The Optimal Quantity of Money Consistent with Positive Nominal Interest Rates

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Abstract

The Friedman rule is strongly immune to most model modifications although it has not actually been observed. The Friedman rule implicitly assumes that a government is perfectly under the control of the representative household. This paper shows that, if a government is not perfectly under the control of the representative household, but also pursues political objectives, the optimal quantity of money generally is accompanied by positive nominal interest and inflation rates through the simultaneous optimization of government and the representative household. The fact that nominal interest and inflation rates are usually positive conversely implies that a government usually pursues political objectives.

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I. INTRODUCTION

The well-known Friedman rule requires that the nominal interest rate be zero and thereby the rate of inflation be negative (see Friedman,1969). Needless to say, nominal interest and inflation rates actually have been positive in most countries and in most time periods, particularly in the period of fiat money after WWII. Very high nominal interest and inflation rates have not been rare. Rather, even in the current low inflation environment, an inflation rate of about 2% has been widely regarded as "desirable." This "2% solution" indicates that, if the real interest rate is 4%, the "desirable" inflation rate is 6 percentage points higher than the "optimal" Friedman inflation rate. These facts therefore suggest that the quantity of money is usually not at the optimal level in the sense of the Friedman rule and make the Friedman rule greatly less compelling.

The Friedman rule, however, is strongly immune to most model modifications and is still thought to be basically correct. Phelps (1973), however, argued that, if distortionary taxes are levied, the Friedman rule is not always optimal. Since Phelps (1973), many economists have pursued this possibility, but the effect of distortionary taxes is presently regarded as quantitatively insufficient to insist that the Friedman rule is not correct; thus, the optimal inflation tax is probably not far from the Friedman rule (e.g., Lucas, 1994; Chari, Cristiano, and Kehoe, 1996; Mulligan and Sala-i-Martin, 1997). On the other hand, some economists have argued that mild inflation may have some benefits in a stochastic environment (e.g., because of short-run rigidity in nominal wages) (see e.g., Akerlof, Dickens, and Perry, 2000). If such benefits really exist, the Friedman rule may not always be desirable. Critics have countered, however, that this explanation is inconsistent with the rational behavior of households. Even though some kind of rigidity may exist, many economists doubt that its effect is so large as to offset the aforementioned discrepancy between the "desirable" and "optimal" inflation rates (e.g., Schmitt-Grohe and Uribe, 2005). As a result, many economists believe that the Friedman rule is approximately correct. Nevertheless, the question remains unanswered: why hasn't the theoretically optimal Friedman rule actually been observed across time periods and countries?[1]

On the other hand, the fiscal theory of the price level (FTPL) has questioned the quantity theory of money on which the Friedman rule is based.[2] If the quantity theory of money is not an appropriate theory to describe the actual world, it may be reasonable that the Friedman rule is not observed. However, many economists are skeptical about the FTPL. A reason for this skepticism may be that the concept of non-Ricardian policy is too general and thus non-Ricardian policies allows too many fiscal policy rules, many of which may be unrealistic and lead to unfavorable and unacceptable consequences. This generality may give the impression that the FTPL is an extreme theory, merely a meaningless and useless gimmick. As a result, the FTPL is not currently regarded as sufficiently satisfactory to be replaced with the quantity theory of money.

Nevertheless, the FTPL provides us a hint for the answer to the question of why the theoretically optimal Friedman rule has not actually been observed. The central concept of the FTPL is that a government does not necessarily care about economic utilities of households. This concept implies that, if a government is an institution that is independent of households and pursues political objectives, the Friedman rule may not be "optimal" because households will behave rationally and optimally under the constraint that the government does not necessarily implement the Friedman rule.[3] Contrary to the FTPL, only the optimization of the

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[1] Recently, Bhattacharya, Haslag, and Martin (2005) argued another possibility that heterogeneity of agents plays an important role in explaining why the Friedman rule does not maximize ex post steady-state welfare.

[2] Advocates of the FTPL are Leeper (1991), Sims (1994, 1998, 2001), Woodford (1995, 2001), and Cochrane (1998a, 1998b, 2005). Critics to the theory include Kocherlakota and Phelan (1999), McCallum (2001, 2003), Buiter (2002, 2004), and Niepelt (2004). See also Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), and Gordon and Leeper (2002).

[3] For simplicity, this paper assumes that a central bank is not independent of government and thus the central bank and government are regarded as a combined institution, not separate independent institutions.

representative households' economic objectives is considered by the Friedman rule, and the optimization of a government's political objectives is neglected because a government is assumed to be perfectly under the control of the representative household. The government is not considered to be an independent active agent but merely a "robot" owned by the representative household; thus, it has no independent will and does not optimize its own utilities but completely obeys the representative household to maximize the representative household's economic utilities. The Friedman rule therefore is always optimal for the representative household because the representative household who demands money also supplies money via the government. The representative household can print money freely as long as it wants (i.e., money is not a scarce resource for the representative household). In this environment, it is optimal for the representative household to supply money up to the saturation point. This nature of the Friedman rule implies that the Friedman rule is always optimal, unless we assume a government has an independent will and is not perfectly under the control of the representative household.

Phelps (1973) argues that the Friedman rule is not always optimal in the case of distortionary taxes. The introduction of distortionary taxes into models is an example of assuming a government that has an independent will and that is not perfectly under the control of the representative household. The government levies distortionary taxes even if the representative household dislikes this practice because of inefficiency caused by distortionary taxes. Distortionary taxes therefore imply that the government is not perfectly under the control of the representative household. Distortionary taxes may be levied for political reasons, for example, to redistribute wealth among people and to enhance political stability. Heterogeneity of households has the same effect as distortionary taxes (e.g., Bhattacharya, Haslag, and Martin, 2005). A government can be under the control of one type of household, but the behavior of the government is automatically exogenous for any other type of household. Hence, the government is not perfectly under the control of the representative household.

The above arguments on distortionary taxes and heterogeneous households suggest that, if a government acts independently from the representative household, there is the possibility that positive nominal interest and inflation rates are optimal.[4] However, these arguments only focus on one aspect of the government's independent will and, as was mentioned above, the effect of distortionary taxes is not thought to be sufficiently large to offset the Friedman rule. A government's independent will may not be limited to distortionary taxes or segmented actions to heterogeneous households. They may also include broader activities that originate in the deeper political motives of government. To answer the question of why the theoretically optimal Friedman rule has not actually been observed across different time periods and countries therefore requires more comprehensive and explicit modeling of the independent political will of government. My purpose here is to construct such a model of government and to present an alternative explanation for the question. The model constructed in this paper indicates that, with simultaneous optimization of the representative household and government, the optimal quantity of money is generally accompanied by positive nominal interest and inflation rates. This result shows a very different picture from the one the Friedman rule gives, but it seems quite natural because nominal interest and inflation rates are generally positive.

The paper is organized as follows. In section II, a model is constructed assuming a Leviathan government in which the representative household maximizes its economic utility and the government also simultaneously maximizes its political utility. In section III, the nature of simultaneous optimization of the representative household and the government is examined, and the law of motion for price as a result of the simultaneous optimization is shown. Section IV shows that, with the simultaneous optimization, the optimal quantity of money is generally accompanied by positive nominal interest and inflation rates. Concluding remarks are offered in

[4] Schmitt-Grohe and Uribe (2004) argue another possibility--unless the social planner has access to a direct 100% tax on monopoly profits, he will always find it optimal to deviate from the Friedman rule. The key here is the friction that the social planner cannot levy a direct 100% tax on monopoly profits even if households want it.

section V.

II. THE MODEL

1. The government budget constraint

The budget constraint of a government in the model in this paper is

$$\dot{B}_t = B_t R_t + G_t - X_t - S_t,$$

where B_t is the accumulated nominal government bonds, R_t is the nominal interest rate for government bonds, G_t is nominal government expenditure, X_t is nominal tax revenue, and S_t is the nominal amount of seigniorage at time t. The tax is assumed to be lump sum. All variables are expressed in per capita terms. The government bonds are long-term, and the returns on the bonds, R_t, are realized only after the bonds are held during a unit period, say a year. Government bonds are redeemed in a unit period, and the government successively refinances the bonds by issuing new ones at each time. R_t is composed of the real interest rate r_t and the expected change of the bonds' price by inflation $\pi_{b,t}^e$ such that $R_t = r_t + \pi_{b,t}^e$. Let $b_t = \frac{B_t}{p_t}$, $g_t = \frac{G_t}{p_t}$, $x_t = \frac{X_t}{p_t}$, and $s_t = \frac{S_t}{p_t}$, where p_t is the price level at time t. Let also $\pi_t = \frac{\dot{p}_t}{p_t}$ be the inflation rate at time t. By dividing by p_t, the budget constraint is transformed to

$$\frac{\dot{B}_t}{p_t} = b_t R_t + g_t - x_t - s_t,$$

which is equivalent to

$$\dot{b}_t = b_t R_t + g_t - x_t - s_t - b_t \pi_t = b_t (R_t - \pi_t) + g_t - x_t - s_t .$$

Because the returns on government bonds are realized only after holding the bonds during a unit period, investors buy the bonds if $\overline{R}_t \geq E_t \int_t^{t+1} (\pi_s + r_t) ds$ at time t where $\overline{R}_t$ is the nominal interest rate for bonds bought at t. Hence, by arbitrage, $\overline{R}_t = E_t \int_t^{t+1} (\pi_s + r_t) ds$ and $\overline{R}_t = E_t \int_t^{t+1} \pi_s \, ds + r_t$ if r_t is constant (i.e., if it is at steady state). This equation means that, during a sufficiently small period between t and $t + dt$, the government's obligation to pay for the bonds' return in the future increases not by $dt\,\pi_t$ but by $dt\, E_t \int_t^{t+1} \pi_s \, ds$. Because $\overline{B}_{t,t+1} - \overline{B}_{t,t} = \overline{R}_t \overline{B}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t}$ where $\overline{B}_{t,t+1}$ is the value of bonds at time $t + 1$, which was issued at time t, then $\dot{\overline{B}}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t}$. If π_t is constant, then $\dot{\overline{B}}_{t,t} = \left(E_t \int_t^{t+1} \pi_s \, ds + r_t \right) \overline{B}_{t,t} \Leftrightarrow \dot{\overline{B}}_{t,t} = (\pi_t + r_t) \overline{B}_{t,t}$, but if π_t is not constant, they are not necessarily equivalent.[5]

Because bonds are redeemed in a unit period and successively refinanced, the bonds the government is holding at t are composed of bonds issued between t - 1 and t. Hence, under perfect foresight, the average nominal interest rate for the total government bonds at time t is the weighted sum of $\overline{R}_t$ such that $R_t = \int_{t-1}^{t} \overline{R}_s \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r_t$.

If the weights $\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv}$ between t - 1 and t are not so different from each other, then approximately $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t$.[6] The average nominal interest rate for the total

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[5] $\dot{\overline{B}}_{t,t} = (\pi_t + r_t) \overline{B}_{t,t}$ has been used for many analyses because π_t is usually assumed to be constant.

[6] More precisely, if π_t is constant, then $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t = \pi_t + r_t$ for any set of weights. If π_t is increasing, then $R_t = \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \left(\frac{\overline{B}_{s,t}}{\int_{t-1}^{t} \overline{B}_{v,t} \, dv} \right) ds + r_t > \int_{t-1}^{t} \int_s^{s+1} \pi_v \, dv \; ds + r_t$ in general because if new bonds are issued at t only for refinancing the redeemed bonds, then $\overline{B}_{t,t} = (1 + \overline{R}_{t-1}) \overline{B}_{t-1,t-1}$. In addition, if π_t

government bonds, therefore, develops by $R_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r_t$. Here, if approximately $\int_{t}^{t+1} \pi_s \, ds = \pi_{t+w}$ for some constant $w \left(0 \le w \le 1\right)$ for any t (i.e., if $\int_{t}^{t+1} \pi_s \, ds$ is represented by π_{t+w} for any t), then $R_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds + r_t = \int_{t-1+w}^{t+w} \pi_s \, ds + r_t$; thus, approximately $\pi_{b,t}^{e}$ indicates a total price change by inflation during a unit period. If π_t is constant, then $\pi_{b,t}^{e} = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds = \pi_t$, but if π_t is not constant, $\pi_{b,t}^{e} = \pi_t$ does not necessarily hold. The equation $\pi_{b,t}^{e} = \pi_t$ is merely a special case of $\pi_{b,t}^{e}$.

2. An economically Leviathan government

A Leviathan government is assumed in the model in this paper.[7] As is known well, there are two extremely different views regarding government's behavior in the literature on political economy: the Leviathan view and the benevolent view (e.g., Downs, 1957; Brennan and Buchanan, 1980; Alesina and Cukierman, 1990). In a Leviathan government, politicians have their own preferences in responding to policy issues. In a benevolent government, politicians desire to behave in accordance with the will of voters, which also ensures that they will be reelected. In the Leviathan view, a government prioritizes pursuing its political objectives whereas, in the benevolent view, a government maximizes the same economic utility as the representative household.

From an economic point of view, a benevolent government maximizes the expected economic utility of the representative household but a Leviathan government does not. Unlike a benevolent government, a Leviathan government is therefore not managed by politically neutral

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is increasing, $\int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds > \pi_t$; thus, $R_t > \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r_t > \pi_t + r_t$. Nevertheless, if weights are nearly equal, then approximately $R_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \; ds + r_t$.

[7] The most prominent reference to Leviathan governments is Brennan and Buchanan (1980).

bureaucrats who are obligated to mechanically maximize the expected economic utility of the representative household at any time and under any political party. It is instead managed by politicians who have strong political wills to achieve their own political objectives by all means.[8] Hence, while the expenditure of a benevolent government is a tool used to maximize the economic utility of the representative household, the expenditure of a Leviathan government is a tool used to achieve the government's policy objectives. For instance, if a Leviathan government considers national security to be the most important political issue, defense spending will increase greatly compared with the case in which a government sees defense as a low priority. If improvement of social welfare is the top priority, however, spending on social welfare will increase dramatically compared with the case in which a government sees social welfare as a low priority.

Is it possible, however, for a Leviathan government to hold office for a long period? It is possible if both economic and political points of view are considered. The majority of people will support a Leviathan government even though they know that the government does not necessarily pursue only the economic objectives of the representative household because people choose a government for both economic and political reasons. A government is generally chosen by the median of households under a proportional representation system, but the representative household usually presumed in the economics literature is basically the mean household.[9] Therefore, the economically representative household is not usually identical to the politically representative household. In other words, the Leviathan government argued here is an

[8] The government behavior assumed in the FTPL reflects an aspect of a Leviathan government. Christiano and Fitzgerald (2000) argue that non-Ricardian policies correspond to the type of policies in which governments are viewed as selecting policies and committing themselves to those policies in advance of prices being determined in markets.

[9] See the literature on the median voter theorem (e.g., Downs 1957). Also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

economically Leviathan government that maximizes the political utility of people whereas the conventional economically benevolent government maximizes the economic utility of people.

The Leviathan view generally requires the explicit inclusion of government expenditure, tax revenue, or related activities in the political utility function of government (e.g., Edwards and Keen, 1996). A Leviathan government derives political utility from expenditure for its political purposes. Hence, the larger the expenditure is, the happier the Leviathan government will be. On the other hand, the Leviathan government knows that raising tax rates will provoke people's antipathy and reduce the probability of being reelected, which makes the government uncomfortable because it expects that it cannot expend money to achieve its purposes if it loses power. The Leviathan government may regard taxes as necessary costs to obtain freedom of expenditure for its own purposes. Expenditure and taxes in the political utility function of the government are analogous to consumption and labor hours in the economic utility function of the household. Consumption and labor hours are both control variables, and as such, the government's expenditure and tax revenue are also control variables. As a whole, the political utility function of government can be expressed as $u_G(g_t, x_t)$.[10] In addition, it can be assumed based on the previously mentioned arguments that $\frac{\partial u_G}{\partial g_t} > 0$ and $\frac{\partial^2 u_G}{\partial g_t^{\ 2}} < 0$, and $\frac{\partial u_G}{\partial x_t} < 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} > 0$.[11] A Leviathan government therefore maximizes the expected sum of these utilities

[10] It is possible to assume that governments are partially benevolent. In this case the utility function of a government can be assumed to be $u_G(g_t, x_t, c_t, l_t)$, where c_t is real consumption and l_t is the leisure hours of the representative household. However, if a lump-sum tax is imposed, the government's policies do not affect steady-state consumption and leisure hours. In this case, the utility function can be assumed to be $u_G(g_t, x_t)$.

[11] Some may argue that it is more likely that $\frac{\partial u_G}{\partial x_t} > 0$ and $\frac{\partial^2 u_G}{\partial x_t^{\ 2}} < 0$. However, the assumption used is not an

discounted by its time preference rate. A Leviathan government pursues political objectives under the constraint of deficit financing. As a whole, an economically Leviathan government should maximize its expected political utility subject to the budget constraint.

3. Optimization problems

3.1 The representative household

The well-known money in utility model of Sidrauski (1967) is used for the optimization problem of the representative household. The representative household maximizes expected economic utility

Max $E_0 \int_0^{\infty} u_P(c_t, m_t) \exp(-\theta_P t) dt$

subject to

$\dot{a}_t = (r_t a_t + w_t + z_t) - [c_t + (\pi_t + r_t) m_t] - g_t$,

where u_P and θ_P are the utility function and the rate of time preference of the representative household, m_t is real money, w_t is real wage, z_t is real government transfers, g_t is real government expenditure, z_t is lump-sum real government transfers, c_t is real consumption, π_t is the inflation rate, $a_t = k_t + m_t$, and k_t is real capital. All variables are expressed in per capita terms. It is also assumed that $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$, $u_P' > 0$, $u_P'' < 0$, $\frac{\partial u_P(c_t, m_t)}{\partial m_t} > 0$, and $\frac{\partial^2 u_P(c_t, m_t)}{\partial m_t^{\,2}} < 0$ where $f(\cdot)$ is the production function. Population is assumed to be constant. The budget constraint means that the output $f(k_t)$ in each period is demanded for private consumption c_t, private investment $\dot{k}_t$, and government expenditure g_t.

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important issue here because $\dfrac{x_t \dfrac{\partial^2 u_G(g_t, x_t)}{\partial x_t^{\,2}}}{\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}} \dfrac{\dot{x}_t}{x_t} = 0$ at steady state, as will be shown in solving the optimization problem later in the paper. Thus, the results are not affected by which assumption is used.

Government expenditure g_t is an exogenous variable for the representative household because it is a Leviathan government. For simplicity, the central bank is not assumed to be independent of the government; thus, the functions of the government and central bank are not separated. It is also assumed that lump-sum government transfers z_t is equal to the seigniorage s_t, and that, although all households receive transfers from a government in equilibrium, when making decisions, each household takes the amount it receives as given, independent of its money holdings.

3.2 An economically Leviathan government

An economically Leviathan government also simultaneously maximizes its expected political utility. The utility function, u_G, of an economically Leviathan government is a constant relative risk aversion utility function. The government's rate of time preference is θ_G. The optimization problem of the government is

$$Max\ E_0 \int_0^\infty u_G(g_t, x_t) \exp(-\theta_G t) dt$$

subject to

$$\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t .$$

The government maximizes its expected political utility considering the behavior of the representative household that is reflected in R_t in its budget constraint.[12]

Note that the time preference rate of government θ_G is not necessarily identical to the time preference rate of the representative household θ_P. This heterogeneity plays an important role later in this study. The rates of time preference are different because of the following: (1) a government is chosen from among many political parties from economic and political points of view whereas the time preference rate of the representative household is related only to

[12] The model can be used to analyze inflation (see Harashima 2004, 2005, 2006, 2007).

economic activities; (2) a government is usually chosen by the median of households under a proportional representation system and the converged policy reflects the median voter—not the mean voter—while an economically representative household is basically the mean household;[13] (3) even though people want to choose a government that has the same time preference rate as the representative household, the rates may differ owing to errors in expectations (e.g., Alesina and Cukierman, 1990); and (4) current voters cannot bind the choices of future voters and, if current voters are aware of this possibility that they cannot bind future voters, they may vote more myopically as compared with their own rates of impatience in private economic activities (e.g., Tabellini and Alesina, 1990). Hence, it seems that the rates of time preference of government and the representative household should usually be heterogeneous. It should be also noted, however, that even though the rates of time preference are heterogeneous, a Leviathan government behaves based only on its own time preference rate, without hesitation.

III. SIMULTANEOUS OPTIMIZATION

1. The simultaneous optimization of government and the representative household

First, the optimization problem of the representative household is examined. Let Hamiltonian H_P be $H_P = u_P(c_t, m_t)\exp(-\theta_P t) + \lambda_{P,t}[r_t a_t + w_t + z_t - c_t - (\pi_t + r_t)m_t - g_t]$, where $\lambda_{P,t}$ is a costate variable, c_t and m_t are control variables, and a_t is a state variable. The optimality conditions for the representative household are

(1) $$\frac{\partial u_P(c_t, m_t)}{\partial c_t}\exp(-\theta_P t) = \lambda_{P,t},$$

(2) $$\frac{\partial u_P(c_t, m_t)}{\partial m_t}\exp(-\theta_P t) = \lambda_{P,t}(\pi_t + r_t),$$

[13] See the literature on the median voter theorem (e.g., also Downs 1957), and also see the literature on the delay in reforms (e.g., Cukierman, Edwards, and Tabellini 1992; Alesina and Drazen 1991).

(3) $\dot{\lambda}_{P,t} = -\lambda_{P,t}\, r_t$,

(4) $\dot{a}_t = (ra_t + w_t + z_t) - [c_t + (\pi_t + r_t)\, m_t - g_t]$, and

(5) $\lim_{t\to\infty} \lambda_{P,t}\, a_t = 0$.

By conditions (1) and (2), $\dfrac{\dfrac{\partial u_P(c_t,m_t)}{\partial m_t}}{\dfrac{\partial u_P(c_t,m_t)}{\partial c_t}} = \pi_t + r_t$, and by conditions (1) and (3),

$-\dfrac{c_t \dfrac{\partial^2 u_P(c_t,m_t)}{\partial c_t^2}}{\dfrac{\partial u_P(c_t,m_t)}{\partial c_t}} \dfrac{\dot{c}_t}{c_t} + \theta_P = r_t$. Hence,

(6) $\theta_P = r_t$;

thus,

(7) $\dfrac{\dfrac{\partial u_P(c_t,m_t)}{\partial m_t}}{\dfrac{\partial u_P(c_t,m_t)}{\partial c_t}} = \pi_t + \theta_P$

at steady state such that $\dot{c}_t = 0$ and $\dot{k}_t = 0$.

Next, the optimization problem of a Leviathan government is examined. Let Hamiltonian H_G be $H_G = u_G(g_t, x_t)\exp(-\theta_G\, t) + \lambda_{G,t}[b_t(R_t - \pi_t) + g_t - x_t - s_t]$, where $\lambda_{G,t}$ is a costate variable. The optimality conditions for the government's problem described above are

(8) $\dfrac{\partial u_G(g_t, x_t)}{\partial g_t}\exp(-\theta_G\, t) = -\lambda_{G,t}$,

(9) $\dfrac{\partial u_G(g_t, x_t)}{\partial x_t}\exp(-\theta_G\, t) = \lambda_{G,t}$,

(10) $\dot{\lambda}_{G,t} = -\lambda_{G,t}(R_t - \pi_t)$,

(11) $\dot{b}_t = b_t(R_t - \pi_t) + g_t - x_t - s_t$, and

(12) $\lim_{t\to\infty} \lambda_{G,t}\, b_t = 0$.

Combining conditions (8), (9), and (10) yields the following equations:

$$\frac{g_t \frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial g_t}} \frac{\dot{g}_t}{g_t} + \theta_G = R_t - \pi_t = r_t + \pi_{b,t}^e - \pi_t \quad \text{and} \quad -\frac{x_t \frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} + \theta_G = R_t - \pi_t = r_t + \pi_{b,t}^e - \pi_t \cdot$$

Here, $\frac{g_t \frac{\partial^2 u_G(g_t,x_t)}{\partial g_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial g_t}} \frac{\dot{g}_t}{g_t} = 0$ and $\frac{x_t \frac{\partial^2 u_G(g_t,x_t)}{\partial x_t^2}}{\frac{\partial u_G(g_t,x_t)}{\partial x_t}} \frac{\dot{x}_t}{x_t} = 0$ at steady state such that $\dot{g}_t = 0$ and $\dot{x}_t = 0$; thus, $\theta_G = r_t + \pi_{b,t}^e - \pi_t$. Because of equation (6), $\theta_G = \theta_P + \pi_{b,t}^e - \pi_t$ and thus

(13) $\pi_{b,t}^e = \pi_t + \theta_G - \theta_P$

at steady state such that $\dot{g}_t = 0$, $\dot{x}_t = 0$, $\dot{c}_t = 0$, and $\dot{k}_t = 0$.[14]

Equation (13) is a natural consequence of simultaneous optimization by a Leviathan government and the representative household. If the rates of time preference are heterogeneous between the government and the representative household, then $\pi_{b,t}^e \neq \pi_t$. Some may find this surprising because it has been naturally conjectured that $\pi_{b,t}^e = \pi_t$. However, this conjecture is a simple misunderstanding because, as was explained above, approximately $\pi_{b,t}^e$ indicates a total price change by inflation during a unit period such that $\pi_{b,t}^e = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds$. On the other hand, π_t indicates the instantaneous rate of inflation at a point such that $\pi_t = \frac{\dot{p}_t}{p_t} = \frac{\lim_{h \to 0} \frac{p_{t+h} - p_t}{h}}{p_t}$. Equation (13) therefore indicates that π_t develops according to the integral equation $\pi_t = \int_{t-1}^{t} \int_{s}^{s+1} \pi_v \, dv \, ds - \theta_G + \theta_P$. The conjecture that $\pi_{b,t}^e = \pi_t$ is true when π_t

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[14] If and only if $\theta_G = -\frac{g_t - x_t - s_t}{b_t}$ at steady state, then the transversality condition (12) $\lim_{t \to \infty} \lambda_{G,t} b_t = 0$ holds. The proof is shown in Appendix 1.

is constant. Because $\pi_{b,t}^{e}=\int_{t-1}^{t}\int_{s}^{s+1}\pi_{v}\,dv\;ds$, if π_t is constant, then the equation $\pi_{b,t}^{e}=\pi_t$ holds. If π_t is not constant, the equation $\pi_{b,t}^{e}=\pi_t$ does not necessarily hold. Equation (13) indicates that the equation $\pi_{b,t}^{e}=\pi_t$ holds only in the case where $\theta_G=\theta_P$ (i.e., a homogeneous rate of time preference). The equation $\pi_{b,t}^{e}=\pi_t$ has generally not been questioned probably because it has been thought that the homogeneous rate of time preference such that $\theta_G=\theta_P$ naturally prevails. However, as argued above, a homogeneous rate of time preference is not usually guaranteed.

2. The law of motion for price

What does equation (13) (or the integral equation $\pi_t=\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\,ds-\theta_G+\theta_P$) indicate? It indicates that inflation accelerates or decelerates when the rates of time preference are heterogeneous. If π_t is constant, the equation $\pi_t=\pi_{b,t}^{e}=\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds$ holds; conversely, if $\pi_t\neq\pi_{b,t}^{e}=\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds$, then π_t is not constant. Without the acceleration or deceleration of inflation, therefore, equation (13) cannot hold in an economy with $\theta_G\neq\theta_P$. That is, inflation accelerates or decelerates as a result of the government and the representative household reconciling the contradiction in heterogeneous rates of time preference.

Here, if $\int_{t}^{t+1}\pi_v\,dv-\pi_t=\theta_G-\theta_P$, then $\pi_t=\pi_0+2(\theta_G-\theta_P)t$. Hence, $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds-\pi_t$ $=\theta_G-\theta_P\neq 0$ suggests that inflation accelerates or decelerates nonlinearly such that $\pi_t=\pi_0+y(\theta_G-\theta_P)t^{z_t}$ where y is a constant and z_t is a variable. To be precise, for a sufficiently small period between $t+1$ and $t+1+dt$, π_{t+1+dt} is determined with π_s $(t-1<s\le t+1)$ that satisfies $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds-\pi_t=\theta_G-\theta_P$, so as to hold the equation $\int_{t}^{t+dt}\int_{s}^{s+1}\pi_v\,dv\;ds$ =

$\int_{t-1}^{t-1+dt}\int_{s}^{s+1}\pi_v\,dv\,ds+\pi_{t+dt}-\pi_t$. Suppose that initially $\theta_G=\theta_P$ but θ_G changes at time 0 and θ_G and θ_P are not identical since then. Because π_t is constant before $t=0$, then $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds=\int_{-1}^{0}\int_{0}^{s+1}(\pi_v-\pi_0)dv\;ds+\pi_0$. Here, for π_t to be smooth at time t = 1, it is assumed that $\pi_t=\pi_0+yt$ for $0\le t<1$ (y is a constant). Thus $\pi_t=\pi_0+6(\theta_G-\theta_P)t$ for $0\le t<1$. After t = 1, π_t gradually departs from the path of $\pi_t=\pi_0+6(\theta_G-\theta_P)t$ upward if $\theta_G>\theta_P$ and downward if $\theta_G<\theta_P$ such that

(14) $\pi_t=\pi_0+6(\theta_G-\theta_P)t^{z_t}$

where $z_t > 1$, so as to hold $\int_{t-1}^{t}\int_{s}^{s+1}\pi_v\,dv\;ds-\pi_t=\theta_G-\theta_P$.

Note that, inflation must be constant without $\theta_G\neq\theta_P$. It is not until being $\theta_G\neq\theta_P$ that inflation can accelerate or decelerate. That is, $\theta_G\neq\theta_P$ bends the path of inflation and makes it nonlinear, which enable inflation to accelerate or decelerate. The many episodes of inflation acceleration and disinflation across time and countries suggest that $\theta_G\neq\theta_P$ is not rare.

IV. THE OPTIMAL QUANTITY OF MONEY

1. Money as a scarce resource

The Friedman rule requires that money should be supplied until the supply reaches the representative household's saturation point. The saturation point is a point such that $\frac{\partial u_P(c^*,m_t)}{\partial m_t}=0$, and $\pi_t+\theta_P=\pi_t+r_t=0$ by equation (7). It is possible to supply money to the saturation point if a government (including its central bank) is under perfect control of the representative household and the representative household demanding the money can supply money indefinitely (i.e., money is not a scarce resource for the representative household).

However, if a government is Leviathan economically and not under the perfect control of the representative household, the representative household can no longer supply money to the saturation point. Combining equations (7) and (14) yields the equation $\frac{\frac{\partial u_P(c_t,m_t)}{\partial m_t}}{\frac{\partial u_P(c_t,m_t)}{\partial c_t}} = \pi_0 + 6(\theta_G - \theta_P)t^{z_t} + \theta_P$ at steady state. Hence, the real quantity of money m_t satisfies the equation such that

$$(15)\quad \frac{\partial u_P(c^*,m_t)}{\partial m_t} = \left[\pi_0 + 6(\theta_G - \theta_P)\, t^{z_t} + \theta_P\right]\frac{\partial u_P(c^*,m_t)}{\partial c^*}$$

at steady state, where c^* is c_t at steady state.

Equation (15) has an important implication. It indicates that, in general, $\frac{\partial u_P(c^*,m_t)}{\partial m_t} \neq 0$ because $\frac{\partial u_P(c^*,m_t)}{\partial c^*} > 0$, and the equation $\pi_0 + 6(\theta_G - \theta_P)\, t^{z_t} + \theta_P = 0$ holds if $\theta_G = \theta_P$ and $\pi_0 + \theta_P = \pi_0 + r_t = 0$, which is the world the Friedman rule assumes. However, as was argued above, it is not necessarily guaranteed *a priori* that equations $\theta_G = \theta_P$ and $\pi_0 + r_t = 0$ hold. Therefore, contrary to the Friedman rule, which requires $\frac{\partial u_P(c^*,m_t)}{\partial m_t} = 0$, the quantity of money is not supplied up to the representative household's saturation point. Because the representative household cannot supply money at will via the government, money is a scarce resource for the representative household and the quantity of money is instead determined endogenously by equation (15). Contrary to the Friedman rule, under which the quantity of money determines the rate of inflation, the rate of inflation determines the quantity of money based on equation (15). The rate of inflation is determined by equation (14), independent of the quantity of money.

2. Positive nominal interest rates

Equation (15) therefore implies an important nature of this simultaneous optimization of utility for the government and the representative household. Because $\frac{\partial u_P\left(c^*, m_t\right)}{\partial m_t} \neq 0$ in general, then $\pi_t + r_t \neq 0$ in general by condition (2); thus, the rate of return on money $-\pi_t$ is not necessarily equal to that on capital r_t. Unlike the Friedman rule, which requires that $\pi_t + r_t = 0$, the simultaneous optimization in this model requires that $\pi_t + r_t \neq 0$ in general; thus, the Friedman rule should not be implemented. What should be emphasized is that this endogenously determined quantity of money implies positive nominal interest and inflation rates. Because $\frac{\partial u_P\left(c_t, m_t\right)}{\partial c_t} > 0$, $\lambda_{P,t} > 0$ by condition (1). By equation (15), $\frac{\partial u_P\left(c_t, m_t\right)}{\partial m_t} > 0$ at steady state if $\theta_G - \theta_P \geq 0$ and $\pi_0 > -\theta_P$. Here, $\frac{\partial u_P\left(c_t, m_t\right)}{\partial m_t} \exp\left(-\theta_P t\right) = \lambda_{P,t}\left(\pi_t + r_t\right)$ by condition (2). Hence, if $\theta_G - \theta_P \geq 0$ and $\pi_0 > -\theta_P$, then

(16) $\pi_t + r_t > 0$

at steady state. Inequality (16) indicates positive nominal interest rates and allows a wide range of positive inflation rates since it is not necessarily guaranteed to be $\pi_0 \leq -\theta_P$ *a priori*. [15] For example, assume that initially $\pi_0 = 0.02$, $\theta_P = r_t = 0.03$, and $\theta_G - \theta_P = 0$. If both households

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[15] In addition, equation (15) predicts the negative marginal quantity of money such that $\frac{\partial m_t}{\partial \pi_{b,t}^e} < 0$ at steady state (the proof is shown in Appendix 2). This feature $\frac{\partial m_t}{\partial \pi_{b,t}^e} < 0$ is consistent with the feature of the well-known money demand function of Cagan (1956); that is, the higher the expected inflation, the lower the demand for real money. However, it should be noted that because inflation rates follow equation (14), given the initial inflation rate, there is only one path for inflation and thus the mechanism of hyperinflation that Cagan (1956) shows does not exist in this model.

and government think that this 2% inflation rate is desirable because it is sufficiently low but works as a buffer in case of stochastic disturbances, then the positive inflation rate $\pi_t = 0.02$ and positive nominal interest rate $r_t + \pi_t = 0.05$ will continue to hold indefinitely because $\theta_G - \theta_P = 0$ will not be changed. Accordingly, the unique quantity of money determined by equation (15) for $\pi_0 = 0.02$, $\theta_P = r_t = 0.03$, and $\theta_G - \theta_P = 0$ is the quantity that satisfies $\frac{\partial u_P\left(c^*, m_t\right)}{\partial m_t} = 0.05 \frac{\partial u_P\left(c^*, m_t\right)}{\partial c^*}$. Hence, positive inflation and nominal interest rates and the quantity of money that satisfies $\frac{\partial u_P\left(c^*, m_t\right)}{\partial m_t} = 0.05 \frac{\partial u_P\left(c^*, m_t\right)}{\partial c^*}$ continue indefinitely.

What should be emphasized is that satisfying equation (15) is optimal even though equation (15) is different from the Friedman rule. The utilities of both government and the representative household are optimized simultaneously at the quantity of money that satisfies equation (15); thus, there is no room for further welfare improvement. Households demand money for transactions that increases their net economic utility. Equation (15) indicates that households demand money up to the point at which the marginal utility derived from using money for transactions is equal to the marginal utility of consumption multiplied by the nominal interest rate. If the government does not supply money to the point at which equation (15) and all the other conditions are satisfied, the optimality conditions for the representative household are violated because of the shortage of money. On the other hand, if the government supplies money beyond that point, the optimality conditions of government are violated because, for equation (15) to hold when there is an oversupply of money, the rate of inflation must be lower than the one the government needs for its optimization by equation (14) owing to the smaller $\frac{\partial u_P\left(c^*, m_t\right)}{\partial m_t}$. Thus, the government's debts are compelled to explode eventually; that is, transversality condition (12) cannot be satisfied. Otherwise, equation (15) is not satisfied and the optimality conditions of the representative household are violated. Implementing the

Friedman rule with over-supplied money therefore means the explosion of government's debts or obstruction of the optimization of the representative household. Hence, the quantity of money that satisfies equation (15) is optimal.

V. CONCLUDING REMARKS

The Friedman rule implicitly assumes that a government is perfectly under the control of the representative household; thus, in practice, the representative household that demands money will supply money indefinitely. Hence, the widely observed positive nominal interest and inflation rates conversely imply that a government is not perfectly under the control of the representative household but pursues its unique political objectives. In this paper, a model with a government that pursues its own political objectives is constructed. The main finding of the paper is that if a government pursues its political objectives, (i.e., if the government is economically Leviathan), the optimal quantity of money is generally accompanied by positive nominal interest and inflation rates through the simultaneous optimization of the government and the representative household. Money is a scarce resource for the representative household. The fact that nominal interest and inflation rates are usually positive is not consistent with the Friedman rule but is consistent with the model developed in the present paper. Conversely, this consistency of the model with the fact strongly implies that usually governments are economically Leviathans.

APPENDIX

1. The transversality condition

By equation (13), $\pi_{b,t}^{e}-\pi_t=R_t-r_t-\pi_t=\theta_G-\theta_P$ at steady state. Hence, $R_t-\pi_t=\theta_G$ by equation (6). Substituting the equation $R_t-\pi_t=\theta_G$ and equation (13) into conditions (10) and (11) and solving both differential equations yield the equation: $\lambda_{G,t}b_t=-\exp\left[(g_t-x_t-s_t)\int\frac{1}{b_t}dt+C^{\#}\right]$ at steady state where $C^{\#}$ is a certain constant. Thereby it is necessary to satisfy $g_t-x_t-s_t<0$ and $\lim_{t\to\infty}\int\frac{1}{b_t}dt=\infty$ for the transversality condition (12) to be held.

Here, by condition (11), $\frac{\dot{b}_t}{b_t}=\theta_G+\frac{g_t-x_t-s_t}{b_t}$ at steady state. Hence, if $\frac{\dot{b}_t}{b_t}=\theta_G+\frac{g_t-x_t-s_t}{b_t}=0$ at steady state, then b_t is constant; thus, $\lim_{t\to\infty}\int\frac{1}{b_t}dt=\infty$. Thereby, the transversality condition holds. However, if $\frac{\dot{b}_t}{b_t}=\theta_G+\frac{g_t-x_t-s_t}{b_t}<0$ at steady state, then b_t diminishes to zero and transversality condition (12) cannot hold because $g_t-x_t-s_t<0$. If $\frac{\dot{b}_t}{b_t}=\theta_G+\frac{g_t-x_t-s_t}{b_t}>0$ at steady state, then $\lim_{t\to\infty}\frac{\dot{b}_t}{b_t}=\theta_G$; thus, b_t increases as time passes and $\lim_{t\to\infty}\int\frac{1}{b_t}dt=\frac{C^{\#\#}}{\theta_G}$, where $C^{\#\#}$ is a certain constant. Thereby transversality condition (12) also cannot hold. ■

2. Proof of $\frac{\partial \boldsymbol{m}_t}{\partial \boldsymbol{\pi}_{b,t}^{e}}<\mathbf{0}$

By equation (6), $\dfrac{\dfrac{\partial u_P(c_t, m_t)}{\partial m_t}}{\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}} = \pi_t + \theta_P$ at steady state. Because $\pi_{b,t}^e = \pi_t + \theta_G - \theta_P$ by equation (13), $\dfrac{\partial u_P(c_t, m_t)}{\partial m_t} = \left[\pi_{b,t}^e + (2\theta_P - \theta_G)\right]\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}$; thus, $\dfrac{\partial^2 u_P(c_t, m_t)}{\partial m_t^{\,2}} = \dfrac{\partial \pi_t^e}{\partial m_t}\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}$ at steady state. Hence, $\dfrac{\partial m_t}{\partial \pi_{b,t}^e} = \dfrac{\dfrac{\partial u_P(c_t, m_t)}{\partial c_t}}{\dfrac{\partial^2 u_P(c_t, m_t)}{\partial m_t^{\,2}}} < 0$ at steady state because $\dfrac{\partial^2 u_P(c_t, m_t)}{\partial c_t^{\,2}} < 0$ and $\dfrac{\partial u_P(c_t, m_t)}{\partial c_t} > 0$. ■

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