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Abstract

A large body of empirical work has established the significance of cash flow in explaining investment dynamics. This finding is further taken as evidence of capital market imperfections. We show, using a perfect capital markets model, that time-to-build for capital projects creates an investment cash flow sensitivity as found in empirical studies that may not be indicative of capital market frictions. The result is due to mis-specification present in empirical investment-q equations under time-to-build investment. In addition, time aggregation error can give rise to cash flow effects independently of the time-to-build effect. Importantly, both errors arise independently of potential measurement error in q. We provide implications and recommendations for empirical work.

JEL classification: D21; E22; E32; G31.

Key words: Investment; Capital market imperfections; Time-to-build.

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1 Introduction

Investment in fixed capital is one of the most important and volatile components of aggregate activity. Understanding investment dynamics is central to the study of aggregate fluctuations. In the neoclassical theory of firm investment with adjustment costs, the firm's market value and investment respond simultaneously to signals about future profitability as encoded in Tobin's q. In this theory, Tobin's q, defined as the expected value of the firm relative to its capital stock becomes a summary statistic for investment. Nevertheless, despite its theoretical appeal the empirical performance of the q theory has been rather disappointing; the explanatory power of q is found to be low and the responsiveness of investment to fundamentals rather weak. Moreover, various measures of internal funds such as profits or cash flow are found significant in explaining corporate investment. This finding is further taken as evidence of capital market imperfections that disturb the firm's investment schedule from the frictionless neo-classical benchmark. This paper uses a neoclassical investment-q model with time-to-build and time-to-plan features for capital and revisits this evidence. We provide a new explanation for the emergence of cash flow effects in empirical investment-q equations that relies on an important technological aspect of capital production.

Time-to-build and time-to-plan are key technological features of investment. A variety of survey (Montgomery (1995) and Koeva (2000)) and firm level (Koeva (2001), Del Boca et al. (2008)) evidence suggests that these technological constraints are important at the firm level. This evidence indicates that the time required for the installation of new equipment and structures ranges from 3 to 4 quarters for equipment and 2 to 3 years for non-residential structures. But as we demonstrate in this paper, the typical investment-q equation that serves as the benchmark for evaluating the capital market imperfections hypothesis, is usually not robust to the presence of time-to-build investment. When time is required to build new capital q is no longer a sufficient statistic for investment. This result arises because under time-to-build an additional state variable significantly affects optimal investment decisions. Investment consists of new and partially-finished projects that have not yet become productive capital. In addition to q the sum of current expenditures on existing incomplete projects belongs to the right hand side of the investment regression. In other words, when the firm decides—on the basis of new information about future investment opportunities—how many new projects to initiate, past projects already under way influence that decision, i.e. they constitute a state variable for this decision. The perfect capital markets model we use allows us to characterize this state variable analytically and show how it induces specification error in the typical investment-q equation. More importantly we show this state variable is strongly correlated with cash flow and thus when not included among the right-hand-side variables of the regression, induces a positive investment cash-flow sensitivity that is nevertheless not indicative of capital market imperfections.

We use the model to calibrate and simulate an industry to the aggregate U.S. manufacturing sector. The specification error we identify and renders q an insufficient summary statistic is the primary driver of cash flow effects in our simulated investment-q regressions. Our results closely corroborate findings recently reported in Eberly et al. (2008) although (as explained below), in contrast to theirs, our findings are free of measurement error in q. Nevertheless as we demonstrate, measurement error magnifies the specification error we identify. Further, our model provides an explanation for the emergence of lagged investment effects in empirical investment-q regressions, in addition to cash-flow effects. The importance of lagged investment effects is a largely overlooked empirical regularity, since most of empirical work focuses almost exclusively on the role of cash flow. But as Eberly et al. (2008) note: "Both cash-flow and lagged-investment effects have been found in virtually every investment regression specification and data sample." In our study—as in Eberly et al. (2008)—we show that the lagged investment rate is an important determinant of current investment because it proxies for an omitted state variable. In Eberly et al. (2008) simulations, lagged investment proxies for a regime-switching component in a firms' demand schedule. In the present model with time-to-build, lagged investment has a different structural interpretation, capturing time-to-build effects for the construction of capital.

We further investigate whether our model can reproduce cross sectional differences in investment cash-flow sensitivities reported in almost all empirical studies that test for capital market imperfections (see for e.g. Fazzari et al. (1988), Gilchrist and Himmelberg (1995), and the survey by Hubbard (1998)). These studies find that firms which are thought *a-priori* to be more vulnerable to imperfections in capital markets, e.g. small, young, with no dividends payout firms, exhibit higher investment cash flow sensitivities compared to firms that are thought to have ample access to external finance, e.g. large, old, distributing dividends firms. We show that the model is capable of reproducing this empirical regularity as long as the former group of (constrained) firms have longer time-to-build investment schedules compared to the latter group of (unconstrained) firms. For this purpose we bring to light evidence from a large sample of Compustat firms that strongly suggests constrained firms to have longer time-to-build investment schedules compared to unconstrained firms.

The presence of mis-specification under time-to-build begs the question of whether and how we can mitigate it when undertaking empirical work within the q framework. We show that we can approximate the omitted state variable with a simple (and easily constructed) variable that is a function of the lagged investment rate and the growth rate of the capital stock. We evaluate the usefulness of this approximation for empirical work in our simulated environment and find that it performs almost as well as its theoretical counterpart, nearly eliminating the cash flow effect from the investment regression. In addition, and *independently* of the time-to-build effect above we show that a cash flow effect can emerge in an investment-q equation when researchers estimate an investment-q regression using annual data—a practice followed in the majority of studies—that are aggregated from more frequent factor input decisions. This time or temporal aggregation error has been highlighted in the context of capital and labor adjustment cost estimates by Hall (2004) but as far as we know the implications in an investment-q framework have not been explored. Finally, it is important to emphasize that our results are not driven by measurement error in q. In the simulated environment we study we use the model consistent measure of expected marginal q.

Recent work by Erickson and Whited (2000), Gomes (2001), Cooper and Ejarque (2003), Alti (2003), Cummins et al. (2006), Abel and Eberly (2003), also cast doubt on the validity of investment cash flow sensitivities as an indicator of capital market imperfections. Erickson and Whited (2000), Gomes (2001) and Cummins et al. (2006) stress that cash flow effects may arise because Tobin's q is measured with error. Cooper and Ejarque (2003) emphasize market power that creates a divergence between average and marginal q while in Alti (2003) Tobin's q is a noisy measure of fundamentals and cash flow is highly informative about long-run profitability. Finally, in Abel and Eberly (2003) cash flow effects arise as a result of specification error induced by changes in the user cost of capital. Yet, our contribution is rather different from all the above. First, in time-to-build, we provide a new and important channel for the emergence of significant cash flow effects in investment-q regressions. Second, in contrast to the studies above our findings do not involve any mis-measurement between average and marginal q and thus are not driven by measurement error.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the solution and calibration. In section 4 results from the simulated version of the model are presented. Section 5 concludes.

2 The Model

We use a model developed in Tsoukalas (2003) suitable for analyzing firms investment decisions in a time-to-build environment. A similar framework has been employed by Zhou (2000) to explain aggregate investment dynamics. The following subsections explain the components that are essential to the framework.

2.1 Firms

2.1.1 Technology

We model an industry which is populated by a continuum of risk-neutral infinitely-lived firms. Firm j produces output, using the following decreasing returns to scale Cobb-Douglas technology:[1]

$$y_{jt} = A_t \omega_{jt} F(K_{jt}, M_{jt}, L_{jt}) = A_t \omega_{jt} K_{jt}^{\alpha} M_{jt}^{\gamma} L_{jt}^{\nu} \quad \gamma + \alpha + \nu < 1$$

where A_t is an aggregate (common) and ω_{jt} an idiosyncratic productivity shock. K_{jt} is capital, L_{jt} is the labor input and M_{jt} is the stock of materials.

The investment technology requires time to build new capital. Specifically, it takes J-periods (stages) to build new productive capacity. This technology implies that in any given period t, firms initiate new projects, s_{Jt}, and complete partially finished projects, s_{it}, $i \neq J$ at stage i. This assumption intends to capture the design and construction (delivery) stages that exist in undertaking investment projects in plant and equipment as suggested by Kydland and Prescott (1982). The assumptions of this time-to-build (TTB) technology are summarized below:

$$s_{it} = s_{i-1,t+1} \quad i = 2, ...J \tag{2.1}$$

$$K_{t+1} = (1-\delta)K_t + s_{1t} \tag{2.2}$$

$$I_t = \sum_{i=1}^{J} \varphi_i s_{it} \tag{2.3}$$

with $0 \leq \varphi_i \leq 1, \quad i = 1, 2, ...J$, and $\sum_{i=1}^{J} \varphi_i = 1$. To clarify notation, s_{Jt} denotes new projects at time t, $s_{J-1,t}$ denotes projects initiated at time $t-1$, that are $J-1$ periods away from completion at time t, and so on. The last stage project, s_{1t} yields productive capital in the following period.

[1] Decreasing returns to scale are necessary for firm size to be well defined. Otherwise firm size is indeterminate and the entrepreneurial sector reduces to just a single producer.

The parameters φ_i determine the fixed fraction of resources allocated to projects that are i periods away from completion, or equivalently the proportion of the value of the project put in place in period i. I_t denotes total investment expenditures at time t and depends on the resources expended for the different incomplete projects. Finally, the capital stock depreciates at rate δ.

New investment projects are subject to adjustment costs. It is assumed that firms face a quadratic cost of adjustment function for investment in new projects, i.e.,

$$G(s_{J,jt}, K_{jt}) = \frac{\eta}{2}(\frac{s_{J,jt}}{K_{jt}} - \delta)^2 K_{jt} \tag{2.4}$$

where the parameter η governs the curvature of G.[2] This function has all the usual properties, i.e., it is convex, with a rising marginal adjustment cost. It also implies a zero adjustment cost in the steady state.

The decision relating to the materials input is as follows. The firm places materials orders d_{jt}, for use in production in period $t+1$, M_{jt+1}. The stock of materials thus evolves according to:

$$M_{jt+1} = (1 - \delta_m)M_{jt} + d_{jt} \tag{2.5}$$

where δ_m denotes the depreciation rate for materials. Note that this timing convention assumes that orders of materials at time t enter the firm after current production has taken place.

Last, firms hire labor from a competitive market at a given (constant) wage rate, w.

2.1.2 The firm's problem

Timing is as follows. At the beginning of period t, the idiosyncratic (ω_{jt}) and aggregate productivity shocks (A_t) are observed. The firm inherits a stock of capital K_{jt}, materials M_{jt}, and partially-completed projects $s_{1,jt}, s_{2,jt}, ...s_{(J-1),jt}$, from the previous period. Then, before A_{t+1} and $\omega_{j,t+1}$

[2] An alternative characterization of the adjustment cost function is to assume that the cost is paid at the time when resources on projects are expended, i.e., $G(s_{1,jt}, ...s_{J,jt}, K_{jt}) = \sum_{i=1}^{J} \varphi_i \frac{\eta}{2}(\frac{s_{i,jt}}{K_{jt}} - \delta)^2 K_{jt}$. We choose to work with the simpler form (2.4) because of the analytical simplicity. We nevertheless present results using this functional form assumption as well.

are observed, the firm chooses new investment projects, $s_{J,jt}$, materials orders, d_{jt}, and labor input, L_{jt}, in order to maximize firm value.

$$\max_{L_{jt}, s_{J,jt}, d_{jt}} E_0 \sum_{t=0}^{\infty} \beta^t div_{jt}$$

where div_{jt} denote dividends and $\beta = \frac{1}{1+r}$ is the market discount factor, where r denotes the risk free rate.[3] Note that the firm's uses and sources of funds equation defines dividends:

$$div_{jt} = A_t \omega_{jt} K_{jt}^{\alpha} M_{jt}^{\gamma} L_{jt}^{\nu} - wL_{jt} - d_{jt} - I_{jt} - G(s_{J,jt}, K_{jt}) \tag{2.6}$$

The laws of motion for the exogenous state variables are described below.

$$lnA_{t+1} = \rho_A lnA_t + \sigma_A \varepsilon_{t+1}^A \quad \varepsilon_t^A \sim N(0,1) \tag{2.7}$$

$$ln\omega_{j,t+1} = \rho_\omega ln\omega_{j,t} + \sigma_\omega \varepsilon_{j,t+1}^\omega \quad \varepsilon_{j,t}^\omega \sim N(0,1) \tag{2.8}$$

Last we note that profits are defined as:

$$\pi_{j,t} = A_t \omega_{jt} K_{jt}^{\alpha} M_{jt}^{\gamma} L_{jt}^{\nu} - wL_{jt} \tag{2.9}$$

The maximization is subject to (2.1)—(2.8) and the given initial values for the state variables.

The firm's problem defined above can be described in 4 plus $J-1$ state variables $(K, M, \{s_i\}_{i=1}^{J-1}, A, \omega)$. Given the high dimensionality of this problem a solution based on a global approach (e.g. policy or value function iteration), will be infeasible for a reasonably accurate characterization of the solution (curse of dimensionality). To circumvent this difficulty a second order approximation method is used.

We can write the Langrangean for the problem,

[3]Notice that the use of the risk free rate in the discount factor implies that the firm's financing choices are irrelevant to its investment decisions. Moreover the presence of materials is inconsequential for our analysis but we have chosen not to maximize out this factor of production in order to be consistent with our calibration procedure.

$$\max_{L_t, s_{Jt}, d_t} E_0 \sum_{t=0}^{\infty} \beta^t \{div_t + q_t(K_{t+1} - (1-\delta)K_t - s_{1t}) + \mu_t(M_{t+1} - (1-\delta_m)M_t - d_t)\}$$

where q_t, μ_t denote the Kuhn-Tucker multipliers associated with the equality constraints (2.2) and (2.5). Note that the multiplier q_t represents the shadow value of installed capital, i.e., marginal q that captures the future expected marginal profitability of a unit of capital (contributing both to profits and the reduction in adjustment costs). The assumptions of our model imply that expected marginal q is a sufficient statistic for investment in new projects, $s_{J,jt}$. Assume that projects require 3 periods to completion. Then it can readily be shown that the optimal investment rate obeys:

$$\frac{I_{j,t}}{K_{j,t}} = \varphi_3 \bigg(-\frac{1}{\eta}(\varphi_3 + \beta\varphi_2 + \beta^2\varphi_1) + \delta \bigg) + \varphi_3 \frac{1}{\eta} \beta^2 E_t(q_{j,t+2}) + \sum_{i=1}^{2} \varphi_i \frac{s_{i,jt}}{K_{j,t}} \tag{2.10}$$

Optimal investment is a function of future expected marginal q, reflecting the fact that capital will become productive with a lag and an additional state variable that represents part of the investment outlays already underway. Thus in this environment q ceases to be a sufficient statistic for investment. To conserve space the complete description of the first order necessary conditions that characterize an optimum for this problem are given in Appendix 1.

3 Solution

The equilibrium of the model is characterized by a set of Euler equations along with the Kuhn-Tucker conditions for the equality constraints and the given initial values for the state variables. This equilibrium is a set of non-linear equations and an analytical solution is infeasible to compute. An approximate solution is calculated by using a second order approximation method around the non-stochastic steady state of the model. The second order Taylor approximation, as described in Schmitt-Grohe and Uribe (2004), can be readily used to calculate the decision rules for new projects, materials orders and labor.[4]

[4]Appendix 1 outlines the essential computational details of the solution.

3.1 Calibration

We calibrate the model using a baseline set of parameter values as in Table 1. We calibrate the parameters needed to simulate our model to characteristics of the U.S. manufacturing sector.

The values for the output elasticity of materials, γ, labor, ν and capital, α are taken from the manufacturing plant level study of Sakellaris and Wilson (2004) (Table 1, p.15, C). These values imply an overall returns to scale equal to 0.98. This value is consistent with Basu and Fernald (1997) estimates of the returns to scale in manufacturing. There is a variety of empirical evidence of time-to-build for capital projects. Regarding equipment investment, Abel and Blanchard (1986) document an average delivery lag for manufacturing firms equal to three quarters (during which time they pay installments for the purchase of the capital good). Mayer and Sonenblum (1955) report that the average time across industries needed to equip plants with new machinery is 2.7 quarters. Montgomery (1995) examines a long series of finely detailed surveys conducted by the U.S. Department of Commerce on TTB patterns for a wide range of firm construction projects. His calculations imply a time-to-build between five to six quarter for non-residential structures. There is still evidence of lengthier construction times for non-residential structures. According to Mayer (1960) and Koeva (2001) it takes approximately two years to complete non-residential structures. A recent study by Del Boca et al. (2008) using Italian firm level data suggests that investment projects require 2-3 years from initial stage to completion, while equipment investment becomes productive within a year. Based on this evidence and given the fact that the model's empirical counterpart is total capital we think that three or four quarters is a reasonable length for the time-to-build assumption. We set the length of the time-to-build equal to three quarters (J=3) in our baseline calibration but we also discuss results varying this value up to four quarters. In terms of the resources spent on each stage of the construction (or installments for delivery) Kydland and Prescott (1982) assume an equal cost distribution. Recently, Zhou (2000) argues that time-to-build is very important for explaining investment dynamics. He estimates φ_i for various values of J and reports that an (approximately) equal distribution of cost for time-to-build investment produces the best fit for aggregate U.S. investment. There also exist estimates (e.g. Del Boca et al. (2008)) particularly for investment in structures that point to initial planning phases with little or no resources spent followed by construction phases with increasing resources as projects near completion. This pattern of spending is known as time-to-plan (TTP). For the baseline calibration we set $\varphi_1 = \varphi_3 = 0.333, \varphi_2 = 0.34$ and explore TTP in the simulations as an alternative scenario. The parameter that governs the convexity of the adjustment cost function, η is set equal to 1.08 at the quarterly rate. This parameter is estimated by Barnett and Sakellaris (1999) using a Tobin's q approach in a panel of manufacturing firms from 1959 to 1987 (see Table 3 p.256). In implementing their approach the authors assume a time-to-build of one year thus closely corresponding to our assumptions. The magnitude of (convex) adjustment costs estimated by Barnett and Sakellaris (1999) and more recently by Cooper and Haltiwanger (2006) seem to be conforming much better to the q theory of investment compared to earlier estimates that produced implausibly large adjustment cost estimates (See for example, Hayashi (1982), or Summers (1981)).[5] We also experiment with several alternative values for η taken from these studies. The subjective discount factor, β, is chosen to match the average risk-free real interest rate over the period 1947 I to 2006 II. The real interest rate is defined as the 3-month U.S. T-bill rate less consumer price inflation. The depreciation rate for materials is calculated as follows. The stock of materials at the end of a quarter is $(1-\delta_m)M_t$. Usage of materials in quarter t is $\delta_m M_t$. Since usage is not available quarterly but only annually we use the following approximation. $usage_q^y = \frac{usage^y}{output^y} output_q^y$, where y denotes year and q quarters. This calculation should be sufficiently accurate since materials usage and output are highly correlated and their ratio will thus be quite smooth in the short-run. The data used for this calculation are available from the Annual Survey of Manufacturers (ASM) and the

[5]We choose to work with these recent (more realistic) estimates for another reason. A higher adjustment cost parameter η would imply a greater positive serial correlation of investment that would (in the presence of autocorrelated productivity) be more strongly correlated with profits, thus making it easier to obtain a significant profit rate coefficient in a mis-specified regression.

NBER manufacturing productivity database. δ_m is then calculated from the restriction $\frac{(1-\delta_m)M_t}{\delta_m M_t} = \frac{\text{materials inventories at end quarter t}}{\text{usage of materials in quarter t}}$. In the data (1962-2000) the ratio is on average equal to 0.33. The calculation implies $\delta_m = 0.75$. We set δ the fixed capital depreciation rate to 0.025 per quarter. We calibrate the process for the idiosyncratic productivity shock, ρ_ω, σ_ω to match the autocorrelation and standard deviation of (cyclical) aggregate manufacturing investment. Finally, we calibrate the process for the aggregate productivity shock, ρ_A, σ_A to match the autocorrelation and standard deviation of (cyclical) aggregate manufacturing output. The data for this calculation (manufacturing investment and output) are taken from the Bureau of Economic Analysis and cover the period 1967 II to 2004 IV.

4 Results

In this section, we present results from the calibrated version of the model. The (approximate) decision rules for the model's variables are simulated and artificial data are generated. Using the artificial data we create a panel of firm level data. We run investment-q regressions on the panel and discuss the role of cash flow.

4.1 Investment-q regressions

Empirical studies that emphasize the role of capital market imperfections in explaining the cyclical behavior of investment utilize firm-level data and typically regress fixed investment to a set of explanatory variables that includes a proxy for changes in net worth. For example in Fazzari et al. (1988) or Gilchrist and Himmelberg (1995) the q theory of investment is augmented (and tested) with measures of internal funds (profits or cash-flow). These studies demonstrate the significance of these measures in explaining fixed investment. A typical finding is a significant cash-flow coefficient along with a very large estimate for the adjustment cost parameter. This is interpreted as evidence of capital market frictions and a rejection of the q theory of investment. This section presents results obtained from our simulated industry, and raises questions about the validity of this interpretation.

We generate a panel of 1000 firms observed over 20 years and demonstrate that a significant cash-flow effect can arise even in a model with perfect capital markets.

Previous work has also reached similar conclusions. Gomes (2001) questions the validity of cash-flow as an indication of capital market imperfections using a structural model. He concludes that a cash-flow effect may arise as a combination of measurement error and identification problems in a linear regression framework. Erickson and Whited (2000) and Cummins et al. (2006) emphasize measurement error in Tobin's q, while Cooper and Ejarque (2003) argue for market power that induces a divergence between marginal and average q. Finally, in Abel and Eberly (2003) cash-flow is a proxy for an (unobserved) time-varying depreciation rate and in Eberly et al. (2008) cash flow arises because it proxies for the regime in demand which is a state variable for investment. In our model of course decreasing returns to scale imply that average q differs from marginal q. But our contribution does not rely on the mis-measurement between average and marginal q. Instead the cash flow sensitivity of investment in our framework has its root in the technology for investment projects and the specification error it creates in the typical investment-q regression. In the analysis below we make use of the theoretically correct measure of marginal q (section 4.4 studies the implications of adding measurement error in q).

We note that empirical studies, including those mentioned above as well as most others we are aware of, typically rely on annual firm level (e.g. Compustat) data, whereas our model is calibrated quarterly. We first present brief results to build intuition using our quarterly model and then aggregate our model to correspond to the annual frequency. This allows to study the role of time aggregation.

To demonstrate the inference-problem associated with reduced form investment equations under TTB, we estimate an OLS regression on the artificial data (for $J = 3$),

$$\frac{I_{j,t}}{K_{j,t}} = \alpha + b_1 E_t(q_{j,t+2}) + b_2 \frac{\pi_{j,t}}{K_{j,t}} + \varepsilon_{j,t} \tag{4.1}$$

where the left-hand-side (LHS) variable is the investment rate, and the right-hand-side (RHS) variables are the expected marginal q along with the profit rate. Expected marginal q is the correct statistic for capturing future investment opportunities under TTB because new investment projects become productive after three periods (see equation 2.10). This is a typical empirical investment equation except that Tobin's q is usually taken as a proxy for the un-observed marginal q.[6] We contrast this equation with the equation implied by the first order condition (FOC) for project starts for $J = 3$,

$$\frac{s_{J,jt}}{K_{j,t}} = -\frac{1}{\eta}(\varphi_3 + \beta\varphi_2 + \beta^2\varphi_1) + \delta + \frac{1}{\eta}\beta^2 E_t(q_{j,t+2}) \tag{4.2}$$

which using the definition of $I_{j,t}$ from (2.3) can be re-written as,

$$\frac{I_{j,t}}{K_{j,t}} = \varphi_3\bigg(-\frac{1}{\eta}(\varphi_3 + \beta\varphi_2 + \beta^2\varphi_1) + \delta\bigg) + \varphi_3\frac{1}{\eta}\beta^2 E_t(q_{j,t+2}) + \sum_{i=1}^{2}\varphi_i\frac{s_{i,jt}}{K_{j,t}} \tag{4.3}$$

Comparing equation (4.1) with (4.3) we can observe (ignoring the constant and error term) that the correct specification under TTB includes $\sum_{i=1}^{2}\varphi_i\frac{s_{i,jt}}{K_{j,t}}$ as a RHS variable. This sum is the part of investment that has responded to old information (about productivity) and is therefore a state variable. The question is whether omitting this variable invalidates the inference drawn on the role of profits from an empirical equation like (4.1). The answer is affirmative if the profit rate is correlated with $\sum_{i=1}^{2}\varphi_i\frac{s_{i,jt}}{K_{j,t}}$. This turns out to be the case with persistent productivity shocks.[7] The intuition is as follows. Suppose that at some time in the past a favorable productivity shock caused a surge in new projects. As time elapses these new projects come closer to completion time and if the shock is persistent then at time t there will be a series of outstanding projects, $s_{1t}s_{2t}, ..., s_{J-1,t}$. Moreover with persistent shocks current profits will also reflect the same past productivity shocks that caused the firm to initiate new projects and are now exactly those projects above that have

[6] A notable exception is Gilchrist and Himmelberg (1995) who construct a proxy for marginal expected q. We note that a typical empirical equation also includes a firm specific effect. In our model firms can only differ in the history of shocks they receive so there is not any ex-ante firm-specific heterogeneity.

[7] To conserve space we present a set of correlations in section 4.2. For the case examined here the correlation between the two series is equal to 0.83.

moved closer to completion. Therefore current profits are correlated with each of these previous capital projects and hence their sum. This implies that profits will proxy for this state variable in an investment-q regression. Of course if q was a sufficient statistic for total investment (as evident from equation (4.2) it is a sufficient statistic only for new projects, s_{Jt}) then profits would not be significant in a regression with investment and q. Table 2 reports the results from estimating equation (4.1) on our artificial panel of firms. We can observe that the profit rate coefficient, b_2 is positive and statistical significant, even though our model was designed without capital market imperfections. Therefore, the profit rate appears as a significant variable and improves the fit of the equation as it proxies for a relevant omitted RHS variable. It is also important to stress that any role for this variable in these regressions does not arise as a result of measurement error since we are using the appropriate (marginal) measure of q. Instead the explanatory role of the profit rate arises as a result of specification error due to TTB for investment.[8]

4.1.1 Quantifying specification and time aggregation error

We wish to know the implications of our TTB assumption and the validity of our simulated regression results on the role for profits when we aggregate our model over time. We previously indicated that most empirical studies use annual firm-level data. We therefore aggregate our artificial data to correspond to the same annual measures used in these studies and make the investment equations directly comparable. Our timing assumptions imply investment decisions that are taken quarterly. The use of annual data on the other hand implies annual factor input decisions. We view the quarterly frequency much more realistic given the flow of information upon which decisions are taken. Hall (2004) argues that decisions concerning factor employment are made more frequently than once a year (quarterly or even monthly) and analyzes how time aggregation biases capital adjustment cost estimates when annual data are used in estimation. In this section we have two goals. First, in view of Hall (2004) findings to explore whether time aggregation can spuriously

[8]As expected, if we estimate the correct specification (4.3) we find no role for the profit rate. We do not report these results for brevity but they are available upon request.

assign a role to cash flow independently of the specification error that is created as a result of TTB. Second, to investigate precisely how the TTB specification error generalizes in this framework. To preview the results we highlight two findings: (i) we identify a time aggregation error that can give rise (independently from the specification error from TTB) to cash flow effects in investment regressions with annual data and (ii) we demonstrate that the TTB specification error generalizes in the annual environment.

To obtain annual from quarterly measures we adopt the same methodology as in the national accounts and employed by Hall (2004). Specifically, we set all the *flow* variables at the annual rate equal to the sum of the corresponding *flow* variables over the quarters, i.e., for *flow* variable x, $x_t^a = \sum_{k=1}^4 x_{t,k}$, where $x = I, \pi, s_i, , i = 1, ...J$ and a denotes annual frequency.

The annual measure for marginal q, is the average over the corresponding quarterly measure. However, it differs slightly depending on the TTB. We use the following definitions[9]

$$J = 1\,, q_t^a = \frac{\sum_{k=1}^4 q_{t,k}}{4} \quad J = 2\,, q_t^a = \frac{\sum_{k=1}^4 E_k q_{t,k+1}}{4}$$

$$J = 3\,, q_t^a = \frac{\sum_{k=1}^4 E_k q_{t,k+2}}{4} \quad J = 4\,, q_t^a = \frac{\sum_{k=1}^4 E_k q_{t,k+3}}{4}$$

Finally, we take the annual capital stock to correspond to the end of year (i.e. fourth quarter) stock.[10]

We then re-estimate the empirical investment equation specified in section 4.1 using the annual measures derived above (for convenience we drop the firm-specific subscript j), for $J = 1, 2, 3, 4$. Here as in the previous section J refers to TTB in quarters, so the maximum length for the construction of capital we consider is one year.

[9] In general $\frac{\sum_{k=1}^4 E_k q_{t,k+J-1}}{4} \neq \frac{\sum_{k=1}^4 q_{t,k}}{4}$. However, with autocorrelated productivity shocks the two measures are highly correlated. We use the marginal expected q for each different J to isolate the omitted variable effect. Our results are broadly similar if we use the same q for each J.

[10] Alternatively, the annual measure for the capital stock can be calculated from $K_{t+1}^a = (1 - \delta^a)K_t^a + s_{1t}^a$. The results presented in this section are insensitive to this alternative definition.

$$\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_2 \frac{\pi_t^a}{K_t^a} + \varepsilon_t^a \tag{4.4}$$

Table 3 reports the results from estimation of (4.4). There are two noteworthy findings. First, the adjustment cost estimate derived from b_1 is upward biased (top panel of Table 3, imposing $b_2 = 0$). Second, adding the profit rate to the regression yields a positive and statistical significant b_2 coefficient (bottom panel). We discuss these findings in detail later in this section. We now focus on the question whether the explanatory power of $(\frac{\pi_t^a}{K_t^a})$ is due to time aggregation and/or specification error resulting from the TTB nature of investment. We can decompose the two sources of error to answer this question. First, notice from Table 2 that for $J = 1$ in the quarterly model, $(\frac{\pi_t}{K_t})$ has no explanatory power. This follows from the fact that for $J = 1$ there is no investment outlay that refers to a decision taken previously ($s_{Jt} = ... = s_{1t} = I_t$) and hence no omitted RHS state variable. Even though the profit rate will be correlated with investment rates, its forecasting role for future investment opportunities is properly accounted for by marginal q. Thus any role for the profit rate in Table 3 in the $J = 1$ column can be solely attributed to the time aggregation error. We can derive an expression for the time aggregation error if we write the first order condition for investment (for $J = 1$),

$$-1 - \eta(\frac{I_k}{K_k} - \delta) + q_k = 0$$

Since this must be satisfied in any quarter $k = 1, 2, 3, 4$, it follows from aggregation over quarters,

$$-4 - \eta\left(\sum_{k=1}^{4}(\frac{I_{t,k}}{K_{t,k}} - \delta)\right) + \sum_{k=1}^{4} q_{t,k} = 0$$

where t denotes years. In Appendix 2 we show that after suppressing the constant terms the equation above can be written as,

$$\frac{I_t^a}{K_t^a} = constant + \left(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}\right) + \frac{1}{\eta^a}q_t^a \tag{4.5}$$

The time aggregation error is the term in parenthesis, $\left(\frac{1}{K_t^a}\sum_{k=1}^4 \frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^4 \frac{I_{t,k}}{K_{t,k}}\right)$. This term will be different from zero except when investment is equal to replacement investment and thus $K_t^a = K_{t,k}$. Moreover, this term will be correlated with the profit rate since it is a function of investment rates. Thus the aggregation of more frequent investment decisions results in a small but positive profit rate coefficient as found in Table 3. On the other hand, the specification error that arises due to the TTB nature of investment can be seen by examining the FOC for optimal investment when $J > 1$. For example, summing the FOC for optimal investment for $J = 3$ we get,

$$-4(\varphi_3 + \beta\varphi_2 + \beta^2\varphi_1) - \eta\left(\sum_{k=1}^{4}(\frac{s_{3t,k}}{K_{t,k}} - \delta)\right) + \beta^2\sum_{k=1}^{4} E_k q_{t,k+2} = 0$$

which after repeating the steps above and using (2.3) we can write as,

$$-(\varphi_3 + \beta\varphi_2 + \beta^2\varphi_1) - \frac{\eta}{4}\left(\sum_{k=1}^{4}(\frac{I_{t,k}}{K_t^a} - \delta)\right) + \frac{\eta}{4}\left(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^{4}\frac{1}{\varphi_3}\frac{I_{t,k}}{K_{t,k}}\right)$$
$$+\frac{\eta}{4\varphi_3}\sum_{k=1}^{4}\frac{\sum_{i=1}^{2}\varphi_i s_{it,k}}{K_{t,k}} + \beta^2\frac{\sum_{k=1}^{4}E_k q_{t,k+2}}{4} = 0$$

Re-arranging this equation to bring $\frac{I_t^a}{K_t^a}$ on the left hand side of the equation we finally arrive at,

$$\frac{I_t^a}{K_t^a} = constant + \left(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \frac{1}{\varphi_3}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}\right) + \frac{1}{\varphi_3}\sum_{k=1}^{4}\frac{\sum_{i=1}^{2}\varphi_i s_{it,k}}{K_{t,k}} + \frac{1}{\eta^a}\beta^2 q_t^a \qquad (4.6)$$

where again we have used, $\sum_{k=1}^4 \frac{I_{t,k}}{K_t^a} = \frac{I_t^a}{K_t^a}$ and $q_t^a = \frac{\sum_{k=1}^4 E_k q_{t,k+2}}{4}$. We see that relative to (4.5), there is an additional RHS variable that reflects the TTB technology. This is given by $\sum_{k=1}^4 \frac{\sum_{i=1}^2 \varphi_i s_{it,k}}{K_{t,k}}$ which is a summation (over quarters per year) of the omitted state variable in equation (4.3), i.e. a linear combination of the latter. The annual profit rate, $(\frac{\pi_t^a}{K_t^a})$, will be the sum of the corresponding quarterly rates and it will be correlated with this state variable since both are sums of the corresponding quarterly measures. Therefore since the profit rate is correlated with the key omitted state variable and the investment rate (see Table 4, lower bottom) regressing $\frac{I_t^a}{K_t^a}$

on q_t^a and the profit rate will result in a statistical significant role for the latter. But this is merely reflecting the omission of an explanatory variable from the RHS of the regression.[11] Since in our model capital markets are perfect, any role for profits must result from this mis-specification.

We now discuss the results reported in Table 3. In the top panel we demonstrate the bias in the adjustment cost parameter, η^a (imposing $b_2 = 0$). This follows from standard econometric results since marginal q and the omitted state variable are correlated (e.g. Judge et al. (1985), p.858). The time aggregation error is illustrated in the bottom panel of Table 3 for $J = 1$. This results in a small positive profit rate coefficient as explained above.[12] We also note that in the regression excluding the profit rate the magnitude of the bias in η^a ranges from roughly 7% for $J = 1$ to 22% for $J = 4$ (top panel, Table 3). Thus lengthier time-to-build technology will produce adjustment cost estimates that imply slower adjustment speeds for capital. Empirical work with the q model has been unsatisfactory (see Chirinko (1993) for a review), producing implausibly large adjustment cost estimates. Our results offer a potential explanation for these estimates since TTB implies an upward bias in the adjustment cost estimate. The increase in the bias reflects almost entirely the differences in the true coefficient —equal to $\frac{1}{\varphi_J}$—of $\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}}$, which is rising as $J \uparrow$ and the fact that the discount factor is also reflected in b_1 (see equation (4.6))—although the latter makes very little difference for the size of the bias. To give a sense of comparison for the bias, we note that when only time aggregation is taken into account (for $J = 1$) our results are consistent with the estimate for the bias in η^a reported in Hall (2004). He analyzes the bias arising—in estimating capital adjustment costs—from aggregating monthly decisions to the annual frequency. Hall (2004) reports biases in η^a from time aggregation in the order of (approximately)

[11] The estimated coefficients in Table 3 reflect both the time aggregation and specification error. However, the former's contribution to the b_2 estimates for $J > 1$ is extremely small. This can be shown by using $\sum_{k=1}^{4} \frac{I_{t,k}}{K_{t,k}}$ instead of $\frac{I_t^a}{K_t^a}$ as the LHS variable in (4.6) thus eliminating the aggregation error. The resulting estimated coefficients are nearly identical to those shown in Table 3 and are not reported but are available upon request. In Table 10 we also report a set of regression results that use the alternative adjustment cost formulation, i.e. $G(s_{1,jt}, ...s_{J,jt}, K_{jt}) = \sum_{i=1}^{J} \varphi_i \frac{\eta}{2}(\frac{s_{i,jt}}{K_{jt}} - \delta)^2 K_{jt}$. Results from Table 10 are qualitatively similar.

[12] If we regress $\sum_{k=1}^{4} \frac{I_{t,k}}{K_{t,k}}$ on $\sum_{k=1}^{4} q_{t,k}$. eliminating the time aggregation term, $\frac{\pi_t^a}{K_t^a}$ looses its significance and explanatory power.

10% (see Table V, p.920)—quite similar to the 7% we obtain in this model. Thus in this simulated environment investment-q regressions that fail to account for the TTB technology, produce upward biased adjustment cost estimates in addition to the cash flow effect. Barnett and Sakellaris (1999) and Del Boca et al. (2008) using US and Italian annual firm level data respectively, provide evidence consistent with our findings, reporting significantly lower adjustment cost parameter estimates when TTB investment is allowed for.

The bottom panel of Table 3 includes the profit rate as an additional RHS variable. Two findings are worth noting. First, the coefficient of the profit rate increases as the TTB length increases. Second, there is a close association between the size of the bias in the adjustment cost estimate and the magnitude of the profit rate coefficient. As we increase the length of TTB (moving from left to right) in the bottom panel of Table 3, we observe a positive relationship between $\widehat{\eta}^a$ (inverse of b_1 reported in the Table) and b_2. For example, as we move from $J = 2$ to $J = 3$ the (inverse of) coefficient on q rises from 0.337 to 0.39, while the profit rate coefficient rises from 0.22 to 0.29. In other words, the regression results indicate that a higher sensitivity of investment to profits is associated with higher adjustment cost estimates as the length of TTB increases. This result can easily be explained since the mean value of both coefficients moves in proportion with the true coefficient of the omitted state variable which equals $\frac{1}{\varphi_J}$ (Appendix 2 provides the details). According to omitted variables result from standard econometrics the mean value of the profit rate coefficient reported in Table 3 will vary proportionately with the true coefficient of $\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}}$ given by $\frac{1}{\varphi_J}$. As long as φ_J falls with J, that is as long as the value put in place at the first stage of the construction falls with time required to completion, the coefficient of the profit rate will rise. This is indeed what the available evidence on TTB suggests.

We now turn to the question of whether our model can replicate the different cross sectional investment-cash flow sensitivity reported in the majority of empirical studies that test the imperfect capital markets hypothesis.

4.2 Cross sectional implications

We have demonstrated that the TTB investment technology will produce a cash flow effect in investment-q regressions with simulated firm level data. We have identified a specification and a time aggregation error, the former due to the TTB technology while the latter arising from aggregating more frequent investment decisions. We now discuss some potential cross sectional implications of TTB. Our model predicts that the cash flow effect will be present across different cross sections of firms as long as all cross sections share the same TTB technology. This will be true for example for small vs. large firms. On the other hand studies that seek to test for the presence of capital market imperfections typically report investment-cash flow sensitivities that vary significantly by cross section. For instance in Fazzari et al. (1988) or Gilchrist and Himmelberg (1995) small firms are estimated to have higher investment cash flow sensitivities compared to large firms and the differences are interpreted as arising from capital market imperfections that affect the former significantly more than the latter. Evidently our model predicts the same cash flow sensitivity for either small or large firms if they are subject to the same TTB technology. The length for TTB however will crucially depend on the type of investment that firms undertake. Consider for example two firms (A and B) that are identical in all other respects except that firm A invests proportionally more in structures and less in equipment compared to firm B. The available evidence discussed in section 3.1 suggests that TTB is considerably longer for structures than it is for equipment and by a wide margin. Therefore firm A will be characterized by a longer TTB technology compared to firm B. Our model then predicts a larger cash flow coefficient for firm A compared to firm B. Looking across Table 3 we note that the profit rate coefficient rises with the length of TTB. For example, the profit rate coefficient rises from 0.03 to 0.22 as we move from 1 quarter to 2 quarters, and from 0.22 to 0.29 as we move from 2 quarters to 3 quarters TTB. Is it then likely that differences in TTB technologies exist among different groups of firms in such a way as to be able to capture the differences in investment-cash flow sensitivities reported in the literature? For this purpose we bring to light evidence that strongly suggests that TTB varies by firm size. We have information from a large sample of Compustat firms (manufacturing sector) that allows us to compute investment spending in structures and equipment. This information is reported in Table 5. Table 5 reports the mean ratio of structures to equipment investment for small and large firms classified as such using the same classification criteria adopted by existing empirical work. The robust feature of Table 5 is that small firms exhibit higher structures to equipment spending ratios compared to large firms. Importantly, the differences are statistical significant at least at the 10% significance level.[13] This evidence from Compustat indicates that small firms invest significantly more on structures per unit of equipment compared to large firms and consequently it is reasonable to be characterized by longer TTB periods for their capital expenditures. The implication of this fact is clear. According to the analysis of the previous section small firms should exhibit higher sensitivity to profits compared to large firms. Even one quarter difference in the TTB technology can produce significant differences in investment–profit sensitivities between firms as Table 3 illustrates. The model is thus capable in replicating the cross sectional differences in investment cash-flow sensitivities documented in empirical work by exploiting differences in TTB technology as suggested by the evidence above.[14]

4.3 Time-to-plan

In this section we consider an alternative characterization of the TTB process. We explore time-to-plan (TTP) effects by altering the fraction of resources (φ_i) that are spent to the different construction (or delivery) stages of the capital projects. In particular, we implement this assumption

[13]The pattern of capital expenditure reported in the Annual Capital Expenditure Survey from the US Census Bureau also shows that in contrast to large firms, small firms (classified by number of employees) invest more in structures compared to equipment. Over the period reported (1995-2006) small firms have an average ratio of structures to equipment expenditure equal to 0.60, while large firms have an average ratio equal to 0.49. The data are for the non farm business sector and cover the period 1995 to 2006. The Annual Capital Expenditure Survey reports capital expenditure separately by structures and equipment for firms with and without employees. The data can be found at: http://www.census.gov/csd/ace.

[14]In Fazzari et al. (1988) the sample splitting criterion is dividend payout. In their sample non dividend paying firms are on average smaller than dividend paying firms. Similarly Gilchrist and Himmelberg (1995) employ additional sample splitting criteria in addition to firm size, such as bond ratings or commercial paper issues. Whereas these splits may identify different cross sections of firms they would nevertheless be strongly correlated with firm size.

by setting $\varphi_J = 0.01$, and distribute the rest of the resources evenly for the remaining φ_i. TTP has been emphasized in Christiano and Todd (1996) and more recently in Edge (2007) as a plausible characterization of the investment process and its ability to explain salient features of the aggregate business cycle better than TTB. TTP effects also seem to be an important feature for investment in structures (see Del Boca et al. (2008) and Koeva (2001)). We solve the model using this alternative calibration and re-estimate the investment equation (4.4) on our artificial panel. Further, because the share of resources that are absorbed by the different stages of the project is now significantly altered as compared to the TTB case we use the alternative adjustment cost formulation whereby adjustment costs are incurred at the time of expenditure of the project and not at the initial stage, i.e. we specify $G(s_{1,jt}, ...s_{J,jt}, K_{jt}) = \sum_{i=1}^{J} \varphi_i \frac{\eta}{2}(\frac{s_{i,jt}}{K_{jt}} - \delta)^2 K_{jt}$. This is a more natural assumption when the majority of resources are spent toward the middle or near project completion. The results are presented in Table 6. As we can see, the regression results are qualitatively very similar to those in Table 3. The most notable finding from the TTP technology is that the role of the profit rate seems to be more important compared to the TTB case. Comparing Tables 3 and 6 we see that the estimated profit coefficients (b_2) are on average larger under TTP for all J, and that the predictive role of the profit rate (as captured by differences in the adjusted R^2) is higher. The finding that the estimated coefficients are larger under TTP follows since in this case the true coefficient of the omitted state variable, $\frac{1}{\varphi_J}$ is considerably larger compared to the TTB case (see the discussion in section 4.2). We also note that this result is consistent with the cross sectional implications we highlighted in the previous section. Given the evidence presented in section 4.2 we would expect small firms investment technology to have a stronger TTP element compared to large firms since the former invest dis-proportionately more in structures compared to the latter. Under TTP investment we would therefore expect differences in cash flow effects to be even more pronounced among firms that differ in size.[15]

[15]We have also experimented with alternative values for the adjustment cost parameter, η, taken from Barnett and Sakellaris (1999) and Cooper and Haltiwanger (2006). More specifically we have used $\eta = 0.7$ from the former and $\eta = 0.455$ from the latter. The results are qualitatively very similar for these alternative

4.4 Implications for empirical work

In light of our findings it is worthwhile investigating the empirical implications and offer some recommendations for empirical work. Specifically we would like to know what particular information from the data can be used in order to estimate a correctly specified investment-q regression under TTB investment. For the remainder of the analysis we focus on the case $J = 3$. It is quite straightforward to generalize for any J. The key state variable that creates the link with cash flow (or more generally any profitability measure) is given by,

$$\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}}$$

We now show that a researcher seeking to estimate an investment-q equation can easily construct a measure that will proxy for the key state variable above. Let us assume that the value of the project put in place in each period is symmetric ($\varphi_1 = \varphi_2 = \varphi_3$). In Appendix 2 we show that the state variable above can be approximated by the following expression,

$$\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}} \approxeq \sum_{k=1}^{4} \left(\frac{I_{t-1,k}}{K_{t,k}} - \varphi_1 (1 - \frac{(1-\delta)}{g_{t,k}})\right) \tag{4.7}$$

where $g_{t,k} = \frac{K_k}{K_{k-1}}$ denotes the quarterly growth rate of capital in year t. For data observed at the annual frequency one can approximate the RHS of the above expression with

$$\frac{I^a_{t-1}}{K^a_t} - 4\varphi_1 (1 - \frac{4(1-\delta)}{g^a_t}) \tag{4.8}$$

where the superscript a denotes annual measures. The expression above involves only observable variables, namely lagged investment rate adjusted by the growth rate of capital, $\frac{I^a_{t-1}}{K^a_t}$ and the growth rate of capital, g^a_t. It follows from the expression above that one need only use $\frac{I^a_{t-1}}{K^a_t}$ and the inverse growth rate of capital $(g^a_t)^{-1}$ as additional RHS regressors in the investment-q regression (the rest of the terms will be subsumed in the constant). Evidently there are two practical advantages of this proxy: (i) it does not require knowledge of the TTB length (i.e. it readily generalizes to any J)

adjustment cost parameters and are not reported for brevity but are available upon request.

and (ii) it is easy to construct as it only requires the lagged investment rate and the growth rate of capital. We now formally evaluate the usefulness of this proxy in our simulated environment. Before we proceed to the regression results we briefly note that the correlation of this measure with the key state variable (for $J = 3$) is equal to 0.97 which is a good indication that it captures the movement of the omitted RHS variable to a great extent. To illustrate the usefulness of this empirical proxy Table 7 reports investment-q regression results augmented with the expression from equation 4.8 as an additional RHS variable (the coefficient of the latter is denoted by b_3). To judge the success of this measure we undertake a comparison with the regression results from Table 3. There are two notable findings. First, compared to Table 3 the inclusion of this proxy rectifies the bias problem with the adjustment cost parameter. Second, and most importantly the profit rate coefficient in Table 7 falls dramatically for all J as compared to the corresponding coefficients from Table 3. For example, for $J = 3$ the profit rate coefficient drops to 0.03 compared to 0.29 when this proxy is not included. The coefficient on the profit rate is still positive—due to the time aggregation error—but the adjusted R^2 does not increase when the profit rate is added to the regression indicating that this variable adds no explanatory power to the regression. Thus the inclusion of this proxy adequately controls for the omitted state variable.[16] In comparing different investment models, Eberly et al. (2008) report empirical results from a Compustat panel of firms and results from a simulated panel of firms that are consistent with the findings from Table 7. Specifically when they add the lagged investment rate in the investment-q regression the latter dominates, and cash flow explains very little of the remaining variation (see Table 2, p.36). In addition the coefficient on cash flow declines by more than half in magnitude compared to a regression that excludes the lagged investment rate. Eberly et al. (2008) stress infrequent regime changes in the firm's demand that makes lagged investment a good indicator of the current regime and thus a state variable. Our analysis by contrast highlights a different structural interpretation. The lagged investment rate in

[16] In the quarterly model regressions (not shown) the profit rate coefficient is essentially zero. This validates our claim that the coefficients on the profit rate reported in Table 7 are an artefact of time aggregation.

our simulated panel of firms proxies for the omitted state variable due to the TTB effects.

We further examine the usefulness of this simple measure when the symmetry cost assumption is invalid. We can explore this scenario by simulating a model when at least a pair of $\varphi's$ differ. Specifically, we simulate the model assuming that (for $J = 3$) $\varphi_1 = 0.45, \varphi_2 = 0.45, \varphi_3 = 0.10$, i.e. most of the project value is put in place during the second and third period of the construction which corresponds to a TTP technology.[17] In this case the difference between the proxy we are proposing as a RHS variable and the true omitted RHS variable is given by (see Appendix 2):

$$\varphi_1(\frac{\varphi_2}{\varphi_1} - 1)\frac{s_{1t,k}}{K_{t,k}} + \varphi_2(\frac{\varphi_3}{\varphi_2} - 1)\frac{s_{2t,k}}{K_{t,k}}$$

Note that the expression above is equal to zero when the symmetry assumption is imposed, i.e. $\varphi_1 = \varphi_2 = \varphi_3$. Table 8 presents the results from this exercise. Most notably, the finding that the coefficient of the profit rate approaches zero is robust even under this alternative calibration with TTP features. Adding the profit rate as an additional RHS regressor in the lower panel does not improve the predictive power of the regression as can be seen by the adjusted R^2 in the bottom panel. Therefore a researcher will correctly conclude that the role of cash flow is un-important in such a regression.

Another serious concern that often arises in empirical work with investment equations is the use of Tobin's or average q calculated from financial market data. Typically researchers are either unable to observe marginal q or the homogeneity assumptions that must be satisfied for the two measures to be equivalent are violated (due to for example market power or decreasing returns to scale). Thus researchers must rely on financial market information and use average (or Tobin's) q to control for future investment opportunities in the RHS of the investment regression. The use of average q has been criticized extensively because of the measurement error it may entail (see Erickson and Whited (2000) and Cummins et al. (2006) among others) but we think it is instructive to assess the regression implications when one has only available this imperfect measure. We would

[17]For $J = 2$ we simulate with $\varphi_2 = 0.10$,$\varphi_1 = 0.90$. For $J = 4$ we use $\varphi_4 = 0.10$,$\varphi_3 = \varphi_2 = \varphi_1 = 0.30$.

like to know how measurement error interacts with the specification error from TTB. In order to evaluate the consequences of using average q we introduce measurement error in our marginal q measure and use this noisy indicator as our q measure,

$$\overline{q_t^a} = q_t^a + \chi_t, \quad \chi_t \sim N(0, \sigma_\chi^2)$$

where χ denotes measurement error and we set σ_χ^2 to 1/10 the variance of marginal q^a implying a signal to noise ratio of 10. We report the results from regressing the investment rate on this noisy measure of q and the profit rate in Table 9. We first note that for $J = 1$ (i.e. when TTB effects are un-important) we obtain a positive and significant coefficient on the profit rate that differs substantially from that in Table 3 where marginal q is used. Thus using a noisy indicator of marginal q makes an irrelevant regressor to appear as explaining the variation in investment. Allowing for the TTB effects in the remaining columns we note that the estimated profit rate coefficients are noticeably larger compared to the corresponding coefficients from Table 3. For example, when measurement error is introduced and TTB equals three quarters the estimated profit rate coefficient equals 0.42. In contrast, when marginal q is used in Table 3 the corresponding profit rate coefficient is only 0.29. These results suggest that the use of a noisy indicator of marginal q magnifies the specification error arising from TTB investment.

4.5 Discussion

Our findings cast some doubt on the interpretation of investment-cash flow sensitivities in empirical investment-q regressions. Nevertheless it is important to clarify that we do not argue against the existence of financial market imperfections, rather we view the investment-q model as an inappropriate framework to test for capital market imperfections. Recently, researchers have undertaken carefully designed tests that are robust to a range of problems associated with profitability measures. Rauh (2006) for example designs an experiment that can identify variation in the availability of internal funds that is by construction orthogonal to future investment opportunities. His results lend support to the existence of capital market imperfections.

Another type of capital that should be less subject to the critique raised in this paper is inventories. Inventories are most likely not subject to TTB effects and have low adjustment costs compared to fixed investment. This is true in our model. Our paper therefore suggests this type of capital to be a better way to test for the perfect capital markets hypothesis. Indeed previous evidence suggests that inventory investment is sensitive to variation in internal funds. This is the approach taken by Carpenter et al. (1994) and Carpenter et al. (1998) for example. Based on the analysis in this paper empirical evidence that focuses on this type of asset rather than fixed investment should be a lot more persuasive.

In our analysis we have used a framework with more frequent factor input decisions than implicit in empirical studies that use annual firm-level data in order to study the significance of time aggregation error. One may question whether our results are sensitive to this assumption. In a model with perfect capital markets if firms make annual decisions and TTB is less than or equal to one year then cash flow should not be found important for explaining investment. In such a model there is neither a time aggregation or specification error and cash flow should not be found important in an investment-q regression. Without needing to re-calibrate our model, we note that this follows (qualitatively) from our results for $J = 1$ in the quarterly model (Table 2). In this case marginal q is a sufficient statistic for investment.

Arguably, if we retain the annual frequency, TTB should not be important for equipment investment. However, TTB will be important for investment in structures since the available evidence clearly indicates a longer than a year construction stage. It is therefore straightforward to think of an extended version of this model with two different types of capital, i.e. equipment and structures where each type of capital is subject to different TTB technologies. This implies that a model with $J = 2$ or $J = 3$ calibrated at the annual frequency, will be a plausible characterization. This model will still predict a role for the profit rate in a the context of a misspecified regression.

Therefore, the more frequent decision choice does not seem to undermine our results, at least qualitatively.

Finally, since our model is designed with perfect capital markets, is not equipped to evaluate the impact of capital market imperfections in the investment-q regressions we have examined. It is entirely possible that at least some of the cash flow effects found in previous empirical work are due to agency costs in capital markets that drive a wedge between the cost of internal and external finance. We can only conjecture that if capital market imperfections coexist with TTB effects will render cash flow sensitivities difficult to interpret as indicators for the severity of financing constraints. An interesting possibility is to examine how the presence of capital market imperfections can interact with the length of TTB. One may reasonably conjecture that small firms may be characterized by lengthier TTB technology because they are constrained in the funds they can extract from the market in order to proceed with the construction (or delivery) stages of their projects. Thus, the evidence presented in section 4.2 suggesting a longer TTB technology for small firms may be due in part to the difficulty they have to obtain outside finance. This is an interesting avenue left for future research.

5 Conclusions

We calibrate an industry with many firms to address the interpretation of an important empirical regularity, namely the finding, established in a large body of empirical work, that cash flow is important in explaining investment dynamics. According to this interpretation, investment is sensitive to internal funds due to capital market imperfections that make external finance costly relative to internal finance. This paper develops a rich decision theoretic model of investment with time-to-build and time-to-plan features for the installation of capital and uses it to evaluate the validity of this view. The central message of our study is that cash flow may be found to be important even if capital markets are perfect and even when future investment opportunities are properly accounted for. Thus investment-cash flow regressions may not be informative for the severity of capital market imperfections. This new explanation relies on the idea and supportive empirical evidence that it takes time to build productive capital. With time-to-build, the simple q framework is inadequate to fully explain optimal investment; an additional state variable defined as the sum of capital projects at different stages away from completion is also relevant. A subset of these projects refer to investment decisions taken in the past and so are part of the information set when new projects are decided upon. This implies that marginal q is not a sufficient statistic for total investment, but only a sufficient statistic for new projects. The subset of projets that refer to past information must be a right hand side variable in an investment regression and cash flow proxies for this omitted right hand side variable in a typical investment equation. We show how a researcher can, under certain assumptions on the time-to-build technology, approximate for this omitted state variable and hence obtain the correct inference from a modified investment-q regression. Our results suggest that investment cash flow sensitivities are not the right framework to evaluate the capital market imperfections view.

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A Appendix 1

This section derives the equilibrium conditions of the model, and describes the perturbation based solution method. A firm i in this industry solves (dropping the subscript):

$$\max_{L_t, s_{Jt}, d_t} E_0 \sum_{t=0}^{\infty} \beta^t div_t \tag{A.1}$$

s.t.

$$div_t = A_t \omega_t K_t^{\alpha} M_t^{\gamma} L_t^{\nu} - wL_t - d_t - I_t - \frac{\eta}{2}(\frac{s_{Jt}}{K_t} - \delta)^2 K_t$$

$$K_{t+1} = (1-\delta)K_t + s_{1t}$$

$$s_{Jt} = s_{J-1,t+1}$$

$$M_{t+1} = (1-\delta_m)M_t + d_t$$

$$lnA_{t+1} = \rho_A lnA_t + \sigma_A \varepsilon_{t+1}^A \quad \varepsilon_t^A \sim N(0,1)$$

$$ln\omega_{t+1} = \rho_\omega ln\omega_t + \sigma_\omega \varepsilon_{t+1}^\omega \quad \varepsilon_t^\omega \sim N(0,1)$$

given the initial values, $K_0, M_0, s_{j0},\ j = 1, ..., J-1; \{\varepsilon_t^A\}_{t=-J+1}^0, \{\varepsilon_t^\omega\}_{t=-J+1}^0$.

Introducing the Kuhn-Tucker multipliers q_t and μ_t we can write the Langrangean for this problem,

$$\max_{L_t, s_{Jt}, d_t} E_0 \sum_{t=0}^{\infty} \beta^t \{div_t + q_t(K_{t+1} - (1-\delta)K_t - s_{1t}) + \mu_t(M_{t+1} - (1-\delta_m)M_t - d_t)\}$$

The first order conditions associated with this problem are:

w.r.t L_t (labor)

$$(\nu A_t \omega_t K_t^{\alpha} M_t^{\gamma} L_t^{\nu-1} - w) = 0$$

w.r.t d_t (deliveries)

$$-1 + \beta E_t \Big\{ A_{t+1}\omega_{t+1}\gamma K_{t+1}^{\alpha} M_{t+1}^{\gamma-1} L_{t+1}^{\nu} + (1-\delta_m) \Big\} = 0$$

w.r.t s_{Jt} (project starts)

$$-\beta^t(\varphi_J + \eta(\frac{s_{Jt}}{K_t} - \delta)) - \beta^{t+1}E_t(\varphi_{J-1}) - \beta^{t+2}E_t(\varphi_{J-2})$$

$$+ ... + \beta^{t+J-1}E_t(-\varphi_1 + q_{t+J-1}) = 0$$

w.r.t K_{t+J} (capital)

$$\beta^{t+J-1}E_t(-q_{t+J-1})$$

$$+\beta^{t+J}E_t\Big\{ A_{t+J}\omega_{t+J}\alpha K_{t+J}^{\alpha-1} M_{t+J}^{\gamma} L_{t+J}^{\nu} + \eta(\frac{s_{J,t+J}}{K_{t+J}} - \delta)\frac{s_{J,t+J}}{K_{t+J}} - \frac{\eta}{2}(\frac{s_{J,t+J}}{K_{t+J}} - \delta)^2 + q_{t+J}(1-\delta) \Big\} = 0$$

$$q_t(K_{t+1} - (1-\delta)K_t - s_{1t}) = 0 \quad q_t \geq 0$$

$$\mu_t(M_{t+1} - (1-\delta_m)M_t - d_t) = 0 \quad \mu_t \geq 0$$

Collecting all the equations above that characterize equilibrium yields:

$$E_t F(y_{t+J}, ..., y_{t+1}, y_t, x_{t+J}, ..., x_{t+1}, x_t) = 0 \tag{A.2}$$

where E_t denotes the mathematical expectations operator conditional on information at time t, x_t denotes the vector of state variables and consists of capital, K_t, materials, M_t partially complete projects, $\{s_{jt}\}_{j=1}^{J-1}$, and the two exogenous precesses for productivity, A_t, and ω_t. The vector y_t denotes the vector of choice variables and consists of labor, L_t, materials orders, d_t, and new projects, s_{Jt}. The solution to the model given in equation $A.2$ can be expressed as

$$y_t = g(x_t, \sigma)$$

$$x_{t+1} = h(x_t, \sigma) + \pi\sigma\varepsilon_{t+1}$$

where g is a function that maps the vector of states, x_t to choice variables, y_t, h is a function that maps the state vector at time t to time $t+1$, π is a vector selecting the exogenous state variables, in this case A_t and ω_t, and $\sigma = [\sigma_A\, \sigma_\omega]$. We want to find a second order approximation of the functions, g, h around the non-stochastic steady state, $(x_t, \sigma) = (\overline{x}, 0)$. The non-stochastic steady state is defined as vectors $(\overline{x}, \overline{y})$ such that $F(\overline{y}, ..., \overline{y}, \overline{y}, \overline{x}, ..., \overline{x}, \overline{x}) = 0$.

To compute the second order approximation around $(x, \sigma) = (\overline{x}, 0)$, one substitutes the proposed policy rules into (A.2) and makes use of the fact that derivatives of any order of (A.2) must equal zero in order to compute the coefficients of the Taylor approximations of the proposed policy functions. The second order solution for all variables of the model is completely characterized by the matrices that collect the first and second order derivatives of the policy (g) and transition (h) functions with respect to the state variables and σ, $g_x, h_x, g_{xx}, h_{xx}, g_{\sigma\sigma}, h_{\sigma\sigma}$. For example, the second order approximation for g and h can be written respectively as (see Schmitt-Grohe and Uribe (2004)),

$$[g(x,\sigma)]^i = [g(\overline{x},0)]^i + [g_x(\overline{x},0)]^i_a (x-\overline{x})_a + \frac{1}{2}[g_{xx}(\overline{x},0)]^i_{ab}(x-\overline{x})_a(x-\overline{x})_b + \frac{1}{2}[g_{\sigma\sigma}(\overline{x},0)]^i[\sigma][\sigma]$$

$$[h(x,\sigma)]^j = [h(\overline{x},0)]^j + [h_x(\overline{x},0)]^j_a (x-\overline{x})_a + \frac{1}{2}[h_{xx}(\overline{x},0)]^j_{ab}(x-\overline{x})_a(x-\overline{x})_b + \frac{1}{2}[g_{\sigma\sigma}(\overline{x},0)]^j[\sigma][\sigma]$$

where $i = L, s_J, d,\ a, b = K, M, \{s_j\}_{j=1}^{J-1}, A, \omega,\ j = K, M, \{s_j\}_{j=1}^{J-1}, A, \omega$. $[g_x]^i_a, [h_x]^i_a$ denote the (i, a) element of the first order derivative of g, h with respect to x and similarly for the second order derivatives. Notice that all the matrices collecting first and second order derivatives above are evaluated at the non-stochastic steady state, i.e. $(\overline{x}, 0)$. In turn the non-stochastic steady state can be easily computed by solving the f.o.c's setting $A_t = A_{t+1} = E(A)$ and similarly $\omega_t = \omega_{t+1} = E(\omega)$ and solving the resulting static system of equations for $\overline{x}, \overline{y}$.

B Appendix 2

Time Aggregation. Derivation of equation 4.5. We begin with equation,

$$-4 - \eta\Bigg(\sum_{k=1}^{4}(\frac{I_{t,k}}{K_{t,k}} - \delta)\Bigg) + \sum_{k=1}^{4} q_{t,k} = 0$$

where t denotes years.

If we add and subtract $\eta\Bigg(\sum_{k=1}^{4}(\frac{I_{t,k}}{K_t^a} - \delta)\Bigg)$ we get

$$-4 - \eta\Bigg(\sum_{k=1}^{4}(\frac{I_{t,k}}{K_t^a} - \delta)\Bigg) + \eta\Bigg(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}\Bigg) + \sum_{k=1}^{4} q_{t,k} = 0$$

The term, $\Bigg(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}\Bigg)$ which will be $\neq 0$ in general, represents the time aggregation error. It is easy to see that this term will be zero only when investment is equal to replacement investment (δK), so that capital in year t, $K_t^a = K_{t,k}$. Similar expressions for the time aggregation error characterize $J = 2, 3, 4$.

Re-writing this equation (dividing by four and using $\sum_{k=1}^{4}\frac{I_{t,k}}{K_t^a} = \frac{I_t^a}{K_t^a}$, $q_t^a = \frac{\sum_{k=1}^{4} q_{t,k}}{4}$,$\frac{1}{\frac{\eta}{4}} = \frac{1}{\eta^a}$) after suppressing all the constant terms yields equation 4.5 in the text,

$$\frac{I_t^a}{K_t^a} = constant + \Bigg(\frac{1}{K_t^a}\sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}K_{t,k} - \sum_{k=1}^{4}\frac{I_{t,k}}{K_{t,k}}\Bigg) + \frac{1}{\eta^a}q_t^a$$

Alternative adjustment cost formulation. We now derive the Euler equation for new projects, s_{Jt}, that corresponds to the following adjustment cost formulation,

$$G(s_{1,jt}, ...s_{J,jt}, K_{jt}) = \sum_{i=1}^{J}\varphi_i\frac{\eta}{2}(\frac{s_{i,jt}}{K_{jt}} - \delta)^2 K_{jt}$$

This is given by,

$$-\beta^t(\varphi_J+\eta\varphi_J(\frac{s_{Jt}}{K_t}-\delta))-\beta^{t+1}E_t(\varphi_{J-1}+\eta\varphi_{J-1}(\frac{s_{Jt}}{K_{t+J-3}}-\delta))-\beta^{t+2}E_t(\varphi_{J-2}+\eta\varphi_{J-2}(\frac{s_{Jt}}{K_{t+J-2}}-\delta))$$

$$+...+\beta^{t+J-1}E_t-(\varphi_1+\eta\varphi_1(\frac{s_{Jt}}{K_{t+J-1}}-\delta))+\beta^{t+J-1}E_tq_{t+J-1}=0$$

The investment-q equation (using (2.3)) that this FOC implies for $(J=3)$ is given by,

$$\frac{I_{j,t}}{K_{j,t}}=\varphi_3\Big(-\frac{1}{\eta}(\varphi_3+\beta\varphi_2+\beta^2\varphi_1)+\delta(\varphi_3+\beta\varphi_2+\beta^2\varphi_1)\Big)\Big(\varphi_3+\beta\varphi_2g^{-1}_{k_{t+1}}+\beta^2\varphi_1g^{-1}_{k_{t+1}}g^{-1}_{k_{t+2}}\Big)^{-1}$$

$$+\varphi_3\frac{1}{\eta}\beta^2E_t(q_{j,t+2})\Big(\varphi_3+\beta\varphi_2g^{-1}_{k_{t+1}}+\beta^2\varphi_1g^{-1}_{k_{t+1}}g^{-1}_{k_{t+2}}\Big)^{-1}+\sum_{i=1}^{2}\varphi_i\frac{s_{i,jt}}{K_{j,t}}$$

where $g_{k_{t+1}}=\frac{K_{t+1}}{K_t}$, $g_{k_{t+2}}=\frac{K_{t+2}}{K_{t+1}}$ denote the growth rates of capital in $t+1$ and $t+2$ respectively.

This is similar to the baseline adjustment cost assumption investment-q equation. Importantly, the implications from the omission of the RHS variable $(\sum_{i=1}^{2}\varphi_i\frac{s_{i,jt}}{K_{j,t}})$ are very similar to those implied by Table 3, i.e. there is also a cash-flow effect. Table 10 summarizes the regression results from this specification (using annual measures as in Table 3).

Empirical proxy. Next, we show the derivation of the empirical proxy in equation 4.7.

$$\sum_{k=1}^{4}\frac{\sum_{i=1}^{2}\varphi_is_{it,k}}{K_{t,k}}=\sum_{k=1}^{4}(\frac{I_{t-1,k}}{K_{t,k}}-\varphi_1(1-\frac{(1-\delta)}{g_{t,k}})+\varphi_1(\frac{\varphi_2}{\varphi_1}-1)\frac{s_{1t,k}}{K_{t,k}}+\varphi_2(\frac{\varphi_3}{\varphi_2}-1)\frac{s_{2t,k}}{K_{t,k}})$$

The RHS of the equation above yields,

$$\frac{I^a_{t-1}}{K^a_t}+\sum_{k=1}^{4}(\frac{I_{t-1,k}}{K_{t,k}}-\frac{I_{t-1,k}}{K^a_t})-\varphi_1\sum_{k=1}^{4}\big(1-\frac{(1-\delta)}{g_{t,k}})\big)+\big(\varphi_1(\frac{\varphi_2}{\varphi_1}-1)\frac{s_{1t,k}}{K_{t,k}}+\varphi_2(\frac{\varphi_3}{\varphi_2}-1)\frac{s_{2t,k}}{K_{t,k}})$$

Equation 4.7 in the text follows from the above when we impose the symmetry assumption of TTB (i.e. $\varphi_1=\varphi_2=\varphi_3$) and use $g_{t,k}\approxeq\frac{1}{4}g^a_t$.

Coefficient bias. We derive the expressions that determine the biases in the coefficients of q and the profit rate in the investment regression.

Consider the regression

$$y = X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + u$$

where $y = \frac{I_t^a}{K_t^a}, X_1 = q_t^a, X_2 = \frac{\pi_t^a}{K_t^a}, X_3 = \sum_{k=1}^4 \frac{\sum_{i=1}^2 \varphi_i s_{it,k}}{K_{t,k}}$. The true coefficient of $\frac{\pi_t^a}{K_t^a}$ will be $\beta_2 = 0$. Now suppose we specify the following regression equation (i.e. equation 4.4)

$$y = X_1\beta_1 + X_2\beta_2 + e$$

where the error e term is now given by

$$e = X_3\beta_3 + u$$

The OLS coefficient vector is given by,

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{bmatrix} \begin{bmatrix} X_1'y \\ X_2'y \end{bmatrix}$$

Using standard matrix formulas this equation can be written as,

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} D_1^{-1} & D_2^{-1} \\ D_3^{-1} & D_4^{-1} \end{bmatrix} \begin{bmatrix} X_1'y \\ X_2'y \end{bmatrix}$$

where $D_1^{-1} = (X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1}$,

$D_2^{-1} = -(X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1}(X_1'X_2)(X_2'X_2)^{-1}$,

$D_3^{-1} = -(X_2'X_2)^{-1}(X_2'X_1)(X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1}$,

$D_4^{-1} = (X_2'X_2)^{-1} + (X_2'X_2)^{-1}(X_2'X_1)(X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1}(X_1'X_2)(X_2'X_2)^{-1}$

The expected value of the OLS coefficients on the profit rate and marginal q will be given by,

$$E(b_2) = [D_3^{-1}(X_1'X_1) + D_4^{-1}(X_2'X_1)]\beta_1 + [D_3^{-1}(X_1'X_3) + D_4^{-1}(X_2'X_3)]\beta_3$$

$$E(b_1) = [D_1^{-1}(X_1'X_1) + D_2^{-1}(X_2'X_1)]\beta_1 + [D_1^{-1}(X_1'X_3) + D_2^{-1}(X_2'X_3)]\beta_3$$

One can show that $D_1^{-1} > 0, D_2^{-1} < 0, D_3^{-1} < 0, D_4^{-1} > 0$ as long as $(X_1'X_1 - X_1'X_2(X_2'X_2)^{-1}X_2'X_1)^{-1} > 0$. Since $X_1 = q_t^a, X_2 = \frac{\pi_t^a}{K_t^a}$ this condition simplifies to $(var(q_t^a) - cov(q_t^a, \frac{\pi_t^a}{K_t^a})var(\frac{\pi_t^a}{K_t^a})^{-1}cov(q_t^a, \frac{\pi_t^a}{K_t^a})) > 0$. This can further be written as $1 > \rho^2_{x_1,x_2}$ which will be always satisfied unless $\rho^2_{x_1,x_2} = 1$. It is easy to see from the expressions above that when this condition is satisfied, $E(b_2)$ will rise with β_3, while $E(b_1)$ will fall with β_3. Therefore as long as φ_J is falling with J this will always be the case.

Table 1: Calibrated parameters

Technology	
γ=0.53	
$\nu = 0.32$	
α=0.13	
Investment	
φ_1=0.33	
φ_2=0.34	
φ_3=0.33	
Depreciation	
δ=0.025	
δ_m=0.75	
Discount factor	
$\beta = \frac{1}{1+r} = 0.99$	
Adjustment cost parameter	
η^a=0.27 (Barnett and Sakellaris (1999))	
Productivity process (common)	
σ_A=0.045	$\rho_A = 0.9$
Productivity process (idiosyncratic)	
σ_ω=0.025	$\rho_\omega = 0.9$

Table 2: Investment regressions–empirical specification

Coefficient	$J = 1$	$J = 2$	$J = 3$	$J = 4$
b_1	0.91	0.55	0.41	0.33
	(0.0001)	(0.001)	(0.002)	(0.002)
b_2	0.0001	0.49	0.54	0.58
	(0.0001)	(0.001)	(0.002)	(0.003)
$\overline{R}^2$	0.99	0.97	0.92	0.87

Notes. The Table reports coefficients of the regression, $\frac{I_{j,t}}{K_{j,t}} = \alpha + b_1 E_t(q_{j,t+2}) + b_2 \frac{\pi_{j,t}}{K_{j,t}} + \varepsilon_{j,t}$ based on the quarterly model. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 3: Investment regressions–empirical specification with annual measures

True η^a = *0.27*				
Coefficient	$J=1$	$J=2$	$J=3$	$J=4$
b_1	3.50	3.37	3.20	3.00
	(0.0001)	(0.004)	(0.008)	(0.01)
$\widehat{\eta}^a = \frac{1}{b_1}$	*0.28*	*0.29*	*0.31*	*0.33*
$\overline{R}^2$	0.99	0.98	0.94	0.88
b_1	3.46	2.97	2.55	2.20
	(0.001)	(0.006)	(0.01)	(0.02)
b_2	0.03	0.22	0.29	0.35
	(0.0007)	(0.003)	(0.004)	(0.006)
$\overline{R}^2$	0.99	0.99	0.95	0.91

Notes. The top panel reports the coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + \varepsilon_t^a$. The bottom panel reports coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_2 \frac{\pi_t^a}{K_t^a} + \varepsilon_t^a$. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 4: Correlations (baseline calibration)

	$J=4$	$J=3$	$J=2$
State variable	$\sum_{k=1}^{4} \frac{\sum_{i=1}^{3} \varphi_i s_{it,k}}{K_{t,k}}$	$\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}}$	$\sum_{k=1}^{4} \frac{\sum_{i=1}^{1} \varphi_i s_{it,k}}{K_{t,k}}$
$\frac{\pi_t^a}{K_t^a}$	0.73	0.85	0.84

$J=3$				
	$\frac{I_t^a}{K_t^a}$	$\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} \varphi_i s_{it,k}}{K_{t,k}}$	$\frac{\pi_t^a}{K_t^a}$	q_t^a
$\frac{I_t^a}{K_t^a}$	1	0.99	0.86	0.97
$\sum_{k=1}^{4} \frac{\sum_{i=1}^{2} s_{it,k}}{K_{t,k}}$		1	0.85	0.93
$\frac{\pi_t^a}{K_t^a}$			1	0.83
q_t^a				1

Notes. This Table reports correlations between the profit rate and the state variable that arises under the TTB assumption. In this Table J denotes TTB in quarters. All statistics are averages over 500 replications.

Table 5: Compustat data (1980-2007)–Evidence for TTB

Firm-year observations	Mean values	Size: Small	Size: Large	Test of equality between mean values
4818	$\frac{I_{str}}{I_{eqp}}$	$0.36^{\flat}$	0.29	$p-value$=0.009
7628	$\frac{I_{str}}{I_{eqp}}$	$0.37^{\dagger}$	0.32	$p-value$=0.086

Notes. Compustat sample of manufacturing firms. Small firms are classified as belonging to the lower 25 percentile using either real sales ($^{\flat}$) or real total assets (†). Large firms are those belonging to the upper 25 percentile of the corresponding distribution. We use the method proposed by Bond and Meghir (1994) to estimate gross investment in structures (I_{str}) and equipment (I_{eqp}). Specifically we use the following calculation: $I_{it} = I_{Tt}\frac{\Delta K_{it}}{\Delta K_{Tt}}$, where i=structures, equipment. I_{Tt} denotes total gross investment (Compustat data item 30), K_{it} capital stock (book value) in i=structures, equipment (Compustat data item 155 and 156) and K_{Tt} total (book value) capital stock (Compustat data item 8).

Table 6: Investment regressions–empirical specification with annual measures and time-to-plan

True η^a = *0.27*			
Coefficient	$J=2$	$J=3$	$J=4$
b_1	3.34	3.19	2.92
	(0.009)	(0.01)	(0.01)
$\widehat{\eta}^a = \frac{1}{b_1}$	*0.30*	*0.31*	*0.34*
$\overline{R}^2$	0.93	0.86	0.75
b_1	2.70	2.35	1.75
	(0.01)	(0.01)	(0.002)
b_2	0.44	0.45	0.50
	(0.006)	(0.007)	(0.01)
$\overline{R}^2$	0.95	0.89	0.80

Notes. The top panel reports the coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + \varepsilon_t^a$. The bottom panel reports coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_2\frac{\pi_t^a}{K_t^a} + \varepsilon_t^a$. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 7: Investment regressions–empirical specification with annual measures and proxy for state variable

True $\eta^a = 0.27$			
Coefficient	$J = 2$	$J = 3$	$J = 4$
b_1	3.63	3.62	3.67
	(0.0006)	(0.0003)	(0.0002)
$\widehat{\eta}^a = \frac{1}{b_1}$	*0.27*	*0.27*	*0.27*
$\overline{R}^2$	0.99	0.99	0.99
b_1	3.68	3.69	3.80
	(0.003)	(0.002)	(0.002)
b_2	0.03	0.03	0.02
	(0.0008)	(0.0008)	(0.0006)
b_3	0.89	0.91	0.93
	(0.002)	(0.001)	(0.0008)
$\overline{R}^2$	0.99	0.99	0.99

Notes. The top panel reports the coefficients of the regression, $\frac{I^a_t}{K^a_t} = \alpha + b_1 q^a_t + b_3(\frac{I^a_{t-1}}{K^a_t} - 4\varphi_1(1 - \frac{4(1-\delta)}{g^a_t})) + \varepsilon^a_t$. The bottom panel reports coefficients of the regression, $\frac{I^a_t}{K^a_t} = \alpha + b_1 q^a_t + b_2 \frac{\pi^a_t}{K^a_t} + b_3(\frac{I^a_{t-1}}{K^a_t} - 4\varphi_1(1 - \frac{4(1-\delta)}{g^a_t})) + \varepsilon^a_t$. In the top panel only coefficient b_1 is reported. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 8: Investment regressions–empirical specification with annual measures and proxy for state variable(robustness to TTB assumptions)

True η^a = *0.27*			
Coefficient	$J = 2$	$J = 3$	$J = 4$
b_1	3.61	3.63	3.62
	(0.0002)	(0.0001)	(0.0001)
$\widehat{\eta}^a = \frac{1}{b_1}$	*0.27*	*0.27*	*0.27*
$\overline{R}^2$	0.99	0.99	0.99
b_1	4.04	1.4	1.6
	(0.003)	(0.004)	(0.003)
b_2	0.02	0.02	0.00
	(0.0009)	(0.001)	(0.001)
b_3	0.49	0.72	0.83
	(0.0006)	(0.0008)	(0.0008)
$\overline{R}^2$	0.99	0.99	0.99

Notes. The top panel reports the coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_3(\frac{I_{t-1}^a}{K_t^a} - 4\varphi_1(1 - \frac{4(1-\delta)}{g_t^a})) + \varepsilon_t^a$. The bottom panel reports coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_2 \frac{\pi_t^a}{K_t^a} + b_3(\frac{I_{t-1}^a}{K_t^a} - 4\varphi_1(1 - \frac{4(1-\delta)}{g_t^a})) + \varepsilon_t^a$. In the top panel only coefficient b_1 is reported. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 9: Investment regressions–empirical specification with annual measures and measurement error in q

Coefficient	$J = 1$	$J = 2$	$J = 3$	$J = 4$
b_1	3.17	2.62	2.23	1.58
	(0.001)	(0.01)	(0.01)	(0.01)
b_2	0.17	0.40	0.42	0.44
	(0.006)	(0.008)	(0.005)	(0.006)
$\overline{R}^2$	0.97	0.96	0.93	0.89

Notes. The Table reports coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 \overline{q_t^a} + b_2 \frac{\pi_t^a}{K_t^a} + \varepsilon_t^a$ where $\overline{q_t^a} == q_t^a + \chi_t$ and χ_t captures measurement error in q. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.

Table 10: Investment regressions–empirical specification with annual measures (robustness to adjustment cost assumption)

True $\eta^a =$ *0.27*				
Coefficient	$J = 1$	$J = 2$	$J = 3$	$J = 4$
b_1	3.48	3.42	3.26	3.18
	(0.001)	(0.004)	(0.008)	(0.01)
$\widehat{\eta^a} = \frac{1}{b_1}$	*0.28*	*0.29*	*0.31*	*0.32*
$\overline{R}^2$	0.99	0.98	0.93	0.89
b_1	3.41	3.01	2.57	2.27
	(0.002)	(0.006)	(0.01)	(0.02)
b_2	0.06	0.27	0.36	0.38
	(0.001)	(0.003)	(0.005)	(0.006)
$\overline{R}^2$	0.99	0.99	0.95	0.91

Notes. The top panel reports the coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + \varepsilon_t^a$. The bottom panel reports coefficients of the regression, $\frac{I_t^a}{K_t^a} = \alpha + b_1 q_t^a + b_2 \frac{\pi_t^a}{K_t^a} + \varepsilon_t^a$. In this Table J denotes TTB in quarters. Standard errors are in parenthesis. All statistics are averages over 500 replications.