Rational bubbles and volatility persistence in India stock market

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Abstracts

This paper employs a combination of unit root tests and fractional integration technique to test for rational bubbles in Bombay Stock Exchange (BSE). It is indicated in the paper that evidence of a unit root in dividend yield is consistent with presence of rational bubbles in the stock prices. The results in the paper strongly support evidence of rational bubbles in BSE. Moreover, the paper also investigates the degree of conditional volatility persistence to show persistence of shocks to stock price volatility is short-termed.

Keywords: Bubbles, Unit roots, Fractional integration, Dividend yield.

1-Introduction:

For the past two decades, emerging markets have been viewed as providers of new menu of opportunities for international investors, who seek extreme gain opportunities and ready to endure extreme loss possibilities. In these events, a crucial issue to be addressed is: to what extent a market is fundamentally a strong? In an efficient market the present value of the expected future dividends of a share represent the fundamental value of the share. This is because in an efficient market stock prices change only in response to a new information about change in fundamentals. When investors purchase shares solely for its future payoff (dividends), stock prices are said to be driven mainly by fundamentals. However, in a market dominated by non-fundamental speculative factors stock price diverge from its fundamental value. Thus, systematic divergence of stock price from its fundamental value is an indication of rational bubble. Blanchard and Watson, 1982, refer to rational bubbles as self-fulfilling expectations that push stock prices towards expected price level, which unrelated to change in the fundamentals of the stock price. Sharma, and Bikhchandani, 2000, attribute rational bubbles to the presence of a large number of investors reacting simultaneously to new information so that an overreaction in aggregate is created. A number of authors (Campbell and Shiller, 1988; Diba and Grossman, 1988; Timmermann, 1995; Nasseh and Strauss, 2003; Koustas and Serletis,2005; Cunado et al 2005) have all investigated the presence of rational bubbles in a number of developed markets by investigating integration of stock prices and dividends. The main difference between the present paper and the above mentioned papers is that in this paper our aim to test rational bubbles in a fast growing major emerging stock market, which is Bombay stock market. Following similar approach as that of Koustas and Serletis, 2005, in this paper we employed the fractional integration technique, which test the order of integration, I(d), when d, takes a fraction value between 0 and 1. The remaining parts of the paper includes the following.

Section two discusses modeling rational bubbles. Section three illustrates ARFIMA(p,d,q) process, while section four illustrates the data and estimation results. The final section concludes the study.

2-Modeling rational bubbles:

In modeling rational bubbles we adopt the same approach as in Campbell and Shiller (1988b). Since stock returns at time t+1, can be defined as the capital gains plus expected dividend yield then:

$$r_{t+1} = \frac{P_{t+1} - P_t}{P_t} + \frac{D_{t+1}}{P_t} \qquad (1)$$

where r_{t+1} is the return of a stock held at the end of the period t+1, P_t and D_t respectively the stock price and the dividends payable at the end of period t. Taking the mathematical expectation on equation (1), based on the available information at time t, and rearranging terms we get:

$$P_t = E_t\left(\frac{P_{t+1} + D_{t+1}}{1 + r_{t+1}}\right) \qquad (2)$$

Now solving equation (2) forward n-periods yield:

$$P_t = E_t\left[\sum_{i=1}^{n}\left(\frac{1}{1 + r_{t+i}}\right)^i D_{t+i}\right] + E_t\left[\left(\frac{1}{1 + r_{t+n}}\right)^n\right] P_{t+n} \qquad (3)$$

To solve for a unique solution, we need to assume that in the long term the last term in equation (3) diminishes to zero so that[1]:

$$E_t\left[\left(\frac{1}{1 + r_{t+n}}\right)^n P_{t+n}\right] \rightarrow 0 \qquad (4)$$

$$as\ n \rightarrow \infty$$

Then from (3) and (4) the fundamental value of the stock price defined as:

$$F_t = E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1 + r_{t+i}}\right)^i D_{t+i}\right] \qquad (5)$$

Taking into account (4) and (5) equation (3) can be re-stated as:

$$P_t = F_t + B_t \qquad (6)$$

where,

$$B_t = E\left(\frac{P_{t+k}}{1 + r_{t+k}}\right) \quad for \quad k = 1,2....$$

Campbell et al (1997) refer to the term B_t in equation (6) as rational bubble, because it is consistent with rational expectation and the time path of the expected return. The time-varying expected stock return component in equation (6) render equation (6) into a nonlinear form. To

[1] This is true for any positive end-period discount rate (i.e., $r_{t+n} > 0$)

simplify equation (6) further, Campbell and Shiller (1988b) suggest a log-linear approximation to equation (1) so that:

$$\begin{aligned}\hat{r}_{t+1} &= \log(P_{t+1} + d_{t+1}) - \log(P_t) \\ &= \hat{P}_{t+1} - \hat{P}_t + \log(1+\exp(\hat{d}_{t+1} - \hat{P}_{t+1})) \qquad (7)\end{aligned}$$

where,

$$\hat{r}_{t+1} = \log(1 + r_{t+1})$$
$$\hat{P}_t = \log(P_t)$$
$$\hat{d}_t = \log(d_t)$$

Equation (7) is a nonlinear function of the log dividend-price ratio. First-order Taylor expansion around the mean, reduce equation (7) to the log-linear approximation:

$$\hat{r}_{t+1} \approx \alpha + \lambda \hat{P}_{t+1} + (1-\lambda)\hat{d}_{t+1} - \hat{P}_t \qquad (8)$$

where λ and α are parameters. Equation (8) is a linear difference equation for the log stock price. Solving forward and imposing the no bubble assumption, $\underset{i\to\infty}{\lambda^{i} p_{t+i}} = 0$, we obtain:

$$\hat{P}_t = \frac{\alpha}{1-\lambda} + \sum_{i=0}^{\infty} \lambda^i \left[(1-\lambda)\hat{d}_{t+1+i} - \hat{r}_{t+1+j}\right] \qquad (9)$$

In a final step, take the mathematical expectation of (9), based on the available information at time t, and solve for the log dividend-price ratio, so that:

$$\hat{d}_t - \hat{P}_t = -\frac{\alpha}{1-\lambda} + E_t\left[\sum_{j=0}^{\infty} \lambda^j [-\Delta\hat{d}_{t+1+j} + \hat{r}_{t+1+j}\right] \qquad (10)$$

Equation (10) implies that when the dividend growth factor, Δd_t, and the log of stock returns are stationary stochastic processes, the log dividend yield is stationary, and thus no rational bubble is holding. As a result, in order to test for rational bubbles, we either test for unit roots in the variables on the right-hand side of equation (10), or alternatively for a unit root in the left-hand side variable, which is the log dividend yield. In this paper we adopt, beside the classical unit root tests, ARFIMA(p,d,q) process to test the order of integration of stock price and dividend yield variables.

3: The ARFIMA(p,d,q) process

$$\phi(L)(1-L)^d(y_t-\mu)=\theta(L)\varepsilon_t \qquad (11)$$

where

$$\phi(L)=\sum_{j=1}^{p}\phi_j L^j\ , \quad \theta(L)=\sum_{j=1}^{q}\theta_j L^j\ ,$$

$$(1-L)^d=\sum_{j=0}^{\infty}\left(j^d\right)(-1)^j L^j=1-dL+\frac{d(d-1)L^2}{2}-.....$$

and L is lag operator, d is fractional differencing parameter, all roots of $\phi(L)$ and $\theta(L)$ assumed to lie out side the unit circle, and ε_t is white noise.

GARCH(p,q) models often used for modeling volatility persistence which have the features of relatively fast decaying persistence. However, in some cases volatility shows very long temporal dependence, i.e., the autocorrelation function decays very slowly. This motivates consideration of Fractionally Integrated Generalized Autoregressive Conditional Hetroskedasticity (FIGARCH) process (Baillie et al, 1996) defined as[2]:

$$\varphi(L)(1-L)^d\varepsilon_t^{\ 2}=w+\{1-\beta(L)\}v_t \qquad (12)$$

where $\varphi(L)$ *and* $\beta(L)$ are respectively the AR(p) and MA(q) vector coefficients and $v_t=\varepsilon_t^{\ 2}-\sigma_t^{\ 2}$,

[2] For the FIGARCH(p,d,q) model to be well defined, and the conditional variance positive for all t, all the coefficients in the ARCH representation must be non-negative.

Following Baillie et al (1996), Bollerslev and Mikkelsen (1996), Granger and Ding (1996), the parameters in the ARFIMA(p,d,q) and FIGARCH(p,d,q) models in (11) and (12) estimated using quasi-maximum likelihood (QMLE) method. In the ARFIMA models, the short-run behavior of the data series is represented by the conventional ARMA parameters, while the long-run dependence can be captured by the fractional differencing parameter, d. A similar result also applies when modeling conditional variance, as in equation (12). While for the covariance stationary GARCH(p,q) model a shock to the forecast of the future conditional variance dies out at an exponential rate, for the FIGARCH(p,d,q) model the effect of a shock to the future conditional variance decay at low hyperbolic rate. As a result, the fractional differencing parameter, d, in the equations (11) and (12) can be regarded the decay rate of a shock to the conditional variance (Bollerslev, 1996). In general, allowing for values of d in the range between zero and unity (or, $0<d<1$) add a flexibility that play an important role in modeling long-run dependence in time series[3].

Bollerslev, 1996, indicates that if $d=0$, the series is covariance stationary and possess short memory process, whereas in the case of $d=1$ the series is non-stationary. However, in the case of $0<d<0.5$, the series even though covariance stationary, its auto-covariance decays much more slowly than ARMA process. If d is $0.5<d<1$ the series is no longer covariance stationary, but still mean reverting with the effect of a shock persist for a long period of time, and in that case the process is said to have a long memory. Given a discrete time series, y_t, with

[3] See Diebold and Rudebuch (1989), Diebold, Husted and Ruch (1991), Lo (1991), and Swell (1992) for a detailed discussion about the importance of allowing for non-integer values of integration when modeling long-run dependence in the conditional mean of time series data.

autocorrelation function, ρ_j , at lag j, Mcleod and Hipel (1978) define long memory as a process

$$\sum_{j=-n}^{n} |\rho_j| \quad as \quad n \rightarrow \infty \qquad (13)$$

characterized as nonfinite. In the non-stationary and in the long memory process a shock e_t at time t, continues to influence future y_{t+k} for a longer horizon, k, than would be the case for the standard stationary ARMA process. While there are varieties of ways to estimate the parameters of (11) and (12), in this paper we employed the maximum likelihood estimator.

4. Data and Estimation Results:

The data employed in this research includes daily, weekly, and monthly aggregates of stock price and dividend yield of Bombay Stock Exchange during the period from 1-Jan- 2002 to 1-Sept-2009. The weekly data corresponds to the averages of the five trading days in each week, whereas the monthly data correspond to the average of the trading days in each month. Before we resort to parametric tests of unit roots in the variables, it may be helpful to investigate the behavior of the ACFs from AR(1) process to see if they behave as stationary process[4]. The non-stationarity condition can be characterized by large non-vanishing spikes in the sample ACF of the original series and insignificant zero ACF for the differenced series[5]. Visual inspection of the plots in figures (1) and (2) indicate the dividend yield and the stock price series are

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[4] The random walk model is a limiting process of the AR(1) process $(1-\phi L) y_t = e_t \ with \ \phi \rightarrow 1$

[5] Wei (1989) indicates that the associated standard errors of ACF and PACF can be calculated respectively, using the formulas: $se_{\hat{\rho}_k} \approx \sqrt{\frac{\sum_{i=1}^{k-1}(1+2\hat{\rho}_i^{\,2})}{n}}$ and $se_{\varphi_k} \approx \sqrt{1/n}$

nonstationary. Figure (3) plots the logarithms of stock prices, and dividend yield of the daily data– which is presented against the second vertical axis. It is evident that there is no apparent trend in the log of the dividend yield. The sharp rise in the dividend yield that occurred during September –December months of 2008 is mainly due to the rapid decline of stock prices after the Lehman brothers bankruptcy announcement. Since the short-termed break in September-December period suggest the possibility of structural change in the trend of the dividend yield, we employed Chow test to check for such possibility. The Chow test results (not reported) indicate no significant structural changes in the trend of the dividend yield. Table (1), reports the unit root test results, using Augmented Dickey-Fuller (1979), Phillip-Perron (1988), and Kwiatkowski et al (1992), known as KPSS, tests[6]. The results of all the three tests, under 5% significance level, reject the stationarity condition in favor of unit root hypothesis, for the daily, weekly, and the monthly series. As indicated earlier, evidence of unit root (non-stationarity) in the log dividend yield is consistent with existence of rational bubbles, which imply persistent deviation of stock prices from its fundamental value, which is the dividend per share value.

However, it is well documented in the literature that the Augmented Dickey-Fuller (ADF) and Phillip-Perron (PP) unit root tests in particular, suffer from very low power against stationary alternative if the roots close to the unit root. Diebold and Rudebusch (1991), and Hassler and Walters (1994) indicate ADF and PP unit root tests have very low power

[6]KPSS test initially was developed to test the null-hypothesis I(0), against the alternative I(1). However, Lee and Schmidt (1996) indicated (Theorem 3, page 291) the KPSS test is consistent with the null hypothesis of short memory, against stationary long memory alternatives, such as I(d) process for $d \in (-0.5, 0.5), d \neq 0$ .Thus, KPSS test can also be used to distinguish short memory and long memory stationary processes.

against fractionally integrated alternative. To account for such a shortfall we investigate the order of integration of the two data sets using fractionally integrated ARMA process. Since the ADF, PP, and KPSS unit root tests restrict the order of integration to the integer values of zero and one, the ARFIMA (p,d,q) process can verify an order of integration of fractional exponent. Results reported in table (2) reject the fractional integration of the log dividend yield and the log price level, for daily, weekly, and monthly time series data. The estimated values of, d, are significantly greater than the stationary range of ($-0.5<d<0.5$). Results of unit root tests and fractional integration test in tables (1) and (2) both suggest rejection of mean reversion hypothesis in the log prices and log dividend yield, in favor of the unit root hypothesis, which imply evidence of rational bubbles in BSE.

The effect of aggregation bias in the data is realized by a number of authors in the literature (Schewart, 1989; Ng 1995; Taylor 2001, 2002) and pointed out that the use of low frequency data increase bias towards random walk process. For instance, Taylor (2001) concludes that if stock price adjustment towards its fundamental value (dividends) is of order of days or weeks, then using monthly data could bias the results towards finding unit roots in the data, and thus concluding existence of rational bubbles. To safeguard against these type of aggregation bias, we conducted Monte Carlo simulation of 2000 replication assuming random walk Data Generating Process. The simulation results in table (3) show the fractional difference parameter, d, is unbiased and therefore complement the significance of the results in table (3), that is the unit root hypothesis of both log dividend yield, and the log price level.

Table (4) present results of volatility persistence of FIGARCH model. The sign and size of the $\hat{d}$ parameter in the FIGARCH model indicate there is no evidence of long memory behavior in the conditional variance of the dividend yield and the stock price. This implies that persistence of shocks to stock price volatility is of short memory.

Table (1): Unit root tests

	Dickey-Fuller (i)	Dickey-Fuller (ii)	Phillip-Perron (i)	Phillip-Perron (ii)	KPSS η_u	KPSS η_r
log price:						
daily data	1.32	2.20	2.13	2.10	81.64	10.41
weekly data	3.44	2.84	2.99	2.69	16.11	5.44
monthly data	1.26	4.32	1.92	3.13	0.48	3.43
log dividends yield:						
daily data	1.21	2.13	1.43	2.28	28.28	3.53
weekly data	3.35	2.88	3.27	3.17	16.99	2.99
monthly data	1.94	2.65	2.64	3.49	0.18	1.17
Critical values (5%) Significance level	4.59	4.68	4.59	4.68	0.463	0.146

Note: (i) with drift only, (ii) with drift and trend. η_u *and* η_r statistics are respectively level stationarity and trend stationarity statistics. The reported KPSS statistics are based on 20 lags for daily, 8 lags for weekly, and 2 lags for monthly data. The optimal lag length order in ADF is selected by Akaike Information Criteria (AIC).

Table (2): Estimation results of ARFIMA(1,d,1)

	Log dividends yield			Log price level		
parameters	daily	weekly	monthly	daily	weekly	monthly
$\hat{d}$ (std.error)	0.99* (0.26E-3)	0.96* (0.0016)	0.75* (0.016)	0.99* (0.96E-4)	0.99* (0.59E-3)	0.95* (0.008)
$\hat{\phi}$ (std.error)	0.0026 (0.002)	0.29* (0.004)	0.52* (0.019)	0.067* (0.001)	0.26* (0.39E-2)	0.33* (0.021)
$\hat{\theta}$ (std.error)	-0.16E-7 (0.17E-7)	-0.174E-6* (0.33E-7)	0.14E-6* (0.93E-7)	0.11E-7 (0.10E-7)	0.66E-8 (0.25E-7)	0.6E-7 (0.15E-6)
c (std.error)	0.0014* (0.75E-4)	0.0047* (0.0003)	0.033* (0.0019)	0.0057* (0.0003)	0.024* (0.17E-2)	0.14* (0.023)
Log-likelihood function	9401	1620	268	10368	1731	231

*significant at 5% significance level.

Table (3): Monte Carlo simulation

	Log dividend yield	Log price
Daily		
$\bar{d}$ ESE STDSE	0.99 0.07 0.001	1.000 0.041 0.0007
weekly		
$\bar{d}$ ESE STDSE	0.96 0.17 0.006	0.99 0.09 0.003
Monthly		
$\bar{d}$ ESE STDSE	0.63 0.36 0.02	0.92 0.20 0.01

Note: $\bar{d}$ is the average parameter estimate. ESE is the average standard error, STDSE is the standard deviation of the standard error.
We used DGP process of ARFIMA(0,d,1):
$(1-L)^d (y_t - u) = e_t$ *where* $e_t = \theta e_{t-1} + \varepsilon_t$ *for* ε_t *is white noise*

Table (4): FIGARCH(1,d,1)

	Log dividends yield			Log price level		
parameters	daily	weekly	monthly	daily	weekly	monthly
$\hat{d}_1$ (std.error)	0.099* (0.039)	0.14* (0.09)	0.39* (0.13)	0.38* (0.030)	-0.49* (0.047)	0.33* (0.15)
$\hat{\phi}_1$ (std.error)	-0.080* (0.043)	-0.069 (0.12)	-0.21* (0.16)	-0.16* (0.035)	0.79* (0.036)	-0.17 (0.17)
$\hat{\theta}_1$ (std.error)	0.050 (1.00)	0.050 (1.00)	0.050 (1.00.)	0.050 (1.00)	0.050 (1.00)	0.05 (1.00)
Log-likelihood function	7477	987	146	10495	1854	254

*significant at 5% significance level

5. Concluding remarks:

This paper has employed a combination of unit root tests and fractional integration techniques to test the order of integration of log dividend yield in Bombay Stock Exchange. The paper shows that the presence of a unit root in the log dividend yield is consistent with the evidence of rational bubble in the stock price level. The paper also investigates the degree of conditional volatility persistence using FIGARCH(p,d,q) model for the log dividend and the log price on daily, weekly, and monthly series, during the period from January-1-2002 to September-1-2009. The results in the paper strongly support evidence of rational bubbles in BSE. Our Monte Carlo simulation results fully support the estimation results and shows no aggregation bias effect on the results. Evidence of rational bubbles in BSE reflect consistent divergence of stock prices from stocks fundamental values. Presence of rational bubbles in BSE can be viewed as indication of herd behavior in the market trading activities, as large number of investors may react simultaneously to new information, and thus creating an overreaction in aggregate.

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Fig 1-The sample ACF of dividend yield

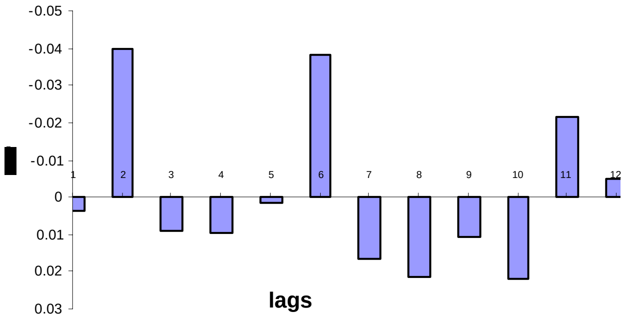

Fig 2-The sample ACF of stock price

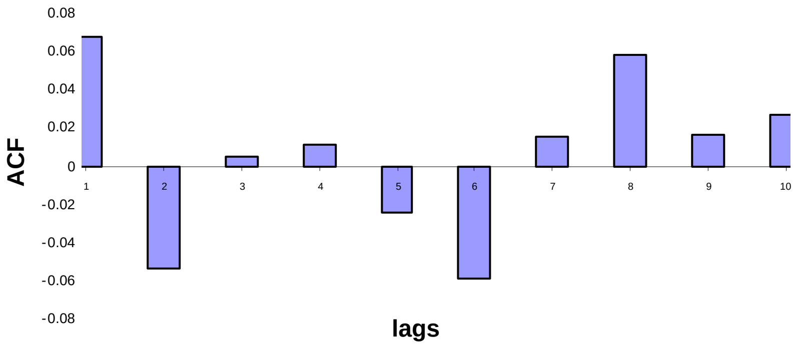

Fig.3:Dividends yield and stock price index