelected, which makes the government uncomfortable because it expects that it cannot expend money to achieve its purposes if it loses power. The Leviathan government may regard taxes as necessary costs to obtain freedom of expenditure for its own purposes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the household. Consumption and labor hours are both control variables, and as such, the government’s expenditure and tax revenue are also control variables. As a whole, the political utility function of government can be expressed as $u_G(g_t, x_t)$. In addition, it can be assumed based on the previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^2} < 0$, and $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^2} < 0$. See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

It is possible to assume that governments are partially benevolent. In this case the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where $c_t$ is real consumption and $l_t$ is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government’s policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$. 

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9 See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

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\( \frac{\partial^2 u_G}{\partial x_i^2} > 0. \) A Leviathan government therefore maximizes the expected sum of these utilities discounted by its time preference rate. A Leviathan government pursues political objectives under the constraint of deficit financing. As a whole, an economically Leviathan government should maximize its expected political utility subject to the budget constraint.

3. The model

The utility function, \( u_G \), of an economically Leviathan government is a constant relative risk aversion utility function. The government’s rate of time preference is \( \theta_G \). The optimization problem of the government is

\[
\text{Max } E_0 \int_0^\infty u_G(g_t, x_t) \exp(-\theta_G t) dt
\]

subject to

\[
\dot{b}_t = b_t(R_t - \pi_t) + \dot{g}_t - x_t - s_t.
\]

The government maximizes its expected political utility considering the behavior of the representative household that is reflected in \( R_t \) in its budget constraint.\(^{12}\)

On the other hand, a representative household maximizes its expected economic utility

\[
\text{Max } E_0 \int_0^\infty u_P(c_t) \exp(-\theta_P t) dt,
\]

where \( u_P \) and \( \theta_P \) are the economic utility function and the rate of time preference of the household. \(^{12}\)

\(^{11}\) Some may argue that it is more likely that \( \frac{\partial^2 u_G}{\partial x_i^2} > 0 \) and \( \frac{\partial^3 u_G}{\partial x_i^3} < 0 \). However, the assumption used is not an important issue here because \( \frac{\partial^2 u_G(g_t, x_t)}{\partial x_i^2} \frac{\dot{x}_t}{x_t} = 0 \) at steady state, as will be shown in solving the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

\(^{12}\) The model can be used to analyze inflation (see Harashima 2004, 2005, 2006).
representative household, subject to the constraint

\[
\dot{k}_t = f(k_t) - c_t - g_t,
\]

where \( f(\cdot) \) is the production function, \( k_t \) is the real capital per capita, and \( c_t \) is the real consumption per capita.\(^{13}\) The constraint means that the output \( f(k_t) \) is demanded for private consumption \( c_t \), private investment \( \dot{k}_t \), and government expenditure \( g_t \). Government expenditure \( g_t \) is an exogenous variable for the representative household because the government is Leviathan. The representative household maximizes its expected economic utility considering the behavior of government reflected in \( g_t \) in its budget constraint. It is assumed that \( u'_p > 0 \) and \( u''_p < 0 \), and the population is constant.

Note that the time preference rate of government \( \theta_g \) is not necessarily identical to the time preference rate of the representative household \( \theta_p \). This heterogeneity plays an important role later in this study. The rates of time preference are different because of the following: (1) a government is chosen from among many political parties from economic and political points of view whereas the time preference rate of the representative household is related only to economic activities; (2) a government is usually chosen by the median of households under a proportional representation system and the converged policy reflects the median voter—not the mean voter—while an economically representative household is basically the mean household;\(^{14}\) (3) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman, 1990); and (4) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility that they cannot bind future voters, they may vote

\(^{13}\) The constraint is equivalent to \( \dot{k}_t = f(k_t) - c_t - \dot{b}_t - \chi_t - \xi_t + b_t \left( R_t - \pi_t \right) \).

\(^{14}\) See the literature on the median voter theorem (e.g., also Downs 1957), and also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).
more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina, 1990). Hence, it seems that the rates of time preference of government and the representative household should usually be heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, a Leviathan government behaves based only on its own time preference rate, without hesitation.

4. A Leviathan government and inflation

The simultaneous optimization of both government and representative household yields clear-cut results. To begin with, the maximization problem of a Leviathan government is solved. Let Hamiltonian $H_g$ be $H_g = u_g(g_t,x_t) \exp(-\theta_G t) + \lambda_{g_t} [b_t(R_t - \pi_t) + g_t - x_t - s_t]$, where $\lambda_{g_t}$ is a costate variable. The optimality conditions for the government’s problem described above are

(1) $\frac{\partial u_g(g_t,x_t)}{\partial g_t} \exp(-\theta_G t) = -\lambda_{g_t}$,

(2) $\frac{\partial u_g(g_t,x_t)}{\partial x_t} \exp(-\theta_G t) = \lambda_{g_t}$,

(3) $\dot{\lambda}_{g_t} = -\lambda_{g_t}(R_t - \pi_t)$,

(4) $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$, and

(5) $\lim_{t \to \infty} \lambda_{g_t} b_t = 0$.

Combining conditions (1), (2), and (3) yields the following equations:

$$g_t \frac{\partial^2 u_g(g_t,x_t)}{\partial g_t^2} \dot{g}_t + \theta_G = R_t - \pi_t = r_t + \pi_{bt} - \pi_t$$

and

$$x_t \frac{\partial^2 u_g(g_t,x_t)}{\partial x_t^2} \dot{x}_t + \theta_G = R_t - \pi_t = r_t + \pi_{bt} - \pi_t$$

Here,

$$g_t \frac{\partial^2 u_g(g_t,x_t)}{\partial g_t^2} \dot{g}_t = 0$$

and

$$x_t \frac{\partial^2 u_g(g_t,x_t)}{\partial x_t^2} \dot{x}_t = 0$$

at steady state such that $\dot{g}_t = 0$. 
and $\dot{x}_i = 0$; thus, $\theta_G = r_i + \pi^e_{b,t} - \pi_r$. Here, by the optimality conditions of the representative household’s maximization problem, $f'(k_i) = r_i - \theta_p$ at the steady state such that $\dot{c}_i = 0$, $\dot{k}_i = 0$, and $\dot{g}_i = 0$. Hence $\theta_G = \theta_p + \pi^e_{b,t} - \pi_r$ and thus
\[
(6) \quad \pi^e_{b,t} = \pi_r + \theta_G - \theta_p
\]
at steady state such that $\dot{g}_i = 0$, $\dot{x}_i = 0$, $\dot{c}_i = 0$, and $\dot{k}_i = 0$.\(^{15}\)

Equation (6) is a natural consequence of simultaneous optimization by a Leviathan government and the representative household. If the rates of time preference are heterogeneous between the government and the representative household, then $\pi^e_{b,t} \neq \pi_r$. Some may find this surprising because it has been naturally conjectured that $\pi^e_{b,t} = \pi_r$. However, this conjecture is a simple misunderstanding because, as was explained above, approximately $\pi^e_{b,t}$ indicates a total price change by inflation during a unit period such that $\pi^e_{b,t} = \int_{t-1}^{t} \int_{s}^{t+1} \pi_r dv \, ds$. On the other hand, $\pi_r$ indicates the instantaneous rate of inflation at a point such that
\[
\pi_r = \lim_{h \to 0} \frac{\dot{p}_t}{p_t} = \frac{\dot{P}_{t+1} - \dot{P}_t}{h}.
\]
Equation (6) therefore indicates that $\pi_r$ develops according to the integral equation
\[
\pi_r = \int_{t-1}^{t} \int_{s}^{t+1} \pi_r dv \, ds - \theta_G + \theta_p.
\]
The conjecture that $\pi^e_{b,t} = \pi_r$ is true when $\pi_r$ is constant. Because $\pi^e_{b,t} = \int_{t-1}^{t} \int_{s}^{t+1} \pi_r dv \, ds$, if $\pi_r$ is constant, then the equation $\pi^e_{b,t} = \pi_r$ holds. If $\pi_r$ is not constant, the equation $\pi^e_{b,t} = \pi_r$ does not necessarily hold. Equation (6) indicates that the equation $\pi^e_{b,t} = \pi_r$ holds only in the case where $\theta_G = \theta_p$ (i.e., a homogeneous rate of time preference). The equation $\pi^e_{b,t} = \pi_r$ has generally not been

\(^{15}\) If and only if $\theta_G = -\frac{g_i - s_i - x_i}{b_i} + \theta_p$, at steady state, then the transversality condition $\lim_{t \to -\infty} \lambda_{G,t} b_t = 0$ holds. The proof is shown in Appendix.
questioned probably because it has been thought that the homogeneous rate of time preference such that \( \theta_G = \theta_P \) naturally prevails. However, as argued above, a homogeneous rate of time preference is not usually guaranteed.

What does equation (6) (or the integral equation \( \pi_t = \int_t^{t+1} \int_s^{t+1} \pi_v dv ds - \theta_G + \theta_P \)) indicate? It indicates that inflation accelerates or decelerates when the rates of time preference are heterogeneous. If \( \pi_t \) is constant, the equation \( \pi_t = \pi_{h,1} = \int_t^{t+1} \int_s^{t+1} \pi_v dv ds \) holds; conversely, if \( \pi_t \neq \pi_{h,1} = \int_t^{t+1} \int_s^{t+1} \pi_v dv ds \), then \( \pi_t \) is not constant. Without the acceleration or deceleration of inflation, therefore, equation (6) cannot hold in an economy with \( \theta_G \neq \theta_P \). That is, inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference.

Here, if \( \int_t^{t+1} \pi_v dv - \pi_z = \theta_G - \theta_P \), then \( \pi_t = \pi_0 + 2(\theta_G - \theta_P)t \). Hence, \( \int_t^{t+1} \int_s^{t+1} \pi_v dv ds - \pi_t \)

\( = \theta_G - \theta_P \neq 0 \) suggests that inflation accelerates or decelerates nonlinearly such that \( \pi_t = \pi_0 + y(\theta_G - \theta_P)t^z \) where \( y \) is a constant and \( z_t \) is a variable. To be precise, for a sufficiently small period between \( t + 1 \) and \( t + 1 + dt \), \( \pi_{t+1+dt} \) is determined with \( \pi_s \) \((t-1<s\leq t+1)\) that satisfies \( \int_t^{t+1} \int_s^{t+1} \pi_v dv ds - \pi_s = \theta_G - \theta_P \), so as to hold the equation \( \int_t^{t+1} \int_s^{t+1} \pi_v dv ds = \int_{t-1}^{t+1+dt} \int_s^{t+1} \pi_v dv ds + \pi_{t+1+dt} - \pi_t \). Suppose that initially \( \theta_G = \theta_P \) but \( \theta_G \) changes at time 0 and \( \theta_G \) and \( \theta_P \) are not identical since then. Because \( \pi_t \) is constant before \( t = 0 \), then

\( \int_t^{t+1} \int_s^{t+1} \pi_v dv ds = \int_{t-1}^t \int_s^{t+1} (\pi_v - \pi_0) dv ds + \pi_0 \). Here, for \( \pi_t \) to be smooth at time \( t = 1 \), it is assumed that \( \pi_t = \pi_0 + yt \) for \( 0 \leq t < 1 \) (\( y \) is a constant). Thus \( \pi_t = \pi_0 + 6(\theta_G - \theta_P)t \) for \( 0 \leq t < 1 \). After \( t = 1 \), \( \pi_t \) gradually departs from the path of \( \pi_t = \pi_0 + 6(\theta_G - \theta_P)t \) upward if \( \theta_G > \theta_P \) and downward if \( \theta_G < \theta_P \) such that
\[ \pi_t = \pi_0 + 6(\theta_G - \theta_p)t z_t \]

where \( z_t > 1 \), so as to hold \( \int_{t-1}^{t} \int_{s}^{t+1} \pi_s dv ds - \pi_t = \theta_G - \theta_p \). However, around \( t = 1 \), approximately

\[ \pi_t = \pi_0 + 6(\theta_G - \theta_p)t \quad \text{and} \]

\[ \pi_t = 6(\theta_G - \theta_p). \]

Note that, inflation must be constant without \( \theta_G = \theta_p \). It is not until being \( \theta_G \neq \theta_p \) that inflation can accelerate or decelerate. That is, \( \theta_G \neq \theta_p \) bends the path of inflation and makes it nonlinear, which enable inflation to accelerate or decelerate. The many episodes of inflation acceleration and disinflation across time and countries suggest that \( \theta_G \neq \theta_p \) is not rare.

### III. The Relationship Between Government and the Central Bank

Equation (7) clearly indicates that there is a possibility of high inflation with Leviathan governments because inflation accelerates if \( \theta_G > \theta_p \). Hence, if a central bank is not independent of a Leviathan government, there is no guarantee that inflation will not accelerate.

Equation (7) therefore strongly implies an independent central bank is necessary to eliminate any possibility of high inflation. It is still not clear how a central bank behaves facing a Leviathan government, however; that is, how does a central bank manage the nominal interest rate considering equation (7)? I examine this question in this section.

1. **A conventional model of inflation**

First, I construct a conventional discrete-time inflation model with random shocks, in which only the central bank controls inflation. This type of inflation model is used for analyses of the short-term adjustment process of inflation deviations caused by random shocks and is based on the backward-looking Phillips curve type of model examined in Svensson (2003). It
consists of an aggregate supply function, an aggregate demand function, and a Taylor-type instrument rule for a central bank.

The aggregate supply function (Phillips curve) is

\[ \pi_{t+1} = \pi_t + \alpha_x x_t + \alpha_z z_{t+1} + \epsilon_{t+1}, \]

the aggregate demand function is

\[ x_{t+1} = \beta_x x_t + \beta_z z_{t+1} - \beta_r (r_t - \bar{r}) + \eta_{t+1}, \]

and the Taylor-type instrument rule for the central bank is

\[ i_t = \bar{r} + \gamma_r (\pi_t - \pi^*) + \gamma_x x_t, \]

where \( \pi_t \) is the rate of inflation; \( x_t \) is the output gap; \( z_t \) is a column vector of exogenous variables; \( r_t \) is the real interest rate; \( \bar{r} \) is the average real interest rate; \( i_t \) is the nominal interest rate; \( \pi^* \) is the target rate of inflation; \( \alpha_x, \beta_x, \beta_r, \gamma_r, \gamma_x \) are constant coefficients; \( \alpha_z \) and \( \beta_z \) are row vectors of constant coefficients; \( \epsilon_t \) and \( \eta_t \) are i.i.d. shocks with zero mean; and \( e_0 = 0 \) and \( \eta_0 = 0 \). Here, \( \bar{r} = \pi^* + \bar{r} \) as is usually assumed, and the real interest rate is defined as follows:

\[ r_t = i_t - \pi_{t+1}, \]

where \( \pi_{t+1} \) is the rate of inflation that is expected in period \( t \) for period \( t + 1 \), and it is assumed that \( r_{t+1} = \bar{r} \) for any \( s = 1, 2, 3, \ldots \).

2. An extended model that incorporates the basic nature of a Leviathan government

I now extend the model to one that incorporates the Leviathan government’s role in inflation to analyze the effects of government on the short-term inflation adjustment processes.\(^{16}\)

The basic nature of a Leviathan government as shown in the previous section is that \( \pi_t \) is a function of \( \theta_0 - \theta_p \) and approximately \( \pi_t = 6(\theta_0 - \theta_p) \) around \( t = 1 \); that is, \( \pi_{t+1} = \pi_t + 6(\theta_0 - \theta_p) \)

\(^{16}\) This extended model is based on Harashima (2005).
in a discrete time model. Here, the equation \( r_t = \theta_p + \mu_t = \bar{r} + \mu_t \) holds at equilibrium in markets with random shocks, where \( \mu_t \) is i.i.d. shocks with zero mean and \( \mu_0 = 0 \). Hence, the law of motion for inflation \( \pi_{t+1} = \pi_t + 6(\theta_G - \theta_p) \) can be rewritten as \( \pi_{t+1} = \pi_t + 6(\theta_G - \bar{r} - \mu_t) \).

Thus, in a discrete-time model with random shocks,

\[
(14) \quad \pi_{t+1} = \pi_0 + 6(\theta_G - \bar{r})(t + 1) - 6 \sum_{v=1}^{t+1} \mu_v + \xi_{t+1},
\]

where \( \pi_0 \) is \( \pi_t \) in period 0 and thus a steady state inflation rate before a shock on \( \pi^* \), and \( \xi_t \) is an i.i.d. shock with zero mean and \( \xi_0 = 0 \). The model here includes equation (14) in addition to equations (10), (11), (12), and (13).

The inclusion of equation (14) implies that either the target rate of inflation \( \pi^* \) or the preference of government \( \theta_G \) is a time-variable endogenous variable. Either the central bank or the government affects the development of inflation. In many existing inflation models, the role of the government is not explicitly separated from the role of the central bank, and the relationship between them is left ambiguous. In contrast, the extended model here separates them clearly, thus allowing a much more lucid examination of how their relationship in the decision-making process of monetary policy affects the development of inflation.

3. The inflation acceleration mechanism in the extended model

The extended model indicates the following important relation between the preferences of an economically Leviathan government and the central bank.

**Proposition:** Approximately \( \theta_G - \bar{r} = \frac{1 - \beta_s}{6 \left( \beta_s (1 - \beta_s) + 1 \right)} \left( \pi^* - \pi_0 \right) \).

**Proof:** By equations (12) and (13),

\[
(15) \quad \pi_t = \pi^* + \frac{1}{\gamma_s} \left( r_t + \pi_{t+1} \right) - \frac{\gamma_s}{\gamma}_x x_t - \frac{\bar{r}}{\gamma_x},
\]
and by equations (10) and (15), \( \pi_x = \pi_x^* + \frac{r_i + \pi_i + \alpha_x x_i + \alpha_z z_{t+1|t} - \bar{\pi} - \gamma_x x_i}{\gamma_x} \). Thereby,

\[
(16) \quad x_t = \frac{1}{\alpha_x - \gamma_x} \left[ (\gamma_x - 1)\pi_x - \gamma_x \pi_x^* - r_i + \bar{\pi} - \alpha_z z_{t+1|t} \right].
\]

By equations (11) and (16),

\[
(17) \quad (\gamma_x - 1)\pi_{t+1} - \gamma_x \pi_t^* - r_{t+1} + \bar{\pi} - \alpha_z z_{t+2|t+1} = \\
\beta_x \left[ (\gamma_x - 1)\pi_t - \gamma_x \pi_t^* - r_i + \bar{\pi} - \alpha_z z_{t+1|t} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right].
\]

By equation (14) and equation (17),

\[
(\gamma_x - 1) \left[ \pi_0 + 6(\theta_G - \bar{\pi})(t + 1) - \sum_{i=1}^{t+1} \mu_v + \bar{\xi}_i \right] - \gamma_x \pi_t^* - (\bar{\pi} + \mu_{t+1}) + \bar{\pi} - \alpha_z z_{t+2|t+1} = \\
\beta_x \left[ (\gamma_x - 1)\pi_t - \gamma_x \pi_t^* - r_i + \bar{\pi} - \alpha_z z_{t+1|t} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right].
\]

because \( \pi_t = \pi_0 + 6(\theta_G - \bar{\pi})t - \sum_{i=0}^{t} \mu_v + \bar{\xi}_i \) by equation (14). Thereby,

\[
6(\gamma_x - 1)(t + 1 - \beta_x) = 6(\gamma_x - 1)(t + 1 - \beta_x) + (1 - \beta_x) \left[ \pi_t - (\bar{\pi} + \mu_{t+1}) + \bar{\pi} - \alpha_z z_{t+2|t+1} \right. \\
\left. + \beta_x \left[ (\gamma_x - 1)\pi_t - \gamma_x \pi_t^* - r_i + \bar{\pi} - \alpha_z z_{t+1|t} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right] \right].
\]

Thus,

\[
\theta_G - \bar{\pi} = \frac{(1 - \beta_x) \left[ \pi_t - (\bar{\pi} + \mu_{t+1}) + \bar{\pi} - \alpha_z z_{t+2|t+1} \right. \\
\left. + \beta_x \left[ (\gamma_x - 1)\pi_t - \gamma_x \pi_t^* - r_i + \bar{\pi} - \alpha_z z_{t+1|t} \right] + (\alpha_x - \gamma_x) \left[ \beta_x z_{t+1|t} - \beta_x (r_i - \bar{\pi}) + \eta_{t+1} \right] \right]}{6(\gamma_x - 1)(t + 1 - \beta_x)}
\]

It is assumed for simplicity that the exogenous variables \( z_t \) play limited roles for inflation and
output gaps; thus, \( \alpha \) and \( \beta \) are near zero and approximately \( \alpha (z_{t+2,y+1} - \beta z_{t+1,y}) + (\gamma - \gamma) \beta z_{t+1,y} = 0 \).

Therefore,

\[
\theta_G - \bar{\gamma} = \frac{(1 - \beta_s) \left[ \bar{\gamma} + \gamma^* - \bar{\gamma} - (\gamma - 1)\pi_0 \right]}{6(\gamma - 1)(t + 1 - \beta_s \gamma)}
\]

\[
+ \left[ \gamma - 1 \left( \sum_{s+1} \mu_s - \xi_{t+1} \right) + \mu_{s+1} - \beta_s \left( \gamma - 1 \left( \sum_{t+1} \mu_t - \xi_t \right) + \mu_t \right) - (\alpha - \gamma) \beta \mu_t - \eta \right] \frac{6(\gamma - 1)(t + 1 - \beta_s \gamma)}{6(\gamma - 1)(t + 1 - \beta_s \gamma)}.
\]

Because it is assumed that \( \eta, \mu_t, \) and \( \xi_t \) are i.i.d. shocks with zero mean, then by taking expectations of both sides of equation (18),

\[
\theta_G - \bar{\gamma} = \frac{1 - \beta_s}{6(\gamma - 1)(t + 1 - \beta_s \gamma)} \left[ \bar{\gamma} + \gamma^* - \bar{\gamma} - (\gamma - 1)\pi_0 \right].
\]

Because \( \bar{\gamma} = \gamma^* + \bar{\gamma} \),

\[
\theta_G - \bar{\gamma} = \frac{1 - \beta_s}{6(\gamma - 1)(t + 1 - \beta_s \gamma)} \left( \gamma^* - \gamma \right).
\]

Q.E.D.

The important implication of proposition is that because \( \frac{1 - \beta_s}{6(\gamma - 1)(t + 1 - \beta_s \gamma)} > 0 \), then

\( \theta_G \leq \bar{\gamma} \) if \( \gamma^* \leq \gamma \) and \( \theta_G > \bar{\gamma} \) if \( \gamma^* > \gamma \).\(^\text{17}\) By equation (14), if \( \theta_G \leq \bar{\gamma} \), then inflation does not accelerate, but if \( \theta_G > \bar{\gamma} \), then inflation accelerates. Hence, equation (19) determines the rate of inflation in the model. Unlike the conventional model, the rate of inflation is determined not only by the target rate of inflation \( \pi^* \) but by the preference of government \( \theta_G \) (i.e., by interactions between the government and the central bank). If \( \gamma^* = \gamma \), then

---

\(^\text{17}\) Note that either the target rate of inflation \( \pi^* \) or the time preference rate of government \( \theta_G \) is a time-variable endogenous variable. For instance, if the target rate of inflation \( \pi^* \) is a time-variable endogenous variable, \( \pi^* \) in equation (18) means \( \pi^*_t \).

\(^\text{18}\) Note again that either \( \pi^* \) or \( \theta_G \) is a time-variable endogenous variable.
\( \theta_G = \bar{r} = \theta_p \) and inflation neither accelerates nor decelerates by equation (14).

If a central bank is not independent, the time preference rate of a Leviathan government is an exogenous variable whereas the target rate of inflation is an endogenous variable. That is, the central bank must set the target rate of inflation consistent with the time preference rate of the Leviathan government.

**Corollary:** If \( \theta_G > \bar{r} \) and \( \theta_G \) is not changed, the target rate of inflation \( \pi^* \) needs to be raised accordingly.

**Proof:** By equation (19), \( \theta_G - \bar{r} = \lim_{t \to \infty} \frac{1 - \beta_x}{6(t(1 - \beta_x) + 1)}(\pi^* - \pi_0) = \lim_{t \to \infty} \frac{\pi^*}{6t} \); thus, \( \lim_{t \to \infty} \frac{\pi^*}{6t} = \) a positive constant.

Q.E.D.

It has been reported that the target rate of inflation during the Great Inflation in the 1960s and the 1970s was high. Clarida, Gali, and Gertler (2000), Favero and Rovelli (2001), and Dennis (2001) conclude that the target rate of inflation in the pre-Volker era was much higher than that in the Volker-Greenspan era. Setting a high inflation target is not a simple policy mistake, but it indicates that a central bank is deliberately committing the “crime” of high inflation. However, corollary implies that the central banks at the time were forced to raise the target rate of inflation because they were not independent of the politically motivated Leviathan governments.

The key mechanism for accelerating inflation lies in how people perceive \( \theta_G \) by observing monetary policy. People cannot directly observe the preference of government, but they can observe how the central bank manipulates the nominal interest rate. If people observe that the central bank sets an inflation target such that \( \pi^* > \pi_0 \), they perceive that \( \theta_G > \bar{r} \) and inflation accelerates by equation (14) and vice versa.

**IV. DISCUSSION**
Proposition implies that a conflict of interest emerges if a central bank is not independent and a Leviathan government sets the target rate of inflation. Because proposition indicates that the target rate of inflation cannot be set independently of the time preference rate of government, the target rate of inflation must be consistent with the time preference rate of government. That is, it must satisfy equation (19), given the time preference rate of government. However, there is no guarantee that the target rate of inflation determined based on the time preference rate of government is identical to the target rate of inflation that the representative household wishes because, as was argued previously, there is no guarantee that the rates of time preference of government and the representative household are identical. Hence, the government must choose whether to set the target rate of inflation subject to its own time preference rate or to change its time preference rate to set the target rate of inflation equal to that of the representative household.

This problem does not trouble a Leviathan government, however, because a Leviathan government has the will to achieve its own political objectives by all means and therefore will decide without hesitation that its own time preference rate should dominate. As a result, there is no guarantee that the target rate of inflation is set as the representative household wishes and that inflation does not accelerate. It is important to note that, in this context, the problem is not that the government is weak, foolish, or untruthful and that people are foolish but that the government is economically Leviathan. If a government is not Leviathan and merely maximizes the representative household’s economic welfare function mechanically, inflation will never accelerate.

How should this problem that there is no guarantee that inflation does not accelerate be solved? Clearly, the link between the political will of a Leviathan government and economic activities needs to be severed in the process of making monetary policy decisions. One practical way of severing the link is to give the authority to set the target rate of inflation to an independent central bank that is not influenced by the political environment. A Leviathan
government may not like this solution because part of its power is transferred to the independent central bank. Once the authority has been transferred to the independent central bank, the time preference rate of the Leviathan government must be changed to one that is consistent with the policy objectives of the independent central bank. That is, the time preference rate of the Leviathan government becomes an endogenous variable that is determined by the independent central bank through equation (19), whereas the target rate of inflation set by the independent central bank is an exogenous variable. If the Leviathan government does not obey the independent central bank, equation (19) does not hold and inflation does not develop as the Leviathan government wants. This means that the knife-edge condition \( \theta_G = \frac{g_t - x_t - s_t}{b_t} \) in Lemma 1) that the Leviathan government wishes to hold based on the presumption that inflation develops according to equation (7) cannot be held. Without changing its time preference rate, therefore, fiscal deficits are expected to explode eventually (i.e., transversality condition (5) does not hold). Even a Leviathan government will not choose this devastating scenario. It thus has no other way than to obey the independent central bank and change its time preference rate to the one that is consistent with the target rate of inflation that the independent central bank sets.\(^{19}\)

V. CONCLUDING REMARKS

The model in this paper escapes the previously mentioned shortcomings of conventional

\(^{19}\) This solution may not be so bad from the Leviathan government’s point of view because only its rate of time preference is changed, and the government can still pursue its political objectives. One criticism of an independent central bank (e.g., Blinder, 1998) is that, since the time-inconsistency problem is more acute with fiscal policy, why aren’t fiscal policies delegated? A Leviathan government, however, will never allow fiscal policies to be delegated to an independent institution because the Leviathan government would then not be able to pursue its political objectives, which in a sense would mean the death of the Leviathan government.
explanations for the necessity of an independent central bank and makes an alternative explanation possible. The starting point of the alternative explanation is equation (6) or (7). These equations indicate that the acceleration and deceleration of inflation are inevitable consequences of reconciling the contradiction in heterogeneity in time preference rates between an economically Leviathan government and the representative household. The conjecture \( \pi^*_t = \pi_t \) is true if \( \pi_t \) is constant because \( \theta_G = \theta_P \), but it is not necessarily true if \( \pi_t \) is not constant because \( \theta_G \neq \theta_P \). Hence, even though a government is not weak, foolish, or untruthful, there is no guarantee that inflation will not accelerate. This result is obtained essentially because both people’s economic and political utilities are considered in the model. I do not argue that all governments are economically Leviathan. The many episodes of high inflation across time and countries suggest, however, that economically Leviathan governments are not rare.

Inflation accelerates because of the strong political will of an economically Leviathan government. Inflation is guaranteed not to accelerate only when the link between the political will of an economically Leviathan government and economic activities is effectively severed. A truly independent central bank severs the link and, for the first time, guarantees that inflation will not accelerate.
APPENDIX

By equation (6), \( \pi_{b,\ell} - \pi_{i} = R_i - r_i - \pi_i = \theta_G - \theta_p \) at steady state. Hence, \( R_i - \pi_i = \theta_G \) because \( \theta_p = r_i \). Substituting the equation \( R_i - \pi_i = \theta_G \) and equation (6) into conditions (3) and (4) and solving both differential equations yield the equation:

\[
\lambda_{G,b} = -\exp\left( (g_i - x_i - s_i) \int_0^b \frac{1}{b_i} dt + C^g \right) \quad \text{at steady state where } C^g \text{ is a certain constant. Thereby it is necessary to satisfy } g_i - x_i - s_i < 0 \text{ and } \lim_{t \to \infty} \int_0^b \frac{1}{b_i} dt = \infty \text{ for the transversality condition (5) to be held.}
\]

Here, by condition (4), \( \frac{\dot{b}_i}{b_i} = \theta_G + \frac{g_i - x_i - s_i}{b_i} \) at steady state. Hence, if \( \frac{\dot{b}_i}{b_i} = \theta_G + \frac{g_i - x_i - s_i}{b_i} = 0 \) at steady state, then \( b_i \) is constant; thus, \( \lim_{t \to \infty} \int_0^b \frac{1}{b_i} dt = \infty \). Thereby, the transversality condition holds. However, if \( \frac{\dot{b}_i}{b_i} = \theta_G + \frac{g_i - x_i - s_i}{b_i} < 0 \) at steady state, then \( b_i \) diminishes to zero and transversality condition (5) cannot hold because \( g_i - x_i - s_i < 0 \).

If \( \frac{\dot{b}_i}{b_i} = \theta_G + \frac{g_i - x_i - s_i}{b_i} > 0 \) at steady state, then \( \lim_{t \to \infty} \frac{\dot{b}_i}{b_i} = \theta_G \); thus, \( b_i \) increases as time passes and \( \lim_{t \to \infty} \int_0^b \frac{1}{b_i} dt = \frac{C^{\#}}{\theta_G} \), where \( C^{\#} \) is a certain constant. Thereby transversality condition (5) also cannot hold. ■
References


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