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Corchon, Luis

Universidad de Alicante - Dep.to Fundamento del Analisis Economico

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Luis CORCHON

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Coordinan: Carmen Herrero
Antonio Villar

REASONABLE CONJECTURES AND THE KINKED DEMAND CURVE(*)

Luis C. Corchon

ABSTRACT.- In this paper we will study the allocations resulting from kinked demand curves. We will see that there are several dificulties with this concept. Some of them, though, can be eliminated by the consideration of reasonable conjectures.

(*) This paper is based upon Chapter 2 of my Doctoral Dissertation. I am indebted with my supervisor, Oliver Hart for his guidance, and with Jose Trujillo for many stimulating discussions on this topic. Financial support from Banco de España is gratefully acknowlwdged. All errors are my own responsibility.

I. INTRODUCTION

The idea of conjectural equilibrium was introduced in economics by Bowley in 1924. Later contributions in the thirties came from Chamberlain (1933, ch. 3), Stackelberg (1934), Swezy (1939) and Hall & Hitch (1939) (see also Robinson (1933, p. 38) and Hicks (1935)). Any of these papers focusses attention on a particular form of the conjecture and derives some conclusions from this assumption. Their models were partial equilibrium models.

Modern contributions were made by Negishi (1961) and Hahn (1978) in a general equilibrium framework. Consistency (or rationality) of the conjectures was introduced as an attempt at narrowing down the class of admisible conjectures. See Hahn (1978), Bresnahan (1980), Kamien & Schwartz (1980), Laitner (1980), Perry (1982) and Uloh (1981) (1982). See Hart (1983) for an overview on some of these contributions.

Finally, recent contributions are Boyer & Moreaux (1982) showing that conjectures are indetermined even if they are required to be consistent; Hart (1982), who works out the notion of reasonable conjectures, and Trujillo (1983), solving some problems associated with Hahn's original approach.

In this paper attention will be focused on kinked demand curves (or conjectures) and reasonable conjectures. More specifically, my goals are:

1. To study the consequences (on prices, allocations, etc.) of assuming that conjectures are kinked.

2. To offer a refinement of the idea of kinked conjectures. This refinement is based on the consideration of **reasonable** kinked conjectures.

The more ambitious question of why conjectures are kinked is not explored in this paper. I just will mention that this class of conjectures are meant to capture price retaliation of some kind (see Hicks (1935)). For example, in markets in which the reaction of competitors is uncertain and firms are risk averse, they expect the worst to happen; if they raise prices, the others will keep theirs (i.e., competitors are price setters). However, if firms decrease prices, competitors will match them (i.e., competitors are quantity setters, for example). In this way, if every firm is uncertain about the behaviour of the competitors and it is risk averse, then kinked demand curves appear as quite natural.

Being kinked conjectures very popular among economists (see Stigler (1978)[1]), it is rather surprising that no formal analysis of the kinked demand curve has been made, as far as I know. The unfortunate consequence of this is that the theory of the kinked demand curve has been misunderstood, and that the so called empirical tests of the theory bears no relationship with the theory at all (compare Stigler (1978) and our Propositions 1,2,3 and 9)).

We present a standard model of oligopoly with differentiated products and apply to it the notion of conjectural equilibrium when conjectures are assumed to be kinked. Our results cast some doubts about the apparent rationality of the kinked demand curve. In particular, we find the following:

1. Kinked conjectures are not easy to handle, since they should be not differentiable.

2. They are not consistent even in a quite moderate sense.

3. They produce too many equilibria. However, a complete characterization of the region in which every possible equilibrium must lie is possible. Hence the theory has some predictive power. Therefore, the "Folk Theorem", which asserts that "the kink may arise everywhere" (i.e., that the kinked demand curve does not have any empirical implication), is false.

4. They are not necessarily good as a coordination device.

5. The Bertrand Nash equilibrium (i.e., a Nash equilibrium in which prices are strategies) is always obtainable as a conjectural equilibrium with kinked conjectures, and there is no way to guarantee that some conjectural equilibrium with

kinked conjectures different from that of Bertrand-Nash exists. However, other solution concepts (price equal to marginal costs, Baumol, limit pricing and sometimes Stackelberg equilibria) give different allocations.

Some comments on 1-5 are in order. The fact that conjectures are not consistent, cannot be taken as a decisive argument against them, as long as they are reasonable (see Hart (1982, p.2)). Point 5 is meant as a reaction against the classical presentation of oligopoly theory: i.e., a verbal discussion and a collection of models without any formal study of their properties and compatibility.

Introducing the idea of reasonable conjectures we see that criticisms (3), (4) and (5) are considerable weakened: the equilibrium set is reduced, the Bertrand-Nash allocation is never an equilibrium and profits are at least as high as any firm could get if it were a Stackelberg leader (all our results for reasonable conjectures are proved for the duopoly case)[e].

In addition:

a. Our "equilibrium-reasonable-conjectures" are also valid if we consider a space of conjectures which is totally general. Hence our results can be visualized as a first step in generalizing the ones obtained by Hart for an homogeneous oligopoly model[a].

b. Our reasonable conjectures are not consistent in the sense mentioned above. Since Hart has proved that under differentiability and interiority conditions any reasonable conjecture is consistent, kinked conjectures are a counterexample to this proposition when differentiability does not hold (recall point 1 before).

c. Our construction gives a new interpretation of the Stackelberg "leader-follower" model, since this is the limit case of the allocations achievable by means of reasonable kinked conjectures.

d. Contrary to the results obtained by Boyer-Moreaux and Ulph, not any allocation can be considered as a reasonable conjectural equilibrium, when conjectures are assumed to be kinked. This may be due to the fact that conjectures are nor differentiable and do not fulfil the Ulph consistency condition (see Def. 11 below).

We also notice that our interpretation of the kinked demand curve is not the only possibility(4). Stiglitz (1979) presented a model in which the kink arises from imperfect information on the consumers' side (see Ryley (1979) also and his notion of 'reactive equilibrium"). Our paper offers a formalization of the kinked demand curve which embraces this last interpretation at least in many parts of the paper. Also a dynamic interpretation in terms of a supergame is possible.

However, in such a situation it is possible that if a firm rises its price, competitors will do exactly the same!. Actually the kind of allocations which can be obtained as an equilibrium with no discounting is enormous (see Rubinstein (1979)), and there are some which cannot be an equilibrium if conjectures are kinked (recall what has been said in the last part of point 3). Hence a supergame framework only catches half of the problem, or relies crucially on discounting.

Finally, we offer a summary of the rest of this paper. In Section II the main definitions are presented. In Section III we will examine the properties of the equilibrium allocations when conjectures are kinked. In Section IV the notion of reasonable conjectures is introduced and worked out. Section V presents a graphical analysis of our approach and Section VI offers our main conclusions

II. THE MAIN DEFINITIONS.

We assume that the number of firms is a given natural number, n. Each firm is completely described by its profit function, $\pi_i = \pi_i(p_i, p_{-i})$, being p_i the price controlled by i and p_{-i} a vector of p_j with $j=1,2,\ldots,n$, but $i \neq j$. Let $p=(p_i, p_{-i})$. All the theorems, conclusions, etc. along the paper can also be obtained (with the appropriate changes) if p_i is interpreted as the output of i. $\pi_i(p)$ can be obtained from the demand function for i, $x_i = f_i(p)$ and the cost function $c_i = c_i(x_i)$, being x_i the output of i, $\pi_i = f_i(p)p_i - c_i(f_i(p))$. We will assume that for every i and $p^*_{-i} \in Rn-1+$, there exists p^*_i such that $\pi_i(p^*) \geq 0$ (possibility of inaction, for instance).

A conjecture for firm i is a function $\phi_{ji}: Rn+1+ \longrightarrow R+$, being $p_j = \phi_{ji}(p_i, p')$ the price conjectured by i if starting at p' moves its price from p'_i to p_i. We will assume that $p'_j = \phi_{ji}(p'_i, p')$, i.e., if p_i does not change p_j does not change either. Finally, let $\phi_i = (\phi_{1i}, \ldots, \phi_{i-1,i}, \phi_{i+1,i}, \ldots, \phi_{ni})$.

DEFINITION 1.- A conjecture is said to be **kinked** if for any $i,j=1,\ldots,n$,

a) $p''_i > p'_i$ implies $p'_j = \phi_{ji}(p''_i, p')$ and

b) $p_i < p'_i$ implies $p'_j > \phi_{ji}(p_i, p')$.

In verbal terms (a) says that if i increases its price it does not expect any reaction from the others; (b) says that if i decreases its price it expects that the others will do the same. In what follows, unless explicitly stated, it will be assumed that conjectures are kinked. However, notice that such conjectures only make sense in case an assumption as gross substitutability holds (see assumption 5 below).

DEFINITION 2.- A **Sweezy equilibrium** is a vector $p*$ such that

a) $p* \in Rn+$

b) for every $i = 1,2,...,n$, $p*_i$ maximizes $\pi_i(p_i, \emptyset_i(p_i, p*))$, i.e., $\pi_i(p*_i, \emptyset_i(p*_i, p*)) \geq \pi_i p_i, \emptyset_i(p_i, p*))$, for any $p_i \in R+$.

In words, a Sweezy equilibrium is a conjectural equilibrium in which the conjectures are assumed to be kinked.

DEFINITION 3.- A **Bertrand-Nash equilibrium** is a vector $p*$ such that

a) $p* \in Rn+$

b) for every $i=1,...,n$, $p*_i$ maximizes $\pi_i(p_i, p*_{-i})$, i.e., $\pi_i(p*) \geq \pi_i(p_i, p*_{-i})$ for any $p_i \in R+$.

A Bertrand-Nash equilibrium is a conjectural equilibrium in which every firm assumes zero conjectural variations.

III. SWEEZY EQUILIBRIA

In this section we study the properties of the Sweezy equilibria (S.E.).

We first analyze the effects on S.E. of the assumption that conjectures are differentiable. Stigler claims that "effects would be minor ,..., if a literal kink were replaced by a very sharp bend" (1978, p.188). Also Negishi (1975, pp. 87-98) assumes differentiable conjectures. However as we will see (Proposition 1), this differentiability restricts severily the kind of allocations which are S.E. In what follows c^1 means "continuously differentiable one time".

Assumption 1.- for any $i=1,2,...,n$, $\pi_i(p)$ is c^1.

Assumption 2.- for any $i=1,2,...,n$, $\pi_i(p)$ is concave on p_i, for every $p_{-i} \in Rn-1+$.

Assumption 3.- for any $i,j=1,2,...,n$, $\phi_{ji}(p_i,p')$ is c^1.

Now we want to analyze the effect of assumption 3 on the kind of allocations we may get. In intuitive terms, the kinked conjecture implies zero slope at the status quo. Hence a concavity assumption will guarantee that this point is a Bertrand-Nash equilibrium. In formal terms we have:

PROPOSITION 1.- Under assumptions 1,2 and 3, if p^* is a S.E., then it is also a Bertrand-Nash equilibrium.

PROOF:

If p^* is a S.E., then

$$\frac{\partial \Pi_i(p^*)}{\partial p_i} + \sum_{j \neq i} \frac{\partial \Pi_i(p^*)}{\partial p_j} \cdot \frac{\partial \phi_{ji}(p^*_i, p^*)}{\partial p_i} \leq 0 \text{ , } i=1,2,\ldots,n$$

and if strict inequality prevails, then $p^*_i = 0$.

Since $\frac{\partial \phi_{ji}(p^*_i, p^*)}{\partial p_i}$ is continuous on p_i, taking a sequence p_i^v converging to p^*_i, with $p_i^v > p^*_i$ for every $v \in N$, we see that $\frac{\partial \phi_{ji}(p^v_i, p^*)}{\partial p_i} = 0$

so, by continuity $\frac{\partial \phi_{ji}(p^*_i, p^*)}{\partial p_i} = 0$

Now we have $\frac{\partial \Pi_i(p^*)}{\partial p_i} \leq 0$, and if strict inequality prevails, then $p^*_i = 0$, so the concavity assumption implies p^* is a Bertrand-Nash equilibrium. Q.E.D.

Proposition 1 puts strong restrictions on the kind of admissible conjectures which produce outcomes different than the Bertrand-Nash ones, and also it provides a link with this kind of equilibria: as the Weierstrass approximation theorem asserts that any continuous function can be approximated by a C^1 function, we may argue that Sweezy equilibria are not robust if conjectures are continuous, since a small perturbation in the kinked conjectures destroys all the outcomes which are not Bertrand-Nash[5]. This also shows that the correspondence which maps conjectures into allocations is not lower-hemicontinuous in the appropriate space.

One may conjecture that any $p^\wedge$ such that $\dfrac{\partial \pi_i(p^\wedge)}{\partial p_i} = 0$ and $\dfrac{\partial^2 \pi_i(p^\wedge)}{\partial p_i^2} \leq 0$ is a Sweezy equilibrium. Figure 1 shows that this is not the case since firm 1 can obtain more profits moving to p_1 and expecting no move from 2.

If the concavity assumption is dropped, then there are Sweezy equilibria which are not Bertrand equilibria. See figure 2.

Now let us define the region in which all possible equilibria must lie.

DEFINITION 4.- $\Omega = \{p \in Rn+/ \ \pi_i(p) \geq 0, \dfrac{\partial \pi_i(p)}{\partial p_i} \leq 0, i=1,\ldots n\}$.

In words, Q is the set of points in the price space for which (a) profits are nonnegative, and (b) the partial derivative of $\pi_i(p)$ with respect to its own price is non positive (Notice that (a) and (b) hold for every firm).

PROPOSITION 2.- Under assumption A.1, if p* is a S.E., then p* ∈ Q.

PROOF:

Suppose that $\dfrac{\partial \pi_i(p^*)}{\partial p_i} > 0$. Then agent i can increase p_i expecting no move from the rest of firms, so p^*_i does not fulfil (b) in def. 2. Hence $\dfrac{\partial \pi_i(p^*)}{\partial p_i} \leq 0$ for every i.

Moreover $\pi_i(p^*) \geq 0$ and them $p^* \in Q$. Q.E.D.

In order to prove the converse to Proposition 2 (i.e., that if $p^* \in Q$, then p* is a S.E.) we need a new assumption.

Assumption 4.- For any i=1,...,n, $p \in Q$, if $p_i^1 < p_i$, then there exists p^1_{-i} such that $\pi_i(p^1) \leq \pi_i(p)$, $p^1 = (p^1_i, p^1_{-i})$.

Assumption 4 postulates that the market is sufficiently homogeneous so that any possible deviation can be punished.

Specifically, if firm i decreases p_i, then the rest of firms can make a combined move such that at the new prices the movement of i results no profitable. In this sense , A.4 can be taken as one of the characteristics of an industry, i.e., a set of firms with strong inner relationships through the demand functions.

PROPOSITION 3.- Under assumptions 1,2 and 4, if $p* \in Q$, then $p*$ is a S.E. for some conjectures.

PROOF:

If it were not the case , then, for some firm i, $p*_i$ would not maximize $\pi_i(p_i,p*_{-i})$ subject to $p*_{-i}=\emptyset_i(p_i,p*)$. In other words, there exists $p\^{}_i$ such that

$$\pi_i(p\^{}_i,\emptyset_i(p\^{}_i,p*)) > \pi_i(p*_i,\emptyset_i(p*_i,p*))$$

Because of the concavity of $\pi_i(p)$ and as $p* \in Q$ it must be that $p\^{}_i < p*_i$. But then, constructing $\emptyset_i$ such that $p\^{}_j = \emptyset_i(p\^{}_i,p*)$, $j \neq i$, then it fulfils A.4, i.e., $\pi(p\^{}) \leq \pi(p*)$. This contradicts that firm i had any incentive to move. Q.E.D.

Propositions 2 and 3 provide testable properties of the S.E., as well as a benchmark in order to elucidate the relationship of the tests actually used with the S.E. (see Stigler (1978), pp. 189-191). This relationship turns out a weak one.

Proposition 3 gives some plausitlility to the claim that a S.E. may be used in order to support sticky prices, in the following sense: a small perturbation of the profit function will leave unaltered "many" old equilibrium prices. Of course, this discussion is rather arbitrary without introducing dynamic considerations.

A characteristic of those conjectures used in Proposition 3 is that they may well be discontinuous (see footnote 8).

Notice that in the proof of Proposition 3 the extreme pessimism of firm i plays a quite prominent role. However, a less drastic construction would give the same conclussions at the cost of some additional technical complications.

If the concavity assumption is dropped, then Proposition 3 no longer holds. As figure 3 makes clear, there are points in the Θ region which can never be equilibria, since the firm will increase its own price (from any price in (a,c] to b), increasing its profits and expecting no move of the rest.

Propositions 2 and 3 destroy the well-spread idea that "there is a fundamental indeterminancy in the location of the kink (and hence in the equilibrium price)" (Stiglitz, 1984); "the theory is not related to prices to be explained",

...,"No scientific function is now performed by the kinked demand curve that would not equally well supplied by the simple argument that changes cannot be made without a cost" (Stigler, 1978).

Proposition 3 gives apparently an existence theorem as a corollary, by noticing that since the Q region is non empty in general, S.E. do exist. Proposition 3 however works only for some conjectures, and as it was remarked before, these ones are rather pessimistic. In particular, it is perfectly possible that if the relationships among firms are not prominent, conjectures are optimistic, and if any deviation from an arbitrary element of Q is not followed by a strong reaction of the rest of firms, many points in Q are not sustainable as equilibria. Then, the question is whether there exists a S.E. when conjectures are kinked but arbitrary.

In Proposition 1 we proved that a differentiability assumption on $\emptyset(\ )$ implies that any S.E. is a Bertrand-Nash equilibrium. In the next proposition, we will prove that, starting with **any** (kinked) conjecture, a Bertrand-Nash equilibrium is a S.E.. Hence, the existence of the former will imply the existence of the latter. In order to do that we will introduce a new assumption.

Assumption 5.- For any $i=1,\dots,n$, $p \leq 0$, $\dfrac{\partial \Pi_i(p)}{\partial p_j} \geq 0$, all $j \neq i$, with at least one strict inequality.

A.5 means that goods are (weakly) gross substitutes.

PROPOSITION 4.- Under A.5, if $p*$ is a Bertrand-Nash equilibrium, then it is a S.E. equilibrium for any (kinked) conjecture.

PROOF:

Suppose not. Then, for some firm i, there exists p^1_i such that $\pi_i(p^1_i, \varnothing_i(p^1_i, p*)) > \pi_i(p*_i, \varnothing_i(p*_i, p*))$.

Obviously $p^1_i < p*_i$ since otherwise by the definition of kinked conjectures the other firm will not react, so that any movement which was unprofitable when no conjectural variation was expected, is also unprofitable under the "kinked conjecture" hypothesis. But now, we have

(1) $\pi_i(p*) \geq \pi_i(p^1_i, p*_{-i})$, and

(2) $\pi_i(p*) < \pi_i(p^1_i, p^1_{-i})$ being $p^1_{-i} = \varnothing_i(p^1_i, p*)$

Also since $p^1_{-i} < p*_{-i}$ it must be that

(3) $\pi_i(p^1_i, p^1_{-i}) < \pi_i(p^1_i, p*_{-i})$ because A.5.

Hence, putting (1),(2) and (3) together we have

$\pi_i(p^1_i, p*_{-i}) > \pi_i(p^1_i, p^1_{-i}) > \pi_i(p*) \geq \pi_i(p^1_i, p*_{-i})$

and i cannot expect any gain by deviating from p^* to p^1_i.
Q.E.D.

Now we have the following proposition:

PROPOSITION 5.- Let us assume A.1,2,5 and that for any $i=1,\ldots,n$, p'_{-i}, $\pi_i(p'_i,p'_{-i}) = 0$ for any $p'_i > p''_i$[6]. Then, for any (kinked) conjecture there exists a S.E.

PROOF:

Under the above assumptions a Bertrand-Nash equilibrium exists and it is a S.E., by Proposition 4. Q.E.D.

Notice that no property on $\phi(\ )$ is assumed in Proposition 5. In particular, we allow for discontinuous conjectures.

We may ask if we cannot go further than Proposition 5, i.e., if we can guarantee the existence of a S.E. equilibrium different from the Bertrand-Nash. The answer is, however, that as long as conjectures are arbitrary, we cannot guarantee that an arbitrary element of Q is a S.E.. This is easily seen since

$$(4)\quad d\pi_i(p) = \sum_{j=1}^{n} \frac{\partial \pi_i(p^\wedge)}{\partial p_j}\, dp_j$$

being dp_j, $j \neq i$ the expected variations in other firms' prices.

For given $\dfrac{\partial \pi_i(p^*)}{\partial p_j}$ j=1,...,n and dp_i, there exists a vector dp_s, $s \neq i$ such that (4) is positive, i.e., a movement from p^* to p^*+dp is profitable from i's point of view, if $\dfrac{\partial \pi_i(p)}{\partial p_i} \neq 0$. Now we want to know more about the set Ω. The strategy we will follow is to study if the solutions to well-known equilibrium concepts ly in Ω. The additional bonus of this procedure is that we will know the compatibility of the kinked demand curve with other possible equilibria, much used in Industrial Organization.

DEFINITION 5.- p is said to be **Industry-Pareto Efficient** if

(a) $p \in Rn+$

(b) there is no $p' \in Rn+$ such that $\pi_i(p') \geq \pi_i(p)$ for all i=1,...,n with at least one strict inequality.

The name "Industry-Pareto Efficient" wants to indicate that only firms are considered in order to evaluate a movement from p to p'. In any other aspect this conceps turns out analogous to the usual definition of Pareto efficiency.

PROPOSITION 6.- Under assumptions 1,2,4 and 5, if $p^\wedge$ is Industry-Pareto Efficient, then it is a S.E. for some conjectures.

PROOF:

Suppose not. Then $\dfrac{\partial \Pi_i(p^\wedge)}{\partial p_i} > 0$ for some i (Prop. 3). But then, increasing $p^\wedge_i$ everybody will be better off or indifferent (because A.5) contradicting that $p^\wedge$ is Industry-Pareto Efficient.

Q.E.D.

Proposition 6 does not validate the claim that the kinked demand curve is some kind of coordination mechanism since its converse does not hold, i.e., there are S.E. with arbitrarily low profits (see figure 4).

In particular notice that the Joint Profit Maximum solution (emphasized by Chamberlain) is a S.E. for some conjectures (see Scherer, 1970, ch. 5 for the contrary view).

In the next proposition we explore the relationship between full efficiency and S.E.

PROPOSITION 7.- Let us assume A.1. Then an allocation for which price is equal to marginal cost for every firm cannot be a S.E. for any conjecture, if S.E. involve positive production for at least one firm.

PROOF:

Let $\pi_i(p) = f_i(p)p_i - c_i(f_i(p))$. Then from Prop. 3 we know that if p is a S.E. then $\frac{\partial \pi_i(p)}{\partial p_i} \leq 0$. But from the profit function we have

$$\frac{\partial \pi_i}{\partial p_i} = \frac{\partial f_i(p)}{\partial p_i} \cdot p_i + f_i(p) - \frac{\partial c_i(x_i)}{\partial x_i} \frac{\partial f_i(p)}{\partial p_i}$$

so in any allocation such that $p_i = \frac{\partial c_i(x_i)}{\partial x_i}$ we have $\frac{\partial \pi_i}{\partial p_i} = f_i(p) > 0$

and then, assuming that p is a S.E. we get a contradiction.
Q.E.D.

Proposition 7 shows that a S.E. is never "totally efficient" since a necessary condition of Pareto efficiency is price equal to marginal cost. As a Corollary of Proposition 7 we obtain that a compatitive equilibrium cannot be sustained as a S.E. for any conjecture which fulfils Definition 2.

For the next two propositions of this section we will assume that n = 2.

DEFINITION 6.- $F_j(p_i) = \{ p_j \,/\, \pi(p_j, p_i) \geq \pi(p'_j, p_i)$, for all $p'_j \in R+ \}$, i, j = 1, 2.

In words, $F_j(p_i)$ is the best reply mapping according to the Bertrand-Nash conjecture of firm j.

DEFINITION 7.- $p^* = (p^*_1, p^*_2)$ is said to be a **Stackelberg equilibrium** with firm 1 as a leader if p^* maximizes $\pi_1(p)$ subject to $p_2 \in F_2(p_1)$.

Assumption 6.- (a) $\pi_2(p)$ is C^2, π_1 is C^1, F_2 is C^1.

(b) $p^*_i > 0$ i=1,2.

(c) $$\frac{\partial^2 \pi_2}{\partial p_2^2} \neq 0$$

PROPOSITION 8.- Under A.2,4,5 and 6, p^* is a S.E iff

$$\frac{\partial^2 \pi_2(p^s)}{\partial p_1 p_2} \gtrless 0^{(7)}$$

PROOF:

If p^* is a Stackelberg equilibrium we have

$$(5) \quad \frac{\partial \pi_1(p^s)}{\partial p_1} + \frac{\partial \pi_1(p^s)}{\partial p_2} \; \frac{\partial F_2(p^s)}{\partial p_1} = 0$$ for 1 and

$$(6) \quad \frac{\partial \pi_2(p^s)}{\partial p_2} = 0$$

Now, from (6) we have

$$\frac{\partial^2 \Pi_2(p^s)}{\partial p_2 p_1} dp_1 + \frac{\partial^2 \Pi_2(p^s)}{\partial p_2^2} dp_2 = 0$$

Because of the second order condition, we have

$$\text{sign}\left[\frac{\partial F_2(p_1^s)}{\partial p_1}\right] = \text{sign}\left[\frac{\partial^2 \Pi_2(p^s)}{\partial p_1 p_2}\right]$$

and because $\partial \Pi_1(p^s)/\partial p_2 > 0$ (by A.5) since

$$\frac{\partial \Pi_1(p^s)}{\partial p_1} \lesseqgtr 0 \qquad \text{iff } p^* \text{ is a S.E. (Propositions 3,4)}$$

it must be that $\dfrac{\partial^2 \Pi_2(p^s)}{\partial p_1 p_2} \gtreqless 0$

Conversely if $\dfrac{\partial^2 \Pi_2(p^s)}{\partial p_1 p_2} \gtreqless 0$, then $\dfrac{\partial \Pi_1(p^s)}{\partial p_1} \lesseqgtr 0$

amd p^* is a S.E. for some conjectures. Q.E.D.

COROLLARY 1.- If p is a Stackelberg disequilibrium point (i.e., 1 and 2 are leaders) then p is a S.E. iff $\dfrac{\partial^2 \Pi_i(p)}{\partial p_1 p_2} \gtreqless 0$, i=1,2.

Corollary 1 is proved along the same lines than Proposition 8. Notice that there are points in Q which are S.E. but are not Stackelberg equilibria (i.e., Bertrand-Nash equilibria).

Stigler (1978, p.193) noticed some similarities between S.E. and limit pricing. We will study now their compatibility.

DEFINITION 8.- The price p^L_1 **limits the entry** (of 2) if for any $p_2 \in R+$, $\pi_2(p^L_1, p_2) \leq 0$:

DEFINITION 9.- The price p^{LP}_1 is a **limit price** if it maximizes $\pi_1(p_1, p_2)$ subject to $\pi_2(p_1, p_2) \leq 0$, for any p_2.

In words the limit price is a price which (a) limits the entry of 2, and (b) maximizes the profits of 1 taking (a) into account.

PROPOSITION 9.- Assume A1, A.5 and that $p^\wedge$ is a S.E. with $\frac{\partial \pi_1(p^\wedge)}{\partial p_1} \neq 0$. Then $p^\wedge$ is not a limit price and viceversa.

PROOF:

As $p^\wedge$ is a S.E., $\frac{\partial \pi_1(p^\wedge)}{\partial p_1} < 0$. But then, decreasing $p^\wedge_1$ π_1 increases, and because A.5 this new p_1 also limits the entry since $0 \geq \pi_2(p^\wedge_1, p_2) > \pi_2(p_1, p_2)$ for any p_2. This contradicts that $p^\wedge_1$ is a limit price. Second part is proved along the same lines. Q.E.D.

Proposition 9 proves that S.E. is not an adequate concept when strategic entry is present, since in this case the kink goes the other way around: a high p_1 will attract entrants, but not a low p_1! However, from the point of view of the entrant firm the expected kink may be the right one, i.e., it may be true that the entry of 2 is followed by an increase in x_2 (compare with Stigler (1978, p. 193)).

Finally we prove that in a S.E. firms are not maximizing sales as stressed by Baumol (see Baumol, 1972).

DEFINITION 10.- p^B is said to be a **Baumol equilibrium relative to** $r \in Rn$ if

(a) $p^B \in Rn+$

(b) for any $i=1,\ldots,n$, p^B maximizes $f_i(p_i,p^B_{-i})p_i$ subject to $\pi_i(p_i,p^B_{-i}) \geq r_i$ (8).

PROPOSITION 10.- Let us assume A.1 and that marginal costs are positive and demand is increasing. Then a Baumol equilibrium is not a S.E. if $p^B_i > 0$ and $\pi_i(p^B) > r_i$ for some i.

PROOF: $$\frac{\partial \pi_i(p)}{\partial p} = \frac{\partial f_i(p)}{\partial p_i} p_i + f_i(p) - \frac{\partial c_i}{\partial x} \frac{\partial f_i(p)}{\partial p_i}$$

And evaluating the previous equation at $p = p^B$

$$\frac{\partial \pi_i(p^B)}{\partial p_i} = - \frac{\partial c_i}{\partial x} \frac{\partial f_i(p^B)}{\partial p_i} > 0$$

So, assuming that p^B is a S.E. we get a contradiction since by proposition 2 $\frac{\partial \pi_i(p^B)}{\partial p_i} \leq 0$, $i = 1,\ldots,n$ Q.E.D.

REMARK.- Under the same assumptions than in proposition 10, if p^* is a S.E., then it is not a Baumol equilibrium.

Proposition 10 is intended to shed some light on the literature concerning sales maximization and the kinked demand curve (see Murphy-Ng, 1974).

It can be shown by means of simple examples (see figure 5) that if condition $\pi_i(p^B) > r$ is relaxed, then proposition 10 is no longer true. However, if restrictions are always binding, sales maximization does not play any particular role and Baumol hypothesis confounds with some kind of "full-cost principle". This "principle" has been advocated as a complement of the kinked demand curve (the so-called "Oxford version", see Stigler, 1978, pp. 187-188) but it is clear that it does not add any new ingredient to S.E..

Finally we explore if kinked conjectures posse some kind of consistency (we will assume here again that $n=2$). In particular we think of the following requirement. Let us assume that firm 2 is wondering whether a change from the status quo (p^o) of p^o_1 to p^1_1 is going to increase its profits. According to 1's conjectures this will cause a shift of p_2 from p^o_2 to p^1_2 according with $p_2 = \phi_1(p_1, p^o)$, i.e., $p^1_2 = \phi_1(p^1_1, p^o)$.

It is true that if 1 changes again from p^1_1 (which is the new status-quo) to p^o_1, then p_2 would change from p^1_2 to p^o_2? In other words: are price changes reversible? or after a movement, perhaps we cannot return to the status quo? This discussion motivates the following definition:

DEFINITION 11.- $\phi_1(p_j, p)$ is said to satisfy the **Ulph consistency condition** (see Ulph, 1981) if $p^\frown_j = \phi_j(p^\frown_1, p^o)$ implies that $p^o_j = \phi_j(p^o_1, p^\frown)$, $i, j = 1, 2$, for every $p^o \in R2{+}{+}$.

PROPOSITION 11.- The kinked conjectures do not satisfy the Ulph consistency condition.

PROOF:

Take $p^\frown_1 < p^o_1$. Then $p^\frown_2 < p^o_2$. However, starting from $p^\frown$

and changing p_1 from $p^{\frown}_1$ to p^{o}_1, $p^{\frown}_2$ does not change. Q.E.D.

Ulph discusses the importance of this consistency condition in order to get "too many" rational conjectural equilibria. In the next section we will explore the set of reasonable equilibria in our case.

IV. REASONABLE SWEEZY EQUILIBRIA

In this section we explore the idea of reasonable conjectures proposed by Hahn (1978) (see also Hart (1982)) in the framework of the kinked demand curve. In others words we want to impose some kind of rationality on the equilibria arising from conjectures which satisfy Definition 1. A motivation for the assumption of "Reasonable Conjectures" is found in Hart (1982, p.1-2).

Let A_i be the space of conjectures of agent i. It includes only those fulfilling definition 1. A_i may be an Euclidean space if conjectures were described by real numbers, or it may be a functional space, etc.. Let $A = A_1 \times \ldots \times A_n$. Also denote by $A^\wedge$ the subset of A for which a S.E. exists. Finally let α be an element of A, $\alpha = (\alpha_1, \ldots, \alpha_n)$.

DEFINITION 12.- A tuple $(p^*_i, \alpha^*_i)_{i=1,2,\ldots,n}$ is said to be a **Reasonable Sweezy equilibria** (R.S.E.) if

a) $\alpha^* \in A^\wedge$

b) p^* is a S.E. relative to α^*

c) for any i=1,...,n there is no a S.E. relative to $\alpha' = (\alpha_i, \alpha^*_{-i})$ with $\alpha'_i \in A_i$ such that if p' is a S.E. relative to α' then $\pi_i(p') > \pi_i(p^*)$.

Condition (a) requires that "equilibrium conjectures" belong to the subset for which a S.E. exists. Condition (c) requires that conjectures are chosen in an optimal way in the following sense: given the conjectures of anybody else, firm i cannot expect any gain by choosing a different conjecture, i.e., α^*_i is a Nash equilibrium of a game with conjectures as strategies and pay-offs defined by profits achieved in S.E. relatives to conjectures. Actually, if several equilibria arise from a given α an optimistic assumption is made, i.e., the most favourable equilibria for firm i is chosen.

In this section we derive some results on the existence and the properties of a R.S.E. for the duopoly case, i.e., n=2. This assumption is keeped along the rest of this section (9)

We begin with the proof of the existence of a R.S.E.. For this we need a new assumption.

Assumption 7.- There is $p^\wedge \in \Omega$ such that if $\pi_i(p') > \pi_i(p^\wedge)$ for some $p' \in \Omega$, then $\dfrac{\partial \pi_j(p')}{\partial p_j} < 0$ for all i,j=1,2.

In words, A.7 requires that if for instance $\pi_2(p') > \pi_2(p^\wedge)$ for some $p' \in \Omega$, then p' does not maximaze $\pi_1(p)$ taking p_2 as given, i.e., p' is not a Nash equilibrium for i given p_2. This last interpretation gives us the clue of why A.7 is needed in our existence proof. Since by Proposition 4 a Bertrand-Nash equilibrium is a S.E. for any (kinked) conjectures if p' is preferred to p^ from 2's point of view and $\frac{\partial \pi_1(p')}{\partial p_1} = 0$ ($\pi_1(p)$ is assumed to be concave), p' is an equilibrium for 1, no matter which conjecture it holds. Hence 2 will choose p^ since it is more profitable for it.

PROPOSITION 12.- Under A.1,2 and 4, A.7 is a sufficient condition for the existence of a R.S.E. Moreover, p^ is a R.S.E.

PROOF:

Take firm 1 (w.l.o.g.). Now construct 2's conjectures in the following way:

For any p' such that $\pi_1(p') > \pi_1(p^\wedge)$, $\alpha^\wedge_e$ is such that 2 finds profitable to change p'_e. Notice that

$$\frac{d\pi_2}{dp_2} = \frac{\partial \pi_2}{\partial p_2} + \frac{\partial \pi_2}{\partial p_1} \frac{dp_1}{dp_2}$$

so for any given $\frac{\partial_2(p')}{\partial p_1}$ there exists a $\frac{dp_1}{dp_2}$ such that $\frac{d\pi_2}{dp_2} < 0$, hence 2 will find profitable to lower its price at p'. Hence p' is not a S.E. relative to $\alpha^\wedge$

$=(\alpha_1, \alpha^\frown_2)$ for any $\alpha_1 \in A$.

For $p'' \neq p^\frown$ such that $\pi_1(p'') \leq \pi_1(p^\frown)$ take any arbitrary conjecture for 2.

For $p = p^\frown$ construct $\alpha^\frown_2$ as in Proposition 3.

Hence, from 1's point of view, $p^\frown$ is an optimal choice since for any other S.É. the resultant prices do not yield greater profits. A similar construction for player 1 gives us the required conjectures. Q.E.D.

Let me give a verbal account of Proposition 12: take the point $p^\frown$. Firm 1 has an incentive to move from $p^\frown$ to any other price p' such that $\pi_1(p^\frown) < \pi_1(p')$. Then we construct 2's conjecture such that p' cannot be an equilibrium for it. Hence firm 1 cannot move to p' since p' is not a S.E. (see part (c) in Def. 12).

In the next proposition we will show that in fact assumption 7 is also a necessary condition. Suppose that p'' is a R.S.E. but there exists p' such that $\dfrac{\partial \Pi_j(p')}{\partial p_j} = 0$ for some j. But then if $\pi_j(p)$ is concave, p' is a S.E. no matter which conjectures j holds. Indeed since p'_j maximizes $\pi_j(p'_i, p_j)$ for p'_i given, p'_j also will maximize $\pi_j(p_i, p_j)$

over any kinked conjecture (this is proposition 4 applied to the case n=1). This contradicts that p" is a R.S.E..

A consequence of these reasonings (which will be explored in Propositions 14 and 15) is that in any R.S.E. any firm can obtain, at least, the profits which would correspond if it were a Stackelberg leader. This is because i has always the option "to push" j to points in j's best reply mapping (see definition 6), no matter which conjectures j holds.

PROPOSITION 13.- Let us assume A.1,2,5. Then if p* is a R.S.E., A.7 also holds.

PROOF:

Take firm 1 (w.l.o.g.). Then if p* is a R.S.E. but A.7 does not hold (and because A.5 there exists p' such that $\pi_1(p') > \pi_1(p^*)$), it must be that $\frac{\partial \pi_2(p')}{\partial p_2} = 0$. But then p'_2 maximizes $\pi_2(p_1,p_2)$ for p'_1 fixed (by A.2). Hence (because proposition 4) it is a S.E. for 2 no matter which conjectures 2 holds. Then 1 will choose p'_1. Hence part (c) in def. 12 is violated. (The conjecture for 1 "supporting" p'_1 is easily constructed).

Q.E.D.

Propositions 12 and 13 imply that under some conditions A.7

is a necessary and sufficient condition for the existence of a R.S.E.. Notice that because of the way in which conjectures were constructed in prop. 12, a R.S.E. is also a reasonable equilibrium, i.e., the construction remains valid if the assumption of kinked conjectures is dropped. Hence Prop. 12 implies that there are reasonable equilibria in the differentiable oligopoly model. We finally remark that it is highly possible that conjectures constructed in prop. 12 are discontinuous. This may be bad in the general case (for instance, Hart (1982) requires continuity in the conjectures he allows), but, as was explained in the previous section, continuity does not fit very well in the framework of the kinked demand curve.

There is another way of charactarizing R.S.E., and the two next propositions are devoted to this. But before, let me introduce some new definitions.

Let π^{s}_{i} the profits achieved by i if it were a Stackelberg leader (def. 7). Then, we call

$$P^{s}_{i} = \{ p \ / \ \pi_i(p) \geq \pi^{s}_{i} \}, \text{ and } \quad T = Q \cap P^{s1} \cap P^{s2}.$$

In words, P^{si} are all the price vectors which give to i more or the same profits as the Stackelberg equilibrium with firm i as a leader. T is the region of prices which are preferred or indifferent by both duopolists to the profits achieved being a Stackelberg leader, and which are possible S.E.[10].

Now we have the following

PROPOSITION 14.- Let us assume A.5. Then, if p^* is a R.S.E., $p^* \in T$.

PROOF:

Suppose not. Hence p^* is a S.R.E. but w.l.o.g. $\pi_1(p^*) < \pi_1(p^{s1})$. However π^s_1 is achievable for any conjecture of 2 (e.g. α^*_2) since maximizing $\pi_1(p)$ assuming that 2 holds α^*_2, should give at least π^s_1 since p^{s1} is a S.E. for any α_2. But then we have a contradiction. Q.E.D.

We can also prove the converse to prop. 14:

PROPOSITION 15.- Let us assume A.1,2,4. Then, if $p^* \in T$, p^* is a R.S.E.

PROOF:

In view of prop. 12 we only need to prove that if $p^* \in T$, then A.7 is fulfilled.

If $p^* \in T$, it follows that T is nonempty, so Q is nonempty. Hence if there is p' such that (w.l.o.g.) $\pi_1(p') > \pi_1(p^*)$ and

$\frac{\partial \Pi_2(p')}{\partial p_2} = 0$, p' would actually be p^{s}_{1}. Hence A.7 is fulfilled and p^* is a R.S.E. Q.E.D.

Somme comments are in order. First, propositions 14 and 15 prove that the Bertrand-Nash equilibria are never R.S.E.[11]. In this sense the rationality embodied in the notion of reasonable conjectures rescues S.E. from being merely another name of Bertrand-Nash. Second, the role of Stackelberg profits π^{s1} and π^{s2} as extremum points is also clear. In this sense Stackelberg leadership emerges as the limit case of R.S.E.. Third, notice that the rationality assumption improves profits a lot, since the profits which can be achieved are at least π^{s}_{1}, a respectable amount in many cases. Hence the role of kinky conjectures as a coordination device is here very clear.

Finally, we study the consistency of the reasonable conjectures arising from a R.S.E.. Consistency here has the following meaning: a conjecture is consistent if (infinitesimally) the expected and real variation in other firms prices coincide when any firm moves its price. Hart has proved that under suitable differentiability and interiority assumptions any reasonable equilibrium is consistent at least if the product is homogeneous (Hart, 1982, p. 19-20). It would be helpful to know if Hart's results are true in our framework.

PROPOSITION 16.- Let us assume A.5. Let (α^*_i, p^*_i), $i=1,2$ be a R.S.E. Let $\alpha' = (\alpha'_1, \alpha^*_2)$ and p' a S.E. relative to α'. Then assuming that $p' = p^* + dp$, we have:

a) $p'_1 < p^*_1$ implies $p'_2 < p^*_2$

b) $p'_2 > p^*_2$ implies $p'_1 > p^*_1$

(Similar results are true if the subscripts 1 and 2 are interchanged).

PROOF:

a) If $p'_1 < p^*_1$ and $p'_2 \geq p^*_2$, α'_1 would give more profits to 1 since $d\pi_1 = \dfrac{\partial \pi_1}{\partial p_1} dp_1 + \dfrac{\partial \pi_1}{\partial p_2} dp_2$ and $\dfrac{\partial \pi_1}{p_1} < 0$ in any R.S.E.

b) If $p'_2 > p^*_2$ but $p'_1 \leq p^*_1$, $dp_1 < 0$ and $dp_2 > 0$ so again 1 will increase its profits moving to α'. Q.E.D.

Prop. 16 asserts that if we move conjectures such that prices change infinitesimally (a) if p_1 decreases, p_2 cannot increase, and (b) if p_2 increases, p_1 also increases. Notice that even when (a) is what we would expect from kinky conjectures (b) goes in an unexpected way. And this is because 2 realises that if 1 moves its conjecture and p_2

increases, p_1 increases also. However we feel the fundamental part of the assumption of kinky conjectures is that a reduction in the price of i will be matched by j, i.e., j will also reduce its price. In any case, conjectures cannot be consistent since 2 observes an increase in both p_1 and p_2. That this fact can happen in a S.E. was first notice by Stigler in 1947 (see Stigler, 1978, p. 189). Here we proved that the introduction of our reasonability assumption on the conjectures does not preclude this fact. However, this cannot be taken as an argument against S.E. (see Introduction), so Stigler criticism cannot be taken as decisive.

Finally we notice that the reason why Hart's result is not true in our framework is because of the lack of differentiability of conjectures and not because of the heterogeneity of the product. This is easily seen as follows. In a conjectural equilibrium (assuming A.1 and that $p_i \neq 0$) we must have

$$\frac{\partial \Pi_i}{\partial p_i} + \sum_{j\neq i} \frac{\partial \Pi_i}{\partial p_j} \frac{\partial p_j^e}{\partial p_i} = 0$$

being the second expression the expected change in i's profits as a consequence of the expected change in $p_i \left[\frac{p_j^e}{p_i} \ \forall j\neq i\right]$

Now suppose that conjectures can be described by real numbers. If the parameter w_i is controlled by i, we have that in a Reasonable Conjectural Equilibrium

$$\frac{\partial \pi_i}{\partial p_i}\frac{\partial p_i}{\partial w_i} + \sum_{j\neq i} \frac{\partial \pi_i}{\partial p_j}\frac{\partial p_j}{\partial w_i} = 0$$

assuming that p_j is a C^1 function of w_i, for all i =1,...,n.

If $\frac{\partial p_i}{\partial w_i} \neq 0$ we have

$$\frac{\partial \pi_i}{\partial p_i} + \sum_{j\neq i} \frac{\partial \pi_i}{\partial p_i} \frac{\frac{\partial p_j}{\partial w_i}}{\frac{\partial p_i}{\partial w_i}}$$

The second expression can be interpreted as the real change in i's profits as a consequence of a change in p_i (induced by a change in w_i).

Hence expected and real variations must be the same. Notice that if n > 2 it is not necessary that the expressions

$\frac{\partial p_j^e}{\partial p_i}$ and $\frac{\frac{\partial p_j}{\partial w_i}}{\frac{\partial p_i}{\partial w_i}}$ be the same, just that the mistakes in the evaluation of real changes in prices cancel each other. Of course if n=2 then both expressions must be the same. Hence we have proved next result:

PROPOSITION 17.- Under differentiability and interiority conditions any reasonable equilibria is consistent, even if the product is not homogeneous

(See figures 6,7 and 8).

Propositions 16 and 17 imply that no c^1 conjecture can "support" a R.S.E.. This is because with c^1 conjectures we only get the Bertrand-Nash equilibrium (prop. 1). But prop. 14 and 15 imply that no Bertrand-Nash equilibrium can be considered as a R.S.E.

V. A GRAPHICAL ARGUMENT

In this section we want to illustrate graphically the arguments of the previous section. In figure 6 we picture the isoprofit lines and the reaction functions under the Nash assumption (recall def. 3). If conjectures are arbitrary then any point in R2+ would be an equilibrium, provided that profits are positive. However, if conjectures are assumed to be kinked, then only the shadowed area in fig. 7 are S.E.[12]. Finally if we consider R.S.E. only the area between the lens (see fig. 8) are equilibria[13]. Notice that the more flatter the isoprofits are, the less important the reasonability assumption is, since with almost flat isoprofits only a few points are discarded as equilibria. The interpretation of this result is that the curvature of isoprofit lines is a measure of the degree of differentiation of the product: the more differentiation we have, the flatter isoprofits are. In the limit we have a pure monopoly if π_i is not sensitive to variations of p_j. In this case the reasonability assumption cannot be of much help. The same happens with a large amount of product differentiation. With no differentiation, isoprofits are "very" curved (since profits depend very much on prices od the competitors). We have a lot of independence and here the reasonability assumption is of much help.

Notice that the Stackelberg disequilibrium may be not a R.S.E.. Moreover it is not true in general that if p is the vector of prices corresponding to the Stackelberg equilibrium with firm 1 (2) as a leader, then p is a R.S.E.. Also notice that generically there are a continuum of equilibria or none, and only exceptionally one.

Finally notice that our Propositions 15 and 16 shed some light on Hahn's famous impossibility result (Hahn, 1978). Basically Hahn proved that if we consider an allocation mechanism sketched by Arrow in 1959 (in "disequilibrium" agents can change prices), there are no rational conjectures. In Hahn's model our π^{ei} is the monopoly point when agent i is maximizing its profits as a monopolist. In Hahn's model T is empty (unless the walrasian equilibrium is on the initial endowments) as a simple diagram can show (see figure 9). In the ligth of the results here obtained, the emptiness of T is what causes his impossibility result, since for any conjecture any agent can be made competitive (e.g., sending him quantity signals such that he is not constrained, hence he cannot change prices).

VI. CONCLUSIONS

In this paper we have shown that:

1. A Nash-Bertrand equilibrium is a Sweezy equilibrium (S.E.) for any conjectures (prop.4). Hence under the same conditions of existence of a Nash equilibrium, a S.E. exists (prop. 5). Also, if conjectures are of the class c^1 any S.E. is a Bertrand-Nash equilibrium (prop. 1).

2. If p* is a S.E., then $\dfrac{\partial \Pi_i(p^*)}{\partial p_i} \leq 0$ for all $i=1,..,n$ and conversely, if this last condition is met, p* is a S.E. (prop. 2 and 3).

3. In a S.E. profits may be big. For instance, that p which maximizes profits of the industry is a S.E. (prop.6); a Stackelberg equilibrium with player 1 as a leader is a S.E. (prop. 8) under some conditions. However, prices yielding arbitrarily low profits can also be supported as equilibria.

4. In a S.E. price and marginal cost are different (prop. 7); S.E. never limits the entry (prop. 9) and it is never a Baumol equilibrium (prop. 10).

5. Kinky conjectures are not consistent in the Ulph sense (prop. 11).

6. Assumption 7 is both necessary and sufficient for the existence of reasonable S.E. (R.S.E.) (prop. 13, 14). This R.S.E. remains as an equilibrium if the conjecture space is not restricted to kinky conjectures.

7. In a R.S.E. profits are at least as big as the ones in the Stackelberg point with one of the firms as a leader (prop. 15, 16). This restricts severely the allocations which are achievable as R.S.E.

8. Reasonable, kinked conjectures are not consisten (prop. 17). Hence, Hart's theorem about the consistency of reasonable equilibria is not true in our framework.

NOTES

(1) But see also Friedman (1982) who judges the idea "without merit" (p. xiv).

(2) Notice that the solution embodies a lot of apparent collusion, but no time.

(3) However, our conjecturea are not necessarily continuous. Hence in this sense we open the space of admissible conjectures (I am indebted with D. Ulph for this observation).

(4) Actually the kink may also arise in Hanh's model. But here the reason is that if some agent is restricted and he wants to trade more of some commodity, he has to change the price of this good. Hence a kink may appear in his budget set.

(5) In this sense we will not worry about the continuity of $\emptyset$ in the rest of the paper, since this is not an adequate requirement in our framework.

(6) This is just a boundness condition. It means that firms are not interested in prices higher than p''_i, since demand is

always zero.

(7) This condition is a strong gross substitution assumption.

(8) The r_i's are the minimum profits which are accepted by the shareholders.

(9) In this section we will assume that if p' contains some zero, then $p' \in Q$.

(10) It can be easily shown that T is not empty if (1) 5 holds and (2) p_i is higher if firm i is a leader than if firm j were leader i,j = 1,2.

(11) Unless $\dfrac{\partial \pi_1(p^*)}{\partial p_2} = 0$ at p*, but this is not allowed by A.5.

(12) Recall Propositions 2 and 3.

(13) Recall Propositions 15 and 16.

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FIGURE I

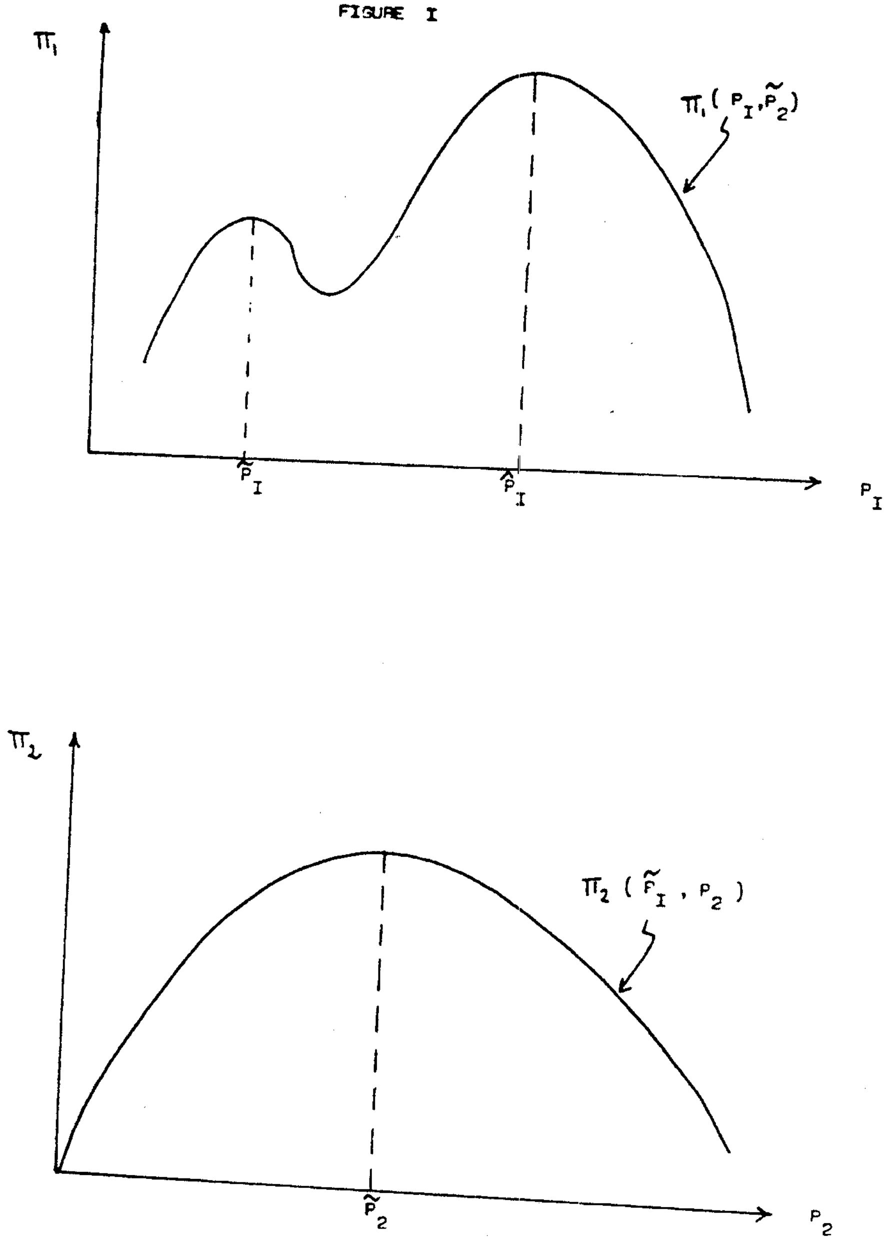
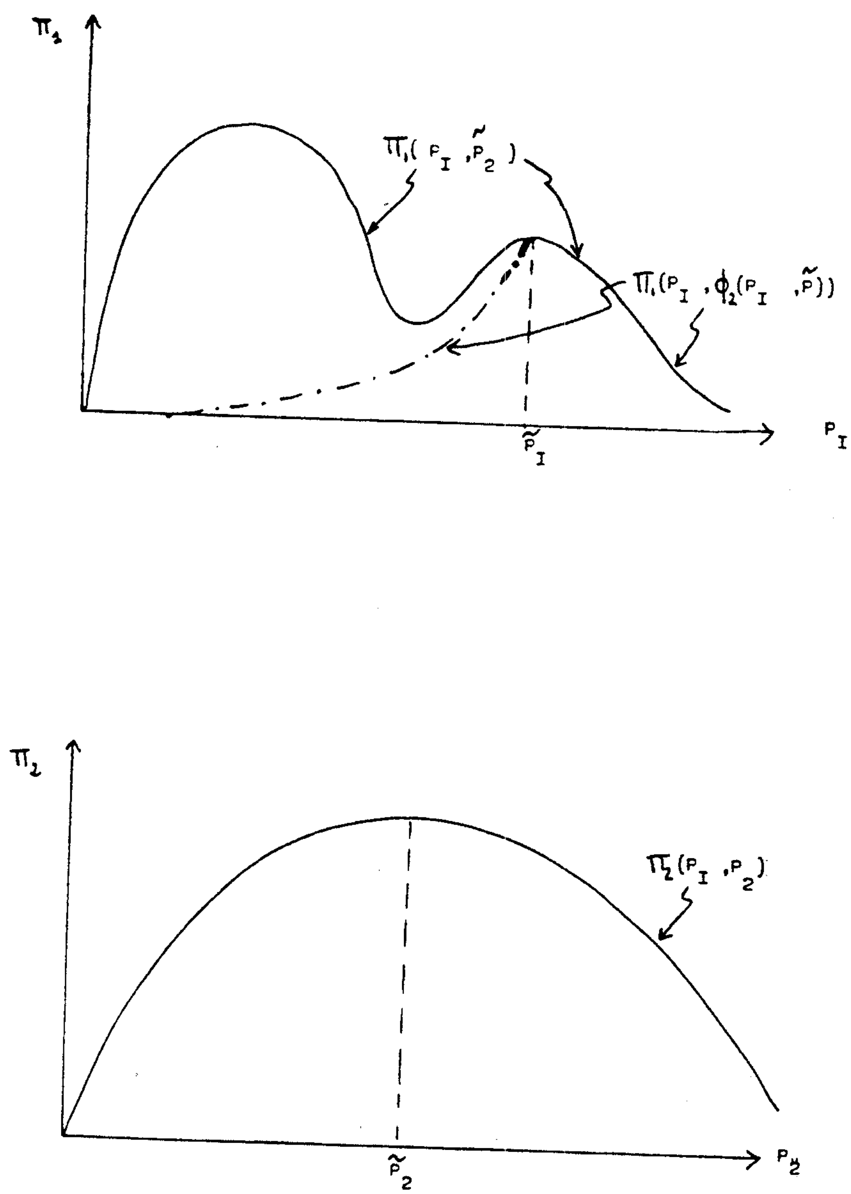


FIGURE 2



π_1
$\pi_1(p_I, \tilde{p}_2)$
$\pi_1(p_I, \phi_2(p_I, \tilde{p}))$
$\tilde{p}_I$
p_I
π_2
$\pi_2(p_I, p_2)$
$\tilde{p}_2$
p_2

FIGURE 3

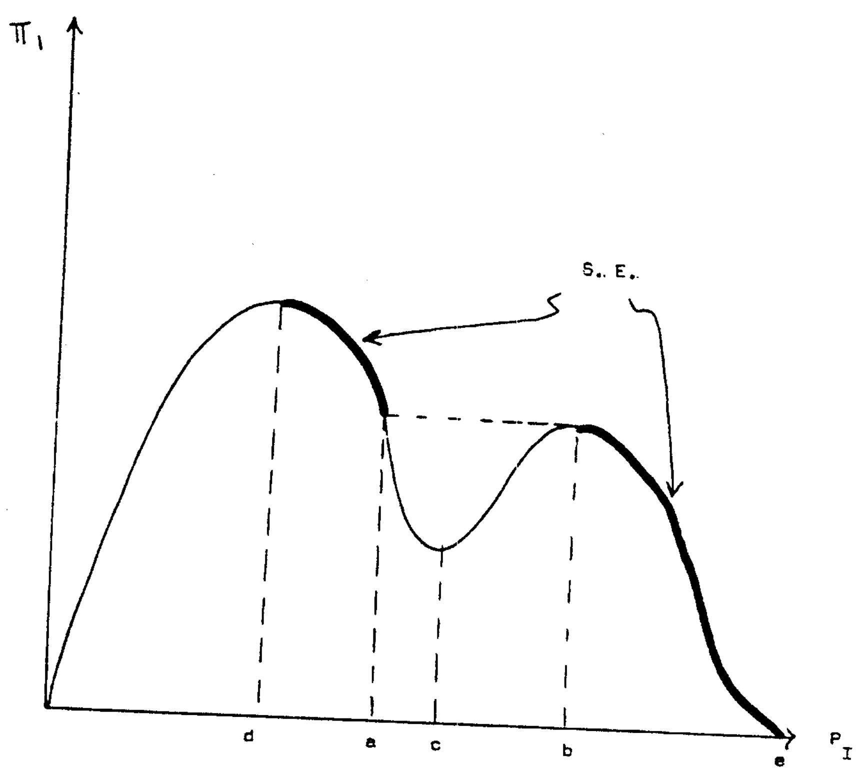


FIGURE 4

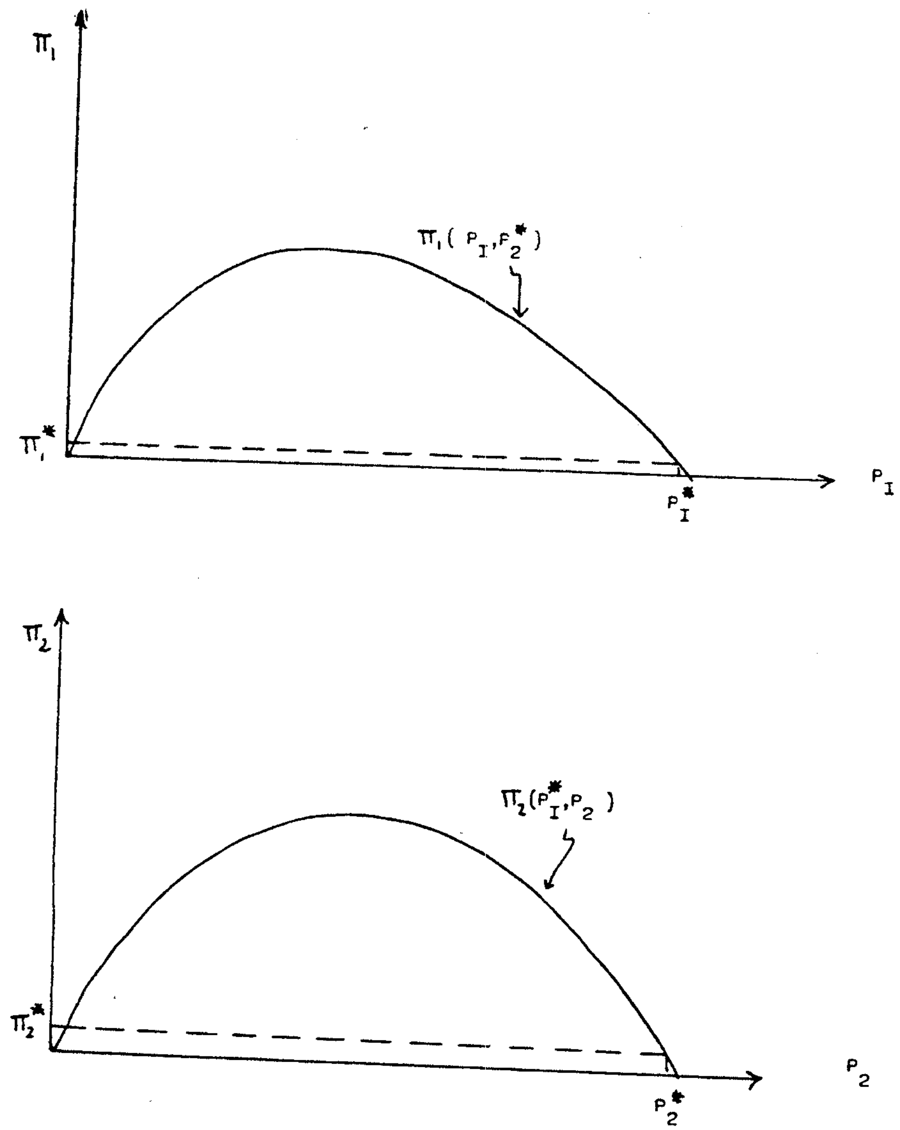


FIGURE 5

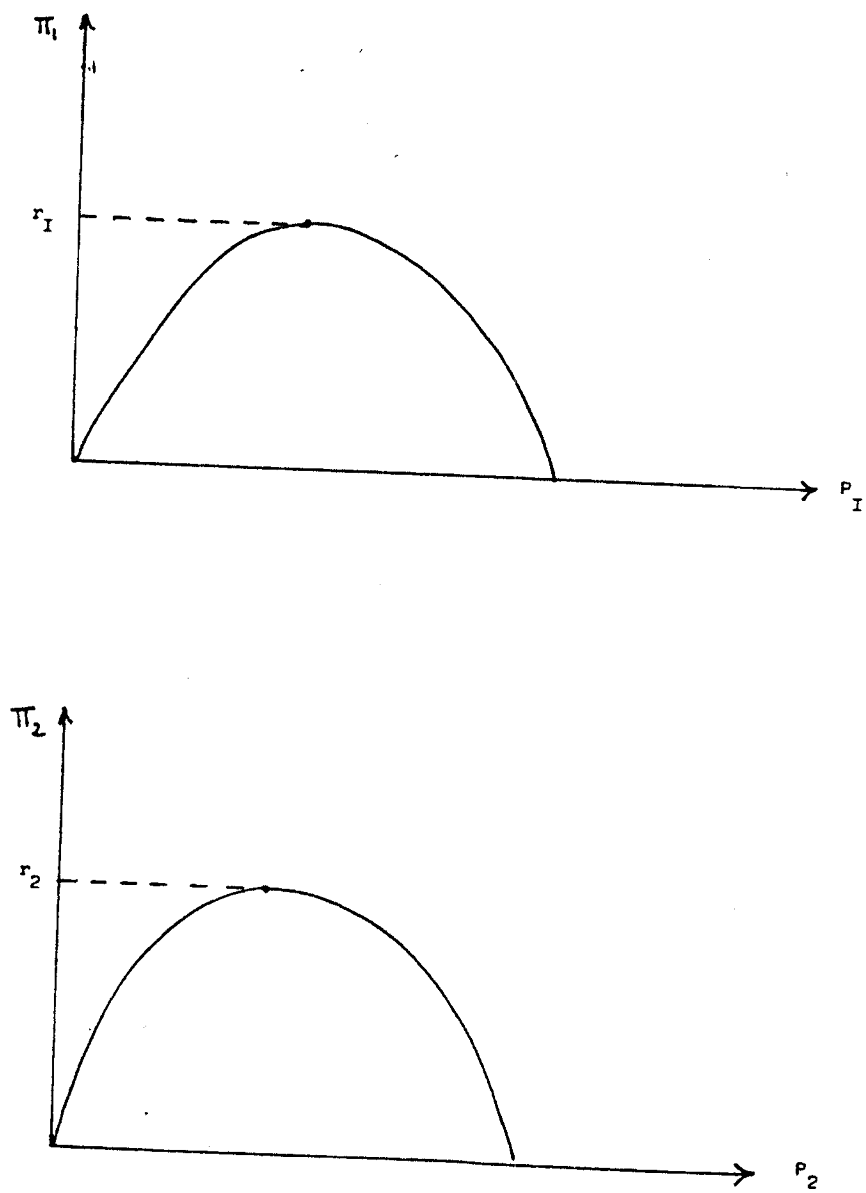


FIGURE 6

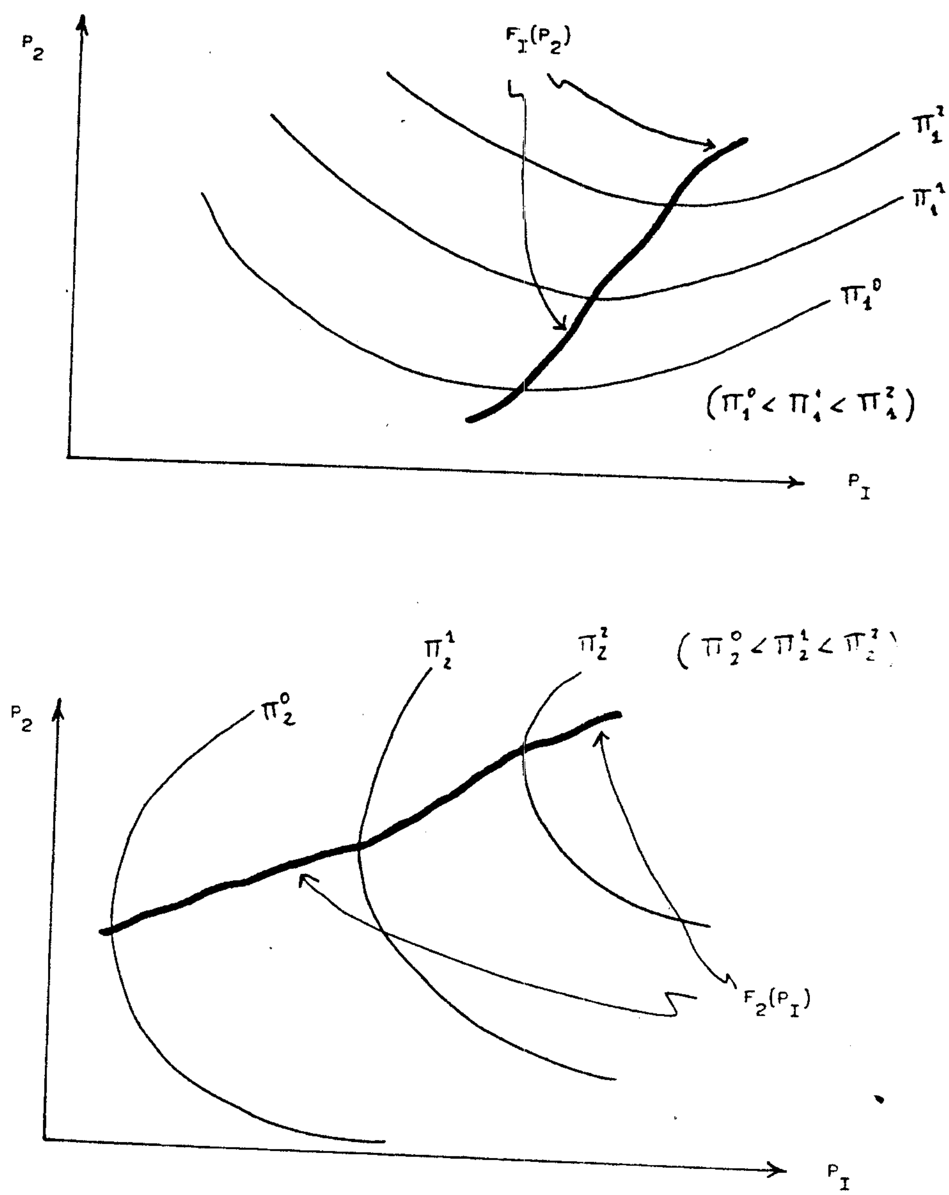

P_2
$F_I(P_2)$
π_1^2
π_1^1
π_1^0
$(\pi_1^0 < \pi_1^1 < \pi_1^2)$
P_I
π_2^1
π_2^2
$(\pi_2^0 < \pi_2^1 < \pi_2^2)$
P_2
π_2^0
$F_2(P_I)$
P_I

FIGURE 7

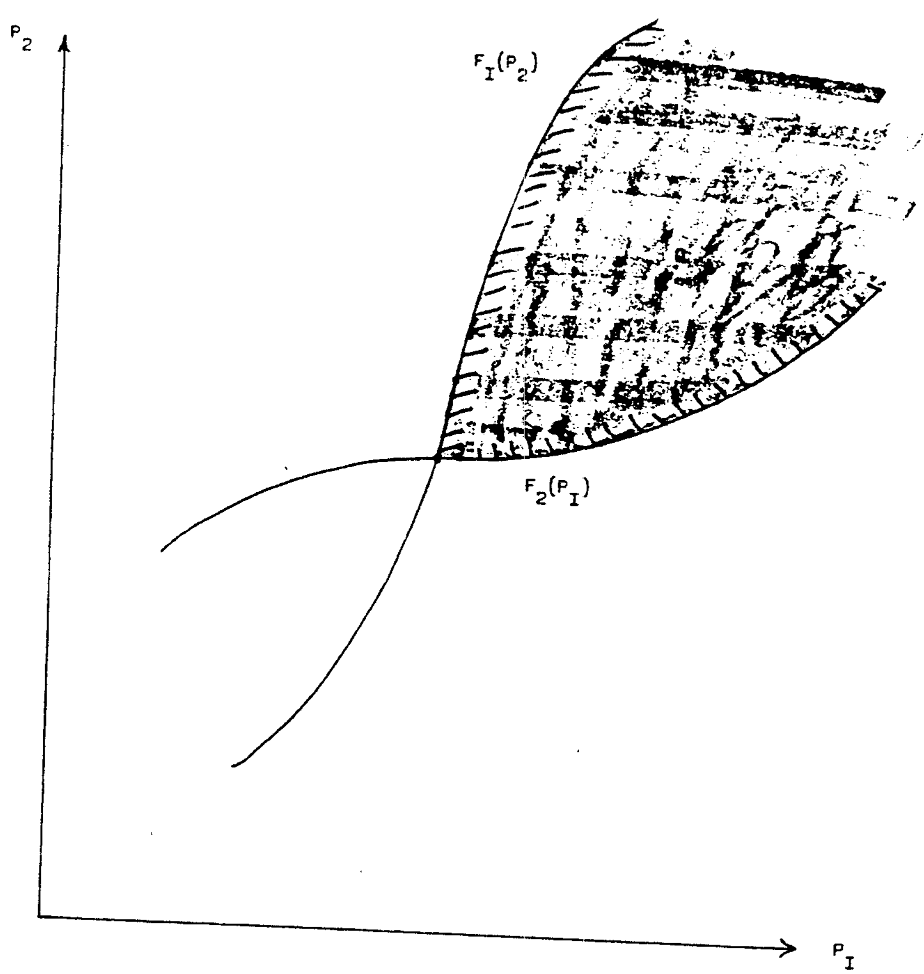


FIGURE 8

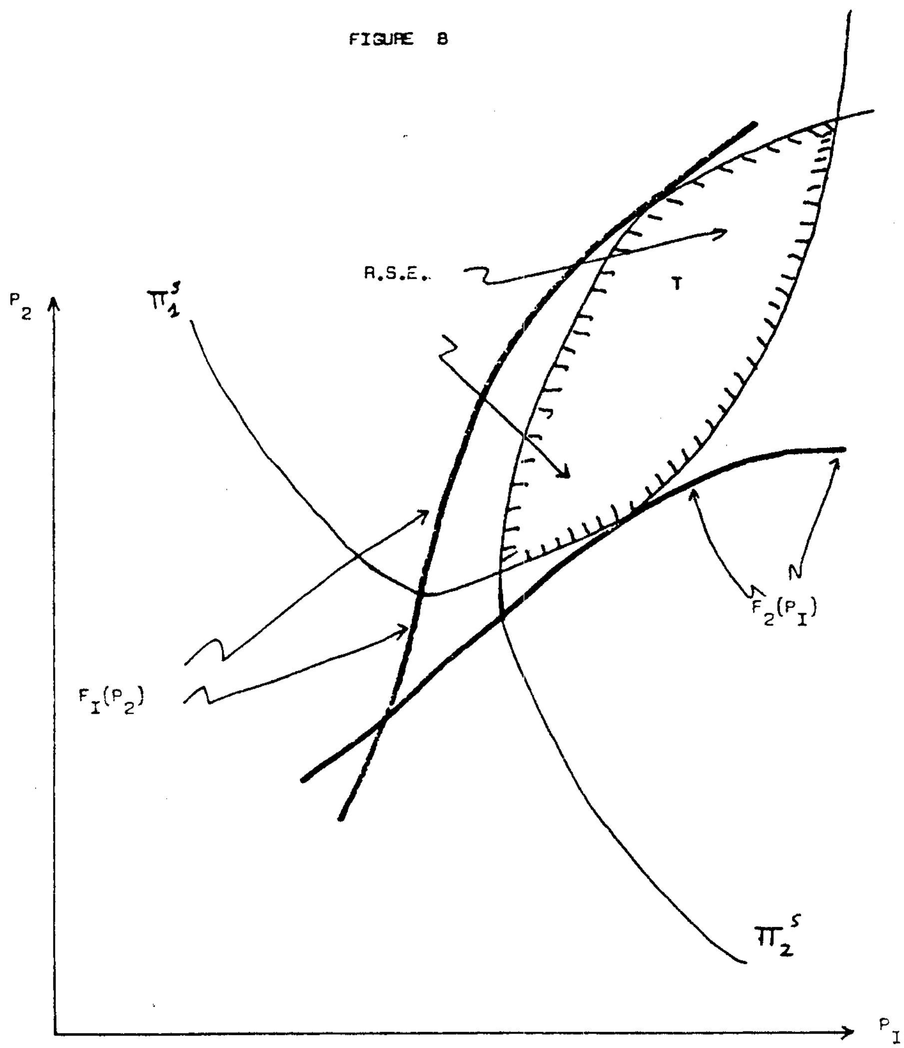
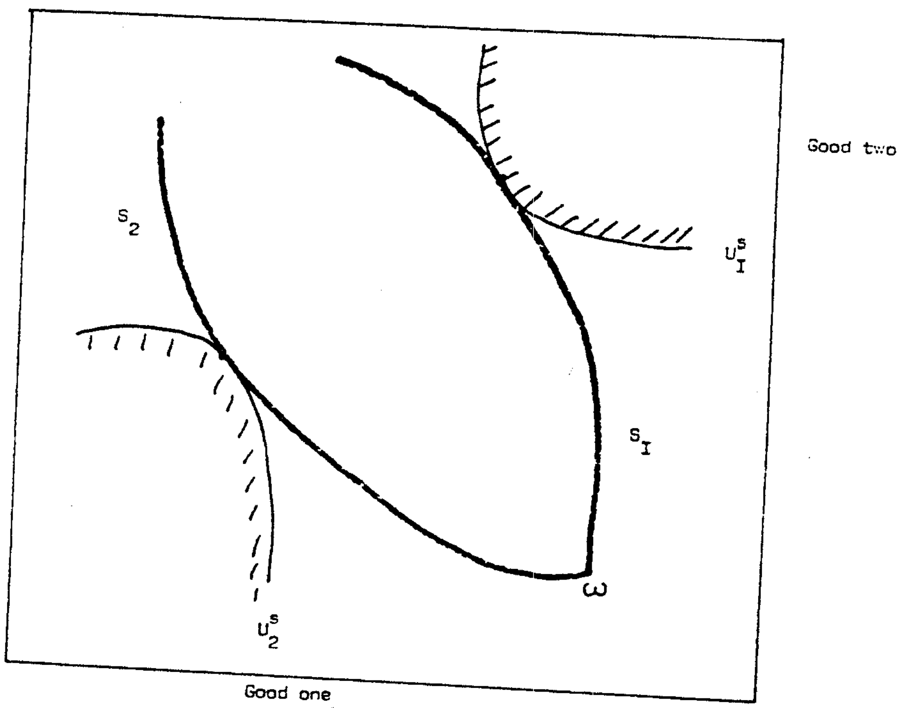


FIGURE 9



S_i = Supply curve of agent i ($F_i()$ in our framework)

U_i^s = Utility level of agent i if he is monopolist (π_i^s in our framework)

ω = Initial endowments