Testing the Equilibrium Exchange Rate Model - Updated

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2006

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Abstract

We find favorable evidence for the textbook equilibrium exchange rate model of Stockman (1987) using Blanchard and Quah's (1989) decomposition. Real shocks are shown to account for more than 90 percent of movements in the real exchange rate between Brazil and the US, and for more than half of nominal exchange rate changes. Impulse response functions also suggest that real shocks alter these countries' relative prices.

Keywords: Equilibrium Exchange Rate Model; Blanchard and Quah's Decomposition
JEL classification: F31, F37, F41, F47

1. INTRODUCTION

The equilibrium approach to exchange rates that commonly stands in course syllabus is that of Stockman (1980). Stockman later replaces his general model with several particular models (Stockman, 1987) to make his case clearer. And this can be further simplified with the use of Cobb-Douglas production and utility functions (Da Silva, 2002). Here we take the latter version to put the equilibrium model to the test.

The simple argument of Stockman is as follows. If real disturbances to demand for goods or supplies of goods, such as preference shifts or productivity shocks, cause relative prices to change, why not extend this to the relative price of foreign goods in terms of domestic goods, i.e. to the real exchange rate defined as the terms of trade? Doing so purchasing power parity cannot be expected to hold. And it makes no sense to think that the central puzzle in international business cycles is volatile and persistent real exchange rates. Stockman thus makes a credible case for real exchange rate behavior to reflect real shocks with permanent components, not price sluggishness.

Tests of Stockman's model are scarce and even non-existent to our knowledge. But Evans and Lothian (1993), Clarida and Gali (1994), and Enders and Lee (1997) model explicitly the relative importance of nominal and real shocks using a structural vector autoregression (VAR) embodying the decomposition suggested by Blanchard and Quah (1989). We find such a methodology appropriate to testing the equilibrium model because there are only two types of shock, and nominal shocks cannot affect the real exchange rate. We thus adopt this technique and decompose the movements of monthly series of nominal and real exchange rates between Brazil and the United States from January 1980 to January 2005. We find evidence in favor of the equilibrium model as a result.

Section 2 employs the Blanchard and Quah's decomposition for the data. Section 3 sums up Stockman's model. Section 4 displays the results of an estimated econometric model. And Section 5 concludes.

2. BLANCHARD AND QUAH'S DECOMPOSITION

Figures 1 and 2 display nominal and real exchange rates between Brazil and the US for the period above, and Figures 3 and 4 show their first differences. (Table 1 presents some descriptive statistics.) Nominal and real rates move together in the short run. But they are turn apart as time goes by. This suggests the presence of two types of shock. One impacts the two series at the same time and one affects

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them distinctly. The first type of shock can be thought of as a real one because it similarly affects the short run path of both nominal and real rates. The second type can be seen as a nominal shock impacting the real exchange rate only temporarily. This picture is at first compatible with any exchange rate model, including the equilibrium one. Indeed it provides an identification constraint that enables one to decompose the series along the lines suggested by Blanchard and Quah (1989).

We first test the stylized fact of unit roots in nominal and real exchange rates. ADF tests (Table 2) confirm conventional wisdom. Yet structural breaks hitting the exchange rates during the period under analysis suggest that the ADF tests' results could be misleading. In such a situation it is prudent to perform Perron's (1997) test. Table 3 shows that the above results are not affected by the breaks, however.

Because the exchange rates are integrated of order 1, we fit a VAR for the series' first differences to check whether the series are cointegrated. We find that the null hypothesis of no cointegration cannot be rejected (Table 4). They can then be written as a bivariate moving average system, i.e.

$$x_t = \begin{bmatrix} \Delta E_t \\ \Delta S_t \end{bmatrix} = B(L)\varepsilon_t = \begin{bmatrix} B_{11}(L) & B_{12}(L) \\ B_{21}(L) & B_{22}(L) \end{bmatrix} \begin{bmatrix} \varepsilon_{rt} \\ \varepsilon_{nt} \end{bmatrix} \tag{1}$$

where E_t stands for the real exchange rate at time period *t*, S_t is the nominal exchange rate at *t*, ε_{rt} is a real shock at *t*, ε_{nt} is a nominal shock at *t*, and $B_{ij}(L)$, i, j = 1, 2, are polynomials of infinite order in lag operator *L*. Innovations are normalized to make sure that $\mathrm{var}(\varepsilon_t) = I$ is non-correlated.

Blanchard and Quah do not relate the structural variables ε_{rt} and ε_{nt} to the pure shocks hitting ΔE_t and ΔS_t. Rather, they consider the latter as endogenous, while ε_{rt} and ε_{nt} are seen as exogenous.

Nominal and real paths following the shocks are determined by the coefficients of $B_{ij}(L)$. The constraint that the long run real rate is not affected by nominal shocks means that the sum of the coefficients in $B_{1,2}(L)$ is nil, i.e. $B_{1,2}(L) = \sum_{j=0}^{\infty} b_{1,2}(j)L^j$ so that

$$B_{1,2}(L) = \sum_{j=0}^{\infty} b_{1,2}(j) = 0 \tag{2}$$

Because $b_{1,2}(j)$ is the effect of ε_{nt} on ΔE_t after *j* periods, the sum of all $b_{1,2}(j)$ gives the cumulative effect of ε_{nt} on ΔE_t. Similarly the long run effect of ε_{nt} on the real exchange rate series in levels is given by

$$E_t = (1-L)^{-1} B_{1,1}(L)\varepsilon_{rt} + (1-L)^{-1} B_{1,2}(L)\varepsilon_{nt} = \sum_{k=0}^{\infty}\sum_{j=0}^{k} b_{1,1}(j)\varepsilon_{rt-j} + \sum_{k=0}^{\infty}\sum_{j=0}^{k} b_{1,2}(j)\varepsilon_{nt-j} \tag{3}$$

where $\sum_{j=0}^{\infty} b_{1,2}(j)$ is now the effect of ε_{nt} on *E* after infinite periods. Thus it represents the long run effect of a nominal shock on the real exchange rate.

Constraint (2) means no cumulative effect of a nominal shock on the real exchange rate in both levels and first differences. While nominal shocks can affect the real exchange rate only temporarily, real shocks can have further long run effects.

To get impulse response functions, the VAR can be inverted to yield a vector moving average (VMA) of (1). Here one has to learn how to constrain the VAR to make sure that $B_{1,2}(L) = 0$ in the VMA representation. One can assume the following VAR model:

$$x_t = \begin{bmatrix} \Delta E_t \\ \Delta S_t \end{bmatrix} = A(L)x_{t-1} + v_t = \begin{bmatrix} A_{11}(L) & A_{12}(L) \\ A_{21}(L) & A_{22}(L) \end{bmatrix} \begin{bmatrix} \Delta E_{t-1} \\ \Delta S_{t-1} \end{bmatrix} + \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} \tag{4}$$

where $v_t = \begin{bmatrix} v_{1,t} & v_{2,t} \end{bmatrix}' = x_t - E\left(x_t \middle| x_{t-\tau}, \tau \geq 1\right)$, and $\mathrm{var}(v_t) = \Sigma$. Inverting the VAR in (4) produces the VMA of x_t, i.e.

$$x_t = \left[I - A(L)\right]^{-1} v_t = K(L)v_t \tag{5}$$

where *I* is an identity matrix of rank two. Stationarity of x_t guarantees the existence of a VMA representation such as that in (5).

Comparing (5) with (1), Blanchard and Quah show that constraint $B_{1,2}(L)=0$ in the VMA is equivalent to a VAR constraint such as

$$\left[I-A_{22}(L)\right]b_{1,2}(0)+A(L)b_{2,2}(0)=0 \tag{6}$$

where $A_{2,2}(L)=\sum_{j=0}^{\infty}a_{2,2}(j)$, and $A_{1,2}(L)=\sum_{j=0}^{\infty}a_{1,2}(j)$.

Coefficients $B(L)$ in (2) give the response to shocks. Because the ortogonalization of ε_t renders it serially and contemporaneously non-correlated, one can relate variance of every *x* to their sources in ε. The forecasting error *t* steps ahead of x_t is given by

$$w_t \equiv x_t - E\left[x_t \,|x_0, x_{-1},...\right] = \sum_{\tau=0}^{t-1} b(\tau)\varepsilon_{t-\tau},$$

its variance is

$$E\left[w_t \quad w_t'\right] \equiv E\left[\sum_{\tau=0}^{t-1} b(\tau)\varepsilon_{t-\tau}, \sum_{\tau=0}^{t-1} b(\tau)\varepsilon_{t-\tau}\right] = \sum_{\tau=0}^{t-1} \text{trace}\, b(\tau)b(\tau)',$$

and its variance generated by innovations of x_j is

$$\sum_{\tau=0}^{t-1} b_{ij}(\tau)^2 \Big/ \sum_{j=1}^{2}\sum_{\tau=0}^{t-1} b_{ij}(\tau)^2 \tag{7}$$

Now we move on to briefly present the equilibrium exchange rate model.

3. THEORETICAL MODEL

Stockman's (1987) model considers two similar countries each producing a different good. Da Silva's (2002) version of the model employs Cobb-Douglas to characterize both utility and production functions. The model equations are as follows.

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}, \quad 0<\alpha<1 \tag{8}$$

$$Y_t^* = K_t^{*\alpha} N_t^{*1-\alpha} \tag{9}$$

$$U_t = Y_t^{\beta}\left(Y_t^*\right)^{1-\beta}, \quad 0<\beta<1 \tag{10}$$

$$E_t = \frac{S_t P_t^*}{P_t} \tag{11}$$

$$E_t = \frac{1-\beta}{\beta}\left(\frac{Y_t}{Y_t^*}\right) \tag{12}$$

$$\frac{M_t}{P_t} = Y_t^{\delta}, \quad \delta>0 \tag{13}$$

$$\frac{M_t^*}{P_t^*} = Y_t^{*\delta}, \quad \delta>0 \tag{14}$$

Equations (8) and (9) represent domestic and foreign production functions. Domestic good, capital, and labor are *Y*, *K*, and *N* respectively (foreign variables are denoted by asterisks). Equation (10) is the shared utility function. Equation (11) provides the definition of the real exchange rate *E* (*S* is still the nominal rate), whereas equation (12) follows from the equilibrium condition that relative prices equal the marginal rate of substitution between domestic and imported goods. Money market equilibria are given by (13) and (14).

The solution to the real exchange rate is

$$E_t = \frac{1-\beta}{\beta}\left(\frac{K_t}{K_t^*}\right)^{\alpha} \tag{15}$$

Thus the real exchange rate depends solely on real variables, i.e. preferences over consumption of domestic and imported goods (β) and productivity shocks affecting the capital stocks. The real exchange rate, for instance, depreciates (i.e. *E* increases) after a domestic productivity rise.

The solution to the nominal exchange rate is

$$S_t = \frac{1-\beta}{\beta}\left(\frac{M_t}{M_t^*}\right)\left(\frac{K_t}{K_t^*}\right)^{\alpha(1-\delta)} \tag{16}$$

This means that the nominal rate depends on the same things affecting the real rate and also on relative money supplies and income elasticity of money demand (δ). Changing preferences and productivity impact the nominal rate in the same fashion as they impact the real rate. And as in monetary models, positive shocks to (domestic) money supply depreciate the nominal exchange rate.

Classical dichotomy holds, i.e. nominal rate changes do not cause real rate changes. This might seem at odds with (11). But S and E are both endogenous; they simultaneously respond to real shocks. Thus the equilibrium model can account for the stylized fact that nominal and real exchange rates are highly correlated. And more importantly (for our purposes in here), the equilibrium model treats real and nominal exchange rates the same way Blanchard and Quah's decomposition does.

The $1-\delta$ term in (16) gives the elasticity of the nominal exchange rate with respect to domestic productivity. That is subject to two opposite effects, namely the relative price effect (from the supply side) and the money demand effect (from the demand side). A domestic productivity increase causes domestic output to rise, thereby increasing the real (and nominal) exchange rate. This is the relative price effect at work. The money demand effect occurs whenever domestic output rises, thereby raising money demand and causing the nominal exchange rate to plummet. For realistic values of the income elasticity of money demand, i.e. $\delta \in (0,1)$, positive shocks to domestic productivity cause the nominal exchange rate to depreciate, because the relative price effect outweighs the money demand effect. Yet this nominal exchange rate rise falls short of the increase in the real rate because the latter is not affected by the money demand effect (equation (15)).

4. ESTIMATED MODEL

To estimate the equilibrium model we have considered 36, 24, 12, 8, 6, 4, and 2 lags. The model with six lags has been chosen by the selection criteria of Akaike and Schwarz. LR test also confirms the six-lag model. Results are displayed in Table 5. We have considered dummies to both the launching of the Brazilian *Real* Plan and the currency crisis of January 1999. The dummies have been tested both jointly and separately, but they were not significant at the 5 percent level.

Then we have taken the variance decomposition under the identification constraint $B_{1,2}(L)=0$ that evaluates the relative contribution of real and nominal shocks to the exchange rate series. Table 6 shows results for the variables in both first differences and levels. More than 90 percent of real exchange rate movements can be explained by real shocks. And though the nominal exchange rate series is more influenced by nominal shocks, the influence of real shocks is more significant. Both results are consistent with the theoretical equilibrium model.

Figures 5 and 6 display impulse response functions of real and nominal rates from the two types of shock. The functions are measured in relation to standard deviation and presented for the variables in levels for best resolution.

As required from the identification strategy, nominal shocks affect the real exchange rate only temporarily, and even then the effect is no greater than 25 percent of a standard deviation. As for the nominal rate, nominal shocks have permanent effects (as expected). The nominal rate overshoots its equilibrium level following the nominal shock, but this evidence of delayed overshooting is not robust because the difference between the peak and the equilibrium level is not significant.

Following the real shock, both real and nominal rates reach the new equilibrium after a little bit more than a year. Because the effects of the real shock on the nominal exchange rate are smaller than those on the real rate, the real shock alters the relative prices between Brazil and the US. This is a result predicted by the theoretical equilibrium model, because the real exchange rate is not subject to the money demand effect that softens the nominal exchange rate increase.

5. CONCLUSION

We employ Blanchard and Quah's (1989) decomposition to monthly series of nominal and real exchange rates between Brazil and the United States from January 1980 to January 2005. In particular, real exchange rate movements are decomposed in components induced by nominal and real shocks. Results are then contrasted with the predictions of Stockman's (1987) equilibrium model. Real shocks are shown to be responsible for more than 90 percent of the real exchange rate movements and for more than half of the nominal exchange rate changes. Impulse response functions show that the effects of real shocks on the nominal exchange rate are smaller than those on the real rate. Thus real shocks alter the relative prices between Brazil and the US. This result can be explained by the money demand effect of the equilibrium model.

Series	Number of Observations	Mean	Standard Deviation	Minimum	Maximum
S	301	0.95737	0.1937888349	0.5811463	1.610464
ΔS	300	0.00034	0.0412158429	–01584403	0.223827
E	301	0.79609	1.0715625592	0.0000000	3.805100
ΔE	300	0.00897	0.0692040579	–0.328200	0.463900

Table 1 Descriptive Statistics

Series	τ_τ	τ_μ	τ
S	–0.264629	–2.158435	0.542558
ΔS	**–6.611432**	**–6.684614**	**–6.445306**
E	–2.244035	–2.388832	–0.297801
ΔE	**–7.301489**	**–7.306592**	**–7.312577**

Table 2 ADF Tests

Note
Bold values mean rejection of the null hypothesis of unit roots at the 5 percent significance level.

Model	S	ΔS	E	ΔE
1	–3.17670	**–8.57850**	–4.42698	**–6.65007**
2	–3.48724	**–9.85743**	–4.41958	**–6.66776**
3	–3.33606	**–8.18515**	–3.62134	**–6.13093**

Table 3 Perron's Unit Root Tests

Note
Bold values mean rejection of the null hypothesis of unit roots at the 5 percent significance level.

H_0	$E = 0$
Eigenvalues	0.02568
λ_{trace}	7.78516
Critical Value at 95%	15.41
λ_{max}	7.77730
Critical Value at 95%	14.07

Table 4 Cointegration Test

Number of Lags	AIC	BIC	LR Test
36	–724.98358	–202.89501	0.99564271
24	–3367.97868	–3013.17939	0.99671861
12	–3591.97818	–3408.83015	0.87723200
8	–3615.66172	–3491.12107	0.48685061
6	–3707.53259	–3619.12668	**0.03990752**
4	–3705.14990	–3609.37682	

Table 5 Lag Length Selection

Note
Bold value means rejection of the null hypothesis at the 5 percent significance level.

Model	First Differences		Levels	
Variable	ΔE	ΔS	E	S
Horizon				
1 month	96.553	62.831	96.553	62.831
3 months	96.652	63.078	96.870	62.232
6 months	96.031	63.093	97.070	58.228
12 months	93.093	63.946	97.792	53.544
24 months	93.064	63.965	97.493	54.557
36 months	93.063	63.965	96.385	50.115

Table 6 Variance Decomposition

Note
Percentage variance of the forecasting error explained by nominal shocks stands for the difference between 100 percent and its corresponding value of the real shock.

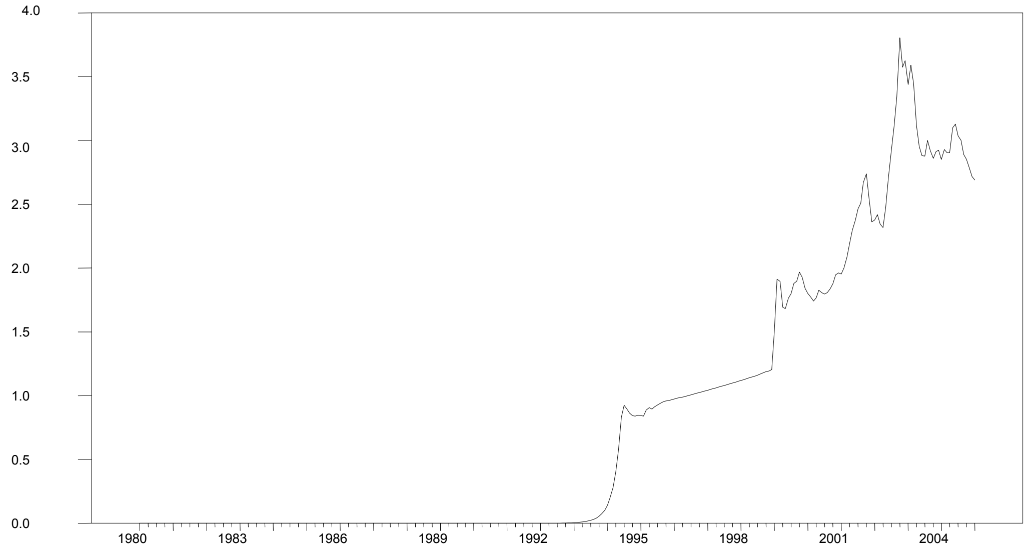

Figure 1 Nominal exchange rate between Brazil and the US from January 1980 to January 2005.

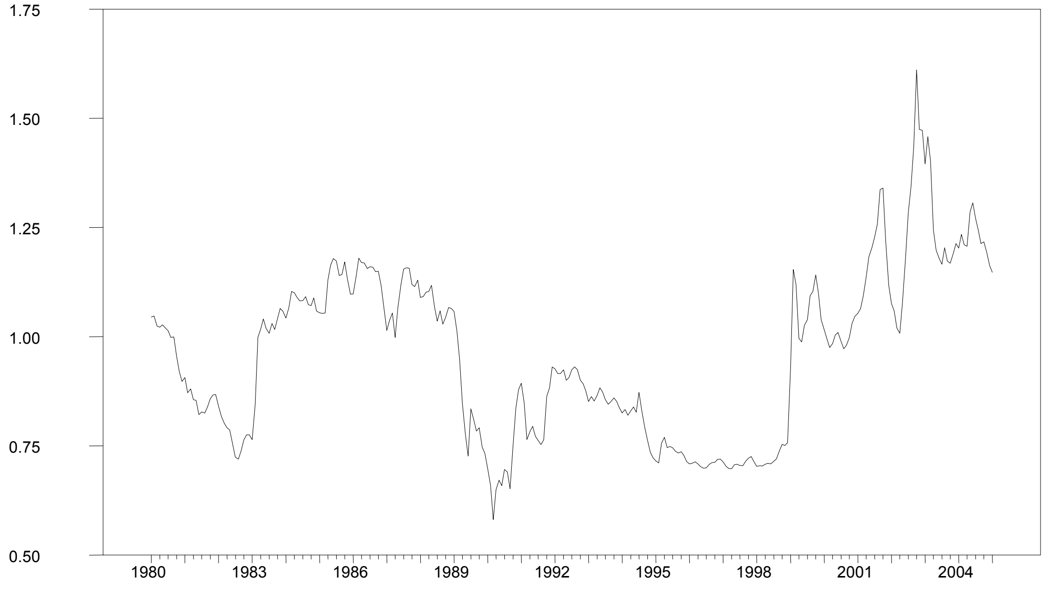

Figure 2. Real exchange rate between Brazil and the US from January 1980 to January 2005.

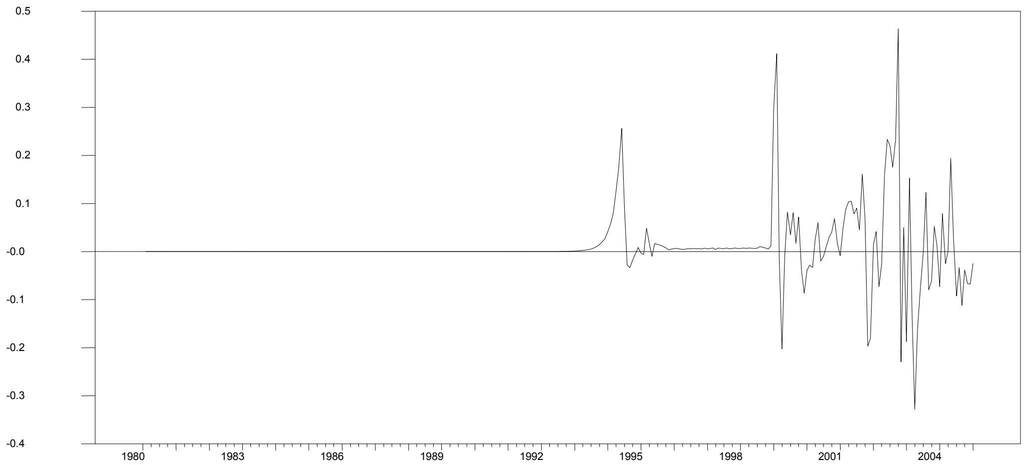

Figure 3. Nominal exchange rate's first differences.

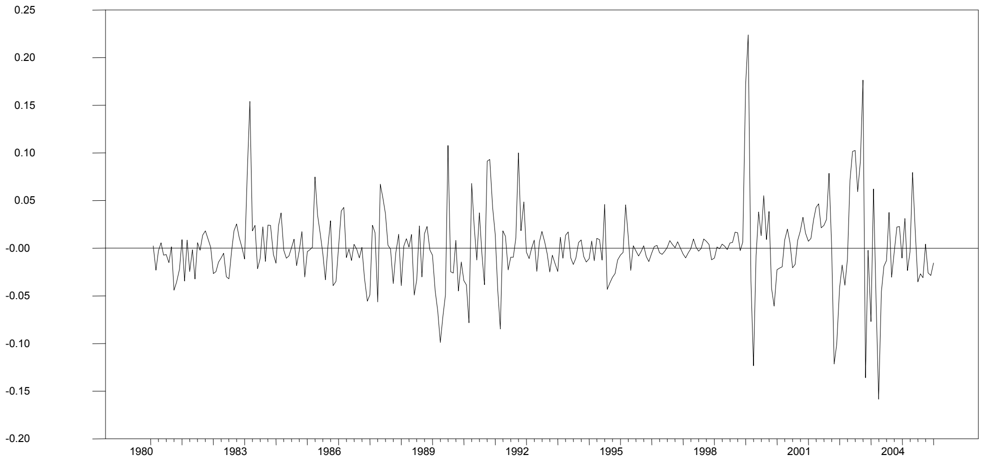

Figure 4. Real exchange rate's first differences.

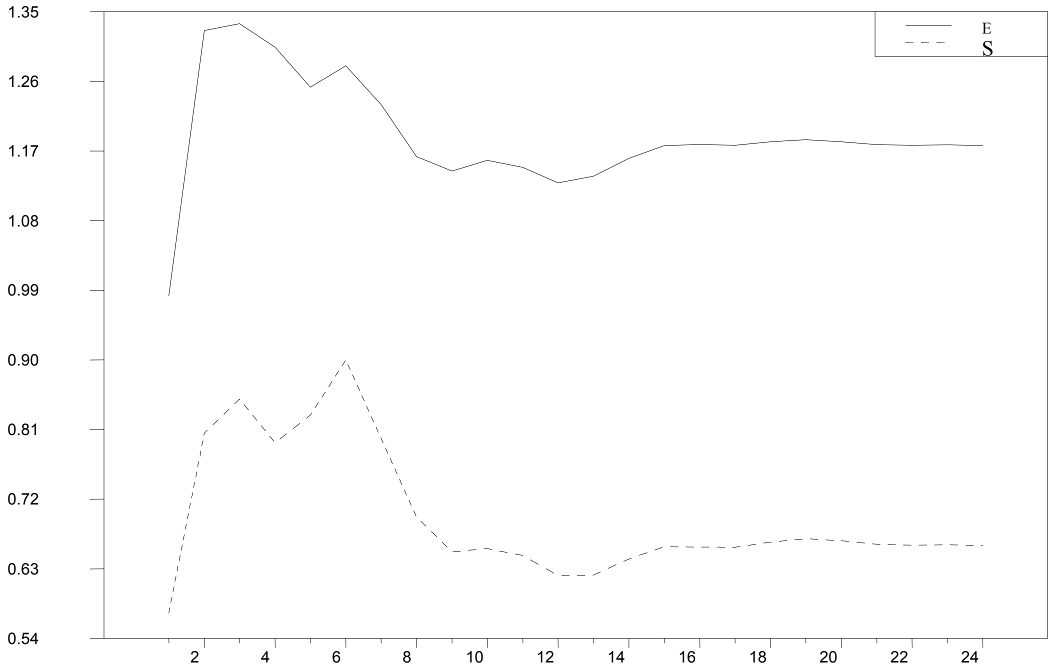

Figure 5. Response to a real shock.

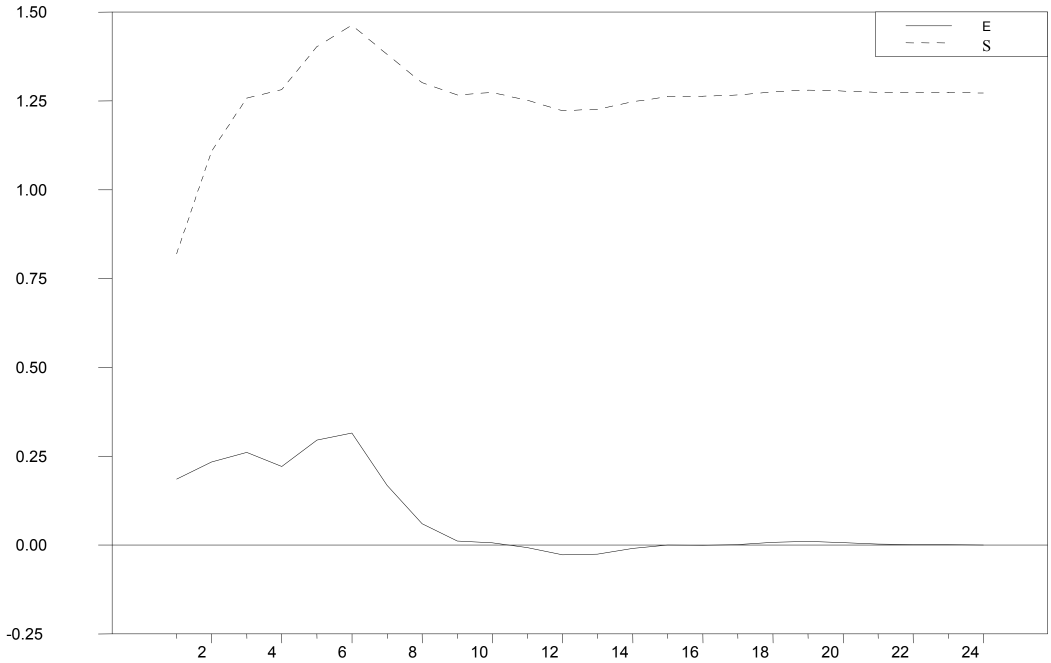

Figure 6. Response to a nominal shock.

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