Collateral in Banking Policy and Adverse Selection

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COLLATERAL IN BANKING POLICY AND ADVERSE SELECTION*

by

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I INTRODUCTION

Recent contributions to the theory of banking behaviour examine the observed phenomenon that borrowers may be rationed in the credit market (Milde, 1980, 1981; Stiglitz and Weiss, 1981; Smith, 1983; Kletzer, 1984). The market mechanism as inherent in the interest rate does not adequately regulate the market. Due to the existence of information asymmetry in the credit market, transactions may be distorted. The potential borrowers of a bank lack signals to demonstrate their varying degrees of default propensities. Although borrowers may possess *a priori* knowledge of their own likelihood of default, lenders are not in a position to sort borrowers into the correct default risk category.

Normally, an excess demand for credit would imply that the interest rate rises. Under an information asymmetry scenario, however, an increase in the interest rate has the undesired adverse selection effect that borrowers with low risk default propensities leave the market. High risk borrowers remain in the bank's credit pool, since they alone are willing to accept a higher interest rate payment. This in turn causes a decrease in the bank's expected return on loans due to the resulting adverse incentive and selection effects of an interest rate increase. It may be rational for a bank to abstain from interest rate increases, the consequence being that the credit market remains non-cleared. Equilibrium credit rationing may persist.

In order to deal with this disequilibrium situation various proposals suggest the use of non-interest items tied to credit contracts in order to achieve market clearing (Baltensperger, 1978). One common proposal is to increase the collateral requirements of credit contracts. This paper presents a theoretical model of lending which emphasizes the role of asymmetric

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information and total debt-service obligations between creditors and debtors. The analytical approach is based upon that of Stiglitz and Weiss (1981); however, emphasis here is placed upon collateral policy aspects of credit contracts as compared to the interest rate policy aspects. It is demonstrated that under certain market constellations, even assuming a completely flexible collateral banking policy, the credit market may be characterized by a disequilibrium situation.

The basic model of lender and borrower behaviour in a credit market characterized by information asymmetry is presented in Section II. The results of Section II are then applied to derive an optimal collateral policy in Section III and some concluding comments follow in Section IV.

II The Basic Model and Adverse Selection

Adverse selection problems are present in many areas of economics characterized by aspects of heterogeneity and imperfect information. Quite often agents on one side of the market exhibit a large degree of heterogeneity, such as may be illustrated by the various default risk categories that may be assigned to groups of borrowers in the credit pool of a bank. Banks, on the other side of the market, are faced with imperfect information concerning the default propensity of their potential customers. It is reasonable to assume that members of the heterogeneous group of borrowers are well informed with regard to their default risk category, whereas the bank's knowledge is limited. The bank faces an optimization problem in which it wishes to reduce this level of information asymmetry.

Under these circumstances, adverse selection arises when banks are forced to treat heterogeneous borrowers as being identical. The usual econ- omic reasoning for identical treatment is that the costs of separating borrowers into homogeneous subgroups are prohibitive. Banks are, however, not quite as helpless as the above argument presumes. For instance, they can attempt to cope with this type of information asymmetry by designing policies, e.g., an optimal collateral policy, so as to elicit the necessary infor- mation for placing borrowers into homogeneous default categories, as will be demonstrated in the following sections using a simple model of lending and borrowing behaviour.

Lender Behaviour. The market structure of the banking sector is assumed to be competitive. The interest rate r and the volume of credit K for an individ- ual bank are taken as given. The decision variable of the bank is c, collaterals. A potential credit customer wishes to obtain a loan to finance an investment project. For simplicity, investment projects are assumed to be 100 per cent loan financed. The gross revenue of the borrower's investment project is a positive stochastic variable $\tilde{R}$ with the density function $f(\tilde{R})$. As usual, the

density function may be described approximately by its first and second moments, $E(\tilde{R}) \equiv \mu$ and $\text{var}(\tilde{R}) \equiv \sigma^2$. Credit repayment consists of either the agreed-upon credit volume plus principal due or the specified collateral requirements. It is assumed that the bank knows the expected value μ of a project but not the standard deviation σ.

The gross revenue return of an investment project $\tilde{R}$ is a continuous stochastic variable. The realization of an effective investment project R determines the amount that the lending bank receives at the end of the repayment period. If R is large enough, then borrowers repay $(1+r)K$. If R is not large enough, they hand over the collateral c together with such revenues as they have earned, i.e., $R + c$ (provided $R + c < (1+r)K$). Hence, at the margin, $R + c = (1+r)K$, and there exists a critical value $R^* \equiv (1+r)K - c$. The critical value R^* may be given the following inter- pretation: in this simple model borrowers face limited liability in the sense that they themselves do not carry the full weight of a possible loan default.

There exists a moral hazard problem since borrowers tend to prefer high-risk projects in comparison to lenders, who are interested in low-risk projects. Hence, the bank's situation at the end of the loan repayment period is described as

$$R^B = \begin{cases} R + c & \text{if } R < R^* \\ (1+r)K & \text{if } R \geqslant R^* \end{cases} \qquad \text{......(1)}$$

The bank's expected profit is thus

$$E(R^B) = c + \mu - \int_{R^*}^{+\infty} (R - R^*) f(R)\, dR \qquad \text{......(2)}$$

It is then helpful to introduce the following standardized stochastic variable $\tilde{u}$,

$$\tilde{u} \equiv \frac{R - \mu}{\sigma} \qquad \text{......(3.1)}$$

characterized by the density function $g(u;\, o,\, 1)$ whereby $E(\tilde{u}) = 0$ and $\text{var}(\tilde{u}) = 1$. Further defining the critical value u^*,

$$u^* \equiv \frac{R^* - \mu}{\sigma} \qquad \text{......(3.2)}$$

one obtains

$$\frac{\partial u^*}{\partial \sigma} = -\frac{u^*}{\sigma} \qquad \text{......(4)}$$

Finally, expected bank profits may be formulated as

$$\Pi^B(c, \sigma) \equiv c + \mu - \sigma \int_{u^*}^{+\infty} (u - u^*) g(u) du \qquad \text{......(5)}$$

Of interest with regard to banking behaviour are the effects of changes in σ and c upon expected profits. The following results are obtained from equation (5):[1]

$$\frac{\partial \Pi^B}{\partial \sigma} = -\int_{u^*}^{+\infty} u g(u)du < 0 \qquad \text{......(6.1)}$$

$$\frac{\partial \Pi^B}{\partial c} = +\int_{0}^{u^*} g(u)du > 0 \qquad \text{......(6.2)}$$

Equation (6.1) simply states that the expected profit of the bank Π^B will be reduced given an increase in risk σ. The standard deviation σ of the density function $f(R)$ is interpreted here as a risk indicator; an increasing σ value implies a higher risk level. Greater variability, or spread, around the common mean may be used as a criterion of risk. The distribution with "more weight in the tails" is thus the more risky.[2] Equation (6.2), on the other hand, illustrates that an increase in the collateral requirements of the credit contract, *ceteris paribus*, raises the bank's expected profit.

Borrower Behaviour: The bank's credit customers are heterogeneous and exhibit risk-neutral behaviour. The volume of credit K, the interest rate r and the collateral requirements c are exogenous variables for each customer. For simplicity, it is assumed that each borrower has a choice of only one investment project, which is fully described by its μ and σ values. The borrower is aware of these project-specific moments. A 100 per cent credit-financed investment project will be undertaken only if the expected return of the project for the credit demander is positive, $E(R^D) > 0$. An individual

[1] The derivative of equation (5) with respect, e.g., to σ is obtained as follows:

$$\frac{\partial \Pi^B}{\partial \sigma} = -\left\{\int_{u^*}^{+\infty} (u - u^*)\, g(u)du + \sigma\int_{u^*}^{\infty}\left(-\frac{du^*}{d\sigma}\right)g(u)du \underset{(4)}{=} -\int_{u^*}^{+\infty} u g(u)du < 0.\right.$$

[2] "More weight in the tails" could occur due to a transfer of density from the middle outwards leaving the mean constant. This method has been proposed by Rothschild and Stiglitz (1970). The mean preserving spread approach may be stated as follows. Assume that F_2 is obtained from F_1 by taking mass from the centre of F_1 and shifting it to the tails in such a manner that $E(x_1) = E(x_2)$. This transformation from F_1 to F_2 illustrates a mean preserving spread in which F_2 is said to be riskier than F_1. Defining x_1 and x_2 on (0, 1), this transformation implies

(i) $\;{}_0\!\int^1 (F_2 - F_1)\, dx = 0.$

(ii) An x_3 exists, such that $F_2 - F_1 < 0$ given $x > x_3$ and $F_2 - F_1 > 0$ given $x < x_3$. These conditions satisfy,

(iii) $\;{}_0\!\int^t (F_2 - F_1)\, dx > 0,\; 0 < t < 1.$

F_2 is dominated by F_1 in the sense of second-order stochastic dominance and $E(x_1) = E(x_2)$. The two cumulative density functions cross at x_3, and this single crossing is the main characteristic of a mean preserving spread.

borrower thus faces the following situation at the end of the investment/borrowing period:

$$R^D = \begin{cases} -c & \text{if } R < R^* \\ R - (1+r)K & \text{if } R \geqslant R^* \end{cases} \qquad \text{......(7)}$$

The borrower's expected profit is then

$$\Pi^D(c,\sigma) \equiv -c + \int_{R^*}^{+\infty} (R - R^*) f(R)dR \qquad \text{......(8)}$$

Standardizing, one obtains

$$\Pi^D(c,\sigma) \equiv -c + \sigma\int_{u^*}^{+\infty} (u - u^*)g(u)du \qquad \text{......(9)}$$

The relevant derivatives of equation (9) are:

$$\frac{\partial \Pi^D(c,\sigma)}{\partial c} = -\int_{0}^{u^*} g(u)du < 0 \qquad \text{......(10.1)}$$

$$\frac{\partial \Pi^D(c,\sigma)}{\partial \sigma} = +\int_{u^*}^{+\infty} u g(u)du > 0 \qquad \text{......(10.2)}$$

In accord with the positive expected return assumption for the commencement of an investment project, the critical value upon which an investment will be founded is $\Pi^D = 0$. This critical value may be transformed for a given σ value into a critical collateral value

$$c^* = \sigma\int_{u^*}^{+\infty} (u - u^*)g(u)du$$

The critical collateral value c^* denotes the necessary collateral requirement capable of insuring that the condition $\Pi^D = 0$ in equation (9) is fulfilled.

A borrower will not apply for a credit as long as the bank demands collateral requirements which are greater than the borrower's critical value c^*. If collateral requirements are, however, lower than the borrower's critical value c^* it is profitable to undertake the investment project and apply for a credit. Due to equation (10.1), the following two cases arise

(a) for $c > c^*$ it follows that $\Pi^D(c,\sigma) < 0$

(b) for $c < c^*$ it follows that $\Pi^D(c,\sigma) > 0$.

Iso-Π^D curves for σ and c may be derived by the total differentiation of $\Pi^D(c, \sigma)$. Substituting into equations (10.1) and (10.2), one obtains the marginal rate of substitution, which is equivalent to the slope of the iso-Π^D curve as

$$\left.\frac{d\sigma}{dc}\right|_{\Pi^D \text{ const.}} = -\frac{\Pi^D_c(c,\sigma)}{\Pi^D_\sigma(c,\sigma)} > 0 \qquad \text{......(11.1)}$$

Graphing such iso-Π^D curves as illustrated below in Fig. 1, the credit pool's drop-out quota may be demonstrated using the instrument c^*, the critical collateral value. With regard to the slope of the iso-Π^D curve (11.1) an intuitive explanation may be seen in the fact that truncating the tail of a high variance distribution increases the conditioned mean by more than does truncating the tail of a low variance distribution.

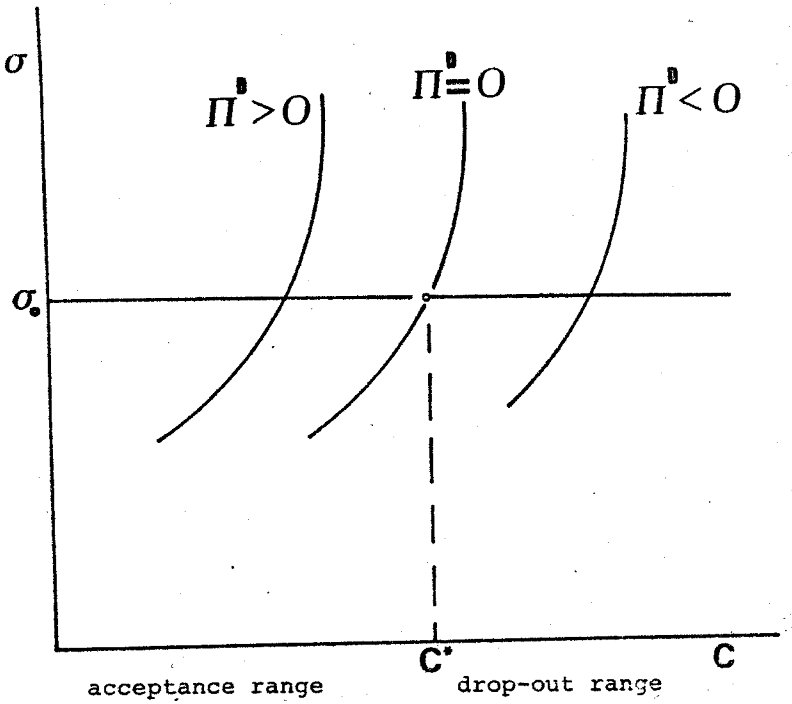

Fig. 1
The Adverse Selection Effect

Fig. 1 may be given the following interpretation. The critical value c^* is defined for a given standard deviation value σ_0 at which $\Pi^D=0$. In situations characterized by $c < c^*$, the borrowers' expected return is positive. Given $c > c^*$, the expected return will be negative. Increasing the value c with a standard deviation σ_0 makes a project appear less and less likely to be profitable in the eyes of a potential borrower. Alternatively, Fig. 1 may be interpreted as saying that, for a given c, higher-risk projects are more likely to be profitable than lower-risk ones.

Analogous to the above transformation (from the borrower's perspective for a given σ-value there exists a critical collateral value c^*), there exists a

critical value σ^* for a given collateral requirement c. σ^* is defined as that standard deviation value under a given collateral requirement c, which is capable of fulfilling the condition $\Pi^D=0$ in equation (9).

Based upon equation (10.2), the following two constellations may occur

(a') for $\sigma < \sigma^*$ it follows that $\Pi^D(c, \sigma) < 0$
(b') for $\sigma > \sigma^*$ it follows that $\Pi^D(c, \sigma) > 0$.

Effective borrowing occurs only when a potential individual investment project is characterized by $\sigma > \sigma^*$ (case (b')). For projects in which $\sigma < \sigma^*$ some borrowers drop out of the credit pool (case (a')). "High"-risk borrowers ($\sigma > \sigma^*$), however, remain in the market, increasing the average risk level of the bank. The critical value of riskiness σ^* is also a positive function of collateral required by the bank ($\sigma^*(c)$). From (10.1) and (10.2) one obtains thus in similar fashion:

$$\frac{d\sigma^*}{dc} = -\frac{\Pi^D_c(c, \sigma)}{\Pi^D_\sigma(c, \sigma)} > 0 \qquad \ldots\ldots(11.2)$$

This positive relationship between average risk and increasing collateral requirements demonstrates "adverse selection".

III Optimal Collateral Policy

In Section II, it was argued that information asymmetry arises from the uncertainty facing the bank concerning the σ-value of an individual credit customer. It was shown that the critical value c^* is determined by a parameter known only to the borrower. Since the bank cannot observe the σ-values, it also does not know the c^*-values. The bank is not capable of discriminating collateral requirements specifically for each customer. The bank must apply a uniform collateral policy for all customers.

The critical collateral value c^* separates the pool of potential borrowers into two categories: those who accept the credit conditions and those who do not. Significant here is the fact that only low-risk borrowers drop out of the bank's credit pool. The loss of low-risk borrowers may be influenced according to equation (11.1) and (11.2) through the collateral policy of the bank. The bank must, however, consider the subsequent effects of any chosen collateral policy: a raising of the collateral requirement c induces further low-risk borrowers to drop out of the credit pool. The average quality of the remaining borrowers decreases—in other words, the average credit risk $\overline{\sigma}$ for the bank rises, thereby lowering the bank's average expected profit $\overline{\Pi}^B(c, \sigma)$. On the other hand, an increase in the collateral requirement c brings about a positive repayment effect. The bank must consider these two contrary effects when contemplating an optimal collateral policy (Wette, 1983).

In calculating average expected profit the bank must observe that a credit contract will come about only with borrowers who are characterized by $\sigma > \sigma^*$. The uncertainty facing the bank with regard to the true σ-values for a specific potential borrower is the origin of the asymmetric information environment of the bank. The entire set of all possible σ-values may be characterized by the density function $h(\sigma)$ defined over the interval $\{\sigma_1, \sigma_2\}$. The lender's expected return on a loan of risk σ is $\Pi^B(c, \sigma)$. Since the lender cannot distinguish between which borrower undertakes the project, the bank must calculate an "average" expected return

$$\overline{\Pi}^B(c, \bar{\sigma}(c)) = \frac{\int_{\sigma^*(c)}^{\sigma_2} \Pi(c,\sigma)h(\sigma)d\sigma}{\int_{\sigma^*(c)}^{\sigma_2} h(\sigma)d\sigma} \qquad \text{......(12)}$$

The next step is to show that $\overline{\Pi}^B(c, \bar{\sigma}(c))$ is not necessarily a monotone increasing function of c. This follows from differentiation of equation (12) with regard to c

$$\frac{\partial\overline{\Pi}^B(c, \bar{\sigma}(c))}{\partial c} = \frac{\int_{\sigma^*(c)}^{\sigma_2} \Pi^B(\cdot)_c h(\sigma)\, d\sigma}{\int_{\sigma^*(c)}^{\sigma_2} h(\sigma)d\sigma} - \frac{(\Pi^B - \overline{\Pi}^B)h(\sigma^*)\sigma^*_c}{\int_{\sigma^*(c)}^{\sigma_2} h(\sigma)d\sigma} \qquad \text{......(13)}$$

The first term in the above equation represents the repayment effect and is positive. The second term characterizes the adverse selection effect and is negative given equation (11.2), $d\sigma^*/dc > 0$, and $\Pi^B > \overline{\Pi}^B$. The expected return on a loan to the bank is a decreasing function of the level of riskiness σ of the investment project financed (compare Theorem 3 in Stiglitz and Weiss, 1981).

IV Concluding Remarks

It follows from equation (13), therefore, that the bank's "average" expected profit $\overline{\Pi}^B$ need not be a monotone increasing function of c, rather the negative adverse selection effect may compensate the positive repayment effect. There exists, however, a specific value $\hat{c}$, which fulfils the condition $\overline{\Pi}^B_C(\cdot) = 0$. This specific value determines the optimal collateral policy for the bank.

Given that the second-order conditions are fulfilled, the bank's optimal collateral policy may be illustrated graphically as shown in Fig. 2 below

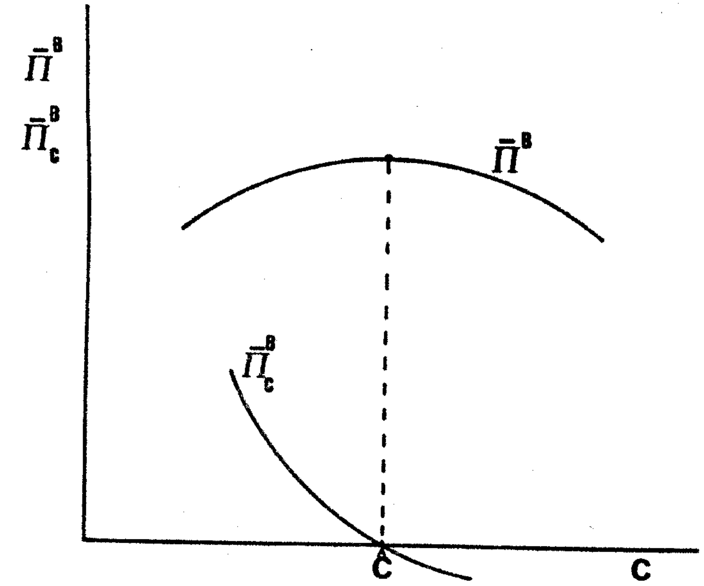

Fig. 2

Optimal Collateral Policy

As Fig. 2 demonstrates, given a situation $\hat{c}$ characterized by an excess demand for credit, some potential borrowers are rationed. However, any increase in the collateral requirements of a loan to the right of $\hat{c}$ in Fig. 2 will be accompanied by a decrease in the bank's average profits. The bank has no incentive to raise collateral requirements above $\hat{c}$, which implies that some potential borrowers remain rationed.

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