Plurality versus proportional electoral rule: which is most representative of voters?

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Abstract

This study compares the representativeness of voters in the proportional electoral system with the situation under plurality rule. Representativeness is commonly measured by comparing parties' received votes with their shares of seats in the Parliament; this implies that proportional rule should always better represent voters. A coalition within the Parliament, however, rules the country without interference and supports the government; when a coalition is formed, the pivotal role of small parties and the proposal right of the *formateur* can significantly impact the distribution of power. Focusing on the coalition formation stage, I demonstrate that the proportional rule is more representative only under very specific conditions. If these conditions are not met, introducing some distortions in the distribution of seats among parties can actually improve representativeness.

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Keywords: Electoral systems; Proportional rule; Plurality rule; Voters' representation

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"Thus there is an inherent conflict between two goals. The ideals of democracy and equality require as proportional representation as possible while efficient government often requires less proportional representation" (Laakso and Taagepera (1981), p. 107).

1 Introduction

Electoral systems differ in terms of government efficiency and representativeness. I focus on representativeness,[1] which is the system's ability to produce laws that reflect the will of voters. In a perfectly representative system, each party's power would be proportional to their share of the votes received.

Under the proportional rule, a Parliament's composition perfectly reflects a party's share of the vote, and thus matches voter preferences. Common wisdom suggests that proportional systems should be the most equitable.[2] This is not necessarily the case: consider for example the experience of Italy during the 2006-2008 Prodi government. A small pivotal party succeeded in heavily influencing his decisions. I will show that the common wisdom is misleading: plurality rule can sometimes better reflects voter preferences.

Two filters impact representativeness and can induce distortions (see Figure 1): the electoral system (filter 1) and the coalition formation stage (filter 2). All electoral systems other than the proportional approach will lead to a distortion in the composition of Parliament. At the coalition formation stage, this distortion is due to two factors: i) some parties are excluded from the government, and ii) the share of power amongst the others differs from the distribution of seats in Parliament, since pivotal parties enjoy disproportionately strong bargaining power. Voters' misrepresentation of preferences depends on the distortions in both filters 1 and 2. When these distortions are of opposite sign, the two distortions will compensate for one another, possibly up to the point where they cancel out.

While several papers have been devoted to each distortion separately,[3] this paper is, to the best of my knowledge, the first to analyse the entire electoral process, from elections to government formation. Within the (possibly infinite) set of potential electoral rules, I focus my attention exclusively on proportional

[1]Efficiency, i.e., the capability to produce well structured laws, that minimise resource wastage, is beyond the scope of this work.

[2]See, for instance, Douglas (1923). However, one drawback of proportional representation is the greater instability and the increased time required to enact laws, compared with majority voting systems. See Laakso and Taagepera (1981), Nurmi (1981) and Schofield (1981).

[3]Besides the well known paper from Taagepera and Shugart (1989), Morelli (2004) and Kestelman (1999) offer a clear and succinct review of the literature on distortions in electoral rules. Baron and Ferejohn (1989), Snyder Jr., Ting, and Ansolabehere (2005) and Kalandrakis (2006) study certain properties of the coalition formation game.

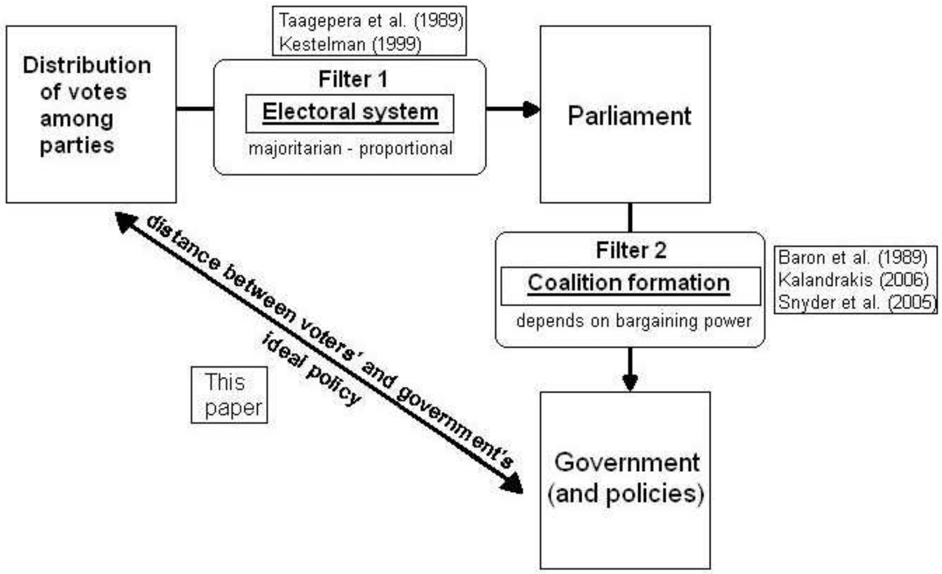

Figure 1: The government formation process

and plurality approaches, which account for most western democracies. I assume that parties act non-cooperatively during government formation. I compute the misrepresentation of voters' preferences in the two systems, defined as the difference between parties' power and their share of the votes received. I derive the conditions under which one system reflects citizens' preferences better than the other.

Not surprisingly, the pivotal position of small parties in coalition formation contexts allows them to enjoy political power that is more than proportional to their share of the seats. This distortion is reduced when the parties are impatient to form a coalition. The distribution of seats under plurality rule is favourable to large parties. Pushing in opposite directions, the distortions compensate; when their magnitude is similar, the voters' preferences are better represented under plurality rule. I conclude that majority voting is preferable when parties are patient, whilst the proportional rule is more representative when the parties are impatient.

Section 2 sets the framework of this study and describes the model used. Section 3 describes my results and discusses the consequences of relaxing certain assumptions. Section 4 validates the model using data from the 2006 and 2008 Italian elections. The last section concludes the paper.

2 The framework and the model

2.1 The political process

The political process begins with elections; negotiations occur only once the distribution of seats in Parliament is known. Parties try to form a coalition that controls the majority of seats; when they succeed, they share power. The government can rule the country if it is supported by a majority: at all times, the parties' shares of power must satisfy their participation constraints.

To form a coalition requires agreeing on a political program and on the distribution of economic benefits. Parties in the winning coalition hold office and enjoy ideological benefits.[4] For expositional convenience, I consider that the winning coalition shares a budget that can be used to implement the political agenda; parties are self-interested and only care about their own share.[5]

Shares depend on bargaining power; they are the outcome of either a cooperative or a non-cooperative game. Cooperative coalition theory applies when forming the *grand coalition* maximises the aggregated profit. Non-cooperative theory applies when players maximise their own payoffs given the others' best responses, which usually occurs if it is convenient to deviate from the cooperative equilibrium. Belonging to the government is necessary to obtain a positive payoff; if a coalition controls the majority of seats, there is no interest in enlarging it. It is thus reasonable to expect that parties will act non-cooperatively.

Coalition formation begins with the selection of a party, called the 'formateur', that will be in charge of leading negotiations.[6] If a single party controls the majority of seats, it will govern the country unimpeded. The method of selection of a formateur is usually not determined by the constitution; larger parties have a greater likelihood of being selected. Baron and Ferejohn (1989) and Austen-Smith and Banks (1988) propose alternative ways of computing the probability of being formateur. Austen-Smith and Banks (1988) assumes that the parties' shares determine the probability of being a successful formateur; the largest party is chosen first, and in the event they fail to form a coalition, the second largest is

[4]Ideological benefits include being able to implement preferred policies. Holding office confers direct and indirect monetary benefits including, for instance, the ability to determine public expenditures in strategic sectors. Some papers concentrate on either office (e.g., Riker (1962) or Baron and Ferejohn (1989)) or ideological (e.g., Schofield (1986)) benefits. Austen-Smith and Banks (1990) analyse both separately, assuming they are orthogonal. In Sened (1996) the two elements are amalgamated.

[5]Alternatively, i) think of orthogonal projects that need to be financed, with each party interested in one project, or ii) assume that parties fix the time devoted to crafting legislative proposals, with power representing the ability to pursue one's own agenda.

[6]For a detailed explanation of the role of the formateur and how it is chosen, see Diermeier and Merlo (2004).

chosen, and so forth. In contrast, Baron and Ferejohn (1989) directly attach to parties a probability of being a successful formateur that is equal to their share of the vote. When the formateur is successful, it always belongs to the winning coalition. The two procedures give similar results; I follow Baron and Ferejohn (1989)'s approach, which is common in the theoretical literature,[7] and performs well in empirical tests.[8]

2.2 The model

I consider a country with 3 parties and 3 groups of homogeneous citizens indexed by i; c_i denote the relative sizes of the groups, i.e., the proportion of votes won by each party; $c_1 + c_2 + c_3 = 1$. Without loss of generality, I order the groups by their size, thus $1 > c_1 \geq c_2 \geq c_3 > 0$. I assume that the number of parties remains unchanged when the electoral rule changes; I do not consider ideological restrictions during coalition formation.[9] Party i's political program maximises the utility of voters in group i. The vector $e = (e_1, e_2, e_3)$ denotes the parties' share of the seats. An electoral system is seen as a function F that transforms parties' shares of votes into shares of seats, i.e., $e = F(c)$. I focus on two systems: proportional and majority voting (also called "plurality rule"). Under proportional rule, the parties' shares of seats equal the shares of votes received, i.e., $e_i = c_i$. Under plurality rule, the country is divided into Q districts (one for each available seat); in each district, the candidate who receives the most votes wins: $e_i = \frac{Q_i(c)}{Q}$, where $Q_i(c)$ is the number of districts in which party i secures the majority of votes cast.

Assumption 1 (No standing-alone) *No party ever obtains the majority of the seats: thus, $e_1 < 0.5$ and, a fortiori, $c_1 < 0.5$.*[10]

The grey area in Figure 2 shows the possible combinations of e_2 and e_3 taking into account the ordering $0.5 > e_1 \geq e_2 \geq e_3 > 0$.

Assumption 2 (Constant coalition value) *The resources available to allocate remains constant. The bargaining issue boils down to the "sharing a dollar" problem, where each party (and its voters) is only interested in its share of the total budget.*

[7]See Baron and Diermeier (2001) or Diermeier, Eraslan, and Merlo (2007).

[8]See Diermeier and Merlo (2004).

[9]Fixing the number of parties is a short-term assumption. Ideological restrictions are not an issue; the model can be understood as the process that occurs when ideologically close parties must negotiate. The exclusion of a party can be reproduced by rescaling the shares of the vote.

[10]Since non-proportional electoral frameworks favour big parties, $c_1 > 0.5$ would imply $e_1 \geq 0.5$.

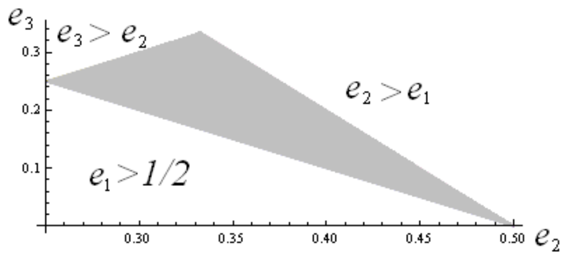

Figure 2: Possible combinations of e_2 and e_3.

When no party secures a majority, the bargaining phase begins. A coalition S is the result of an agreement between 2 parties regarding how to share the budget. Let Z denote the set of all feasible allocations, $Z = \{z \in \Re^3_+ : \sum_{i=1}^{3} z_i \leq 1\}$, z_i is the budget share of party i. The agents' utility, linear in z_i, is independent of z_j, i.e., $U_i(z) = z_i$. A winning coalition has to be supported by at least half of all Parliamentarians. $D \subseteq 2^{\{1,2,3\}}$ is the set of all possible winning coalitions; with 3 parties, D does not depend on e: any pair of parties can secure a majority. Given the asymmetry among the parties implied by the role of the formateur, we henceforth denote the formateur by the first element in a coalition; thus coalitions (i, j) and (j, i) are different.

At time $t = 0$ a party, called the formateur, is randomly chosen. To form a coalition $S \in D$, the formateur proposes a vector z of shares to the parties in S;[11] if z is accepted by S the game ends: the government is formed and the budget is shared according to z. Otherwise, in the next period a formateur (possibly the same one) is randomly chosen and the game continues until an agreement is reached. I use the notation z_i^j to indicate the i^{th} element of vector z when j is the formateur.

Assumption 3 (Recognition probability) *As in Baron and Ferejohn (1989), the recognition probability π_i of being a successful formateur is equal to party i's share of seats (i.e., $\pi_i = e_i$).*

Following Kalandrakis (2006) and Snyder Jr., Ting, and Ansolabehere (2005), I consider a bargaining game à la Rubinstein-Ståhl (Rubinstein (1982)). The continuation value v is the vector of the parties' expected utility during the next period. I concentrate on stationary proposal strategies with no delay: in each period a party behaves the same way and proposes a share vector such that all parties belonging to the proposed coalition accept without delay.

[11]To have a winning coalition, all parties in S should obtain a positive share. There is no reason to leave a positive share to parties outside the coalition.

Assumption 4 *Parties discount the future (they care about the time needed, after the elections, to form a government). The patience rate, $\delta < 1$, is the same for all parties.*[12]

At time t, the utility of obtaining z_i in $t+k$ is: $U_i(z,k) = \delta^k z_i$; the parties' outside opportunity is zero. Party i's continuation value $v_i = \delta \sum_{h=1}^{3} \pi_h z_i^h$, is the discounted expected utility of i, conditional on forming a coalition in the subsequent period; the uncertainty concerns the formateur's identity, and the value of v depends on the recognition probabilities.

Given the vector of seat shares e and the time discount factor δ, a game is denoted by $\Gamma(\delta, e)$. When an agent serves as formateur, its action consists of proposing a division $z^i \in Z$ of the budget, and the others' action space consists of accepting the formateur's proposal or not. A Stationary, Subgame Perfect, Pure Strategy (SSPPS) equilibrium for game $\Gamma(\delta, e)$ is a set z^i of stationary strategies and acceptance strategies. In equilibrium, SSPPS implies no delay: the formateur proposes a share that is immediately accepted. A SSPPS equilibrium requires the share of all parties in the coalition to be such that $z_i \geq v_i$;[13] the existence of a SSPPS Nash equilibrium is not an issue for game $\Gamma(\delta, e)$ according to the arguments of Banks and Duggan (2000). Other equilibria may exist; I focus on the stationary ones in pure strategies.

Voters preferences. Preferences cannot be directly observed; people's votes can. If voters act strategically, we cannot deduce their preferences from their vote; a change in the electoral system may impact the voting strategy. I assume sincere voting; note that, in addition to its use in the theoretical literature, recent empirical studies have shown that the hypothesis of non-strategic voting cannot be rejected: for instance, Hooghe, Maddens, and Noppe (2006) find evidence that the aggregate behaviour of the voters did not change significantly after the last change of government in Belgium. They interpreted this as a signal of myopic/sincere voting, which is reasonable under majority voting conditions when the parties' policy vectors are orthogonal or at least sufficiently different not to be seen by voters as substitutes. Furthermore, it is costly to be informed about politics (programmes, performances, the electoral system, other voters' expected behaviours), and voters may prefer to vote sincerely. Assuming sincere voting allows us to infer the voters' preferences from their votes.

Measuring misrepresentation. The goal of the model is to relate voter preferences to government policies.[14] Given the voters' choice, I determine the winning

[12]Note that only the smallest party's δ impacts the results, so to account for different discount rates, simply replace δ with the discount rate of the smallest party in the coalition.

[13]For more details, see Kalandrakis (2006), p. 444.

[14]Policies are interpreted in the context of party resources, which are proportional to the share z_i.

coalition and compute the parties' shares. I compare the representativeness of the rules, measuring misrepresentation through M^y, where $y = \{PR; MV\}$ denotes the electoral system (PR=proportional rule, MV=majority voting).

Let $\Pr(h, j)$ be the probability of forming coalition $\{h, j\}$. The electoral system y determines parties' shares of seats, impacting the probability that a coalition will form ($\Pr(h, j)$) and influencing parties' budget shares (z_i^h).

$$M^y = \sqrt{\sum_{i=1}^{3} \left(\sum_{h=1}^{3} \sum_{j=1}^{3} \Pr(h, j) \cdot z_i^h - c_i \right)^2} \tag{1}$$

Equation (1) represents the Euclidean distance between parties' expected budget share and the optimal one; i.e., it computes the distance between the parties' expected power (discounted by the probability of forming each possible coalition) and their shares of votes c_i.

Equation 1 takes large values when parties are either under- or over-represented. Over-representation occurs when a party, being pivotal for a coalition, obtains a share of benefits larger than the share of the population that it represents.

To compute M^y, both c_i and e_i are necessary.[15] The relation between c_i and e_i depends on the electoral system. Taagepera and Shugart (1989) show that, by properly choosing the parameter τ, every electoral system can be approximated through a function $e_i = F(c_i) = \frac{c_i^\tau}{\sum_{j=1}^{n} c_j^\tau}$. For proportional representation, $\tau = 1$ and $c_i = e_i$. Under plurality, the share of seats depends on the geographical distribution of voters' preferences over districts; for single-member district systems and a normal population, $\tau \approx 3$ (Qualter (1968)), hence the name 'cube rule'.[16] According to Taagepera and Shugart (1989), $\tau = 2.5$ better suits modern western societies with plurality-based single-member district systems, while $\tau = 8$ would be a better approximation for the actual system in the USA. In the literature, the cube rule (with $\tau = 3$) is considered a good approximation in the two-party case and it also fits the data for the three-party case; the precision of this measure declines with more than 3 parties.

Assumption 5 (Cube rule) *To compute the share of seats of a party under majority voting, I assume that we observe the aggregate number of votes for each party (c_i) and the cube rule holds, thus* $e_i = \frac{c_i^3}{\sum_{j=1}^{3} c_j^3}$.[17]

[15] We need e_i to compute $Pr(S) \cdot z_i^S$.

[16] J. P. Smith formulated this relationship in 1909, Duverger (1954) developed and publicised it. For a discussion of its drawbacks, see Riker (1982), Blau (2001) or Rogowski and Kayser (2002).

[17] The results can be replicated for other values of τ, to fit a specific country's electoral system and any given geographical distribution of preferences. Section 4.1 discusses this issue further.

From the previous assumption, (1) can be rewritten as follows:

$$M^{PR} = \sqrt{\sum_{i=1}^{3}\left(\sum_{j=1}^{3} c_j z_i^j - c_i\right)^2} \tag{2a}$$

$$M^{MV} = \sqrt{\sum_{i=1}^{3}\left(\sum_{j=1}^{3} \frac{c_j^3}{c_1^3 + c_2^3 + c_3^3} z_i^j - c_i\right)^2} \tag{2b}$$

3 Main predictions

In this section, after deriving the parties' shares of power, I compute the voters' misrepresentation (Equation 1) under both proportional and plurality systems. I show that the distortion under the former is greater than that under the latter when δ is sufficiently large.

A coalition of 2 out of 3 parties always secures a majority. The formateur compares its utility in each possible coalition in order to choose the other member. Ex ante, eight scenarios may occur, depending on the identity of the formateur.

Proposition 1 (Sharing rule) *In equilibrium, coalitions always include only two parties. Formateur i proposes to party j its continuation value v_j, and nothing to the other one. The share vector $z^i = \left(z_i^i; z_j^i; z_x^i\right)$ takes the following values $\left(1 - z_j^i; \frac{\delta}{1-\delta e_i}(e_j z_j^j + e_x z_j^x); 0\right)$. Party x, excluded from the coalition, receives 0; party j obtains the present value of what it would get (in discounted expected terms) in the next period.*

Proof. See Appendix A. ■

Proposition 2 (Minimal winning coalition) *In the SSPPS equilibrium, the formateur always forms a coalition with the smallest party.*[18] *The ex-ante unique equilibrium coalitions are {(1,3),(2,3),(3,2)}, where the first element of a pair denotes the formateur. The equilibrium shares depend on the formateur: a priori, $z_j^i \neq z_i^j$.*

Proof. See Appendix B. ■

Corollary 2.1 (Probability of forming a coalition) *From Assumption 3 regarding recognition probabilities, the probability $Pr(S)$ of coalition $S = (i, j)$ is $Pr(1, 3) = e_1$, $Pr(2, 3) = e_2$ and $Pr(3, 2) = e_3$.*

Corollary 2.2 *Table 1 summarises the parties' equilibrium shares z^i.*

$z^1 = \left(\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}; 0; \frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} \right)$
$z^2 = \left(0; \frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}; \frac{(1-\delta e_2 - \delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} \right)$
$z^3 = \left(0; \frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3}; \frac{(1-\delta)(1-\delta e_2)+\delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3} \right)$

Table 1: The equilibrium vectors z^i.

Proposition 2 means that small parties are "cheaper"; the formateur prefers to form a coalition with them. From Table 1, the discount factor δ plays a key role in the budget share. The formateur is the residual claimant: it pays its partners their continuation value, which is an increasing function of their patience. The shares are, *a priori*, different for each party. When δ is close to one (the parties are patient), the formateur is forced to give almost the entire share to the other party.[19]

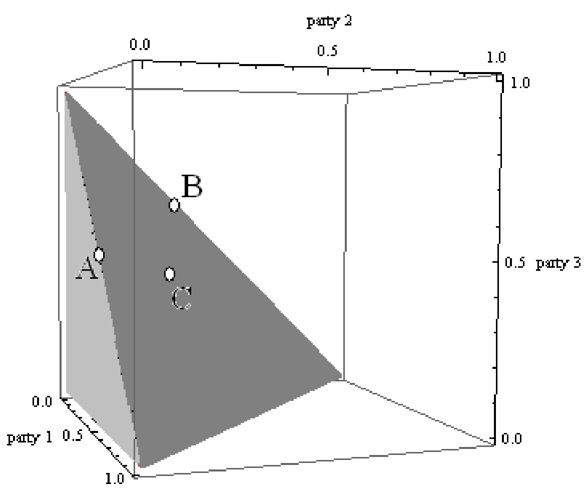

Figure 3: Coalitional space Z

Figure 3 shows feasible combinations of budget share amongst 3 parties. Each axis represents one party's share of the votes and of the budget. The simplex dark side is the set Z; a socially optimal share would attribute to each party exactly the budget that corresponds to the share of votes received. This would be independent

[18]This result is in line with the empirical evidence that parties tend to form minimal winning coalitions, to reduce coordination costs and increase efficiency. Theoretical (Riker (1962)) and empirical (Martin and Stevenson (2001)) studies confirm this.

[19]This differs from Snyder Jr., Ting, and Ansolabehere (2005), where the equilibrium shares are all the same, since the authors solve for one of the mixed strategy equilibria (players compete on z_i, to belong to the winning coalition; the formateur can extract more surplus from them).

of δ and the point would lie on the simplex front face (e.g., point C). In equilibrium, a party is always excluded (Proposition 1); share points lie on an axis (e.g., points A and B) and the location depends on δ: the larger the value of δ, the farther the equilibrium is from the formateur's axis.

Using Table 1 in Equation 1, I compute voters' misrepresentation under the proportional (M^{PR}) and plurality (M^{MV}) rules. Equation 1 becomes:

$$\begin{gathered} M^y = \left[\left(e_1\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3} - c_1\right)^2 + \right. \\ \left(e_2\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3} + e_3\frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3} - c_2\right)^2 + \\ \left.\left(\frac{(e_1+e_2)(1-\delta e_2-\delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3} + e_3\frac{(1-\delta)(1-\delta e_2)+\delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3} - c_3\right)^2\right]^{0.5}, \end{gathered} \tag{3}$$

where each line is a function of the distance between a party's share of the vote and its power. By definition, under proportional representation $c_i = e_i$: Equation 3 becomes

$$M^{PR} = \frac{\delta c_1 c_3}{(1-\delta+\delta^2 c_1 c_3)}\left[(1-\delta+\delta c_1)^2 + (\delta c_2)^2 + (1-\delta c_3)^2\right]^{0.5}. \tag{4}$$

Using the cube rule (Assumption 5) to obtain the relationship between c_i and e_i, under majority voting conditions, Equation 3 becomes

$$\begin{gathered} M^{MV} = \frac{1}{x}\left[\left[c_1^3(1-\delta)\left(\sigma-\delta c_3^3\right) - c_1 x\right]^2 + \right. \\ \left.\left[c_2^3(1-\delta)\sigma - c_2 x\right]^2 + \left[c_3^3\left(\sigma-\delta\left(c_1^3+c_2^3\right)\right) - c_3 x\right]^2\right]^{0.5}, \end{gathered} \tag{5}$$

where $\sigma = \sum_{i=1}^{3} c_i^3$ and $x = (1-\delta)\sigma^2 + c_1^3 c_3^3 \delta^2$.

The difference in misrepresentation between the two electoral systems is

$$\begin{gathered} MM(c_1,c_2,c_3,\delta) = M^{PR} - M^{MV} = \\ \frac{\delta c_1 c_3}{(1-\delta+\delta^2 c_1 c_3)}\left[(1-\delta+\delta c_1)^2 + (\delta c_2)^2 + (1-\delta c_3)^2\right]^{0.5} - \\ \frac{1}{x}\left[\left[c_1^3(1-\delta)\left(\sigma-\delta c_3^3\right) - c_1 x\right]^2 + \left[c_2^3(1-\delta)\sigma - c_2 x\right]^2 + \right. \\ \left.\left[c_3^3\left(\sigma-\delta\left(c_1^3+c_2^3\right)\right) - c_3 x\right]^2\right]^{0.5}. \end{gathered} \tag{6}$$

The fact that $MM(c_1, c_2, c_3, \delta) > 0$ means that the difference between the parties' expected and optimal shares is larger under proportional than plurality rule; when Equation 6 is positive, plurality rule better represents voters' preferences (i.e., each party's expected budget share is closer to its share of the vote).

Proposition 3 (Role of the discount factor) *Regardless of the distribution of seats among parties, two thresholds exist ($\underline{\delta} \approx 0.108, \bar{\delta} \approx 0.780$) for the discount factor such that: a) majority voting is preferable when $\delta > \bar{\delta}$, and b) proportional rule is preferable when $\delta < \underline{\delta}$. When the value of δ is between the two thresholds, the parties' relative shares of seats determine which voting system is preferable.*

Proof. See Appendix C. ■

From Proposition 3, either system is always preferred outside the interval $(0.108, 0.780)$; within the interval, the outcome depends on the relative shares of seats. After the elections, a coalition forms: the budget share that the formateur gives to its partner is equal to the partner's continuation value. When δ is sufficiently small (parties are impatient), it is cheap to persuade a partner: indeed, as δ tends to 0, the formateur's share tends to one and parties' expected utility tends to their shares of seats, and the proportional electoral system is the best.[20] Since filter 2 (Figure 1) disappears for δ going to zero, there is no reason to distort the mechanism at the filter 1 level.

The share that the formateur has to give its partner equals the partner's discounted expected earnings. As δ gets larger (the parties are patient), distortion becomes apparent at the filter two level: the formateur has to give to its partner a larger portion of future earnings; the small parties' expected shares become greater than their shares of the votes received. A plurality system distorts election results in the opposite direction (reducing small parties' shares of seats); when δ is sufficiently large ($\delta > 0.78$), filter two's distortion level is large and majority rule becomes desirable. When $\delta \in (0.108, 0.78)$, small parties' bargaining power is limited: according to the parties' relative shares of seats, the distortion in plurality rule may be larger than what is necessary to counter-balance the distortion associated with coalition formation (i.e., the distortion at the filter 1 level induced by plurality rule is larger than the one at the filter 2 level). In particular, for $\delta \in (0.108, 0.78)$, with c_3 and c_2 sufficiently large, majority voting is better than proportional rule.

[20] Analytically, $\delta \to 0$ implies $z_j^i \to 0$, thus $z_i^i \to 1$ and $EU_i(z_i^S, 0) = \sum_{h \in D} \pi_h z_i^h \to \pi_i$. Since $\pi_i = e_i$, to minimise the difference between the share of votes (c_i) and the expected share of budget for a party ($EU_i(z_i^S, 0)$) we need $e_i = c_i$, which is the case under the proportional electoral system.

3.1 Relaxing certain assumptions

This subsection briefly discusses certain assumptions. Limiting the number of parties to 3 allows us to obtain closed-form results. It is possible to solve the model for $n > 3$ parties, but additional restrictions must be introduced. It would be necessary to model the formateur's trade-off between increasing the number of parties and forming a coalition that includes larger parties. Proposition 2 may not hold: a coalition that includes the smallest party may be not sufficient to secure the majority. According to which kind of coalition is formed, the thresholds for δ would change, but the results would qualitatively remain the same.

Relaxing Assumption 1 can lead to two scenarios: if a party controls the majority of seats regardless of the electoral system, then all rules are equally representative of voter preferences. Additional, non-trivial analysis is required for the case in which one party controls the majority only under one regime.

If (contrary to Assumption 2) the coalition's value depends on the identity of the partners (e.g., because of ideological affinities), as for the number of parties, the specific form of the utility function would heavily affect results, which would have been assumption driven. A model with total incompatibility between two parties is equivalent to the model with $n - 1$ parties.

As regards the common value of δ (Assumption 4), the discount factor of the non-formateur party determines the parties' shares; all results can be extended, replacing δ by the discount factor of the non-formateur party in the winning coalition. The value of δ must always be strictly less than 1 to ensure that the solution to the problem exists and is unique.

Section 4.1 discusses the role of Assumption 5. In particular, it explains the role of τ and how the results change for $\tau < 3$.

4 An application with data from the Italian elections

In Italy, over the two last legislatures, the smallest parties in the winning coalitions commanded great power compared to their shares of seats in Parliament. In this section I use the Italian election results to illustrate the model.[21]

The Italian Parliament has two houses: the Lower house and the Senate. All adults (older than 18 years of age) can vote for the former, and those over 25 can vote for both. The electoral law permits region-specific rules for each house. Out of 20 regions, in 18 (19 for the Lower house) the electoral law is based on proportional

[21]Election results can be found on the web page of the *Ministero degli interni*, on the web page of each of the two houses of Parliament, and on many independent web pages.

representation. For my calculations, I used the numbers of votes received, in order to eliminate local specificities.

In 2006 a centre-left coalition elected Romano Prodi as prime minister. The winning coalition officially included eight parties in the Senate; most of them were created *ad hoc* before the election, to take advantage of certain unusual features of the electoral law; only few of them had their own leaders and programmes independent of the main party. I focus on the small independent party UDEUR. This party represented about 1% of all citizens on a national basis. It was pivotal and when in 2008 it left the coalition, the government lost the majority and new elections were called.

Coalition members were aware of the consequences of this party leaving: the majority controlled three more senators than the opposition coalition, including the external support received from certain "senatori a vita"[22]. During the last months of Mr. Prodi's government, UDEUR's senators used their influence on the media and their pivotal positions (by threatening to leave) to push through some major changes in several laws, especially in the "Finanziaria".[23] They pursued their own agenda and clearly showed that their real power within the coalition amounted to more than 1%.

In what follows, I consider the left and the right coalitions as two individual parties (named CL and CR) and UDEUR as a third independent party that can form a coalition with either party.[24] The first column of Table 2 summarises the 2006 Senate seats under proportional representation. The second column is the "cube rule" estimate of the shares of seats under a plurality system.

	Seats - Proportional Representation	Seats - Plurality System
CL	49.5%	49.999%
CR	49.5%	49.999%
UDEUR	1%	0.002%

Table 2: Seat Share - 2006 Italian Senate

The budget share depends on the discount factor: Figure 4 depicts the equilibrium share for each party for different levels of δ. The straight line refers to the majority voting system; the dotted line refers to proportional representation; the dashed line represents the parties' shares of votes; each chart corresponds to one party: (from top left) CR, CL and UDEUR. Under plurality rule, both CR's and

[22] "Senatori a vita" are senators who are not elected (e.g., former Republic Presidents). They sit in Parliament for life.

[23] The "Legge Finanziaria" is one of the most important Italian laws; it determines the forthcoming year's public expenditures.

[24] This would have been politically plausible; UDEUR is a centre party, its leader had already formed certain coalitions with the centre-right party and he ran in the 2009 European Parliament elections.

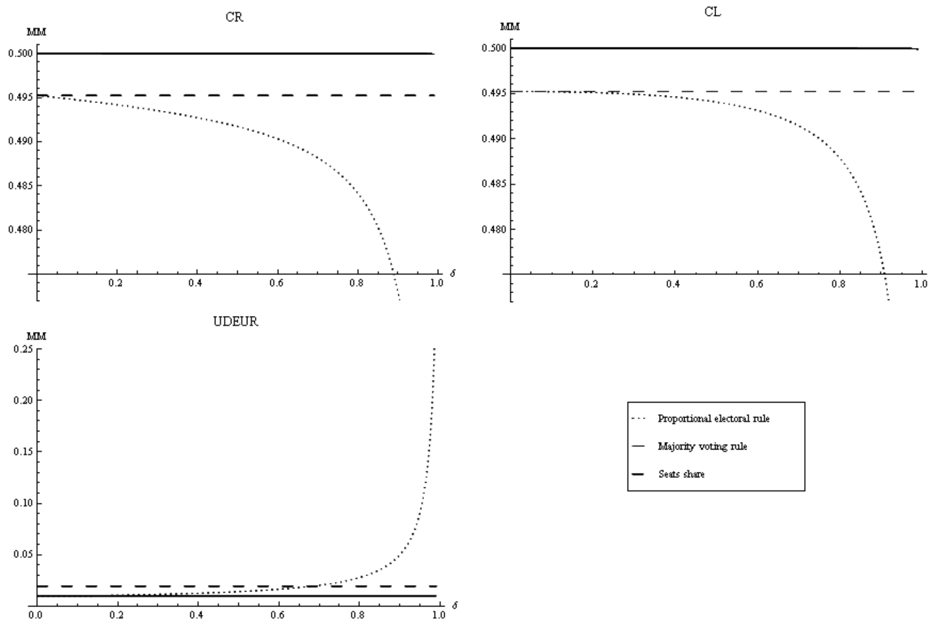

Figure 4: Budget share depending on δ

CL's shares increase at the expense of UDEUR, whose share is extremely large under the proportional context, if the discount factor is large (e.g., for $\delta = 0.99$, under proportional representation its expected share is 33%, while under plurality rule it is 0.02%).

Equation 1 measures the Euclidean distance between a party's average power and the optimal scenario, given voter preferences; the larger its value, the greater the difference between the distribution of power according to the government and voters' preferences. Using the 2006 election data, the difference in misrepresentation (MM, Equation 6) is shown in Figure 5 as a function of δ. For $\delta > 66.7\%$, a plurality system ensures a better representation of voters; the opposite is true for $\delta < 66.7\%$.

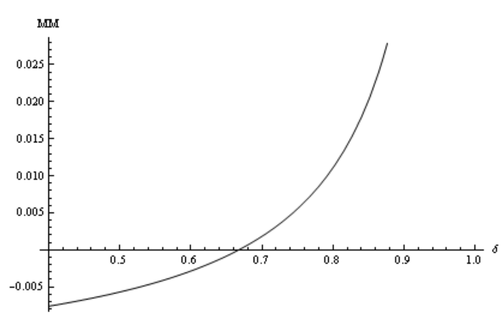

Figure 5: Misrepresentation as a function of δ

After the 2008 elections, due to a change in the political strategies of the two main parties, only four parties are now represented in Parliament: PD-IDV (the centre-left party),[25] PDL (the centre-right party), Lega Nord and UDC, the smallest party.[26] Table 3summarises the situation.

	Seats - Proportional Representation	Seats - Plurality System
PD-IDV	41.7%	49.8%
PDL	41.5%	49.2%
Lega Nord	10.5%	0.8%
UDC	6.2%	0.2%

Table 3: Seats Share - 2008 Italian Congress

UDC did not obtain enough seats to form a two party coalition, so it became a "dummy player", that is: regardless of the coalition, its contribution will always be irrelevant; it never belongs to a winning coalition. The successful coalition was the centre-right group (PDL with Lega Nord), and Mr. Berlusconi was elected prime minister. Lega Nord had already proved that it would not accept the coalition's decisions without negotiations. PDL had already withdrawn its own law proposals more than once because they were not consistent with Lega Nord's platform, and promoted others, against the will of most of the parliament (including several PDL leaders).[27]

Figure 6 shows the share of budget of each party, according to the value of δ for PD, PDL and Lega Nord. Under proportional rule, Lega Nord secures a share of budget that is considerably larger than its share of votes received (e.g., with a share of the vote of 10.5% and for $\delta = 80\%$ the expected budget share for Lega Nord is 26.8%, while under majority voting this value would be 2.5%). Considering aggregate data and the level of misrepresentation calculated using Equation 6, Figure 7 shows that, for $\delta \geq 64.64\%$ the majority rule is preferable to the proportional system.

4.1 Comments on the cube rule

I assumed that the cube rule would hold with $\tau = 3$. Although empirically valid, the cube rule lacks a theoretical foundation. The actual share of seats depends on

[25]PD and IDV, through a pre-electoral agreement, ran together and shared both a single platform and a single candidate for prime minister.

[26]To be more precise, one more party (SVP) is represented. SVP is a local party from a cross-border region where the majority of citizens speak German. Aimed at protecting linguistic minorities, a special electoral rule allowed SVP to obtain 2 seats in the Parliament (equivalent to 0.3%).

[27]For instance, in April 2009 Lega Nord, by threatening to leave the coalition, secured from Mr. Berlusconi's party permission to change the day of a referendum, at an estimated cost of 400 million Euros.

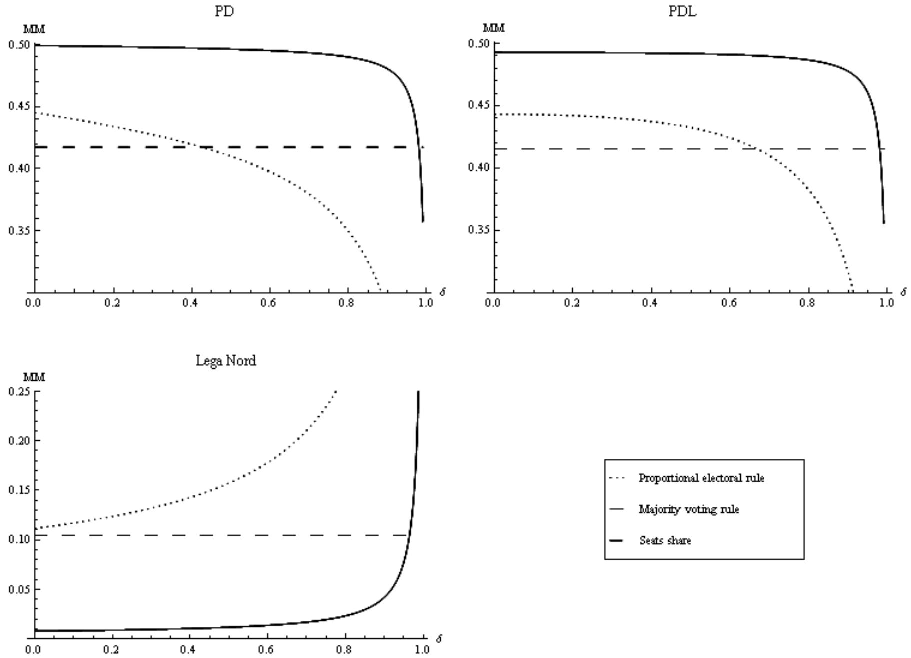

Figure 6: Budget share depending on δ

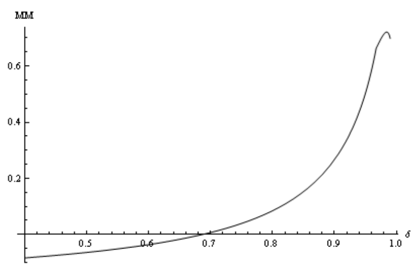

Figure 7: Misrepresentation as a function of δ

the distribution of preferences across districts.[28] For some countries, $\tau = 3$ may be a poor proxy; different values for τ account for idiosyncratic differences in the electoral system, the distribution of voter preferences, etc.[29]

I wish to explore how the previous results change as functions of δ and τ. In the 2006 case, we have two big parties and one very small one; in the 2008 case, the smallest party is relatively large. Figure 8 shows how the misrepresentation index changes with δ and τ. Whatever the value of τ, non-proportional voting systems perform better if the parties are patient.

[28] For instance, in a country where, in all districts, the parties' shares are $(40\%, 30\%, 30\%)$, under a plurality system the first party obtains 100% of seats, while the cube rule predicts a share $(54\%, 23\%, 23\%)$.

[29] As a general rule, we should expect τ to be larger in a country with one big party and many small local parties, while τ should be less than 3 for countries with heterogeneous districts, strong local parties and no large national parties.

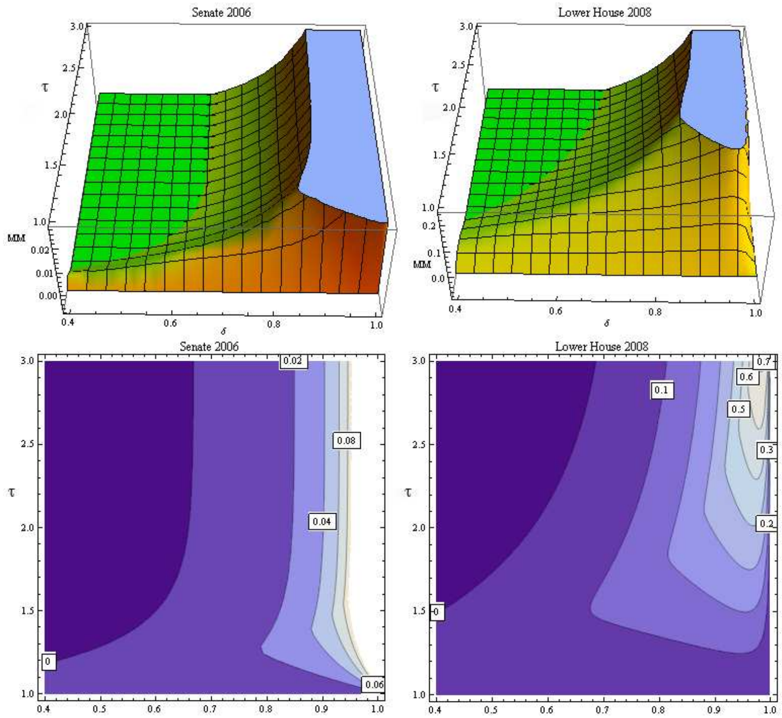

Figure 8: Changes both in δ and τ

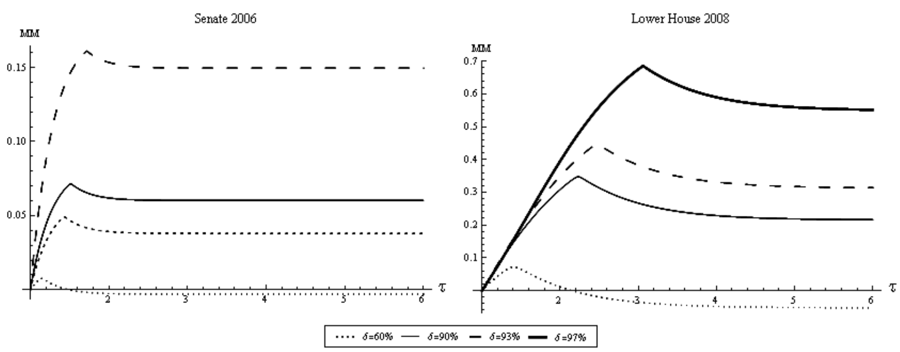

Figure 9: Effect of τ for different levels of δ

For low levels of δ, non-proportional systems perform better only when the value of τ is small. In Figure 9 we can see that the peak is close to one for low levels of δ (each line corresponds to a different level of δ, and lower lines are for lower values of δ). When the smallest party is very small, the majority rule might cause too much distortion and a small value of τ would be preferable (this can be obtained artificially, with a mixed electoral system, or it can simply be a consequence of the geographical distribution of preferences). On the other hand, when the smallest party obtains a large share of votes, the majority voting distortion is smaller and it is preferable to use a τ value closer to three.

Figure 10 shows how the level of misrepresentation changes as a function of δ

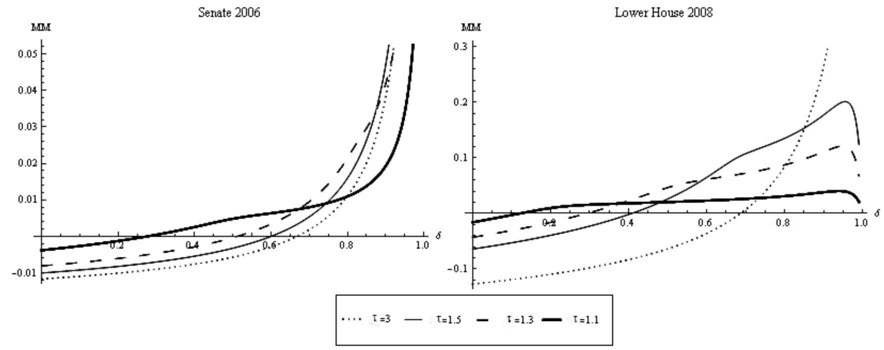

Figure 10: Effect of δ for different levels of τ

for different levels of τ. The value of δ for which proportional and majority voting rule are equivalent is an increasing function of τ: the smaller the value of τ, the more it is likely that the introduction of distortions in favour of large parties will prove beneficial.

5 Conclusions

Electoral systems are a social compromise. Many countries (e.g., Italy) use proportional representation, while others (e.g., the U.K. or the U.S.A.) rely on a plurality-based system. Most countries have adapted their systems to meet local needs. My work focuses on the two basic electoral systems (i.e., purely proportional versus plurality systems), disregarding local specificities.

Proportional electoral rules are costly in term of governance: the number of represented parties in the winning coalition tends to increase, the expected duration for which governments will hold power falls and the average time to introduce structural changes increases, because of extended negotiation times. According to advocates of proportional representation, however, decisions reflect citizens' preferences since, by definition, the Parliament's composition precisely reflects voters' preferences.

People forget that decisions in a proportional government are mainly made by the government and by the Parliamentary majority. Coalitions form to support a government, and parties' shares of power depend on their role in the coalition, not on their shares of seats in Parliament. Given the distortion due to negotiation and the importance of bargaining during the coalition formation stage, it is pointless to measure the degree of representativeness of Parliament. What matters is the relationship between voter preferences and the parties' power in government.

I have shown that, especially when parties are patient during the coalition formation stage, the distortion resulting from the negotiation process (filter 2) increases the small parties' power; at the election stage (filter 1), the plurality rule distorts Parliament's representativeness; the two distortions have opposite signs. If the parties are impatient, filter 2 distortion will be negligible; thus a non-distorting electoral system is better. However, when parties are patient, the magnitude of the distortion increases and it is more beneficial to use a non-proportional electoral system. My model demonstrates that, under a proportional system, governments are not always more representative; a plurality voting system can be preferable in terms of representativeness.

The Italian example is instructive: during the 15th legislature, a party representing 1% of voters, threatening the government, managed to substantially change part of the 2008 "Finanziaria" law and this led to the government's downfall in January 2008. Similarly, during the first year of the 16th legislature, Lega Nord (11% of votes at the 2008 elections) strongly influenced the government's decisions regarding controversial issues, such as reforms of the justice system, of immigration laws and of federalism. With a less proportional system (for instance, under a plurality system), the role of small parties would decrease and more decisions would be taken by parties that represent a larger fraction of the population.

Plurality rule may excessively reduce the smallest party's power. Preliminary results from section 4.1, and in particular the study of misrepresentation for different values of τ (that is, the degree of distortion within an electoral system), suggest that the proportional representation is never the most representative approach. Better results can be achieved by introducing certain distortions that will increase the power of large parties.

Appendix

A Proof of Proposition 1

There is no reason to form a coalition with more than one party, since the value of a coalition is constant and two parties are sufficient to control the majority of seats. The formateur, namely the residual claimant, minimises j's share (z_j) and proposes to it exactly its reservation value. The cheapest price that a party accepts is its next period discounted profit, i.e., its continuation value $v_j = \delta(e_i z_j^i + e_j z_j^j + e_x z_j^x)$. Thus, it follows directly from $z_j^i = v_j = \delta(e_i z_j^i + e_j z_j^j + e_x z_j^x)$ that $z_j^i = \frac{\delta}{1-\delta e_i}(e_j z_j^j + e_x z_j^x)$.

B Proof of Proposition 2

The generic shares of the budget with three parties are shown in Table 4.[30]

	Formateur		
Shares	1	2	3
z_1	$1 - z_2^1 - z_3^1$	$\frac{\delta}{1-\delta e_2}(e_1 z_1^1 + e_3 z_1^3)$	$\frac{\delta}{1-\delta e_3}(e_1 z_1^1 + e_2 z_1^2)$
z_2	$\frac{\delta}{1-\delta e_1}(e_2 z_2^2 + e_3 z_2^3)$	$1 - z_1^2 - z_3^2$	$\frac{\delta}{1-\delta e_3}(e_1 z_2^1 + e_2 z_2^2)$
z_3	$\frac{\delta}{1-\delta e_1}(e_2 z_3^2 + e_3 z_3^3)$	$\frac{\delta}{1-\delta e_2}(e_1 z_3^1 + e_3 z_3^3)$	$1 - z_1^3 - z_2^3$

Table 4: Generic shares with three parties

By solving the system of equations, we can (for each of the eight scenarios) compute the continuation value for each party. For the case {(1,3), (2,3), (3,2)}, the results are as follows:

$$z_1^1 = 1 - z_3^1 \tag{7a}$$
$$z_2^2 = 1 - z_3^2 \tag{7b}$$
$$z_3^3 = 1 - z_2^3 \tag{7c}$$
$$z_3^1 = \delta(e_1 z_3^1 + e_2 z_3^2 + e_3 z_3^3) \tag{7d}$$
$$z_3^2 = \delta(e_1 z_3^1 + e_2 z_3^2 + e_3 z_3^3) \tag{7e}$$
$$z_2^3 = \delta(e_2 z_2^2 + e_3 z_2^3). \tag{7f}$$

Noticing that 7d=7e, we obtain $z_1^1 = z_2^2$ and $z_3^1 = z_3^2$. Combining 7b with 7e

[30]Note that, according to the coalition formed, some of the cells in the table will take the value zero.

and 7f with 7c, we obtain:

$$z_2^2 = 1 - \frac{\delta e_3}{1 - \delta e_1 - \delta e_2} z_3^3 \tag{8a}$$

$$z_3^3 = 1 - \frac{\delta e_2}{1 - \delta e_3} z_2^2. \tag{8b}$$

Solving the system, we obtain

$$z_2^2 = \frac{1 - \frac{\delta e_3}{1-\delta e_1 - \delta e_2}}{\frac{1-\delta+\delta^2 e_1 e_3}{(1-\delta e_1-\delta e_2)(1-\delta e_3)}} \tag{9a}$$

$$z_3^3 = \frac{\frac{1-\delta e_2-\delta e_3}{1-\delta e_3}}{\frac{1-\delta+\delta^2 e_1 e_3}{(1-\delta e_1-\delta e_2)(1-\delta e_3)}}. \tag{9b}$$

After some simplifications and using the property that $\delta e_1 + \delta e_2 + \delta e_3 = \delta$, we obtain the results summarised in Table 5.

	Formateur		
Shares	1	2	3
z_1	$\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}$	0	0
z_2	0	$\frac{(1-\delta)(1-\delta e_3)}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta)\delta e_2}{1-\delta+\delta^2 e_1 e_3}$
z_3	$\frac{(1-\delta e_2-\delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta e_2-\delta e_3)\delta e_3}{1-\delta+\delta^2 e_1 e_3}$	$\frac{(1-\delta)(1-\delta e_2)+\delta^2 e_1 e_3}{1-\delta+\delta^2 e_1 e_3}$

Table 5: Shares at equilibrium

The continuation value depends on the coalition and on the identity of the formateur. For a stationary equilibrium, each party always chooses to form the same coalition when it is the formateur. For stationarity, its choice must be the best response to the other players' behaviour at each time period, and the strategy always has to be the same. Committing to a given strategy allows parties to modify their continuation value when they are not the formateur.

Within the eight scenarios, we look for Nash simultaneous stationary subgame perfect equilibria in pure strategies (SSPPS). Each player has two possible actions (consisting in forming a coalition with either of the remaining parties). Comparing expected payoffs of each party in each situation (through a reduced form game matrix of payoff), we notice that only scenario $\{(1,3),(2,3),(3,2)\}$ is SSPPS. If for example party 2 always form a coalition (2,3) and party 3 a coalition (3,2), then for party 1 it is a best-reply strategy to form a coalition (1,3).

To check that this scenario really is an equilibrium, take the generic recognition probabilities (a, b, c). From the definition of the continuation value, $v_j^* = az_j^1 + bz_j^2 + cz_j^3$; thus

$$
\begin{aligned}
v_1 =& \frac{a\left(1-\delta\right)\left(1-\delta c\right)}{1-\delta+\delta^2 ac} + 0 + 0 && (10)\\
v_2 =& 0 + b\frac{\left(1-\delta\right)\left(1-\delta c\right)}{1-\delta+\delta^2 ac} + c\frac{\left(1-\delta\right)\delta b}{1-\delta+\delta^2 ac} = \frac{b\left(1-\delta\right)}{1-\delta+\delta^2 ac} && (11)\\
v_3 =& \left(a+b\right)\frac{\left(1-\delta b-\delta c\right)\delta c}{1-\delta+\delta^2 ac} + c\frac{\left(1-\delta\right)\left(1-\delta b\right)+\delta^2 ac}{1-\delta+\delta^2 ac} = \frac{\left(1-\delta c-\delta b\right)c}{1-\delta+\delta^2 ac}, && (12)
\end{aligned}
$$

and thus $v = \left(\frac{a(1-\delta)(1-\delta c)}{1-\delta+\delta^2 ac}; \frac{b(1-\delta)}{1-\delta+\delta^2 ac}; \frac{(1-\delta c-\delta b)c}{1-\delta+\delta^2 ac}\right)$.

For $a = e_1$, $b = e_2$, $c = e_3$, and knowing that $z^i_j = \delta v_j$ for $i \neq j$, we recover the results in Table 5.

We now confirm that no player wishes to deviate: we refer to an equilibrium E via the corresponding coalition that is formed when a given party is the formateur. We call E^* the equilibrium proposed above (that is, $\{(1,3),(2,3),(3,2)\}$) and we define E^i as the alternative candidate equilibrium if party i deviates.

Focusing on stationary pure strategy equilibria, to show that no player wants to deviate, I show that a) $E^* \succeq_1 E^1 = (\{1,2\},\{2,3\},\{3,2\})$, b) $E^* \succeq_2 E^2 = (\{1,3\},\{2,1\},\{3,2\})$ and c) $E^* \succeq_3 E^3 = (\{1,3\},\{2,3\},\{3,1\})$.

a) $E^* \succeq_1 E^1$ if and only if the continuation value of party 3 when the equilibrium is E^* is smaller than that of party 2 in the equilibrium E^1, that is iff $v_3(E^* < v_2(E^1)$,, which means $\frac{(1-\delta e_2-\delta e_3)e_3}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta e_2-\delta e_3)e_2}{1-\delta+\delta^2 e_1 e_2}$. Thus $e_3\left(1-\delta+\delta^2 e_1 e_2\right) < e_2\left(1-\delta+\delta^2 e_1 e_3\right)$. Since $e_3 < e_2$, it is clear that $e_3\left(1-\delta\right) < e_2\left(1-\delta\right)$.

b) $E^* \succeq_2 E^2$ iff $v_3(E^*) < v_1(E^2)$, which means $\frac{(1-\delta e_2-\delta e_3)e_3}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta e_1-\delta e_3)e_1}{1-\delta+\delta^2 e_2 e_3}$. From $0.5 > e_1 > e_2 > e_3$, it is a matter of simple algebra to show that the left hand side is always smaller than the right hand side.

c) $E^* \succeq_3 E^3$ iff $v_2(E^*) < v_1(E^3)$, which means $\frac{(1-\delta)e_2}{1-\delta+\delta^2 e_1 e_3} < \frac{(1-\delta)e_1}{1-\delta+\delta^2 e_2 e_3}$. The result follows directly.

C Proof of Proposition 3

Solving equation 6 for δ, a plurality system is preferable if and only if $MM(c_1, c_2, c_3, \delta)$ is positive. The proof consists of three parts: the first shows the existence of a value of δ for which Equation 6 equals zero. This guarantees that (independent of any relevant parameters) for all δ below a threshold the proportional representation system is preferred, while above a possibly different threshold, the plurality rule is preferred. The second part of the proof, through a counterexample, shows that for some values of δ, the sign depends on the value of the parameters c_1, c_2 and c_3. The two parts together are sufficient to conclude that there exists i) a threshold for δ under which the proportional rule is preferred, ii) another, higher threshold above which the plurality rule is preferred, and iii) an interval between the two thresholds for which the preferred electoral system depends on the relative size of the groups of voters. $MM(c_1, c_2, c_3, \delta)$ is defined implicitly; it is impossible to compute the value of the 2 thresholds, but the third part of the proof shows the graphical results of the numerical simulation to identify the value of the 2 thresholds.

First part: From the Bolzano Theorem, a threshold $\bar{\delta}$ exists. In particular,

$$MM(c_1, c_2, c_3, 0) = -\frac{1}{\sigma^2}\left[\left(\sigma c_1^3 - c_1\sigma^2\right)^2 + \left(\sigma c_2^3 - c_2\sigma^2\right)^2 + \left(\sigma c_3^3 - c_3\sigma^2\right)^2\right]^{0.5} \leq 0,$$

with strict inequality for all $c_i \neq \frac{1}{3}$. On the other hand,

$$MM(c_1, c_2, c_3, 1) = \left[c_1^2 + c_2^2 + (c_1 + c_2)^2\right]^{0.5} - \left[c_1^2 + c_2^2 + \left(\frac{c_3^3}{c_1^3} - c_3\right)^2\right] 0.5 \geq 0.$$

The function crosses the axis at least once, since the function is continuous on $\delta \in [0, 1]$, $MM(c_1, c_2, c_3, 0) \leq 0$, and $MM(c_1, c_2, c_3, 1) \geq 0$. This part of the proof is independent of the c_i, while the threshold depends on these data.

Second part: Solving Equation 6 for $\delta = \frac{1}{2}$, the plurality rule is better for $c_1 = .4$, $c_2 = .3$ and $c_3 = .3$ ($MM(0.4, 0.3, 0.3, 0.5) \approx 0.02$); the proportional rule is better for $c_1 = .4$, $c_2 = .35$ and $c_3 = .25$ ($MM(0.4, 0.35, 0.25, 0.5) \approx -0.02$). Therefore the preferred electoral rule, when $\delta = 0.5$, depends on the relative sizes of the parties. The first part of the proof shows that, for δ being sufficiently low, the proportional approach is always better; we conclude that there is a threshold $\underline{\delta} < \frac{1}{2}$ below which c_i does not matter and the proportional situation is preferred. Similarly, there exists a threshold $\bar{\delta} > \frac{1}{2}$ above which the plurality system is always preferred. Finally, in the neighbourhood of $\delta = \frac{1}{2}$, the result depends on the value of certain relevant parameters.

Third part: As previously suggested, the equation can not be solved explicitly, and the values $\underline{\delta}$ and $\bar{\delta}$ must be derived numerically.

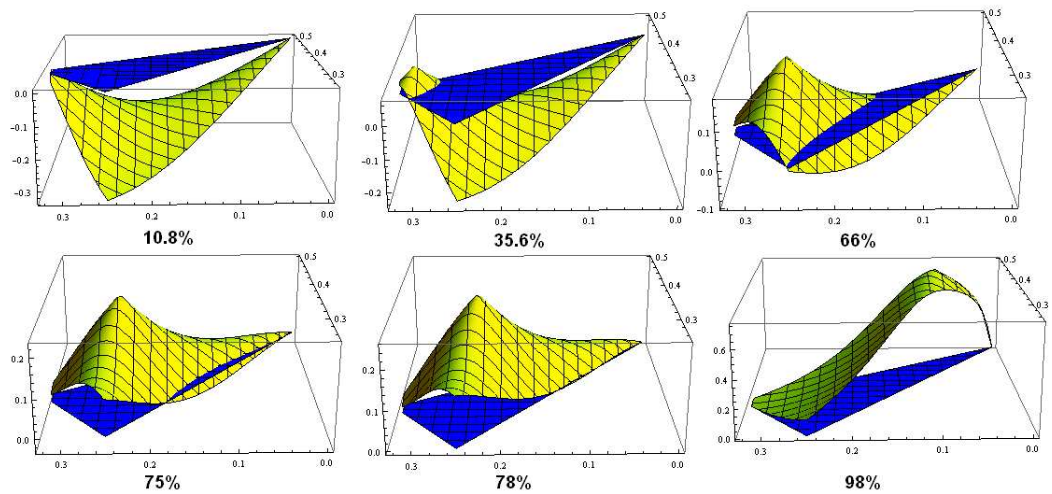

Figure 11: The impact of δ

Figure 11 describes the behaviour of Equation 6, showing its shape for six values of δ. The horizontal axis depicts the share of seats e_3 of party 3, while e_2 is shown on the depth axis. The dark surface corresponds to the zero plan; the light surface depicts Equation 6. Given δ and the combinations of different shares of seats, the light surface is above the dark one if the majority voting is preferable. For instance, for $\delta = 0.78$ and $\delta = 0.98$, the light surface is above the dark surface regardless of the distribution of seats among parties.

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