Education, Utilitarianism, and Equality of Opportunity

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Education, Utilitarianism, and Equality of Opportunity[1]

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Abstract

We analyze in this paper the impact of different policies on the investment of the families in the education of their children. Families make decisions on the level of human capital of their offsprings regarding the future income that this capital entails (under the assumption that higher education levels yield higher expected income). The families' optimal investment in education depends on their preferences (summarized by their time discount and risk aversion parameters) and their circumstances (initial wealth, parents' education, and children' natural abilities). The public authority designs a balanced tax/subsidy scheme in order to maximize aggregate welfare. We compare the case of a purely utilitarian planner with one that cares about the equality of opportunity.

Keywords: Equality of Opportunity; Investment in Education; Policy design.

JEL classification: D31; D63.

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1. Introduction

This paper deals with the design of an equality of opportunity policy when agents' outcomes are random variables that depend on their previous choice of effort. That choice of effort is a function of the agents' characteristics and the policy variables. We aim at determining the implications of the policy that leads to equality of opportunity, relative to the standard maximization of aggregate welfare. In order to fix ideas we focus on the case in which agents are families who decide on the education of their children.

We consider a society made of a finite number of heterogeneous agents (families) who make decisions *today* on how much to invest in the education of their children, bearing in mind that such an investment is positively correlated to the children's income *tomorrow*.[3] Investment today is the effort decision whereas the human capital achieved by children tomorrow is the outcome. We assume, according to the empirical evidence, that labour income is positively related to the level of studies (quantity and quality of the human capital accumulated), so that investing in education amounts to determine the children's future expected income (see Card (1999)). The action of the public authority consists of affecting today's decisions of families via taxes and subsidies.

We assume that parents are altruistic with respect to their offsprings and, therefore, they are ready to invest in education in order to provide them with the best possible chances in life, given their circumstances. The model only considers the family's aggregate investment in education, without paying attention to the composition of that expenditure. Note, however, that investment in education can take a number of alternative forms with different implications: providing studies that go beyond compulsory education (e.g. university degrees), making expenses that are complementary to public schooling (extra-curricular activities, choice of private schools, etc.), or investment in early education, say.[4] Yet, the analysis of those implications is beyond the scope of this paper.

Following the equality of opportunity approach [eg. Roemer (1993, 1998)], we assume that individual outcomes (future income in our case) can be regarded as the result of two different effects: effort and opportunity (see also Roemer et al (2003), Ruiz Castillo (2003) and Villar (2005)). Effort has to do with responsibility (that here refers to the investment in the children's human capital) whereas opportunity alludes to the agents' external circumstances (that here translates into

[3] Needless to say, in real life some of those decisions involve not only the parents and the students but also the schools (on this interplay see the model by de Fraja et al. (2008)).

[4] There is evidence on the relevance of the education in early stages (Heckman (2006)) because, at that time, children not only acquire some basic abilities (cognitivie skills) but also conform their attitudes (non-cognitive skills). The interplay between cognitive and non-congnitive skills explains much of the future achievements of the young (Heckman & Rubinstein (2001), Carneiro et al (2003), Heckman (2007)).

the parents' wealth and education and the children's natural abilities). The bottom line of this approach is that a fair society should compensate agents for differences in opportunity but not for those differences derived from autonomous decisions. Therefore, the public policy is oriented to the equalization of agents' circumstances by altering the initial wealth trying to help the children of the less favoured families. Roemer (2006) presents a model in this vein focussing on the political equilibrium in which families vote for the different policies; Hanushek et al. (2003) analyze the redistribution through education with respect to other policies.

Contrary to the usual equality of opportunity models, effort is here an explicit decision variable and not a residual that explains the different achievements. Moreover, our set up is non-deterministic. That is, the children's future income is a random variable (a function of education, ability and luck, so to speak). We focus on the case of perfect information: the public authority observes the families's effort and applies a policy that maximizes social welfare under the equality of opportunity constraint.[5]

The work is organized as follows. Section 2 presents the model and explains the behaviour of both the families and the public authority in a conventional setting. Each family is characterized by a two-period separable utility function, the family background (initial wealth and education of the parents), and the child's natural ability. In order to simplify the analysis, we shall assume most of the time that agents' utilities are equal, except for a "concern parameter" (a sort of time discount factor that measures the substitutability between today's disposable wealth and tomorrow's income of the children) and the risk aversion coefficient. Therefore, families will differ in their "concern" for the future, their risk attitudes, their initial wealth and cultural background, and their children's natural abilities. We first determine the optimal investment of the families, for a given tax subsidy scheme. The equilibrium level of the family's investment increases with the concern parameter and with the subsidy (as it reduces the opportunity cost of education). The relationship between effort and natural abilities, on the one hand, and effort and risk aversion, on the other hand, turn out to depend on whether the coefficient of risk aversion is greater or smaller than unity. Once individual decisions have been characterized, we analyze the behaviour of a utilitarian planner.
Section 3 deals with the optimal policy from an equality of opportunity viewpoint and relates these results to those under the utilitarian regime. To do so we generate a partition of the population in different types, according to the agents' initial wealth, cultural background, and natural abilities. Then we propose an index that allows to compare degrees of investment effort across types. The equality of opportunity policy aims at equalizing the expected income of those children whose families exerted a comparable degree of effort.
Our results show that both the utilitarian and the equality of opportunity policies

[5] See Lefranc et al (2007) for a model with random outcomes and non-observable effort. A dynamic version of this setting, with incomplete information is analyzed in Calo-Blanco (2008).

coincide in that they imply transfers from rich to poor people. Similarly, the more concerned the parents for the future of their children, the larger the subsidy. Those policies differ in the treatment of productivity. The utilitarian policy gives more subsidies to those who are more productive (as they contribute more to the aggregate welfare), whereas the equality of opportunity policy implies transferring wealth to those who have less natural ability, worse cultural background, and to those who exert a higher relative effort.
A few final comments are gathered in section 4.

2. The basic model

Consider a society consisting of a public authority (also referred to as `the social planner') and a set of agents (families), in a two-period scenario. Families make decisions in the first period concerning the education of their children, as a function of their preferences, the parental background (human capital previously accumulated and wealth), and the children's abilities. Children land on the second period with a level human capital that results from their parents' decisions and determines their expected earnings. The planner may alter the initial distribution of wealth by taxing and subsidizing the families, under the restriction of aggregate budget balance.

2.1. The family's decision

Our society consists of a set $\mathcal{M} = \{1, \ldots, m, \ldots, M\}$ of families. To make things simpler we assume that each family is made exactly of the parents and one child. Families are heterogeneous concerning their preferences and their external circumstances, that we associate with their cultural background (human capital previously accumulated), denoted by H_m, and their wealth, denoted by $w_m \in \mathbb{R}_+$, for all $m \in \mathcal{M}$. The wealth of family m can be modified by the presence of a tax/subsidy scheme that results in an amount $\gamma_m \in \mathbb{R}$ (γ_m is the subsidy that the planner gives to agent m, when positive, or the wealth tax when negative).
Each family makes a decision in the first period concerning the investment in the education of the child. We assume that the amount invested by family m, denoted by e_m, is a point in the interval $[e^o, e_m^+]$, where e^o corresponds to the basic cost of the public compulsory education (that we assume to be equal for all agents, for the sake of simplicity) and we let $e_m^+ \geq w_m/2$. [6] The *disposable wealth* of family m with an investment e_m, is thus given by:

[6] This upper bound is established in order to ensure the consistency with the formulation below.

$$w_m^D = w_m + \gamma_m - e_m$$

The uncertain future is described by a set of S states of nature, with probabilities $\pi_1, \pi_2, \ldots, \pi_S$. The investment in education of the parents results in a given level of human capital for the child in the second period, denoted by h_m (that we assume to be independent on the state of nature, for the sake of simplicity). The technology of production of human capital can be described by a function:

$$h_m = (\alpha_m)^p (H_m)^q \cdot e_m$$

where H_m is the parental cultural background (human capital formerly accumulated), and α_m is a coefficient that reflects the child's natural ability. The coefficients p and q control the impact of those variables on the production of human capital. We assume that all first derivatives are positive.

The income that the child of family m will obtain in state s, y_m^s, is a linear function of the human capital: $y_m^s = \phi(s) h_m$, where $\phi(s)$ is a coefficient that describes how human capital is transformed into income depending on the state of nature. Therefore, we can write:

$$y_m^s = \lambda_m^s e_m$$

In this way we make it explicit the dependence of the child's future income on the decision of the families concerning the private investment in education (with $\lambda_m^s \coloneqq \phi(s)(\alpha_m)^p (H_m)^q$ for $s = 1, 2, \ldots, S)$. It is therefore assumed that the human capital achieved by the child will be proportional to the investment in education, with a proportionality coefficient that depends on the child's natural ability, the parental cultural background, and the state of nature.[7]

The expected labour income of the child of family m associated with an educational investment e_m is thus given by:

$$E(y_m(e_m)) = \sum_{s=1}^{S} \pi_s \lambda_m^s e_m = \widehat{\lambda}_m e_m$$

where $\widehat{\lambda}_m = \sum_{s=1}^{S} \pi_s \lambda_m^s$ denotes the child's *average productivity* (the expected return associated with one unit of investment). We assume that $\lambda_m^s > 1$, for all s, that is, investing in education always yields non-negative returns.

[7] One may also consider the effect of the level of public expenditure in education, Here we implicitly assume that this variable is fixed during the analysis, so that it can be omitted from the production function of human capital.

Family m derives utility from today's net wealth and from the child's future income, according to the following separable utility function:

$$U_m(\gamma_m, e_m) = u_m(w_m^D) + \delta_m \sum_{s=1}^{S} \pi_s u_m(y_m^s(e_m))$$

where u_m, is assumed to be monotone, differentiable and concave. The parameter $\delta_m \in [0,1]$ denotes the family's concern about the future income of the child. This is a sort of time discount factor that summarizes the relationship between today's income of the family and tomorrow's expected income of the child. The family decides the optimal investment in education as a solution to the following program:

$$\left.\begin{array}{c} Max_{e_m}\ u_m(w_m + \gamma_m - e_m) + \delta_m \sum_{s=1}^{S} \pi_s u_m(\lambda_m^s e_m) \\ s.t.:\ e_m \in [e_0,\ e_m^+] \end{array}\right\}$$

The first order conditions of this program, assuming interior solutions, are given by:

$$\frac{\partial u_m}{\partial(w_m + \gamma_m - e_m)} = \delta_m \sum_{s=1}^{S} \pi_s \lambda_m^s \frac{\partial u_m}{\partial(\lambda_m^s e_m)}, \quad \forall\, m \in M \qquad [1]$$

Those conditions establish, not surprisingly, that each family's marginal utility in the first period equals the expected marginal utility in the second period. The optimal choice of effort (investment) is, therefore, dependent on the family's characteristics (wealth, human capital of the parents, marginal utility, concern parameter, and average productivity of the child) and the tax/subsidy policy. The educational investment clearly increases with the concern parameter: an increase in δ_m requires an increase in investment so that the left hand side levels the higher r.h.s. Also note that optimal investment increases with the family's wealth and the subsidy. Therefore, an increase in the subsidy leads to a higher education. The reason is that the marginal utility in the first period will decrease as a result of the higher subsidy (due to the concavity of the utility function), so that the investment in education should increase in response. The intuition is that the subsidy reduces the opportunity cost of the investment.
The relationship between optimal investment and the child's productivity is more complex and can be linked to the degree of concavity of the utility function (that is measured by the elasticity of the marginal utility). The following result summarizes that relation:

- **Proposition 1.-** ***The investment in education increases (resp. decreases) with the productivity coefficient*** λ_m^s ***if and only if*** $\rho_m^s < 1$

(resp. $\rho_m^s > 1$), where ρ_m^s is the elasticity of the marginal utility of income in state s .

Proof.-

Let us consider the derivative of the r.h.s. of the equilibrium condition $[1]$ with respect to a change in the productivity coefficients:

$$\frac{\partial\left(\delta_m \pi_s \lambda_m^s \frac{\partial u_m}{\partial \lambda_m^s e_m}\right)}{\partial \lambda_m^s} = \delta_m \pi_s \frac{\partial u_m}{\partial(\lambda_m^s e_m)} + \delta_m \pi_s \lambda_m^s \frac{\partial^2 u_m}{\partial(\lambda_m^s e_m)^2} e_m$$

Observe now that, as $\lambda_m^s e_m$ is the income of agent m in state s, the first term of the r.h.s of this equality is positive (by monotonicity) whereas the second one is negative (by concavity). Therefore, the derivative turns out to be positive if and only if,

$$\delta_m \pi_s \frac{\partial u_m}{\partial \lambda_m^s e_m} + \delta_m \pi_s \lambda_m^s \frac{\partial^2 u_m}{\partial(\lambda_m^s e_m)^2} e_m > 0$$

that is, if and only if

$$\frac{\partial u_m}{\partial \lambda_m^s e_m} > -\frac{\partial^2 u_m}{\partial(\lambda_m^s e_m)^2} \lambda_m^s e_m$$

or, put differently,

$$1 > -\frac{u_m''}{u_m'} \lambda_m^s e_m = \rho_m^s$$

Therefore, when $\rho_m^s < 1$ an increase in the productivity coefficient will induce an increase in the r.h.s of the equilibrium condition $[1]$. That implies, in order to keep the equality, an increase of the marginal utility in the l.h.s. of $[1]$, which requires an increase of the investment.

A similar reasoning applies for $\rho_m^s > 1$. **Q.e.d.**

This result simply says that when the marginal utility is inelastic, higher productivity is associated with higher investment in education. And viceversa: when the marginal utility changes more than proportionally with respect to the income, then higher productivity translates into a smaller investment in education. When $\rho_m^s = 1$ the investment turns out to be independent on the productivity.

From this result we obtain the following:

Corollary *When the marginal utility is elastic, $\rho_m^s > 1$, higher parental cultural background or higher natural ability imply lower family investment in education.*

The interpretation of this corollary is very simple: when $\rho_m^s > 1$ parental background and the child's natural ability are substitutes of the family's investment in education.

2.2. Concern and risk aversion

The results above suggest that the concern parameter and the degree of concavity of the utility function can be singularized as the most relevant traits of the families' preferences, in order to explain their investment effort. We therefore develop the ensuing analysis focussing on utilities that depend explicitly on those two parameters. Bearing in mind that the elasticity of marginal utility corresponds to the Arrow-Pratt coefficient of relative risk aversion, we assume that u_m is drawn from the following family of functions:

$$u(x) = \frac{x^{1-a} - 1}{1 - a}, \qquad a > 0$$

where a is the coefficient of relative risk aversion (constant for all x) . This coefficient controls the curvature of the function (degree of concavity), from straight lines (for $a = 0$) to right angles (for $a = +\infty$). The case $a = 1$ boils down to the usual logarithmic utilities: $u(x) = \ln x$.
Within this setting agents' preferences are characterized by two different parameters, the concern parameter and the coefficient of risk aversion (that we assume to be independent). The optimal investment of family m is obtained from the maximization of the following function:

$$U_m(\gamma_m, e_m) = \frac{(w_m + \gamma_m - e_m)^{1-a_m} - 1}{1 - a_m} + \delta_m \sum_{s=1}^{S} \pi_s \frac{(\lambda_m^s)^{1-a_m} e_m^{1-a_m} - 1}{1 - a_m}$$

The first order conditions, when solutions are interior, are given by:[8]

$$e_m^* = \frac{b_m^{1/a_m}}{1 + b_m^{1/a_m}} (w_m + \gamma_m) \qquad [2]$$

where $b_m = \delta_m \sum_{s=1}^{S} \pi_s (\lambda_m^s)^{1-a_m}$ represents the expected level of human capital associated with one unit of investment, weighted by agent m 's risk aversion and concern. This term is a summary of the family's features: parental background,

[8] We assume implicitly that $\gamma_m + w_m > e_0$, for all m. Needless to say, $e_m^* = e_0$ when $e_m^* = \frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w_m + \gamma_m) < e_0$, and $e_m^* = e_m^+$ when $e_m^* = \frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w_m + \gamma_m) > e_m^+$.

concern, risk aversion and the child's natural ability. Note that productivity is considered an expression of the child's natural ability and the family background; it is therefore part of her "external circumstances". Concern and risk aversion, on the contrary, can be regarded as expressions of the family's responsibility (we return on this later on).

It is easy to check that optimal investment in education is an increasing function of the concern parameter, the initial wealth, and the subsidy. It increases or decreases with the productivity coefficients depending on whether a_m is smaller or greater than one (Proposition 1 and its corollary). Trivially, $a_m = 1$ implies $b_m^{1/a_m} = \delta_m$ and investment turns out to be independent on the child's productivity.

The relation between optimal investment and the coefficient of risk aversion is more complex and depends on whether b_m is smaller or greater than unity. We can establish the following on this respect:

Proposition *The optimal investment is an increasing function of the coefficient of risk aversion provided* $a_m > 1$.

Proof.-

We have to check the sign of the derivative of e_m^* with respect to a_m :

$$\frac{\partial e_m^*}{\partial a_m} = \frac{\partial}{\partial a_m}\left(\frac{b_m^{1/a}}{1 + b_m^{1/a}}(w_m + \gamma_m)\right)$$

Consider first the following:

$$\begin{aligned}
\frac{\partial b^{1/a_m}}{\partial a_m} &= \frac{\partial}{\partial a}\left[\delta_m \sum_{s=1}^{S} \pi_s(\lambda_m^s)^{1-a_m}\right]^{1/a_m} \\
&= b_m^{1/a_m}\left[\frac{-1}{a_m^2}\ln b_m + \frac{\delta_m \sum_{s=1}^{S} \pi_s(\lambda_m^s)^{1-a_m}(-1)\ln\lambda_m^s}{b_m}\frac{1}{a_m}\right] \\
&= -\frac{b_m^{1/a_m}}{a_m^2}\left[\ln b_m - \frac{a_m}{b_m}\delta_m \sum_{s=1}^{S} \pi_s(\lambda_m^s)^{1-a_m}\ln\frac{1}{\lambda_m^s}\right]
\end{aligned}$$

The sign of this expression depends on whether b_m is smaller or greater than one. Clearly, $\partial b^{1/a_m}/\partial a_m > 0$ whenever $b_m \leq 1$.

If we let $\phi \coloneqq -\frac{1}{a_m^2}\left[\ln b_m - \frac{a_m}{b_m}\delta_m \sum_{s=1}^{S} \pi_s(\lambda_m^s)^{1-a_m}\ln\frac{1}{\lambda_m^s}\right]$ and assume that $b_m \leq 1$, we would have:

$$\frac{\partial e_m^*}{\partial a_m} = \frac{b^{1/a_m}\phi(w_m+\gamma_m)(1+b^{1/a_m}) - b^{1/a_m}(w_m+\gamma_m)b^{1/a_m}\phi}{[1+b^{1/a_m}]^2}$$
$$= \frac{b^{1/a_m}\phi(w_m+\gamma_m)}{[1+b^{1/a_m}]^2} > 0$$

Therefore, $\partial e_m^*/\partial a_m > 0$ provided $b_m \leq 1.$

Finally, observe that $a_m > 1$ is a sufficient condition for $b_m \leq 1.$ Q.e.d.

Proposition 2 establishes that an increase in the parameter of risk aversion induces a higher investment in education when the marginal utility is elastic (in this case an increase in a_m makes it "cheaper", in utility terms, investing in education). The impact of the increase is controlled by the concern coefficient δ_m.

The equilibrium disposable wealth of agent m in the first period is:

$$\widetilde{w}_m^D = \frac{1}{1+b_m^{1/a_m}}(w_m+\gamma_m) \qquad [3]$$

(we shall use this expression subsequently to analyze the impact of the different policies).

2.3. The standard social choice problem

Consider now a standard utilitarian social planner, while keeping the formulation presented above. The planner has to find the subsidies that maximize aggregate welfare, subject to budget balance and the participation constraint. That can be formulated as follows:[9]

$$\left.\begin{array}{c} \max_{\{\gamma_m\}} \sum_{m=1}^{M}\left(\frac{(w_m+\gamma_m-e_m)^{1-a_m}-1}{1-a_m} + \delta_m \sum_{s=1}^{S} \pi_s \frac{(\lambda_m^s)^{1-a_m} e_m^{1-a_m}-1}{1-a_m}\right) \\ s.t.:\ e_m = \frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w_m+\gamma_m) \\ \sum_{m=1}^{M}\gamma_m = 0 \end{array}\right\} \qquad [4]$$

[9] We assume interior solutions in order to simplify the discusion. Otherwise the first restricion of program $[4]$ should be: $e_m = \max\left\{e_0,\ \min\left\{\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(\theta_m+\gamma_m),\quad e_m^+\right\}\right\}.$

The first order conditions entail:

$$\mu = \frac{(w_m + \gamma_m)^{-a_m}}{(1+b^{1/a_m})^{-a_m}} \qquad \forall\, m$$

(where μ stands for the Lagrangean multiplier associated to the budget balance restriction). That is, the change in social welfare, due to an increase in the subsidy of agent $m,$ is inversely proportional to the agent's equilibrium disposable wealth (see equation $[3]$) and obviously corresponds to the marginal utility of income of family m . Therefore, in the social optimum we get:

$$\left(\widetilde{w}_m^D\right)^{a_m} = \left(\widetilde{w}_h^D\right)^{a_h}, \qquad \forall\, m,h \in \mathcal{M} \qquad [5]$$

That is, the utilitarian planner generates a distribution of taxes and subsidies that equalizes the families' disposable wealth, exponentially weighted by their coefficients of risk aversion.

From this relation it follows that the wealthier the family the smaller the subsidy, other things equal, so that the optimal policy typically implies a transfer from rich to poor families. Similarly, the higher the concern of the family the larger the subsidy (as more concerned families are more willing to invest today into their children's future income). Also note that the utilitarian planner gives relatively higher subsidies to those families whose children are more productive (i.e. families with a better cultural background or with more able children).

Expression $[5]$ can be rewritten in terms of optimal efforts as follows:

$$(e_m^*)^{a_m}\frac{1}{b_m} = (e_h^*)^{a_h}\frac{1}{b_h}, \qquad \forall\, m,h \in \mathcal{M} \qquad [6]$$

so that the utilitarian policy implies equalizing investments across families, scaled down bye their corresponding concern parameters, productivity coefficients and risk aversion coefficients.

The case of logarithmic utilities ($a = 1$), gives us particularly simple expressions of equations $[5]$ and $[6].$ Namely:

$$\widetilde{w}_m^D = \widetilde{w}_h^D \;\; \forall\, m,h \in M \qquad [5']$$

$$\frac{1}{\delta_m}e_m^* = \frac{1}{\delta_h}e_h^*, \; \forall\, m,h \in M \qquad [6']$$

That is, in this particular case the utilitarian planner generates a distribution of taxes and subsidies such that: (i) The disposable wealth becomes equal for all families; and (ii) The investment in education, scaled down by the corresponding concern parameters, becomes equal for all families. In this particular case the policy does not depend on the productivity of the children.

3. Equality of opportunity

Suppose now that the planner aims at implementing an equality of opportunity policy, instead of the utilitarian one. Here we have to find the tax/subsidy system that equalizes the expected income achieved by those children whose families realized a similar *degree of effort* (where effort corresponds to investment in education). In order to analyze this problem we have to specify the different types of families and the way of making comparisons among them. Both steps imply a number of compromises and simplifications and determine the kind of differences the social planner is willing to compensate for.

3.1. The setting

Assume that there is a set $\mathbf{Q} = \{1, 2, ..., Q\}$ different family *types*, defined by their external characteristics, that we associate with their initial wealth and cultural background of the parents and the natural ability of their children. That is to say, all families of type q have the same initial wealth, $w(q)$, the same parental human capital, $H(q)$, and children with the same natural abilities $\alpha(q)$. Therefore, all children from the families of type q exhibit the same productivity coefficients and, consequently, identical average productivity, $\widehat{\lambda}(q)$, for all $q \in Q$. Call $\mathcal{M}(q)$ the set of families of type q, with $\cup_{q=1}^{Q} \mathbf{M}(q) = \mathbf{M}$, $\mathbf{M}(q) \cap \mathbf{M}(q') = \varnothing$, for all $q \neq q'$ in Q.

Let $m, h \in \mathcal{M}(q)$ denote two families within the same type. By definition:

$$
\begin{aligned}
w_m &= w_h = w(q) \\
\lambda_m^s &= \lambda_h^s, \ \forall\, s \\
\sum_{s=1}^{S} \pi_s \lambda_m^s &= \sum_{s=1}^{S} \pi_s \lambda_h^s = \widehat{\lambda}(q)
\end{aligned}
$$

According to this formulation, two families of the same type will choose different investments whenever their concern parameters or risk aversion coefficients differ. Those are their responsibility features.

We cannot directly compare investment levels between types because the distribution of this kind of effort variable is a type-dependent characteristic. We can define the family m 's *degree of investment effort*, $\widehat{e}_m^q$, for $m \in \mathcal{M}(q)$, as a function that depends on the investment of family m and a representative value of the investment within her type. Let $e(q)$ denote such a representative value (e.g. the average, the median, or the maximum investment level); then define agent m 's *degree of investment effort* as:

$$\widehat{e}_m^q = \frac{e_m^q}{e(q)}$$

(i.e. the percentage of the representative investment of her type).
The equality of opportunity principle requires that the children of any two families with the same degree of investment effort should achieve the same expected income in the second period.[10] That is, $\widehat{\lambda}(q)e_m = \widehat{\lambda}(q')e_{m'}$ whenever $\frac{e_m}{e(q)} = \frac{e_{m'}}{e(q')}$, with $m \in \mathcal{M}(q), \quad m' \in \mathcal{M}(q')$.
The following result shows that equality of opportunity amounts to equalizing all types' expected income associated with the reference investment level.

- **Proposition 2.-** ***A planner implements the equality of opportunity policy if and only if it allocates taxes and subsidies so that***

$$\widehat{\lambda}(q)e(q) = \widehat{\lambda}(q')e(q') \quad \forall\, q,q' = 1,2,\ldots,Q$$

Proof.-
(i) Consider the case of two agents of different types, $m \in \mathcal{M}(q)$ and $m' \in \mathcal{M}(q')$, that exert the same level of effort; that is, $\widehat{e}_m^q = \widehat{e}_{m'}^{q'}$. The individual optimality condition $[2]$ implies in that case,

$$\frac{(w(q) + \gamma_m)}{(w(q') + \gamma_{m'})} = \frac{\frac{b_{m'}^{1/a_{m'}}}{1+b_{m'}^{1/a_{m'}}}e(q)}{\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}e(q')} \qquad [7]$$

The equality of opportunity policy establishes that both agents get the same expected human capital. That is, the subsidy policy should be such that:

$$\widehat{\lambda}(q)\frac{b_m^{1/a_m}}{1 + b_m^{1/a_m}}(w(q) + \gamma_m) = \widehat{\lambda}(q')\frac{b_{m'}^{1/a_{m'}}}{1 + b_{m'}^{1/a_{m'}}}(w(q') + \gamma_{m'}) \qquad [8]$$

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[10] When there is uncertainty, equality of opportunity is defined in terms of expected outcomes, provided luck is "even handed" across types (see the discussion in Lefranc et al (2007)). This amounts to saying that families should not be compensated for the good or bad luck (the realization of the random variable that affects the outcomes).

Or, put differently,

$$\frac{(w(q)+\gamma_m)}{(w(q')+\gamma_{m'})} = \frac{\hat{\lambda}(q')\frac{b_{m'}^{1/a_{m'}}}{1+b_{m'}^{1/a_{m'}}}}{\hat{\lambda}(q)\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}} \qquad [8']$$

Therefore, it follows from $[7]$ and $[8']$:

$$\hat{\lambda}(q)e(q) = \hat{\lambda}(q')e(q') \quad \forall\, q,q' \in Q \qquad [9]$$

(ii) Suppose now that the planner has arranged the tax/policy scheme so that the effort levels induced yield precisely equation $[9]$. Let us now see that this implies equal expected human capital for the children of those families with equal degree of effort. If two families exert a comparable degree of effort chosen optimally, we have:

$$\frac{e_m^q}{e(q)} = \frac{\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w(q)+\gamma_m)}{e(q)} = \frac{\frac{b_{m'}^{1/a_{m'}}}{1+b_{m'}^{1/a_{m'}}}(w(q')+\gamma_{m'})}{e(q')} = \frac{e_{m'}^{q'}}{e(q')}$$

Multiplying the l.h.s of equation $[9]$ by $\frac{\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w(q)+\gamma_m)}{e(q)}$ and the r.h.s. by $\frac{\frac{b_{m'}^{1/a_{m'}}}{1+b_{m'}^{1/a_{m'}}}(w(q')+\gamma_{m'})}{e(q')}$, we get:

$$\hat{\lambda}(q)\frac{b_m^{1/a_m}}{1+b_m^{1/a_m}}(w(q)+\gamma_m) = \hat{\lambda}(q')\frac{b_{m'}^{1/a_{m'}}}{1+b_{m'}^{1/a_{m'}}}(w(q')+\gamma_{m'})$$

which is precisely the expected human capital for their children.

Q.e.d.

This Proposition provides us with a precise recipe to implement the equality of opportunity policy: equalize the expected human capital associated with the reference investment level for all types. Or, put differently, this policy implies equalizing all types reference investment levels, weighted by their corresponding average productivity (compare with equation $[6]$) . Therefore, those types with relatively lower productivity will be induced to invest relatively more. This implies, in particular, that the planner compensates for the differences in cultural backgrounds and natural abilities across types.

Note that the equilibrium condition $[9]$ can be rewritten as:

$$\frac{\widehat{\lambda}(q) b_m^{1/a_m}}{\widehat{e}_m^{q}} \widetilde{w}_m^{D} = \frac{\widehat{\lambda}(q') b_{m'}^{1/a_{m'}}}{\widehat{e}_{m'}^{q'}} \widetilde{w}_{m'}^{D} \forall\, m, m' \in \mathcal{M} \qquad [9']$$

That is, the equality of opportunity policy equalizes the families' disposable wealth weighted by average productivity, relative investments and the b_m coefficients.

3.2. The equality of opportunity policy

The planner that implements an equality of opportunity policy looks for a system of taxes and transfers that solves the following program:

$$\left.\begin{array}{c} \max_{\{\gamma_m\}} \sum_{m=1}^{M} \left(\frac{(w_m + \gamma_m - e_m)^{1-a_m} - 1}{1-a_m} + \delta_m \sum_{s=1}^{S} \pi_s \frac{(\lambda_m^s)^{1-a_m} e_m^{1-a_m} - 1}{1-a_m} \right) \\ s.t.:\ e_m = \frac{b_m^{1/a_m}}{1+b_m^{1/a_m}} (w_m + \gamma_m) \\ \sum_{m=1}^{M} \gamma_m = 0 \\ \widehat{\lambda}(q) e(q) = \kappa \quad \forall\, q \in \mathcal{Q} \end{array}\right\} \qquad [10]$$

Now the planner's program contains three types of restrictions: the participation constraint (all agents choose their optimal effort levels), the budget balance constraint, and the equality of opportunity constraint. Therefore, the solution to this program will typically induce a level of social welfare below that corresponding to the pure utilitarian planner, that only involves the first two constraints. One may regard that difference as the welfare cost of egalitarianism. The equilibrium associated with that tax/subsidy system under equality of opportunity is thus a second best solution. Efficiency may fail, in particular, when "rich" families have more able children or exhibit a higher concern parameter relative to the "poor" ones; in that case transferring one euro from a poor to a rich person may result in a higher aggregate income, that can be shared so that both agents get better.
Trivially, when agents only differ in their initial wealth, the egalitarian solution coincides with the utilitarian one and is, therefore, efficient. In that case the tax/subsidy scheme is such that it equalizes all agents' disposable wealth and, consequently, it brings about an equal expected income.

The equality of opportunity policy implies transferring wealth to the less favoured families, either because they are poorer or have children with lower productivity (parents with lower education and/or children with less natural abilities). It also implies transferring wealth to those families that exert a higher relative effort. This scheme clearly differs from the utilitarian one in which types play no role whatsoever and the policy tends to favour the families with more productive children (see equation $[6]$) . Under the utilitarian regime, two families with the same risk aversion end up with the same disposable wealth, no matter what. Under equality of opportunity, two families with the same risk aversion end up with different disposable wealth, depending on their children's average productivity and their relative investment effort.

The case of logarithmic utilities illustrates well the differences between both policies. When $a_m = 1$ for all $m \in \mathcal{M},$ the income distribution that results from a utilitarian policy reflects, precisely, the differences in productivity and concern. It follows from equations $[5']$ and $[6']$ that:

$$E^U(y_m) = \widehat{\lambda}_m \delta_m K_1, \ \ \forall\, m \in \mathcal{M}$$

(where $K_1 = \widetilde{w}_m^D$ is constant and $E^U(y_m)$ stands for the expected income of child m under the utilitarian policy).[11] The income distribution derived from the equality of opportunity policy reflects the degrees of effort. It follows from equation $[9]$ that:

$$E^{Eop}(y_m) = \widehat{e}_m^{\,q} K_2, \ \ \forall\, m \in \mathcal{M}$$

(where $K_2 = \widehat{\lambda}(q) e(q)$ is constant and $E^{Eop}(y_m)$ is the expected income of child m under equality of opportunity).

4. Final comments

We have presented a model in which children's future income depends on their natural abilities, the family background, and the investment made by their parents. Such an investment is in turn a function of the parents' wealth and education, the concern parameter and the coefficient of risk aversion. The public authority affects family decisions through a simple policy variable (money transfers). The model shows that the investment of families in human capital for their children depends positively on the initial wealth, the concern parameter and the subsidy. In

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[11] In the general case we have: $E^U(y_m) = \widehat{\lambda}_m (b_m K_0)^{1/a_m}$ with $K_0 = 1/\left(\widetilde{w}_m^D\right)^{a_m}$.

particular, higher subsidies imply higher investment levels due to the reduction in the opportunity cost of education they induce. The relationship between investment in the children's education and their productivity depends on the elasticity of marginal income (or, alternatively, the coefficient of relative risk aversion). The case of logarithmic utilities is specially simple because it neutralizes the effect of average productivity on the investment decisions. The relation between optimal investment and risk aversion is positive, provided the marginal utility of income is elastic.

We have considered two different tax/subsidy schemes, depending on the nature of the social planner. The maximization of social welfare for a utilitarian planner entails a series of money transfers that compensate the differences in the agents' initial wealth, taking into account their risk aversion coefficients. This policy benefits those families with more productive children. The planner who cares for equality of opportunity compensates not only for the differences in initial wealth but also for the differences in natural abilities, parental cultural background, and relative investment efforts. As a consequence, the income distribution that results from a utilitarian policy reflects the family differences in productivity, concern and risk aversion, whereas the income distribution derived from the equality of opportunity policy reflects the degrees of investment efforts.

The model contemplates three variables related to opportunity: the wealth and education of the parents, on the one hand, and the children's natural abilities, on the other hand. One can also consider further opportunity variables (race, gender, etc.) Introducing those external circumstances would affect the model by modifying the generation of types, which implies altering the schedule of relative investments and, consequently, the allocation of subsidies. The model makes it clear the key role of the choice of the reference variable that is used to compare degrees of effort between the types (Proposition 3 shows that those reference variables actually define the equality of opportunity condition).

The concern parameter and the coefficient of risk aversion are the variables related to responsibility in this model. One may argue that this approach implies making grown up children responsible for the attitudes of their parents, which might be unfair. Yet this is partly an interpretation issue. Our model is very simple and small children in the first period blow up to adults in the second one. Yet, children grow and assume responsibilities on their future progressively so that the concern coefficient may be regarded as incorporating the share of the child's responsibility in its own education (think of the case of deciding about engaging university studies). And the same happens with the risk aversion coefficient, that is to be interpreted as the whole family's attitude towards the risk, that includes the child's preferences.[12]

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[12] A more natural approach would be to add an intermediate period to the model in which children start making decisions on their own future as a function of their parents former and present investment and their preferences. In that context one may discuss some other aspects of the problem, such as the the role of early education in the configuration of non-cognitive skills, the role of schools, or the interplay between altruistic parents, egotistic children and policy variables. The meaning and implications of the egalitarian policy in this context is much less obvious. That is a relevant topic that is left for future research.

Neither the utilitarian planner nor the planner who cares for equality of opportunity compensate agents for luck. Equality of outcomes, conditional on comparable degrees of investment, is formulated in expected terms so that the particular realization of the random variable is not considered ethically relevant. This corresponds implicitly to the case in which luck is even-handed across types and makes sense in a static model. The case in which luck is type-dependent may call for a reformulation of the model (e.g. weighting expected incomes by the standard deviations within types). This reformulation is inevitably much deeper when effort is not observable and we consider a dynamic context with repeated interactions between the planner and the agents (see Calo-Blanco (2008)).

Finally, observe that the formal decision model developed here is one in which agents make costly investments in order to achieve future rewards. It might thus be applicable to very different scenarios in which equality of opportunity matters. As an example we can think of the case of a Federal State in which the agents are the states, effort refers to share of the GDP invested in R&D, and the outcomes correspond to the regional per capita GDP or growth rates. The model would help to analyze the transfer policy that would implement the equality of opportunity principle in this context.[13]

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[13] Closer to the focus of this paper is the case in which one analyzes the schooling performance. In a different paper the authors analyze the results of the PISA study in the Spanish Regions. There we take the PISA scores as the outcomes, the regional expenditures per student as the revealed effort, the per capita GDP as the initial income, and the % of the working age population with non-compulsory studies as an index of the human capital accumulated. By setting $a = 1$ the model allows us to estimate the concern parameter and the average productivity of the different regions (see Calo-Blanco & Villar, 2009).

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