Informational Lobbying and Competition for Access

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Competition for Access

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Abstract

In *competition for access*, interest groups provide contributions to a politician and those that provide the highest contributions win access. Groups with access present information that may influence the politician's beliefs about the socially optimal policy. Because equilibrium contributions are chosen endogenously, the politician learns about the information quality of all interest groups, even when he grants access to only some of the groups. Contribution limits reduce the signaling power of the equilibrium contributions, resulting in a less informed politician, and *strictly* reducing expected social welfare. (*JEL* D72, D44, D78)

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Money doesn't buy...a position. But it will definitely buy you some access so you can make your case.

- Thomas Downey (former US Congressman)

Doing what's right isn't the problem. It's knowing what's right.

- Lyndon B. Johnson

1 Introduction

In the United States, interest groups and lobbyists provide political contributions in an effort to gain access to politicians. Access allows a contributor to present information and arguments in favor of its preferred policy. Contributions typically are *not* provided in a quid pro quo exchange for policy favors. These statements not only summarize the views of interest groups and policy makers (e.g., James F. Herndon, 1982; Martin Schram, 1995),[1] they are also strongly supported by empirical evidence (e.g., Laura I. Langbein, 1986; Jeffrey Milyo et al., 2000; Stephen Ansolabehere et al., 2002).[2] Even campaign finance reform advocates argue that the current system is bad for society because it favors wealthy interests who are more likely to secure *access* to politicians compared with less-well-financed interest groups and individuals (Larry Makinson, 2003). [3]

Despite evidence in support of the idea that political contributions are largely motivated by the possibility of gaining access rather than in the explicit exchange for policy favors, the theoretical literature has focused almost exclusively on the latter motivation for providing contributions. For example, Gene M. Grossman and Elhanan Helpman (1994, 1996), Andrea Prat (2002), and Stephen Coate (2004) develop models in which politicians intentionally choose policies that are not favored by their constituents in order to attract larger interest group contributions. Others, including

[1]For example, in Herndon (1982, p1000), an anonymous interest group representative stated: "About all you get [in exchange for a contribution] is a chance to talk to them... If you have a good case you can win them over. But you have to be able to talk to them." Former US Senator Howard Metzenbaum said "Those who contribute may have more ready access and may at least be able to present their arguments with you whether you agree with them or not," and former US Senator Dennis DeConcini said "What they got our of me for that contribution is access to come in .. and to tell me why ... it's good 'for America'" (Schram 1995).

[2]See also Larry Sabato (1984), Richard L. Hall and Frank W. Wayman (1990), John Wright (1990), and Dan Clawson et al. (1992).

[3]This view of political contributions and access is not limited to the United States. For example, Lee Rhiannon, an Australian politician, and Norman Thompson argue the following about their country: "Access is power, and money buys access to politicians in our country. This means large donors can influence governmental decisions, which benefit them and their companies" (Rhiannon and Thompson, 2006).

Gordon Tullock (1980), Michael R. Baye et al. (1993), and Yeo-Koo Che and Ian L. Gale (1998), model the competition between interest groups for an explicit policy favor such as a government contract or favorable legislation. These models ignore the issue of access completely. The few papers that do incorporate access into their models (David Austen-Smith, 1995; Susanne Lohmann, 1995) assume that interest group information is completely unverifiable.[4] In these models, presenting unverifiable information to a politician does nothing to the politician's beliefs about the best policy. The only way the interest groups can influence a politician is by providing contributions to signal the intensity of their preferences.[5] Access in these models is clearly inconsistent with the idea that it allows an interest group to present evidence or arguments in favor of its preferred policy.[6]

This paper models *competition for access* in a way that is consistent with the lobbying process as described by interest groups and politicians. The model makes three fundamental assumptions. First, interest groups have verifiable (or at least persuasive) information that can influence policy makers' beliefs about the best course of action. If interest group information is completely unverifiable, or if the groups have no private information, then they have no incentive to gain access to the politician since access does not enable them to influence a politician's beliefs. Second, interest groups compete for access by providing political contributions, where the probability an interest group receives access is increasing in the relative size of its contribution. Third, politicians do *not* provide access to all interest groups. This limit may be due to time constraints or other costs. If all groups receive access, they have no incentive to compete for access by providing contributions.

[4]The vast majority of informational lobbying models assume that interest group information is completely unverifiable. See Grossman and Helpman (2002) for an excellent overview.

[5]In this way, these previous "access" models are similar to the signaling models of Austen-Smith (1994) and Joan Esteban and Debraj Ray (2006) in which interest groups provide contributions or engage in costly lobbying in order to provide a signal to a policy maker regarding their preferences. In these signaling models information is unverifiable, and access is ignored. Other papers consider the disclosure of private, verifiable information to a decision maker, but ignore the issue of access all together. Paul Milgrom and John Roberts (1986) consider the decision of interest groups to disclose private verifiable information to a decision maker. Morten Bennedsen and Sven E. Feldmann (2002) extend Milgrom and Roberts (1986) to a multiple-policy maker setting. Bennedsen and Feldmann (2006) and Matthias Dahm and Nicolas Porteiro (2006a, 2006b) combine a simple verifiable information disclosure framework with a quid pro quo exchange of contributions and policy favors.

[6]Austen-Smith (1995) and Lohmann (1995) both develop "access" models that assume interest groups have private, unverifiable information in support of or against their favorite policy, and that the groups can use contributions to send signals to a politician about their information. However, because information is *completely unverifiable*, revealing it to the politician cannot influence the politician's beliefs about the best policy, and therefore access itself does nothing in these models. In reality, interest groups expect their information (not just their contributions) to influence the decision maker's beliefs.Austen-Smith (1995) shows how this limitation of the model is eliminated when he assumes that politicians do *not* know the policy preferences of the interest groups. He argues that if we find such an assumption unreasonable (which I do), then we may have to reject the belief that interest groups seek access in order to share information. I show that this conclusion is incorrect. In this paper's competition for access model, access is present even though the politician knows the policy preferences of the interest groups.

These assumptions reflect the reality of the lobbying system in the United States; however, to my knowledge, no other paper makes all three assumptions.

By using the above assumptions as a foundation, this paper develops a more realistic framework to analyze political contributions and lobbying. In traditional models of rent-seeking, contributions (or bids or effort) directly determine who wins the prize.[7] When applied to lobbying, these models require that a politician be willing to award prizes or favors to the highest contributors, even if the action hurts their constituents. In contrast, competition for access allows the politician to always undertake the actions he thinks are best for his constituents (or society as a whole). Interest groups still provide the politician with contributions, but these contributions help secure *access* to the politician, not direct favors. Groups with access can influence the politician's beliefs by presenting information or arguments in favor of their preferred policies. The politician uses the presented information to update his beliefs before implementing the policies he *believes* are best.

The model shows how politicians do not have to sacrifice social welfare in order to collect contributions from interest groups. Groups continue to provide money to politicians even when the money only buys access, not explicit favors. Even more interesting is that interest groups with better information are willing to provide larger contributions than those with worse information. The strict monotonicity of the equilibrium contribution function means that the rational politician learns an interest group's information quality by observing its contribution, even if he does not grant the group access.[8] By observing contributions, the politician learns about the information quality of *all* interest groups, even when he provides access to only *some* of the groups. The politician then implements the policy profile that maximizes social welfare.

When interest groups compete for access (as they do in this paper), political contributions enable the politician to better identify and implement the welfare-maximizing policy. This is in contrast to the papers in which interest groups provide contributions in exchange for policy favors (e.g., Grossman and Helpman, 1994, 1996). In the other models, the exchange of policy favors for contributions tends to reduce social welfare as politicians trade off policies preferred by their

[7] See for example other applications of the all-pay auction by Charles A. Holt (1979), Holt and Roger Sherman (1982), Baye et al. (1993, 1996), Simon P. Anderson et al. (1998), Che and Gale (1998), and Benny Moldovanu and Aner Sela (2001).

[8] In equilibrium, interest groups do not have an incentive to increase their contributions in order to signal higher quality information than they actually have. This is because increasing one's contribution also increases the probability that the interest group submits one of the highest contributions and receives access. When a group receives access, the politician learns their information quality with certainty.

constituents for policies that increase total contributions, and the results suggest that limiting or even banning political contributions may benefit society.[9] However in the competition for access framework, contributions provide signals about interest group information quality, enabling the politician to better identify and implement socially beneficial policies. Expected social welfare is maximized when there are *no* limits.

Up to this point, the discussion has not directly considered the most popular argument in favor of contribution limits: that they level the playing field between wealthy interest groups and less-wealthy groups. Section 4 incorporates interest group wealth differences into the model to specifically consider this argument. The analysis shows that rich interest groups tend to realize higher equilibrium payoffs compared to poor groups, and that wealth differences may result in an equilibrium policy profile that is biased in favor of rich-group policy preferences. Contribution limits can eliminate the rich-group advantage. However, I show that just because the contribution limit levels the playing field between rich and poor groups, this does not imply that the limit improves social welfare (although it can do so under certain conditions). Just as in the case without wealth differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about information. This can result in a less informed politician and a policy profile that is worse for society. Although campaign finance reformers are correct in their claim that the exchange of contributions for access tends to favor wealthy interest groups, they are incorrect in concluding that contribution limits must therefore improve social welfare. Contribution limits do level the playing field between rich and poor interests, but they may do so at the cost of reducing overall welfare.

The paper proceeds as follows. Section 2 develops and solves an informational lobbying game in which interest groups provide contributions in a competition for access. Section 3 incorporates a contribution limit into the model and shows how it strictly reduces expected social welfare. Section 4 allows interest groups to differ in terms of their wealth. I show that although contribution limits can improve social welfare when there is wealth inequality, they often decrease welfare even when they also eliminate biases that favor rich groups. Section 5 considers additional extensions of the model. Section 6 concludes the paper with a brief discussion of the results, policy implications, and

[9] Coate (2004) shows how the negative impact of contributions may be (at least partially) offset if the politicians use the contributions to communicate information about themselves during an election.

suggestions for future extensions and applications of the model.

2 Informational Lobbying Game

2.1 Model

There are N independent policy issues. There is a risk-neutral politician who, for each issue, must choose a policy from a single-dimensional policy space defined by the interval $[-1, 1]$. There are a total of $2N$ interest groups, where for each of the N issues one interest group prefers policy -1, and one group prefers policy 1. An interest group is denoted by the issue it is concerned with and its policy preference; therefore (n, j) refers to the interest group concerned with issue $n \in \{1, ..., N\}$ and policy preference $j \in \{-1, 1\}$. Where it is clear which issue a group is concerned with, I refer to it as group j.

At the beginning of the game, each interest group draws private information in support of its preferred policy. The quality of (n, j)'s information is denoted by I_n^j and is the independent realization of a random variable distributed on the continuum $[0, 1]$. A higher I_n^j can be thought of as interest group (n, j) having stronger evidence or a better argument in support of its preferred policy. The distribution of information quality is denoted by function F, with density function f, and is common knowledge.[10]

After the interest groups realize their information qualities, they independently provide contributions to the politician. Group (n, j) provides contribution $b_n^j \geq 0$. Interest groups receive access if they provide one of the K largest contributions, where $K \in \{1, ..., 2N - 1\}$.[11] If the Kth and $(K + 1)$th largest contributions are equal, then all of the groups that provide this same contribution receive access with equal probability.

Interest groups with access present their information to the politician. When a group presents

[10]The body of the paper assumes that the distribution of information quality is the same for all interest groups. This does not have to be the case. Alternatively, F_n^j could define the distribution of group (n, j)'s information. So long as the different distributions are common knowledge, the results of the analysis do not change.

Assuming that the information quality of the different interest groups is independently drawn simplifies the analysis. However, an alternative model can be developed in which the information quality of the two groups concerned with the same issue is correlated. For example, when one of the groups concerned with an issue has very strong information, it may be more likely that the other group concerned with the same issue has relatively low information. Under reasonable assumptions, such an alternative model would not change the results of the analysis.

[11]If $K = 0$ or $K = N$, then interest groups provide no contributions in equilibrium. The basic model assumes that K is determined independently of the model. Section 6 discusses the case where K is endogenous. As Section 6 shows, the politician will commit to provide a positive amount of access ($K \geq 1$) when K is endogenous.

its information, the politician becomes fully informed of the group's information quality. Assuming that interest groups with access *must* present their information greatly simplifies the description of the game. However, the results do not change if groups are allowed to reject access.[12] Let Ω denote the vector of interest group information qualities revealed to the politician through access. After observing the contributions of all interest groups and the information quality of those with access, the politician chooses a policy for each of the N issues (by maximizing a payoff function defined later). Let $p_n^* \in [-1, 1]$ denote the policy implemented by the politician for issue n, and let $p^* = \{p_1^*, ..., p_N^*\}$ denote the policy profile chosen for all issues.

The politician is not corrupt, and does not sell policy favors. Contributions determine whether an interest group receives access, but do not directly influence the policy choice of the politician. Contributions are non-refundable, and are not contingent on being granted access. Therefore, the exchange of access for political contributions is an *all-pay auction*: all bidders (interest groups) pay their bids (contributions) before the prizes (access) are allocated to the highest bidders.[13]

The politician is *fully informed* about an issue if he is certain about the information quality of both interest groups involved with the issue. When a politician is fully informed about an issue, he can determine the welfare-maximizing policy choice for that issue. Let p_n^o denote the *socially optimal policy* for issue n, and the vector $p^o = \{p_1^o, ..., p_n^o\}$ define the vector of socially optimal policies across all issue. For simplicity, I assume p_n^o takes the form $p_n^o = I_n^1 - I_n^{-1}$. It is straightforward incorporate a more complicated socially optimal policy function into the analysis, and reasonable changes to the function do not change the paper's results.[14]

[12]It can be shown that interest groups with access will always accept. If a group rejects access, the politician believes that the group had lower information quality than he expected. This causes him to update his beliefs and lower his expectation. This results in an unravelling of beliefs until the politician believes any interest group that rejects an offer of access has the lowest possible information quality.

[13]I use an all-pay auction to model competition for access because it seems the most realistic framework. The results continue to hold so long as the probability of winning access is non-decreasing in the size of a group's contribution, which results in an equilibrium contribution function that is strictly increasing in an interest group's information quality. The model could alternatively assume that the politician allocates access through another type of auction, a lottery in which one's probability of gaining access is proportional to the relative size of a group's contribution, or even if all groups receive access with equal probability independent of their contributions.

[14]For any issue n, the analysis requires that $p_n^o\left(I_n^{-1}, I_n^1\right)$ be strictly decreasing in I_n^{-1}, strictly increasing in I_n^1, additively separable, and such that $-1 \le p_n^o(1,0) \le p_n^o(0,1) \le 1$. The function p_n^o does not have to be linear in information quality. So long as the function is additively separable in terms of I^{-1} and I^1, the asymmetries between the impact of the two groups' information on the optimal policy may be accounted for through a transformation of their information distribution functions F_n^j. As stated previously, allowing for asymmetric distribution functions does not change the results of the analysis.

The model can be adapted to allow for a biased politician by changing the definition of the socially optimal policy. For each issue n, let the fully informed politician implement policy $p_n^o = \beta_n^1 I_n^1 - \beta_n^{-1} I_n^{-1}$, where each $\beta_n^j \in (0, 1]$ is common knowledge. When $\beta_n^j > \beta_m^i$, interest group (n, j) will have a greater impact on the implemented policy than

Payoffs

Social welfare depends on the difference between the implemented policy and the socially optimal policy across all issues. The parameter $\gamma_n > 0$ represents the relative weight society places on issue n. Social welfare is

$$W(p^*, p^o) = -\sum_{n=1}^{N} \gamma_n \times |p_n^* - p_n^o|.$$

Welfare is maximized when the politician implements the socially optimal policy profile $p^* = p^o$. The politician can determine p^o with certainty only when he is fully informed about all issues.

The politician is concerned with social welfare and collecting political contributions. The parameter $\rho > 0$ represents how much the politician cares about revenue generation relative to social welfare, and b represents the profile of contributions from all interest groups. His payoff is

$$U^P(p^*, p^o, b) = W(p^*, p^o) + \rho \sum_{n=1}^{N} \left(b_n^1 + b_n^{-1}\right).$$

An interest group's payoff is decreasing in the size of its contribution and the distance between its preferred policy and the implemented policy for the issue it cares about. Group (n, j)'s payoff is

$$U_n^j\left(p_n^*, b_n^j\right) = V(|p_n^* - j|) - b_n^j.$$

The function V defines interest group policy utility where $V'(\cdot) < 0$, and $V(0) = 0$. Since interest group (n, j) prefers policy j, $|p_n^* - j|$ denotes the distance between the implemented policy and the group's preferred policy.[15]

States and Beliefs

The realized state of the world is defined by the vector of realized information qualities $\left\{I_n^j\right\}_{\forall(n,j)}$. Let S denote the set of all potential states of the world, and $s \in S$ denote an arbitrary state within the state space S. Each s assigns a value $I \in [0, 1]$ to each interest group. Let $I_n^j(s)$ denote the information quality of (n, j) in state s. The function $\mu(\cdot \mid b, \Omega)$ defines the politician's beliefs about the state of the world given the contribution vector b and the vector of information revealed through access Ω. These beliefs may be fully represented by the vector of all updated density

interest group (m, i) by revealing the same quality information to the politician.

[15] All interest groups have the same policy utility function. The results do not change if the utility functions differ, so long as they are common knowledge.

functions $\left\{f_n^j\left(\cdot \mid b, \Omega\right)\right\}_{\forall(n,j)}$, where $\mu\left(s \mid b, \Omega\right) = \prod_{(n,j)} f_n^j\left(I_n^j\left(s\right) \mid b, \Omega\right)$. Also the operator E represents the expectations given the ex ante distribution of states, and E_μ represents the politician's expectations given his beliefs μ.

Solution Concept

The analysis solves for the symmetric Perfect Bayesian Equilibrium of the game, which I label the *contribution equilibrium.* A complete description of the equilibrium must include the strategy profiles for the interest groups and the politician, as well as the politician's beliefs about the state of the world at the time he chooses a policy profile. The politician's beliefs must be consistent with using Bayes' Rule on the ex ante distribution of information quality given the strategies of the interest groups. Each player's strategy must be a best response to the strategies of the other players, given the player's beliefs.

In the contribution equilibrium, all interest groups share the same contribution function $B : I \rightarrow b$, where $B\left(I\right)$ defines the equilibrium contribution for an interest group with information quality I. The value $P_n^*\left(\mu\right)$ defines the politician's equilibrium policy choice given his beliefs. A description of $P_n^*\left(\mu\right)$ for all possible μ and each n fully describes the politician's equilibrium strategy.[16]

2.2 Equilibrium

I first determine the politician's policy choice at the conclusion of the game, then analyze the all-pay auction in which interest groups choose the size of their contributions, and the highest contributors receive access.

[16]A formal definition of a contribution equilibrium requires some additional notation. Let $\Omega\left(b_n^j, s; B\right)$ define the vector of information qualities presented by interest groups with access in state s when group (n, j) contributes b_n^j, and all other groups contribute according to B. Let $\hat{\mu}\left(s \mid I\right)$ denote the probability that an interest group puts on the world being in state $s \in S$ given that its own information quality is I.

Definition 1 *The interest group contribution function B and politician strategy $\left\{P_n^*\left(\mu\right)\right\}_{n=1}^N$ and beliefs μ constitute a **contribution equilibrium** if*

1. *For all $I_n^j \in [0, 1]$,*
   $B\left(I_n^j\right) \in \arg\max_{b_n^j} \int_{s \in S} \hat{\mu}\left(s \mid I_n^j\right) U_n^j\left(P_n^*\left(\mu\right), b_n^j\right) ds$
2. *For any possible b and Ω,*
   $\left\{P_n^*\left(\mu\right)\right\}_{n=1}^N \in \arg\max_{p^*} \int_{s \in S} \mu\left(s \mid b, \Omega\right) U^{PM}\left(p^*, \left\{p_n^o\left(I_n^1\left(s\right), I_n^{-1}\left(s\right)\right)\right\}_{\forall n}, \left\{B\left(I_n^j\left(s\right)\right)\right\}_{\forall(n,j)}\right) ds$
3. *Beliefs μ meet the requirements of Perfect Bayesian Equilibrium, given the equilibrium strategy profile.*

For a detailed description of Perfect Bayesian Equilibrium belief requirements, see Fudenberg and Tirole (1991, pp. 324-326).

Policy Choice

At the time the politician chooses policy, the interest groups have already given their contributions. This means the policy choice can only impact social welfare. The politician chooses the policy profile that maximizes expected social welfare given his beliefs. For each issue n, his policy choice p_n^* is defined by the function $P_n^*(\mu) = E_\mu p_n^o = E_\mu I_n^1 - E_\mu I_n^{-1}$, where E_μ denotes the politician's expectations given beliefs μ.[17] When the politician is fully informed, $E_\mu I_n^j = I_n^j$, and $E_\mu p_n^o = p_n^o$. A politician who is fully informed about all issues implements the socially optimal policy profile and maximizes social welfare.

Interest Group Contributions

In equilibrium, all interest groups contribute according to the contribution function B. Similar to in the last section, I start with the assumption that the contribution function is strictly increasing in a group's information quality. After solving for B, I show that this assumption holds. Since B is strictly increasing, it is invertible, where $I(b) = B^{-1}(I)$, and there is a one-to-one mapping between the group's contribution and its information quality. It immediately follows that a rational agent can determine an interest group's information quality if he observes its contribution.

To solve for the contribution function, the analysis considers the contribution decision of interest group (n, j) assuming that all other groups contribute according the the equilibrium function. Because the other groups contribute according to B, the politician is certain regarding all other groups' information qualities. Let $\Theta\left(I; I_n^{-j}\right)$ be the probability that fewer than K other interest groups have information quality greater than I, given that group $(n, -j)$ has I_n^{-j}. Therefore, $\Theta\left(I(b); I_n^{-j}\right)$ denotes the probability that group (n, j) receives access given contribution b. Interest group (n, j) chooses its contribution b to maximize the following equation:

$$\int_0^1 f\left(I_n^{-j}\right)\left(\left(1-\Theta\left(I(b); I_n^{-j}\right)\right) V\left(1 - I(b) + I_n^{-j}\right) + \Theta\left(I(b); I_n^{-j}\right) V\left(1 - I_n^j + I_n^{-j}\right)\right) dI_n^{-j} - b. \tag{1}$$

With probability $\Theta\left(I(b); I_n^{-j}\right)$ group (n, j) receives access and presents its information to the politician who then chooses policy $I_n^j - I_n^{-j}$. This results in policy utility $V\left(1 - I_n^j + I_n^{-j}\right)$ for the interest group. Alternatively, with probability $\left(1 - \Theta\left(I(b); I_n^{-j}\right)\right)$ the group does not receive access and the politician believes the interest group has information quality $I(b)$ rather than its

[17] Also, $E_\mu I_n^j = \int_{s \in S} \mu(s \mid b, \Omega) I_n^j(s)\, ds$.

true information quality I_n^j. This results in interest group policy utility $V\left(1 - I(b) + I_n^{-j}\right)$.

First order conditions for the interest groups' problem are given by

$$\int_0^1 f\left(I_n^{-j}\right)\left(\begin{array}{c}\left(1-\Theta\left(I(b);I_n^{-j}\right)\right)V'\left(1-I(b)+I_n^{-j}\right)\frac{\partial I}{\partial b}(-1)\\ +\frac{\partial\Theta}{\partial I}\frac{\partial I}{\partial b}\left(V\left(1-I_n^j+I_n^{-j}\right)-V\left(1-I(b)+I_n^{-j}\right)\right)\end{array}\right)dI_n^{-j}-1=0.$$

The first row of notation represents the marginal impact of a change in an interest group's contribution on the politician's beliefs about the group's information quality provided that it does not win access, and the corresponding change in the group's policy utility. The second row represent the marginal impact of a change in a group's contribution on the probability the group wins access.

In equilibrium, all interest groups contribute according to the function B, which implies $I(b) = I_n^j$ for all (n,j). Strict monotonicity of the function means $\left(\frac{\partial I}{\partial b}\right)^{-1} = \frac{\partial B}{\partial I}$. The first order conditions simplify to

$$\frac{\partial B(I)}{\partial I} = -\int_0^1 f\left(I_n^{-j}\right)\left(\left(1-\Theta\left(I_n^j;I_n^{-j}\right)\right)V'\left(1-I_n^j+I_n^{-j}\right)\right)dI_n^{-j}.$$

It is straightforward to show that $\frac{\partial B}{\partial I}$ is positive.[18] Therefore, the contribution function B is strictly increasing in a group's information quality. This also means that a group's information quality is increasing in the size of its equilibrium contribution, and that the politician can correctly infer an interest group's information quality by observing its contribution.

The closed-form solution for the contribution function is

$$B(I) = -\int_0^I\int_0^1 f\left(I_n^{-j}\right)\left(1-\Theta\left(y,I_n^{-j}\right)\right)V'\left(1-y+I_n^{-j}\right)dI_n^{-j}dy.$$

In equilibrium, for any information quality I, the benefit an interest group receives from bidding more than $B(I)$ in an attempt to convey higher-quality information is completely offset by the cost of doing so.

Game Equilibrium

The above analysis derives the unique contribution equilibrium of the game. The first lemma summarizes the results.

Lemma 1 *In the contribution equilibrium,*

[18]This follows because $f(I) > 0$, $(1-\Theta(I)) \geq 0$ (with strict inequality for some I), and $V'(\cdot) < 0$.

1. $b_n^j = B\left(I_n^j\right)$ *for all* (n, j)

2. $P_n^*(\mu) = E_\mu p_n^o = E_\mu I_n^1 - E_\mu I_n^{-1}$ *for all* $n \in \{1, ..., N\}$*, and*

3. the politician's beliefs μ *are such that for any* (n, j)*,* $f_n^j\left(I_n^j \mid b, \Omega\right) = 1$ *if group* (n, j) *has access, and* $f_n^j\left(I\left(b_n^j\right) \mid b, \Omega\right) = 1$ *if group* (n, j) *does not have access.*

In the contribution equilibrium, all interest groups contribute according to the same function B, and the politician chooses the policy profile that he believes maximizes social welfare. Because the monotonicity of the contribution function allows the politician to learn the information quality of all interest groups, the politician knows the socially optimal policy profile at the time he chooses polities. It immediately follows that the politician implements the socially optimal policy profile. This result is stated by the first proposition.

Proposition 1 *In the contribution equilibrium,* $p_n^* = p_n^o$ *for all* $n \in \{1, ..., N\}$*.*

The first lemma and proposition follow directly from the above analysis.

Previous models of political contributions imply that contributions result in policies that benefit special interests, but decrease overall welfare (e.g., Grossman and Helpman, 1994). This paper suggests that contributions can have the opposite impact on social welfare. In competition for access, political contributions can have a positive impact on overall welfare because they enable the politician to recognize and implement better policies.

Importance of Access

In equilibrium, the politician becomes fully informed about the information quality of all groups by observing their contributions alone. This does not imply that the politician can provide *no* access. If the politician does not provide access to any group, then the contributions become uninformative. Without access, all interest groups face the same incentives when choosing their contributions; groups with high qualifications are no longer willing to provide larger contributions than groups with low qualifications. The politician recognizes this and does not take the size of the contributions into account when updating his beliefs. This means that $E_\mu I_n^j = EI_{n'}^j$ for all (n, j), and the politician chooses $p_n^* = 0$ for all n. Because contributions have no impact on the politician's beliefs, the interest groups are unwilling to contribute anything, and the politician receives nothing.

If the politician provides no access, the interest groups provide no contributions and the politician does not learn anything about the groups' information. However, providing access to at least one group allows the politician to become fully informed about the information quality of all groups. Section 5 endogenizes the amount of access.

3 Contribution Limits

The previous section assumes that there are no limits to the maximum size of interest group contributions. This section considers how the analysis changes if contributions are constrained.[19] The parameter $\bar{b}$ denotes the maximum allowed size of a contribution, where $b_n^j \in [0, \bar{b}]$ for all (n, j). Assume $0 < \bar{b} < B(1)$, which implies that the contribution limit is lower than the maximum possible contribution in the game without contribution limits, and high enough such that contributions exist.[20]

When there is a contribution limit, the politician continues to implement the policy profile he believes is best for society, or $P_n^*(\mu) = E_\mu I_n^1 - E_\mu I_n^{-1}$ for each n. However, because contribution limits change the politician's beliefs μ, they change the resulting policy profile.

Impact of Contribution Limits

If there did not exist a contribution limit, then groups with high-enough information quality would contribute more than $\bar{b}$ in equilibrium. With a limit in place, groups with high I prefer to provide the maximum contribution $\bar{b}$ compared to a lower amount; although some groups may prefer to contribute more than $\bar{b}$ if it was allowed. Groups with relatively low I prefer to contribute less than $\bar{b}$.

The appendix formally derives the equilibrium of the game with a contribution limit. Here, I describe the results. The equilibrium contribution function $B_{CL} : I \rightarrow b$ is a discontinuous function comprised of two parts: a continuous function $\tilde{B}$ for low enough I, and the constant $\bar{b}$ for higher I. Let $\bar{I}$ denote the information quality of the interest group that is indifferent between contributing

[19] Che and Gale (1998) consider the impact of a contribution limit in a game where bidders in an all-pay auction compete for a policy favor (e.g., a government contract) rather than for access (as is the case in this paper). They show that even in the competition for policy favors, contribution limits can have a negative impact on social surplus.

[20] When more interest groups provide the maximum contribution than the politician provides access to, I assume that the politician allocates access between each of the groups with equal probability. Alternatively, he could provide access to the interest groups involved with the issues that he cares the most about (those with the largest γ_n's). This alternative assumption does not change the results, although it complicates the analysis.

according to $\tilde{B}$ and contributing amount $\bar{b}$. Therefore,

$$B_{CL}(I) = \begin{cases} \tilde{B}(I) & \text{for } I \in [0, \bar{I}) \\ \bar{b} & \text{for } I \in \left[\bar{I}, 1\right] \end{cases}$$

The function $\tilde{B}$ is derived in the same way that B was derived in the game without contribution limits (I formally derive $\tilde{B}$ in the appendix). $\tilde{B}$ is strictly increasing in a group's information quality. Therefore $\tilde{B}$ is invertible and the politician becomes fully informed of the information quality of any group that provides a contribution according to this function. In contrast, when the politician observes contribution $\bar{b}$, he cannot infer which value of $I \in \left[\bar{I}, 1\right]$ resulted in such a contribution. The politician only learns with certainty the information quality of an interest group that provided the maximum contribution if he grants that group access. It is possible that more interest groups provide the maximum contribution than the politician grants access to. When this happens, the politician remains less than fully informed about the information quality of some groups, and cannot determine the policy profile that maximizes social welfare.[21]

An example of a contribution function B_{CL} is provided by Figure 1.

[[[Insert Figure 1]]]

The politician may no longer know each group's information quality with certainty. The following proposition interprets this difference in terms of social welfare. Expected social welfare is strictly lower when there is a contribution limit than when there is no limit.

Proposition 2 *In the informational lobbying game, $EW(p^*, p^o)$ is strictly higher when there is no contribution limit than if there exists a contribution limit $\bar{b} \in [0, B(1))$.*

Since the politician provides access to fewer than the total number of interest groups ($K < 2N$), there is a positive probability that the number of interest groups with $I \geq \bar{I}$ (the number that contribute $\bar{b}$) is larger than the number of groups that receive access. Since the politician only learns the information quality of a group that provides $\bar{b}$ if the group receives access, there is a

[21] In the equilibrium of the no-limit game, all interest groups are indifferent between gaining access and not gaining access to the politician after they submitted their contribution. This is because their contributions communicate their information quality to the politician, and gaining access does not allow them to further impact the politician's beliefs. This is *not* the case in the game with a contribution limit. The politician acts as if all interest groups that provide the maximum contribution (and do not gain access) have the same expected information quality. The groups that have information quality above this expected level are made better off if they gain access, since access results in the politician learning that their information quality is higher than his expectations.

positive probability that the politician is less than fully informed when he chooses a policy profile. A less than fully informed politician almost certainly chooses a policy profile that is different from the social optimal. Therefore, contribution limits strictly reduce expected welfare. This does not mean that the realized welfare is necessarily lower. Rather, contribution limits never improve realized social welfare, and they reduce realized welfare with positive probability.

4 Interest Group Wealth Differences

The previous sections assume that interest groups all have the same wealth, preference intensity, and distribution of information quality. In reality interest groups are not homogeneous. In this section, I allow interest groups to differ in terms of their wealth. I focus on wealth differences instead of preference intensity, information distribution, or other possible heterogeneities because they allow me to address issues central to the policy debate.[22]

Campaign finance reform advocates often argue that the contributions-for-access system favors wealthy interest groups relative to less-wealthy interest groups and individuals. They claim that limiting or banning contributions reduces the rich-group advantage and therefore increases social welfare. The analysis in this section illustrates the flaw in this logic. Although a contribution limit can eliminate the advantage rich groups tend to have over poor groups, this is not evidence that the limit also improves social welfare. Under certain conditions, a limit may improve welfare (in contrast to Section 3 in which it strictly reduces expected welfare). However, this is not generally true, and a contribution limit often reduces social welfare even though it eliminates the biases in favor of the rich groups.

The model with wealth differences differs from the game presented in Section 2 as follows. Each interest group is rich with probability α, and poor with probability $(1-\alpha)$, where the parameter α is common knowledge. A poor group faces a binding budget constraint such that its contribution must be less than wealth ω.[23] Each interest group knows its own wealth, but does not know the wealth of other interest groups. The rest of the model is unchanged.

The first subsection describes the equilibrium of the game, first for when there is no contribution

[22]It is possible to adapt the analysis in this section to consider other types of heterogeneity.

[23]For the budget constraint to be binding, ω must be less than the amount an interest group with the highest quality information ($I_n^j = 1$) would want contribute in equilibrium if there was no budget constraint. Formally, $\omega < B(1)$ where B is the contribution function derived in Section 2.

limit, then for when there is a limit. The second subsection shows how rich groups tend to receive higher payoffs compared to similar poor groups, and how a contribution limit can eliminate this payoff inequality. In the third subsection, I show that contribution limits often reduce social welfare even when they eliminate the rich group advantage. The details regarding the analysis is provided in the appendix, Section 7.2.

4.1 Equilibrium Contribution Functions

No Contribution Limit

I consider the symmetric Perfect Bayesian Equilibrium of the game, which I label the *contribution equilibrium with wealth differences.* A description of the new equilibrium requires an explanation of two different contribution functions: one for rich groups, and one for poor groups. The following lemma describes the contribution functions of the game with wealth differences. The functions B_P and B_R respectively denote the poor and rich group contribution functions.

Lemma 2 *In the contribution equilibrium with wealth differences, there exists cut off values* $\bar{I}_a \geq 0$ *and* $\bar{I}_b \in \left(\bar{I}_a, 1\right)$, *and functions* B_a *and* B_b, *where* $B_a\left(0;\omega\right) = 0$, $B_b\left(\bar{I}_b;\omega\right) = 0$, $\frac{\partial B_a}{\partial I} > 0$, *and* $\frac{\partial B_b}{\partial I} > 0$ *such that*

$$B_P\left(I;\omega\right) = \begin{cases} B_a\left(I;\omega\right) & \text{for } I \in [0, \bar{I}_a) \\ \omega & \text{for } I \in [\bar{I}_a, 1] \end{cases}$$

$$B_R\left(I;\omega\right) = \begin{cases} B_a\left(I;\omega\right) & \text{for } I \in [0, \bar{I}_a) \\ \omega & \text{for } I \in [\bar{I}_a, \bar{I}_b) \\ \omega + B_b\left(I;\omega\right) & \text{for } I \in [\bar{I}_b, 1] \end{cases}$$

Figure 2 illustrates example rich and poor group contribution functions.

[[[Insert Figure 2]]]

The wealth constraint has a similar impact on the poor group contribution function B_P as a contribution limit had on the contributions of all interest groups in Section 3. For low enough I, a wealth constrained interest group's contribution is strictly increasing in its information quality, and for higher I the group contributes ω.[24] This means that when the politician observes a

[24] The cutoff value $\bar{I}_a \geq 0$ represents the information quality at which an interest group is indifferent between providing a contribution according to function B_a, and providing contribution ω. If the group provides contribution $B_a\left(\bar{I}_a\right)$, then the politician learns the group's information quality with certainty. If instead the group provides

contribution not equal to ω, he can accurately infer the interest group's I. However, when he observes a contribution equal ω but does not grant the interest group access, he is uncertain about the true information quality of the group and acts as if the group has $I = E_\mu (I \mid \omega)$.[25]

For $I \leq E_\mu (I \mid \omega)$ rich groups provide the same contributions as poor groups with similar information in the symmetric equilibrium. This is because rich and poor groups only differ in terms of the budget constraint. Therefore, a group's preferred contribution is independent of its wealth, and when a poor group prefers its chosen contribution to any other $b \geq 0$, a rich group with the same I prefers to contribute the same amount. Alternatively, for $I > E_\mu (I \mid \omega)$ both rich and poor groups prefer to provide more than ω, but only the rich group can afford to do so. Therefore, rich interest groups with high enough I contribute more than similar poor groups. Poor interest groups cannot afford a higher contribution, therefore poor groups with $I > E_\mu (I \mid \omega)$ provide ω in equilibrium instead. It follows that $\bar{I}_b = E_\mu (I \mid \omega)$.

Contribution Limit

In considering relative payoffs between the different wealth types, I focus on the case where contribution limit $\bar{b}$ is no greater than ω. This ensures that all interest groups can provide the maximum contribution if they choose to do so. Focusing on this range of limits is most consistent with the policy debate in which limits are intended to eliminate the ability of wealthy interest groups to outspend less wealthy groups. The appendix describes the impact that a higher limit has on the contribution functions.

Since $\bar{b} \leq \omega$, both rich and poor interest groups now face the same contribution constraint. This means that in the symmetric equilibrium, the contribution function is independent of wealth, and is the same as B_{CL} in Section 3. See Figure 1 for an illustration. There exists a cut off value $\bar{I} \in [0, 1)$ such that an interest group with $I < \bar{I}$ contributes $\tilde{B}(I)$, and an interest group with $I \geq \bar{I}$ contributes $\bar{b}$.

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contribution $\omega > B_a (\bar{I}_a)$, then the politician learns the group's true information quality only if the group receives access. If the group does not receive access, which happens with positive probability, the politician overestimates the interest group's information quality. For an interest group with information quality $\bar{I}_a$, the benefit from contributing ω is completely offset by the cost of doing so.

[25] The paper generally assumes that the highest K contributors receive access. This assumption makes the analysis more straightforward. Similar results will follow from an analysis that allows interest groups who contribute ω to receive access before groups who contribute more than ω.

4.2 Rich Group Advantage

When there is no contribution limit, in equilibrium, poor interest groups with information quality greater than $\bar{I}_b$ prefer to contribute more than ω (just as the rich groups do), but they are prevented from doing so by their budget constraint. Therefore, rich groups with high-quality information have a higher probability of communicating their information to the politician compared with poor interest groups with similar quality information. This can result in policies that are on average biased in favor of the rich groups, and it does result in rich groups having higher expected payoffs than similar poor groups.

If a rich interest group and a poor interest group have opposite policy preferences involving the same issue, the politician will tend to choose a policy that favors the rich group, all else equal. This is stated by the following lemma.

Lemma 3 *If (n, j) is rich, $(n, -j)$ is poor, and $I_n^j = I_n^{-j}$, then $Ep_n^* > 0$ when $j = 1$ and $Ep_n^* < 0$ when $j = -1$.*

Furthermore, a group's expected payoffs are higher when it is rich than when it is poor. This is because rich interest groups are always able to provide the contribution that maximizes their expected payoffs, but poor interest groups face a budget constraint that prevents them from doing so for some values of I. This result is stated by the following proposition.

Proposition 3 *EU_n^j is at least as great when (n, j) is rich than when (n, j) is poor, and is strictly greater for some $I_n^j \in [0, 1]$.*

A contribution limit can eliminate the bias in favor of rich groups. When there is a contribution limit, an interest group's contribution and the probability that the group receives access are independent of its wealth. This means that policies do not tend to favor rich groups, and that an interest group's expected payoff is independent of its wealth. This is stated by Proposition 4.

Proposition 4 *If $\bar{b} \leq \omega$, then (1) $Ep_n^* = 0$ for all n and (2) EU_n^j is independent of wealth for all (n, j).*

A contribution limit can completely eliminate the rich group advantage. This is consistent with the argument made by many campaign finance reform advocates. However, it is important to recognize that these results says nothing about social welfare. I discuss the welfare implications in the following section.

4.3 Impact on Welfare

This section considers the impact that a contribution limit has on expected social welfare when interest groups have different levels of wealth. I show that under certain parameter values, contribution limits *can* improve expected welfare. However, this is not generally the case. Just because a contribution limit eliminates the impact of wealth on interest group payoffs, this does not imply that the same limits improve social welfare.

In the game with wealth differences, as in the game without differences, a contribution limit tends to reduce the number of interest groups for which the politician is certain about their I. Without a limit, the politician learns with certainty the I of any rich group with high enough information, or any group (rich or poor) with $I < \bar{I}_a$ even if these groups do not receive access. A contribution limit can reduce the range of I for which an interest group's contributes according to a strictly increasing function, and for which the politician learns a group's information quality regardless of access. This means that a limit can result in the politician being less informed when he chooses a policy profile, which tends to reduce social welfare.

A limit does not necessarily have this affect, and could improve welfare under the certain parameter values. For example, a contribution limit at $\bar{b} = \omega$ can increase the probability that a group who gave ω in the no limit game receives access. Depending on the model parameters, the benefit of increasing the probability that the groups who had given ω receive access may be greater than the expected costs of not always learning the information quality of the rich groups that gave more than ω before the limit. This is because when there is no contribution limit, the politician may tend to have significantly inaccurate beliefs about the I of those that give ω and do not receive access, and when there is a contribution limit, the politician may have relatively accurate beliefs about the I of groups for whom he does not learn the I with certainty but for whom he would have learned I if contributions were not limited (the rich interest groups that give more than ω when there is no limit). When this is true, the contribution limit can result in the politician having more accurate beliefs and choosing better policies, on average. Although this is possible, the conditions that must be met for a limit to improve social welfare are often very specific, and one should not conclude that contribution limits generally have a positive impact on welfare. Instead, the opposite seems to be true.

To illustrate the potential welfare impact of a contribution limit, I consider a very simple competition for access game that allows for the explicit solution of different equilibrium variables and payoffs. Suppose that information quality is uniformly distributed on $[0, 1]$, and that interest group policy utility is linear, or $V\left(1 - I^j + I^{-j}\right) = -\left(1 - I^j + I^{-j}\right)$. There is a single issue with two interest groups, and one groups receives access, or $K = 1$. The appendix provides details regarding the solution of this game. Here, I briefly describe the results.

In this simplified game, there exists cut off values α', $\omega'_L(\alpha)$, and $\omega'_H(\alpha)$ such that contribution limit $\bar{b} \leq \omega$ improves expected social welfare if and only if $\alpha \in (\alpha', 1)$, $\omega \in [\omega'_L(\alpha), \omega'_H(\alpha))$, and $\bar{b} \in \left[\frac{1}{4}, \omega\right]$. The values $\omega'_L(\alpha)$ and $\omega'_H(\alpha)$ depend on the value of parameter α, and determining the values α' and $\omega'_H(\alpha)$ require the calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. $\omega'_H(\alpha)$ is greater than $\omega'_L(\alpha)$ only when α is high enough, implying that α' is approximately 0.750427. This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any α, the range of ω for which a limit can have a positive impact is even more restrictive. For any $\alpha > \alpha'$, $\omega'_L(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260199, and $\omega'_H(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260751. At its maximum, the difference between $\omega'_H(\alpha)$ and $\omega'_L(\alpha)$ is approximately 0.00215, meaning that for any value α, only a very narrow range of ω result in the contribution limit being beneficial. Furthermore, because $\omega'_H(\alpha)$ is close to $\frac{1}{4}$, even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected social welfare.

The results have two important implications. First, unlike in the game without wealth differences (Section 3), a contribution limit *can* improve social welfare when there is wealth inequality. For the limit to actually do so may require that a large enough portion of interest groups are poor, and that the contribution limit is high enough (among other possible restrictions). Imposing a low limit, or banning all contributions, has a strictly negative impact on expected social welfare in the simple example above. Second, the result that a contribution limit eliminates the advantage that rich groups tend to have over poor groups does not imply that the limit improves social welfare. This means that the logic behind the popular argument in support of contribution limits is flawed. As the above example illustrates, the contribution limit often reduces welfare even when it eliminates the bias in favor of wealth groups.

5 Other Extensions

The competition for access model developed in this paper is robust to a variety of refinements and complications. This section discusses some of these changes to the model. The discussion focuses on extending the standard model that was presented and analyzed in Sections 2 and 3. Incorporating the extensions into the model of wealth differences presented in Section 4 should not change on the results.

5.1 Endogenous Access

The previous sections assume that the number of interest groups that receive access K is determined exogenously. In this section, I relax this assumption, and allow the politician to choose the amount of access, where $K \in \{0, 1, ..., K_{\max}\}$.[26] The politician commits to K at the beginning of the game, before the interest groups provide contributions.[27]

First, I show that the politician always provides access to some groups ($K \geq 1$). If the politician provides no access ($K = 0$), there is no possibility an interest group gets caught if it signals higher quality information than it actually has. When there is no possibility of receiving access, interest groups with high quality information no longer have an incentive to provide larger contributions than groups with low quality information. Instead, contributions are independent of information quality and the politician can no longer infer anything about a group's information quality by observing its contribution. This eliminates any incentive for the interest groups to provide contributions. Therefore, if the politician chooses no access, he learns *nothing* about the interest groups' information, and receives *no* contributions. By choosing a positive K he receives contributions and

[26] Moldovanu and Sella (2001) consider the optimal choice of K in a traditional all-pay auction in which bidders benefit from receiving a prize (not from the act of bidding, which is the case here).

[27] If the politician chooses K following the contribution decisions of the interest groups, there exists $K_{\max} + 1$ contribution equilibria. In each of these equilibria, the politician becomes fully informed of all interest groups' information quality, and implements the policy profile that maximizes social welfare. When there are contribution limits, the politician is less than fully informed with positive probability. This means that contribution limits strictly reduce expected citizen welfare, independent of which contribution equilibrium is achieved.

learns about group information quality (as I show in the previous sections). Thus, the politician will always commit to providing a positive amount of access.

When there are no contribution limits and interest groups are certain about their information quality, the politician becomes fully informed about the information quality of all interest groups independent of the amount of access (see Section 2). In this case, the amount of access he chooses to provide only impacts total contributions since he becomes fully informed so long as $K \geq 1$. One can show that expected total contributions are strictly decreasing in the number of interest groups that receive access.[28] Therefore the politician chooses the minimum, positive amount of access ($K = 1$), which results in the politician maximizing contributions while becoming fully informed about interest group information quality.

This result is inconsistent with what we observe in reality: politicians provide access to more than one interest group. The result is driven by the simplifying assumptions of the basic model, and is eliminated when there are contribution limits (Section 3), interest groups have different levels of wealth (Section 4), or interest groups are uncertain about their information quality (Section 5.1). When the model makes any of these more realistic assumptions, the politician does not necessarily choose the lowest amount of access. When the politician lowers K, he receives higher contributions, but also learns with certainty the information quality of fewer interest groups. The politician's access decision trades off contributions for information, and it is not possible to determine the equilibrium choice of K without making further assumptions regarding the weight the politician places on contributions relative to social welfare, or the ex ante distribution of information quality. Although we do not know which K the politician will choose in the game with contribution limits, it is likely more than one. Just as in the previous sections, it is straightforward to show that a contribution limit strictly reduces expected social welfare.

Furthermore, when politicians spend time meeting with a greater number of interest groups, they may do so at the cost of spending less time in their home districts meeting with constituents.

[28] Access can be interpreted as the politician's monitoring of interest group contributions in order to limit the groups' ability to overrepresent their true information quality. When an interest group increases its contribution, it benefits from the increase in the information quality the politician believes it has only when it *does not* win access. When the politician reduces the number of groups that receive access, the action increases the probability an interest group does not win access given any contribution, thereby increasing the potential benefit to an interest group from increasing its contribution. This is true for all groups independent of their information quality. Therefore, when the politician reduces the number of interest groups that receive access, all groups increase their contributions, except for those that already provide the maximum contribution amount (in the case of contribution limits). This is also the reason why revenue equivalency (of auctions) does not apply to in a competition for access.

When the politician can choose the amount of access to provide, he may choose to meet with a greater number of interest groups when there are contribution limits compared with when there are no limits. To the extent that this choice reduces the amount of time the politician spends meeting with constituents, contribution limits may further reduce social welfare. However, I do not explicitly model this trade off here.

5.2 Uncertain Information Quality

Up to this point, the analysis assumes that interest groups are certain regarding the quality of their own information. This is not essential. Instead of observing their actual information quality, interest groups could observe signals that correspond to the expected value of their information quality. Under this alternative assumption, in equilibrium, the politician learns an interest groups signal by observing its contribution, but cannot be certain regarding a group's actual information quality with certainty unless he provides the group access. Therefore, contributions lead to the politician having more accurate expectations about the socially optimal policy than he would without contributions. However, society is better off the greater the amount of access that is granted. Similar to the case when the groups know their I values with certainty, contribution limits decrease the expected accuracy of the politician's beliefs, and result in lower expected social welfare than when there are no contribution limits.

5.3 One Interest Group Per Issue

Throughout the paper, I assume that two interest groups are concerned with each issue, where one group prefers policy 1 and one group prefers policy -1. However, there may be issues for which there exist a single interest group that represents one of the extreme policies. In this case, there is no interest group to present information in support of the other extreme policy. For example, consider a revised game in which for any issue n, there is only one interest group, and the politician chooses some policy $p_n^* \in [0, 1]$. The interest group prefers policy 1, and the politician prefers to implement the socially optimal policy $p_n^o = I_n$, where I_n is the interest group's realized information quality. It is straightforward to show that in equilibrium an interest group's contribution function is strictly increasing in its information quality, and that the politician becomes fully informed about the information quality of all interest groups. Imposing a contribution limit strictly reduces expected social welfare, just as it did in the previous sections.

6 Discussion

Past models of lobbying assume that interest groups provide contributions in a quid pro quo exchange for policy favors (e.g., Grossman and Helpman 1994). When this is the case, politicians implement policies preferred by interest groups in exchange for political contributions. Such models suggest that political contributions have a negative impact on social welfare, and that limiting or banning contributions can increase welfare. These models rely on the assumption that politicians are corrupt, or at least willing to choose policies that hurt their constituents in exchange for money. They also tend to ignore the role of access in the lobbying process.

The *competition for access* framework developed in this paper represents a more realistic model of lobbying and political contributions. Interest groups compete for access to a politician by providing contributions. The groups that provide the largest contributions can reveal their private information to the politician in an effort to influence the politician's decision. By observing the contributions of the interest groups in the competition for access, a politician becomes fully informed about the socially optimal course of action, and can implement it in equilibrium. This result has the opposite implications regarding contributions compared with when contributions buy policy favors. This paper's model shows how contribution limits or bans may *strictly* reduce expected social welfare.

This result is not irrefutable evidence that special interests should be allowed to provide unlimited amounts of money to politicians. When I incorporate interest group wealth differences into the model, a contribution limit *can* improve social welfare. Although, in the simple example I consider, a limit is only beneficial for very narrow ranges of the parameter values, this does suggest that a carefully selected limit may be beneficial in some situations. Furthermore, there are likely a number of other factors that impact an interest group's contribution decision, but which I ignore. Future work should incorporate the competition for access framework into more complete models of the political process. For example, in reality, interest groups not only seek access to influence a politician's policy preferences, they also seek access in order to influence the legislative agenda (the issues for which legislation is proposed). Additionally, interest groups often provide political contributions to a politician who already supports their cause, in order to help the politician win an election (Ansolabehere et al., 2002). Also, the model assumes there is only one politician who is already in office, and interest groups make one-time contributions. In reality, there are usually multiple politicians who work together to choose policies, politicians are often biased in favor of one interest group or ex ante policy position over another, and politicians and interest groups often interact on a repeated basis. In order to maintain the generality of the competition for access model, I do not incorporate these complications into the paper. This paper's framework may be expanded in these other directions.

7 Appendix

7.1 Contribution Equilibrium with Limits

Here, I derive the details regarding the contribution equilibrium with limit $\bar{b} < B(1)$, where $B(1)$ is the contribution of an interest group with the highest-possible quality information in the equilibrium without a limit. Let $B_{CL}\left(I;\bar{b}\right)$ describe the equilibrium contribution of an interest group with information quality I, when there is a contribution limit $\bar{b}$. Since $\bar{b} < B(1)$, a group with high-enough I prefers to contribute more than $\bar{b}$ but cannot do so. Instead, it provides the maximum contribution $\bar{b}$. There exists a cut off value $\bar{I}\left(\bar{b}\right) \in [0,1)$ such that for any $I \in \left[\bar{I}\left(\bar{b}\right),1\right]$ an interest group provides contribution $\bar{b}$ in equilibrium. For any $I \in [0, \bar{I}\left(\bar{b}\right))$, an interest group's contribution is strictly increasing in I. There exists a function $\tilde{B}$, where $\tilde{B}\left(0;\bar{b}\right) = 0$ and $\frac{\partial \tilde{B}}{\partial I} > 0$, such that in the contribution equilibrium with limit $\bar{b}$

$$B_{CL}\left(I;\bar{b}\right) = \begin{cases} \tilde{B}\left(I;\bar{b}\right) & \text{for } I \in [0, \bar{I}\left(\bar{b}\right)) \\ \bar{b} & \text{for } I \in \left[\bar{I}\left(\bar{b}\right),1\right] \end{cases}.$$

For the rest of the analysis, I assume $\bar{b}$ is fixed, and ignore its value when writing the fuctions. So, $B_{CL}\left(I;\bar{b}\right) = B_{CL}(I)$ and $\tilde{B}\left(I;\bar{b}\right) = \tilde{B}(I)$. Let $\tilde{I}(b) = \tilde{B}^{-1}(I)$.

In equilibrium, for every issue the politician chooses the policy he expects will maximize social welfare, or $p_n^* = E_\mu I_n^1 - E_\mu I_n^{-1}$ for every n. A complete description of the equilibrium must also describe the politician's beliefs μ. In equilibrium, if interest group (n,j) receives access, then $E_\mu I_n^j = I_n^j$. If (n,j) does not receive access and provides contribution $\bar{b}$, then $E_\mu I_n^j = \frac{1}{1-F\left(\bar{I}\left(\bar{b}\right)\right)} \int_{\bar{I}\left(\bar{b}\right)}^1 f(y)\, y dy$. If (n,j) does not receive access and provides contribution $b_n^j < \bar{b}$, then $E_\mu I_n^j = \tilde{I}\left(b_n^j\right)$.[29]

The function $\tilde{B}$ is derived just as B was derived in the no-limit game. Interest group (n, j) chooses a contribution b to maximize

$$\int_0^1 f_{CL}\left(I_n^{-j}\right)\left(\left(1-\Theta_{CL}\left(\tilde{I}(b);I_n^{-j}\right)\right)V\left(1-\tilde{I}(b)+I_n^{-j}\right)+\Theta_{CL}\left(\tilde{I}(b);I_n^{-j}\right)V\left(1-I_n^j+I_n^{-j}\right)\right)dI_n^{-j}-b. \tag{2}$$

This differs from a group's no-limit maximization problem in that b must now be on the interval $\left[0,\bar{b}\right]$, the functions Θ_{CL} and f_{CL} take into account the fact that all other interest groups with $I \geq \bar{I}\left(\bar{b}\right)$ provide the same contribution $\bar{b}$. The function $\Theta_{CL}\left(I(b);I_n^{-j}\right)$ defines the probability that (n, j) is granted access in equilibrium given that it contributes b. I leave the formal derivation of Θ_{CL} to the reader, as it is not required for the analysis. Note that $\Theta_{CL}\left(I;I_n^{-j}\right)$ is increasing in I for all $I \leq \bar{I}$. In equilibrium, all groups with $I \in \left[\bar{I}, 1\right]$ have the same ex ante probability of being offered access. I denote this probability by $\bar{\Theta}_{CL}$ when nothing is assumed about the information quality of another interest group, and by $\bar{\Theta}_{CL}\left(I^{-j}\right)$ when one other group has information quality I^{-j}. The value $f_{CL}(I)$ denotes the ex ante probability that $E_\mu I_n^{-j} = I$. Therefore,

$$f_{CL}(I)=\begin{cases} f(I) & \text{for } I\in[0,\bar{I}) \\ \bar{\Theta}_{CL}f(I) & \text{for } I\in\left[\bar{I},1\right]\cap I\neq\int_{\bar{I}}^1 f(y)\,y\,dy \\ \left(1-\bar{\Theta}_{CL}\right)\int_{\bar{I}}^1 f(y)\,dy & \text{for } I=\int_{\bar{I}}^1 f(y)\,y\,dy \end{cases}.$$

Using the interest group's above maximization problem, one can solve for $\tilde{B}$ using the technique from Section 3. It follows that

$$\tilde{B}(I)=-\int_0^I\int_0^1 f_{CL}\left(I_n^{-j}\right)\left(\left(1-\Theta_{CL}\left(y;I_n^{-j}\right)\right)V'\left(1-y+I_n^{-j}\right)\right)dI_n^{-j}dy.$$

The cut-off information quality $\bar{I}\left(\bar{b}\right)$ is the information quality at which an interest group is indifferent

[29]Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of b that are on the path of play for any $I \in [0,1]$ (Fudenberg and Tirole,1991, pp. 324-326). The concept of Perfect Bayesian Equilibrium does not restrict beliefs off the equilibrium path of play. For those b that are never played in equilibrium, such assumptions regarding the politician beliefs guarantees that a pure strategy equilibrium exists; other less restrictive beliefs produce the same result.

between contributing according to $\tilde{B}$ and contributing $\bar{b}$, and solves the following equation for $\bar{I}$

$$\int_0^1 f_{CL}\left(I_n^{-j}\right) V\left(1-\bar{I}+I_n^{-j}\right) dI_n^{-j} - \tilde{B}\left(\bar{I}\right)$$

$$= \int_0^1 f_{CL}\left(I_n^{-j}\right) \begin{pmatrix} \left(1-\bar{\Theta}_{CL}\left(I_n^{-j}\right)\right) V\left(1-\frac{1}{1-F(\bar{I})}\int_{\bar{I}}^1 f(y)\, y dy + I_n^{-j}\right) \\ +\bar{\Theta}_{CL}\left(I_n^{-j}\right) V\left(1-\bar{I}+I_n^{-j}\right) \end{pmatrix} dI_n^{-j} - \bar{b}$$

For an interest group with information quality $\bar{I}$, the left hand side of the equality denotes the group's expected utility from providing a contribution according to the increasing equilibrium function $\tilde{B}$, and the right hand side of the equality denotes the expected utility from providing the maximum contribution. It follows that $\tilde{B}\left(\bar{I}\left(\bar{b}\right)\right) < \bar{b}$. In equilibrium, the group with the cut-off information quality is indifferent between the two actions. If the solution to this equality is negative, then $\bar{I}\left(\bar{b}\right) = 0$ and all interest groups contribute $\bar{b}$ independent of their information qualities.

Next, I show that interest groups that do not have an incentive to deviate from the contribution function B_{CL}. For an interest group with $I < \tilde{I}\left(\bar{b}\right)$, providing $\bar{b}$ rather than $b < \bar{b}$ gives an expected benefit of

$$\begin{aligned} &\int_0^1 f_{CL}\left(I_n^{-j}\right) \begin{pmatrix} \left(1-\bar{\Theta}_{CL}\left(I_n^{-j}\right)\right) V\left(1-\frac{1}{1-F(\bar{I})}\int_{\bar{I}}^1 f(y)\, y dy + I_n^{-j}\right) \\ +\bar{\Theta}_{CL}\left(I_n^{-j}\right) V\left(1-I+I_n^{-j}\right) \end{pmatrix} dI_n^{-j} \\ &-\bar{b} - \left(\int_0^1 f_{CL}\left(I_n^{-j}\right) V\left(1-I+I_n^{-j}\right) dI_n^{-j} - \tilde{B}\left(I\right)\right) \end{aligned}$$

which is strictly increasing in an interest groups information quality. To see this, take the derivative of the benefit with respect to I.

$$\int_0^1 f_{CL}\left(I_n^{-j}\right)\left(1-\bar{\Theta}_{CL}\left(I_n^{-j}\right)\right) V'\left(1-I+I_n^{-j}\right) dI_n^{-j} + \frac{\partial \tilde{B}\left(I\right)}{\partial I}$$

Since $\bar{\Theta}_{CL}\left(I_n^{-j}\right) - \Theta_{CL}\left(I; I_n^{-j}\right)$ for any $I < \tilde{I}\left(\bar{b}\right)$, simplifying gives

$$-\int_0^1 f_{CL}\left(I_n^{-j}\right)\left[\bar{\Theta}_{CL}\left(I_n^{-j}\right) - \Theta_{CL}\left(I; I_n^{-j}\right)\right] V'\left(1-I+I_n^{-j}\right) dI_n^{-j} > 0.$$

From the derivation of $\tilde{B}$, we already know that an interest group with $I < \bar{I}\left(\bar{b}\right)$ prefers to provide $\tilde{B}\left(I\right)$ rather than any other $b < \bar{b}$.

From the derivation of $\tilde{B}$, it also follows that any group with $I \geq \tilde{I}\left(\bar{b}\right)$ prefers to provide $b \to \bar{b}$ instead of any other $b < \bar{b}$. The expected benefit to an interest group with $I > \tilde{I}\left(\bar{b}\right)$ from giving $\bar{b}$ instead of $b < \bar{b}$ is

$$\lim_{b \to \bar{b}} \int_0^1 f_{CL}\left(I_n^{-j}\right) \left(\begin{array}{c} \left(1 - \bar{\Theta}_{CL}\left(I_n^{-j}\right)\right) V\left(1 - \frac{1}{1-F(\bar{I})} \int_{\bar{I}}^1 f\left(y\right) y dy + I_n^{-j}\right) \\ + \bar{\Theta}_{CL}\left(I_n^{-j}\right) V\left(1 - I + I_n^{-j}\right) \end{array} \right) dI_n^{-j} - \bar{b}$$
$$- \int_0^1 f_{CL}\left(I_n^{-j}\right) \left(\begin{array}{c} \left(1 - \Theta_{CL}\left(\tilde{I}\left(b\right); I_n^{-j}\right)\right) V\left(1 - V\left(1 - \tilde{I}\left(b\right) + I_n^{-j}\right) + I_n^{-j}\right) \\ + \Theta_{CL}\left(\tilde{I}\left(b\right); I_n^{-j}\right) V\left(1 - I + I_n^{-j}\right) \end{array} \right) dI_n^{-j} + b$$

which is strictly increasing in the group's I. To see this, take the derivative with respect to I:

$$\lim_{b \to \bar{b}} - \int_0^1 f_{CL}\left(I_n^{-j}\right) \left[\bar{\Theta}_{CL}\left(I_n^{-j}\right) - \Theta_{CL}\left(\tilde{I}\left(b\right); I_n^{-j}\right)\right] V'\left(1 - I + I_n^{-j}\right) dI_n^{-j} > 0.$$

Since the benefit of providing $\bar{b}$ rather than $b < \bar{b}$ is strictly increasing in a groups information quality, and $\bar{I}\left(\bar{b}\right)$ is the information quality at which a group is indifferent between the two actions, it follows that interest groups with $I < \bar{I}\left(\bar{b}\right)$ strictly prefer to contribute according to $\tilde{B}$, and groups with $I > \bar{I}\left(\bar{b}\right)$ strictly prefer to contribute $\bar{b}$.

7.2 Contribution Equilibrium with Wealth Differences

In this section, I derive the contribution equilibrium of the model presented in Section 5. Poor interest groups have wealth constraint such that their contributions must be no greater than ω, and rich group contributions are unconstrained. Assume $\omega < B\left(1\right)$. A poor group with high-enough I prefers to contribute more than ω but cannot do so. Instead, it provides ω.

The functions B_P and B_R respectively denote the equilibrium poor and rich group contribution functions. There exists cut off values $\bar{I}_a \geq 0$ and $\bar{I}_b \in \left(\bar{I}_a, 1\right)$, and functions B_a and B_b, where $B_a\left(0; \omega\right) = 0$,

$B_b\left(\bar{I}_b;\omega\right)=0$, $\frac{\partial B_a}{\partial I}>0$, and $\frac{\partial B_b}{\partial I}>0$ such that

$$B_P\left(I;\omega\right)=\begin{cases} B_a\left(I;\omega\right) & \text{for } I\in[0,\bar{I}_b) \\ \omega & \text{for } I\in[\bar{I}_a,1] \end{cases}$$

$$B_R\left(I;\omega\right)=\begin{cases} B_a\left(I;\omega\right) & \text{for } I\in[0,\bar{I}_a) \\ \omega & \text{for } I\in[\bar{I}_a,\bar{I}_b) \\ \omega+B_b\left(I;\omega\right) & \text{for } I\in[\bar{I}_b,1] \end{cases}$$

The cut off values are dependent on poor group wealth ω, and the probability that a group is poor, α. Also, through the rest of the analysis, I take ω as fixed and ignore it in writing the contribution functions $B_P\left(I\right)$, $B_R\left(I\right)$, $B_a\left(I\right)$, and $B_b\left(I\right)$. Let $I_a\left(b\right)=B_a^{-1}\left(I\right)$ and $I_b\left(b\right)=B_b^{-1}\left(I\right)$.

In equilibrium, the politician chooses the policy profile he expects will maximize social welfare, or $p_n^*=E_\mu I_n^1-E_\mu I_n^{-1}$ for every n. A complete description of the equilibrium must also describe the politician's beliefs μ. In equilibrium, if interest group (n,j) receives access, then $E_\mu I_n^j=I_n^j$. If (n,j) does not receive access and provides contribution ω, then

$$E_\mu I_n^j=E\left(I\mid\omega\right)=\frac{\left(1-\alpha\right)\int_{\bar{I}_a}^1 f\left(y\right)ydy+\alpha\int_{\bar{I}_a}^{\bar{I}_b} f\left(y\right)ydy}{\left(1-\alpha\right)\left(1-F\left(\bar{I}_a\right)\right)+\alpha\left(F\left(\bar{I}_b\right)-F\left(\bar{I}_a\right)\right)}.$$

If (n,j) does not receive access and provides contribution then $E_\mu I_n^j=I_a\left(b_n^j\right)$ if $b_n^j<\omega$, and $E_\mu I_n^j=I_b\left(b_n^j\right)$ if $b_n^j>\omega$.[30]

The function B_a is derived just as B was derived in the no-limit game. Interest group (n,j) chooses a contribution b to maximize

$$\int_0^1 f_{WL}\left(I_n^{-j}\right)\left(\left(1-\Theta_{WL}\left(I_a\left(b\right);I_n^{-j}\right)\right)V\left(1-I_a\left(b\right)+I_n^{-j}\right)+\Theta_{WL}\left(I_a\left(b\right);I_n^{-j}\right)V\left(1-I_n^j+I_n^{-j}\right)\right)dI_n^{-j}-b.$$

This differs from the maximization problem in Section 2 since Θ_{WL} and f_{WL} take into account the fact that all other interest groups contribute according the the contribution function described above. The value $\Theta_{WL}\left(I\left(b\right);I_n^{-j}\right)$ defines the probability that (n,j) is granted access in equilibrium given that it

[30] Such a belief system is required by the concept of Perfect Bayesian Equilibrium for all values of b that are on the path of play for any $I\in[0,1]$. For those b that are never played in equilibrium, such assumptions regarding the politician beliefs guarantees that a pure strategy equilibrium exists; other less restrictive beliefs produce the same results.

contributes b. In equilibrium, all groups that provide contribution ω have the same ex ante probability of being offered access. I denote this probability by $\bar{\Theta}_{WL}$ when nothing is assumed about the information quality of another interest group, and by $\bar{\Theta}_{WL}\left(I^{-j}\right)$ when one other group has information quality I^{-j}. The value $f_{WL}\left(I\right)$ denotes the ex ante probability that $E_{\mu}I_n^{-j} = I$.

Using the interest group's above maximization problem, one can solve for $\tilde{B}$ using the technique from Section 2. It follows that

$$B_a\left(I\right) = -\int_0^I \int_0^1 f_{WL}\left(I_n^{-j}\right)\left(\left(1-\Theta_{WL}\left(y; I_n^{-j}\right)\right) V'\left(1-y+I_n^{-j}\right)\right) dI_n^{-j} dy.$$

Similarly, B_b is derived from the maximization problem of an interest group with $I > \bar{I}_b$.

$$B_b\left(I\right) = -\int_{\bar{I}_b}^I \int_0^1 f_{WL}\left(I_n^{-j}\right)\left(\left(1-\Theta_{WL}\left(y; I_n^{-j}\right)\right) V'\left(1-y+I_n^{-j}\right)\right) dI_n^{-j} dy.$$

The cut-off information quality $\bar{I}_a$ is the information quality at which an interest group is indifferent between contributing according to B_a and contributing ω. The equilibrium value of $\bar{I}_a$ solves the following equation for $\bar{I}$

$$\int_0^1 f_{WL}\left(I_n^{-j}\right) V\left(1-\bar{I}+I_n^{-j}\right) dI_n^{-j} - B_a\left(\bar{I}\right) =$$

$$\int_0^1 f_{WL}\left(I_n^{-j}\right)\left(\begin{array}{c}\left(1-\bar{\Theta}_{WL}\left(I_n^{-j}\right)\right) V\left(1-I_{WL}\left(\omega\right)+I_n^{-j}\right) \\ +\bar{\Theta}_{WL}\left(I_n^{-j}\right) V\left(1-\bar{I}+I_n^{-j}\right)\end{array}\right) dI_n^{-j} - \omega$$

where $I_{WL}\left(\omega\right)$ denotes the politicians equilibrium expectations about the information quality of an interest group that provides ω and does not receive access. If the solution to the equality is negative, then $\bar{I}_a = 0$ and all poor interest groups contribute ω independent of their information qualities.

The cut-off information quality $\bar{I}_b$ is the information quality at which an interest group is indifferent between contributing according to B_b and contributing ω. In equilibrium,

$$\bar{I}_b = E\left(I \mid \omega\right) = \frac{\left(1-\alpha\right)\int_{\bar{I}_a}^1 f\left(y\right) y dy + \alpha \int_{\bar{I}_a}^{\bar{I}_b} f\left(y\right) y dy}{\left(1-\alpha\right)\left(1-F\left(\bar{I}_a\right)\right) + \alpha\left(F\left(\bar{I}_b\right) - F\left(\bar{I}_a\right)\right)}.$$

Just as I do in the previous section of the appendix where I derive the equilibrium for the case with contribution limits, it is straightforward to show that none of the interest groups have an incentive to deviate in the equilibrium. Groups with information quality $I > \bar{I}_b$ strictly prefer to contribute $\omega + B_b(I)$ compared to ω, and ω compared to any amount less than ω. Groups with $I > \bar{I}_a$ strictly prefer to provide ω than any amount less than ω, and those with $I \in \left(\bar{I}_a, \bar{I}_b\right)$ strictly prefer to provide ω compared with any amount greater than or less than ω. Similarly, groups with $I < \bar{I}_a$ strictly prefer to provide $B_a(I)$ to any other contribution.

With Contribution Limits

Using the techniques developed earlier in this paper, one can incorporate contribution limits into this game. I describe the equilibrium contribution function here, but leave the details of the derivation to the reader.

The paper focuses on the case where $\bar{b} \leq \omega$, which is consistent with the contribution limits proposed in the policy debate. In this case, the contribution limit has the same impact on the equilibrium contribution functions as the limit would have in the game without wealth differences. See Sections 3 and 7.1 for a derivation and illustration.

Although the body of the paper does not discuss the other cases, I describe them here. When $\bar{b} > \omega$, there are two cases. First, if the limit is high-enough, then the contribution limit impacts function B_b in the same manner that the limit impacted B in the case without wealth differences. In this case, rich interest groups with $I > E(I \mid \omega) = \bar{I}_b$ contribute $\omega + B_b(I)$ for relatively low I (as $I \longrightarrow \bar{I}_b$) and contribute $\bar{b}$ for higher I (as $I \longrightarrow 1$). The contributions of the poor interest groups are not impacted. This case is illustrated in Figure 3.1. Second, if $\bar{b} > \omega$ and the limit is relatively close to ω, then there exists a function $\tilde{B}_a$ that is strictly increasing in I and where $\tilde{B}(0) = 0$, and values $\bar{I}, \bar{I}'_b \in (0,1)$ such that when (n,j) is rich, $b^j_n = \tilde{B}\left(I^j_n\right)$ for $I^j_n \in [0, \bar{I})$, $b^j_n = \omega$ for $I^j_n \in [\bar{I}, \bar{I}'_b)$, and $b^j_n = \bar{b}$ for $I^j_n \in \left[\bar{I}'_b, 1\right]$, and when (n,j) is poor, $b^j_n = \tilde{B}\left(I^j_n\right)$ for $I^j_n \in [0, \bar{I}'_a)$, $b^j_n = \omega$ for $I^j_n \in \left[\bar{I}'_a, 1\right]$. It can be shown that $\bar{I} < \bar{I}'_b < \bar{I}_b < E(I \mid \omega)$, where $\bar{I}_a$ and $\bar{I}_b$ are the cutoff values when there is no contribution limit, and $E(I \mid \omega)$ is the politician's expectations about an interest group's information quality when that group contributes ω but does not receive access. This case is illustrated by Figure 3.2.

Impact of limits on welfare in a simple game

Here, I solve the simple example from Section 4.3. Information quality is uniformly distributed on $[0,1]$, interest group policy utility is linear, or $V\left(1 - I^j + I^{-j}\right) = -\left(1 - I^j + I^{-j}\right)$, and both $N = 1$ and $K = 1$. Using the methods described previously, it is straightforward to solve for the game's equilibrium.

For the case when there is no contribution limit. First, one can find that $B_a(I) = I - \frac{1}{2}I^2$, and calculate the expected payoff to an interest group from contributing less than ω, which equals $\frac{1}{2}I^2 - \frac{3}{2}$. Note that ω must be less than $\frac{1}{2}$ to have an impact on contributions. If the same group contributed ω instead, its expected payoff equals

$$\begin{aligned} -\int_{\bar{I}_b}^{1} \left((1-\alpha)\left(1-\bar{I}_b+I'\right) + \alpha\left(\frac{1}{2}\left(1-I+\bar{I}_b\right) + \frac{1}{2}\left(1-\bar{I}_b+I'\right)\right)\right) dI' \\ -\int_{\bar{I}_a}^{\bar{I}_b} \left(\frac{1}{2}\left(1-I+\bar{I}_b\right) + \frac{1}{2}\left(1-\bar{I}_b+I'\right)\right) dI' - \int_0^{\bar{I}_a} \left(1-I+I'\right) dI' - \omega. \end{aligned}$$

This simplifies to

$$-\frac{3}{2} - \frac{1}{4}\bar{I}_a^2 + (1-\alpha)\left(\bar{I}_b - \frac{3}{4}\bar{I}_b^2\right) + \frac{1}{4}\alpha + \frac{1}{2}I\left(\bar{I}_a + \alpha + \bar{I}_b(1-\alpha)\right) - \omega.$$

Because $\bar{I}_b = E_\mu(I \mid \omega)$ it follows that $\bar{I}_b = \frac{1+\bar{I}_a}{1+\alpha}$. When an interest group has $I = \bar{I}_a$, these expected payoffs from contributing ω equals the expected payoff from giving $B_a(I)$. Therefore, one can solve for $\bar{I}_a$, where

$$\bar{I}_a = 1 - \frac{(1+\alpha)\sqrt{2(1-2\omega)}}{\sqrt{2-\alpha+3\alpha^2}}$$

when $1 - (1+\alpha)\sqrt{2(1-2\omega)}/\sqrt{2-\alpha+3\alpha^2} \geq 0$. If the expression is negative, $\bar{I}_a = 0$. Solving for $B_b(I)$ is not required for the results.

When there is a contribution limit, $\tilde{B}(I) = B_a(I)$.[31] Since rich and poor groups give b for the same range of I, it follows that $\bar{I}'_b = \frac{1+\bar{I}}{2}$, where $\bar{I}'_b = E\left(I \mid \bar{b}\right)$. Solving for $\bar{I}$ in the same way that $\bar{I}_a$ was found above gives

$$\bar{I} = 1 - \frac{2\sqrt{1-\bar{b}}}{\sqrt{3}}$$

when $1 - \left(2\sqrt{1-\bar{b}}\right)/\sqrt{3} \geq 0$, and $\bar{I} = 0$ otherwise.

[31] This is true because V is linear.

When there is no contribution limit, expected social welfare is

$$-\alpha^2 \int_{\bar{I}_a}^{1}\int_{\bar{I}_a}^{1} \left|I_1 - \bar{I}_b\right| dI_1 dI_2 - 2\alpha\left(1-\alpha\right)\int_{\bar{I}_a}^{\bar{I}_b}\int_{\bar{I}_a}^{1}\left(\frac{1}{2}\left|I_1-\bar{I}_b\right| + \frac{1}{2}\left(\left|\bar{I}_b - I_2\right|\right)\right) dI_1 dI_2$$
$$- 2\alpha\left(1-\alpha\right)\int_{\bar{I}_b}^{1}\int_{\bar{I}_a}^{\bar{I}_b}\left|I_1-\bar{I}_b\right| dI_1 dI_2 - \left(1-\alpha\right)^2 \int_{\bar{I}_a}^{\bar{I}_b}\left(\int_{\bar{I}_a}^{\bar{I}_b}\left(\bar{I}_b - I_1\right) dI_1 + \int_{\bar{I}_b}^{1}\left(\bar{I}_b - I_2\right) dI_1\right) dI_2$$
$$- \left(1-\alpha\right)^2 \int_{\bar{I}_b}^{1}\int_{\bar{I}_a}^{\bar{I}_b}\left(\bar{I}_b - I_1\right) dI_1 dI_2.$$

Substituting in for $\bar{I}_b$ and $\bar{I}_a$, when $1-\left(1+\alpha\right)\sqrt{2\left(1-2\omega\right)}/\sqrt{2-\alpha+3\alpha^2} > 0$ expected social welfare simplifies to

$$EW_{\text{no limit}} = -\frac{2\alpha\left(1-2\omega\right)\left(1+\alpha\right)\sqrt{2\left(2-\alpha+3\alpha^2\right)\left(1-2\omega\right)}}{\left(2-\alpha+3\alpha^2\right)^2}. \tag{3}$$

If $1-\left(1+\alpha\right)\sqrt{2\left(1-2\omega\right)}/\sqrt{2-\alpha+3\alpha^2} \leq 0$, then $EW_{\text{no limit}} = -\frac{\alpha}{(1+\alpha)^2}$.

When there is a contribution limit, expected social welfare is $-\int_{\bar{I}}^{1}\int_{\bar{I}}^{1}\left|I_1 - \bar{I}'_b\right| dI_1 dI_2$. When $1-\left(2\sqrt{1-\bar{b}}\right)/\sqrt{3} > 0$,

$$EW_{\text{limit}} = -\frac{2\left(1-\omega\right)^{2/3}}{3\sqrt{3}}.$$

If $1-\left(2\sqrt{1-\bar{b}}\right)/\sqrt{3} \leq 0$, then $EW_{\text{ limit}} = -\frac{1}{4}$.

When both $\bar{I}_a$ and $\bar{I}$ are 0, the contribution limit never improves expected social welfare since $EW_{\text{no limit}} = -\frac{\alpha}{(1+\alpha)^2} > EW_{\text{ limit}} = -\frac{1}{4}$ for $\alpha \in (0,1)$. Similarly, one can show that when $\bar{I}_a > 0$ and $\bar{I} = 0$, the contribution limit cannot improve welfare since $-\frac{2\alpha(1-2\omega)(1+\alpha)\sqrt{2(2-\alpha+3\alpha^2)(1-2\omega)}}{(2-\alpha+3\alpha^2)^2} > -\frac{1}{4}$. There are no parameter values for which $\bar{I}_a = 0$ and $\bar{I} > 0$.

When $1-\left(1+\alpha\right)\sqrt{2\left(1-2\omega\right)}/\sqrt{2-\alpha+3\alpha^2} > 0$ and $1-\left(2\sqrt{1-\bar{b}}\right)/\sqrt{3} > 0$, both $\bar{I}_a$ and $\bar{I}$ are positive. In this case the contribution limit improves expected social welfare when

$$-\frac{2\left(1-\omega\right)^{2/3}}{3\sqrt{3}} > -\frac{2\alpha\left(1-2\omega\right)\left(1+\alpha\right)\sqrt{2\left(2-\alpha+3\alpha^2\right)\left(1-2\omega\right)}}{\left(2-\alpha+3\alpha^2\right)^2}.$$

Using Mathematica, I solve for the numerical values for which all three of these conditions as well as the priors on α, ω, and $\bar{b}$ are simultaneously met. There exists cut off values α', $\omega'_L\left(\alpha\right)$, and $\omega'_H\left(\alpha\right)$ such that contribution limit $\bar{b} \leq \omega$ improves expected social welfare if and only if $\alpha \in \left(\alpha', 1\right)$, $\omega \in \left[\omega'_L\left(\alpha\right), \omega'_H\left(\alpha\right)\right)$, and $\bar{b} \in \left[\frac{1}{4}, \omega\right]$. The values $\omega'_L\left(\alpha\right)$ and $\omega'_H\left(\alpha\right)$ depend on the value of parameter α, and determining the values α' and $\omega'_H\left(\alpha\right)$ require the calculating the root of high-degree polynomials for which a non-numeric solution is not possible. To overcome this issue, I use Mathematica to numerically determine the cut off values. $\omega'_H(\alpha)$ is greater than $\omega'_L(\alpha)$ only when α is high enough, implying that α' is approximately 0.750427. This means that most interest groups must be poor in order for a contribution limit to potentially improve expected welfare. Additionally, given any α, the range of ω for which a limit can have a positive impact is even more restrictive. For any $\alpha > \alpha'$, $\omega'_L(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260199, and $\omega'_H(\alpha)$ takes on values between $\frac{1}{4}$ and 0.260751. At its maximum, the difference between $\omega'_H(\alpha)$ and $\omega'_L(\alpha)$ is approximately 0.00215, meaning that for any value α, only very specific values of ω result in the contribution limit being beneficial. Furthermore, because $\omega'_H(\alpha)$ is close to $\frac{1}{4}$, even when the other conditions are met, there only exists a small range of contribution limits that benefit society. Whenever the above conditions do not hold, the contribution limit strictly reduces expected social welfare.

7.3 Proofs

Proof (Lemma 1). Follows immediately from analysis in paper. ■

Proof (Proposition 1). Follows immediately from analysis in paper and Lemma 1. ■

Proof (Proposition 2). Where $W(p^*, p^o) = -\sum_{n=1}^{N} \gamma_n \times |p^* - p^o|$, let $w_n = -\gamma_n \times |p^*_n - p^o_n|$ for each n. When there are no contribution limits, the politician chooses $p^*_n = p^o_n$ for all n, and social welfare $W = 0$, and $w_n = 0$ for all n.

Consider the case when there are contribution limits. The parameter M denotes the realized number of interest groups that contribute $\bar{b}$ in equilibrium. The ex ante probability of any $M \in \{0, 1, ..., 2N\}$ equals $\varphi(M) = \frac{2N!}{2N-M!M!} F(\bar{I})^{2N-M} \left(1 - F(\bar{I})\right)^{M}$. Therefore, $M > K$ with probability $\sum_{m=K+1}^{2N} \varphi(m)$. For each $m \in \{K+1, ..., 2N\}$, $\varphi(m) > 0$ since $\bar{I} \in (0,1)$ and $F(\bar{I}) \in (0,1)$ for any $\bar{b} \in (0, B(1))$. Thus, $\sum_{m=K+1}^{2N} \varphi(m) > 0$, the politician is less than fully informed with positive probability. With probability $\sum_{m=K+1}^{2N} \varphi(m) > 0$ there exists at least one interest group for which the politician knows $I^j_n \in [\bar{I}, 1]$, but does not know I^j_n when he chooses a policy profile. Without loss of generality assume this is group $(1,1)$. The politician implements policy $p^*_1 = E_\mu I^1_1 - E_\mu I^{-1}_1$, where $E_\mu I^1_1 = \int_{\bar{I}}^{1} I \frac{f(I)}{1-F(\bar{I})} dI$. Given the continuous distribution of I, both $E_\mu I^1_1 \neq I^1_1$ and $E_\mu I^1_1 - E_\mu I^{-1}_1 \neq I^1_1 - I^{-1}_1$ with probability one. Social welfare attributable to issue 1 is $w_1 = -\gamma_n \times \left| E_\mu I^1_1 - E_\mu I^{-1}_1 - I^1_1 + I^{-1}_1 \right|$. Since $E_\mu I^1_1 - E_\mu I^{-1}_1 \neq I^1_1 - I^{-1}_1$ with probability one, it follows that $w_1 < 0$. For all other issues $m \in \{2, ..., N\}$, $w_m \leq 0$, and $W(p^*, p^o) = \sum_{n=1}^{N} w_n < 0$.

When there are contribution limits, $W(p^*, p^o) < 0$ with probability $\sum_{m=K+1}^{2N} \varphi(m) > 0$, and $W(p^*, p^o) = 0$ with probability $1 - \sum_{m=K+1}^{2N} \varphi(m)$. Therefore, expected social welfare is strictly negative and therefore strictly less than welfare when there are no contribution limits. ■

Proof (Lemma 3). See Section 7.2 for derivation of the equilibrium. Assume $j = 1$, and $I = I_n^1 = I_n^{-1}$. Consider three cases: $I \in [0, \bar{I}_a)$, $I \in [\bar{I}_a, \bar{I}_b]$, and $I \in (\bar{I}_b, 1]$. If $I \in [0, \bar{I}_a)$, then $b_n^1 = b_n^{-1} = B_a(I)$, and $p_n^* = I - I = 0$. So, when $I \in [0, \bar{I}_a)$, $Ep_n^* = 0$.

If $I \in [\bar{I}_a, \bar{I}_b]$, then $b_n^1 = b_n^{-1} = \omega$. Let $\bar{\Theta}_2$ be the equilibrium probability an interest group that provides ω receives access when $b_n^1 = b_n^{-1} = \omega$. Therefore, when $I \in [\bar{I}_a, \bar{I}_b]$, $Ep_n^* = \bar{\Theta}_2 I + (1 - \bar{\Theta}_2)\bar{I}_b - \bar{\Theta}_2 I - (1 - \bar{\Theta}_2)\bar{I}_b = 0$.

If $I \in (\bar{I}_b, 1]$, then $b_n^1 > \omega$ and $b_n^{-1} = \omega$. Let $\bar{\Theta}_1$ be the equilibrium probability an interest group that provides ω receives access when $b_n^1 > \omega$ and $b_n^{-1} = \omega$. Therefore, when $I \in (\bar{I}_b, 1]$, $Ep_n^* = I - \bar{\Theta}_1 I - (1 - \bar{\Theta}_1)\bar{I}_b = (I - \bar{I}_b)(1 - \bar{\Theta}_1) > 0$.

For all values of I, $Ep_n^* = (1 - F(\bar{I}_b))(I - \bar{I}_b)(1 - \bar{\Theta}_1) > 0$. If $j = -1$, the same method shows that $Ep_n^* < 0$. ■

Proof (Proposition 3). See Section 7.2 for derivation of the equilibrium. Because $B_P(I) = B_R(I)$ for any $I \leq \bar{I}_b$, it follows that b_n^j is independent of (n, j)'s wealth when $I_n^j \in [0, \bar{I}_b]$. Given b_n^j, the probability $\Theta_{WL}\left(I\left(b_n^j\right), I_n^{-j}\right)$ is independent of (n, j)'s wealth. Therefore, if $I_n^j \in [0, \bar{I}_b]$, then (n, j) faces the same expected payoff EU_n^j independent its wealth.

Alternatively, if $I \in (\bar{I}_b, 1]$, then $B_R(I) > B_P(I)$. Since $B_R\left(I_n^j\right) = \arg\max_b EU_n^j(b)$ and $B_P\left(I_n^j\right) = \omega$ is a feasible contribution for a rich group, it follows that $EU_n^j\left(B_R\left(I_n^j\right)\right) > EU_n^j\left(B_P\left(I_n^j\right)\right)$. Therefore, when (n, j) is rich, EU_n^j is the same as if he was poor when $I_n^j \in [0, \bar{I}_b]$, and EU_n^j is strictly greater than if he was poor when $I \in (\bar{I}_b, 1]$. ■

Proof (Proposition 4). Notation is consistent with Section 7.2. Let $\bar{\Theta}_2$ be the equilibrium probability that (n, j) receives access if $b_n^j = b_n^{-j} = \bar{b}$, and let $\bar{\Theta}_1$ be the equilibrium probability that (n, j) receives access if $b_n^j = \bar{b}$ and $b_n^{-j} < \bar{b}$. Therefore,

$$
\begin{aligned}
Ep_n^* = & \int_0^{\bar{I}} \int_0^{\bar{I}} \left(I^1 - I^{-1}\right) dI^{-1} dI^1 + \int_0^{\bar{I}} \int_{\bar{I}}^1 \left(I^1 - \bar{\Theta}_1 I^{-1} - \left(1 - \bar{\Theta}_1\right) \int_{\bar{I}}^1 f(I)\, I dI\right) dI^{-1} dI^1 \\
& + \int_{\bar{I}}^1 \int_0^{\bar{I}} \left(\bar{\Theta}_1 I^1 + \left(1 - \bar{\Theta}_1\right) \int_{\bar{I}}^1 f(I)\, I dI - I^{-1}\right) dI^{-1} dI^1 \\
& + \int_{\bar{I}}^1 \int_{\bar{I}}^1 \left(\bar{\Theta}_2 I^1 + \left(1 - \bar{\Theta}_2\right) \int_{\bar{I}}^1 f(I)\, I dI - \bar{\Theta}_2 I^{-1} - \left(1 - \bar{\Theta}_2\right) \int_{\bar{I}}^1 f(I)\, I dI\right) dI^{-1} dI^1
\end{aligned}
$$

which reduces to $Ep_n^* = 0$.

Rich and poor groups both contribute to B_{CL}. Because a group's contribution is independent of wealth for any I, it follows that the realization of Θ_{CL} is independent of wealth. Therefore EU_n^j is independent of wealth. ■

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