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Abstract

Applying an overlapping-generations model with lifetime uncertainty, we examine in this paper China's partially funded public pension system. The findings show that the individual contribution rate does not affect the capital-labor ratio but the firm contribution rate does. The optimal firm contribution rate depends on the capital share of income, social discount factor, survival probability, and population growth rate. The simulation results indicate that the optimal firm contribution rate rises with China's life expectancy but, surprisingly, falls with the population growth rate. We demonstrate that the optimal firm contribution rate should be cut when the effect of falling population growth rate is greater than that of rising life expectancy and that the rate is much more sensitive to the population growth rate than to life expectancy. This paper also solves the optimal interval to cope with China's population aging peak in the 2030s.

Key Words: Public pension, population growth rate, life expectancy

I. Introduction

The Chinese State Council Document 38 of 2005 – "Decision on Improving the Basic Pension System for Enterprise Employees" – introduced a new urban public pension program in China. Under the program, the government establishes an individual account for each employee plus a social pool for all employees and retirees. Each firm contributes 20% of its payroll to the social pool, while each employee contributes 8% of her wage to the individual account. The social pool fund employs a pay-as-you-go (PAYG) pension system to cover the benefits of current retirees.[1] The accumulation in the individual account is used to cover the financial need of the individual participant at retirement. Each retiree thus receives pension benefits from her individual account and PAYG pension benefits from the social pool.

In China, population aging is very rapid because of rising longevity and falling population growth rate. It is expected that the population aging peak will appear in the 2030s. What

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[1] For the sake of simplicity, the author uses only "she" and its variations in this paper.

policies should the Chinese government adopt to deal with the pension, rising longevity and population aging issues?

We find literature on the relationship between lifetime uncertainty, population growth rate and public pension. Yaarri (1965) pioneers studies on consumer allocation over time concentrating on the lifetime uncertainty and ignoring the other uncertainties by a continuous time model. Some of the literature on public pension with lifetime uncertainty studies PAYG pension systems (e.g., Pecchenino and Utendorf, 1999; Fuster, 2000; Zhang and Zhang, 2001; and Pecchenino and Pollard, 2002). Several studies analyze fully funded pension (e.g., Abel, 1987; and Karni and Zilcha, 1989). Some of the literature examine both PAYG and fully funded pensions (e.g., Sheshinski and Weiss, 1981; Pecchenino and Pollard, 1997; and Zhang et al., 2001). However, no papers in the literature on public pension with lifetime uncertainty are known to have examined a partially funded public pension system. Hence, this paper investigates this issue using China as a case.

In the literature mentioned above, pensions are financed only by wage taxes. However, in most of the countries that have public pension systems, the government levies pension taxes on each employee's wage and on each enterprise's payroll using a proportional taxation schedule. So does the Chinese government.

As a means to generate an overlapping-generations model with lifetime uncertainty, we examine China's partially funded public pension combining social pool and individual accounts. By doing so, we attempt to find optimal pension contribution rates.

The rest of this paper is organized as follows: Section 2 presents the model in market economy. Section 3 derives the formula to compute the optimal contribution rate. Section 4 simulates three cases. Section 5 concludes the study.

II. The Model

This model extends those of Pecchenino and Pollard (1997) and Pecchenino and Pollard (2002) by considering a partially funded public pension financed by individual contributions and firm contributions. We define that a closed economy is composed of numerous individuals and firms and a government. The generation born at the beginning of period *t* is called generation *t*. The population grows at the rate of $n = N_t / N_{t-1} - 1$, where N_t is the population size of generation *t*.

Individuals. Each individual survives to the end of her working period certainly. The person's survival probability in the retirement period is $p \in [0,1]$. In the working period, each individual earns wage by supplying inelastically one unit of labor and makes pension contributions. It is possible for the worker to inherit some unintentional bequests from her parent with probability $(1-p)$. The worker consumes part of income and saves the rest. If she outsurvives the working period, then the retiree consumes her savings with accrued interest, funded pension benefits and PAYG pension benefits. If the person dies at the beginning of the retirement period, then her savings with accrued interest and funded pension benefits are inherited equally by the surviving children as unintentional bequests.

Each individual derives utility from her working-period consumption c_{1t} and possible retirement-period consumption c_{2t+1}. The utility is described by an additively separable logarithmic function. Thus, each individual solves the following maximization problem:

$$\max_{\{c_{1t}, c_{2t+1}, s_t\}} U_t = \ln c_{1t} + p \ln c_{2t+1}, \tag{1}$$

$$\text{s.t. } c_{1t} = (1-p)b_t + (1-\tau)w_t - s_t\,, \tag{2}$$

$$c_{2t+1} = (1+r_{t+1})s_t + F_{t+1} + P_{t+1}\,, \tag{3}$$

$$(1+n)b_{t+1} = (1+r_{t+1})s_t + F_{t+1}\,, \tag{4}$$

where w_t is the wage, τ the individual contribution rate, s_t the savings, and r_{t+1} the interest rate, F_{t+1} the funded pension benefits, where $F_{t+1} = (1+r_{t+1})I_t$, I_t the individual account principal per worker, P_{t+1} the PAYG pension benefits, and b_{t+1} the bequests inherited by each children. The relation between I_t and τ is explained in below in this section.

Substituting equations (2)-(4) into equation (1) and differentiating it with respect to s_t yields the first-order condition:

$$p(1+r_{t+1})c_{1t} = c_{2t+1}\,. \tag{5}$$

This familiar expression implies that the utility loss from reducing one unit of working-period consumption is equal to the utility gain from increasing $(1+r_{t+1})$ units of possible retirement-period consumption with probability *p*.

Firms. Firms produce a single commodity in competitive markets. The production is described by a Cobb-Douglas function $Y_t = AK_t^{\alpha}N_t^{1-\alpha}$ or $y_t = Ak_t^{\alpha}$, where Y_t is the output in period *t*, *A* the productivity, K_t the capital stock, *α* the capital share of income, $k_t = K_t/N_t$ the capital-labor ratio, and y_t the output-labor ratio.

Firms make pension contributions at the rate of $\eta \in (0,1)$ on their payroll. According to the product distribution, one can get $AK_t^{\alpha}N_t^{1-\alpha} = r_tK_t + (1+\eta)w_tN_t$. Firms act competitively, renting capital to the point where the marginal product of capital is equal to its rental rate, and hiring labor to the point where the marginal product of labor is equal to $(1+\eta)w_t$:

$$r_t = \alpha Ak_t^{\alpha-1}\,, \tag{6}$$

$$w_t = \frac{(1-\alpha)Ak_t^{\alpha}}{1+\eta}\,. \tag{7}$$

The Government. The accumulation in the individual account is used to pay the individual when she retires in the next period. The pension benefit is funded as $(1+r_{t+1})\tau w_t = F_{t+1}$ or:

$$I_t = \tau w_t\,, \tag{8}$$

The social pool fund is paid to the retirees in the current period and offers PAYG pension benefits of $pN_{t-1}P_t = N_t\eta w_t$ or:

$$P_t = \frac{1+n}{p}\eta w_t\,. \tag{9}$$

Equilibrium. Let us assume that the savings and the individual account principal of the young in period *t* generate the capital stock in period *t*+1:

$$s_t + I_t = (1+n)k_{t+1}. \tag{10}$$

A competitive equilibrium for this market economy is a sequence as $\{c_{1t}, c_{2t+1}, s_t, w_t, r_{t+1}, I_t, P_{t+1}, b_t, k_t\}_{t=0}^{\infty}$ that satisfies equations (1)-(10) for all *t*, given the initial condition k_0 and the values of parameters *τ* and *η*.

Substituting equations (2)-(4) and (6)-(10) into equation (5) gives a dynamic system:

$$\begin{aligned} & p\left(1+\alpha A k_{t+1}^{\alpha-1}\right)\left[\frac{1-\alpha}{1+\eta} A k_t^{\alpha} - (1+n)k_{t+1} + (1-p)k_t\left(1+\alpha A k_t^{\alpha-1}\right)\right] \\ & = (1+n)k_{t+1}\left(1+\alpha A k_{t+1}^{\alpha-1}\right) + (1+n)\frac{\eta}{p}\frac{1-\alpha}{1+\eta} A k_{t+1}^{\alpha} \end{aligned}. \tag{11}$$

Assume that there exists a unique, stable and nonoscillatory equilibrium. The stability condition of this system is shown in Appendix A.

The individual contribution rate does not exist in equation (11). It implies that the individual contribution rate has no effect on the capital-labor ratio. This is because that the mandatory savings (individual contributions) crowd out private savings by one-for-one, which is reflected by equation (10).

III. Social Equilibrium

The optimal firm contribution rate can be defined by comparing a decentralized equilibrium with the social optimum. The social welfare function in this paper is defined as the sum of the weighted utilities of all current and future generations:[2]

$$W = p \ln c_{20} + \sum_{i=0}^{\infty} \rho^i \left(\ln c_{1i} + p \ln c_{2i+1}\right). \tag{12}$$

where ρ is the social discount factor, which reflects the preference of the social planner. It indicates how much the social planner weights the utilities of different generations in his or her social welfare calculations. It is assumed to be in the interval of (0, 1); that is, the social planner cares less about future generations.[3]

The resource constraint is:

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[2] Samuelson (1968), Blanchard and Fischer (1989) and Groezen et al. (2003) also use analogous social welfare functions.

[3] $\rho \in (0,1)$ implies that the social planner gives diminishing weights on the utilities of future generations, i.e., weight 1 on the utilities of generation 0 and surviving generation -1, weight ρ on the utility of generation 1, weight ρ^2 on the utility of generation 2, etc.

$$k_i + Ak_i^{\alpha} = (1+n)k_{i+1} + c_{1i} + \frac{pc_{2i}}{1+n}, \tag{13}$$

for given k_0. The social planner maximizes the social welfare subject to the resource constraint and the initial condition. Thus, the first-order conditions for the social welfare maximization problem are (also refer to Appendix B):

$$(1+n)c_1^* = \rho c_2^*, \tag{14}$$

$$1+\alpha A\left(k^*\right)^{\alpha-1} = \frac{1+n}{\rho}, \text{ or } k^* = \left(\frac{1+n-\rho}{\rho\alpha A}\right)^{\frac{1}{\alpha-1}}, \tag{15}$$

where the superscript (*) denotes the optimal steady state values of variables. When the capital-labor ratio satisfies equation (15), the ratio is optimal and the social welfare reaches the optimal level.

$$\eta^* = \frac{\frac{p}{\rho}\frac{1-\alpha}{\alpha}(1+n-\rho)+\left[p(1-p)\frac{1+n}{\rho}-(1+p)(1+n)\right]}{\frac{1}{p}\frac{1-\alpha}{\alpha}(1+n-\rho)-\left[p(1-p)\frac{1+n}{\rho}-(1+p)(1+n)\right]}. \tag{16}$$

Equation (16) is the formula to compute the optimal firm contribution rate. The active factors in the formula are the survival probability in retirement period, social discount factor, capital share of income, and population growth rate. Partially differentiating η^* with respect to p and n, respectively, gives that the effects of life expectancy and population growth rate on the optimal firm contribution rate are ambiguous. We check the effects of it using simulations.

IV. Simulations

In this section, we simulate the effects of life expectancy and population growth rate on the optimal firm contribution rate so that we can find the optimal firm contribution rate. The simulations is performed in two steps: The first step is to estimate the baseline parameter values. The second step is to simulate three cases: (1) rising life expectancy; (2) falling population growth rate; and (3) rising life expectancy and falling population growth rate.

Estimation of Parameter Values. We estimate the baseline parameter values in this subsection. The capital share of income, α, is usually to be estimated as 0.3 in developed countries (e.g., Zhang et al., 2001; Pecchenino and Pollard, 2002; and Barro and Sala-I-Martin, 2004). The labor in China is comparatively cheaper, and thus the labor share of income is lower, while the capital share of income is higher than that in developed countries. Hence, we assume that α in China could be 0.35.

There are several calibers for population statistics in China. Because the public pension system in urban area is different from that in rural area and because only the former is examined in this paper, we select the caliber of "Urban Population." The population growth rate in the period from 1978 to 2004 is computed at n =2.148 based on the data in China Statistical Yearbook – 2005.[4]

The survival probability in retirement period is estimated by life expectancy. By virtue of the China Statistical Yearbook, life expectancy in 2000 is 71.40 years old. The survival life-span in retirement period is 19.4 (or 71.40 – 26 $\times$ 2) years. (One period includes 26 years. Even if childhood period is omitted in the model, it must be taken into account when we practically compute the survival life-span in retirement period). Hence, the survival probability is p=19.4/26=74.615%. This computation approach is equivalent to:[5]

$$(1-p)\times 26\times 2+p\times 26\times 3=71.4\ .$$

The social discount factor reflects the preference of the social planner. It indicates how much the government weights different generations in its social welfare calculations. It should thus be estimated in compliance with government regulations. Based on the Chinese State Council Document 38 of 2005, one can get that the optimal firm contribution rate decided by the government is 20%. Substituting the above parameter values into equation (16) and calculating repeatedly until the difference between the two sides becomes 0, one can get that ρ=0.547.

The Optimal Firm Contribution Rate. In this subsection, we examine the effects of life expectancy and population growth rate on the optimal firm contribution rate by simulating the mentioned three cases. The white paper, "China's Social Security and Its Policies" (Information Office of China's State Council, 2004), points out that "China is now an aging society. As the aging of the population quickens, the number of elderly people is becoming very large. This trend will reach its peak in the 2030s." Our simulation is based on this anticipation. In the simulations, the values of α and ρ are constant.

<u>Case 1 (Rising Life Expectancy)</u>. According to the UN (2003), the life expectancy in China during 2030-2035 will be 74.7 years old. Simulating by substituting the values into the formula to compute the survival probability $(1-p)\times 26\times 2+p\times 26\times 3=74.7$ gives:

$$p=(74.7-26\times 2)/26=87.308\%.$$

In the computation, life expectancy is assumed to start and remain at the level of 74.7 years old. Simulating by substituting 1+n=3.148 and p=87.308% into equation (16) yields that η^* = 22.65 %.

The UN (2003) also forecasts that the life expectancy in China during 2035-2040 will be 75.5 years old. Simulating analogously gives that η^* = 22.90 %. In the simulation, life expectancy is assumed to start and remain at the level of 75.5 years old. The simulation results are shown in Table 1. It is shown that the rise in life expectancy induces the increase in the optimal firm contribution rate.

[4] There are two reasons to choose a period length of 26 years in this model. One is that the length is usually in the interval of 25-30 years in the literature on OLG model. The other is the structure of the data in China.

[5] Although the choice of period length is arbitrary, it has to obey the following rule: Three times of the period length should be longer than or equal to life expectancy to ensure $p \le 1$.

Table 1: The Effect of Life Expectancy on η^*

Year	Life Expectancy	p	$1+n$	η^*
2030-2035	74.7	87.308%	3.148	22.65%
2035-2040	75.5	90.385%		22.90%

Table 2: The Effect of Population Growth Rate on η^*

The Line Decided by	$1+n$	p	η^*
(1996, 1.0606) and (2040, 1)	2.207	74.615%	15.39%
(2004, 1.0364) and (2040, 1)	1.794		11.47 %

Table 3: The Effects of Life Expectancy and Population Growth Rate on η^*

Life Expectancy	p	$1+n$	η^*
74.7	87.31%	2.207	17.03%
		1.794	12.31%
75.5	90.38%	2.207	17.05%
		1.794	12.16%

Case 2 (Falling Population Growth Rate). The China Population Society and China Family Planning Association (2005) announced that "China will realize the zero growth of population by the year of 2040." For simplification, it is assumed that the urban population growth rate will also become zero in 2040. We estimate the value of (1+*n*) in period 2005-2030 by a line decided by two points. One of the points is (2040, 1). The other point can be found in China Statistical Yearbook – 2005.

Let l_j denote the population growth rate in year *j*, and then the values of $(1+l_j)$ from 1990 to 2004 are computed as shown in Appendix C. The maximum is $(1+l_{1996})$, which is equal to 1.0606. The value of $(1+l_{2004})$ is equal to 1.0364.

The two points, (1996, 1.0606) and (2040, 1), decide the following line:

$$(1+l_j) = 1.0606 - \frac{0.0606}{44} \cdot (j - 1996). \tag{17}$$

Using Line (17), we predict the values of $(1+l_j)$ from 2005 to 2030 (refer to Appendix C).

The product of $\left(1+l_j\right)$ from 2005 to 2030 is the value of (1+n) in period 2005-2030, which is equal to 2.207. Simulating by substituting 1+n=2.207 and p=74.615% into equation (16) yields η^*=15.39 %, which is shown in Table 2.

Similarly, the two points, (2004, 1. 0364) and (2040, 1), decide the following line:

$$\left(1+l_j\right)=1.0364-\frac{0.0364}{36}\cdot\left(j-2004\right). \tag{18}$$

Simulating analogously gives that the value of (1+n) in period 2005-2030 is 1.794, furthermore, the optimal firm contribution rate is η^*=11.47 % (shown in Table 2).

The simulation shows that the fall in the population growth rate leads to a decrease in the optimal firm contribution rate. It can be explained as follows. The fall in the population growth rate has two opposite effects: the first is to decrease the social pool benefits directly; and the second is to increase the wage indirectly. When the direct effect dominates the indirect one, the social pool benefits will fall with the population growth rate. Because the firm contribution rate has a positive relation with the social pool benefits, the optimal firm contribution rate is decreasing in the population growth rate.

<u>Case 3 (Rising Life Expectancy and Falling Population Growth Rate)</u>. Using the forecasted life expectancy, 74.7 or 75.5 years old, and the predicted (1+n) in period 2005-2030, 2.207 or 1.794, we get four combinations shown in Table 3. Simulating by substituting the combinations of values into equation (16) gives the optimal firm contribution rate under each combination. It is shown that the optimal firm contribution rate should be cut when the effect of falling population growth rate dominates that of rising life expectancy.

Tables 1-3 show that the optimal firm contribution rate is much more sensitive to the population growth rate than to life expectancy. In Table 3, for example, if (1+n) is 2.207, the change in the optimal firm contribution rate is only from 17.03% to 17.05% as life expectancy rises from 74.7 to 75.5 years old. However, if life expectancy is 74.7 years old, the optimal firm contribution rate changes from 17.03% to 12.31% when (1+n) falls from 2.207 to 1.794.

If life expectancy rises to 74.7-75.5 years old in the 2030s as forecasted by the UN (2003) and the population growth rate falls to zero by the year of 2040 as anticipated by the China Population Society and China Family Planning Association (2005), then the optimal firm contribution rate should be cut from 20% to the interval of (12.16%, 17.05%). This is the optimal interval of firm contribution rate to cope with the population aging peak in China in the 2030s.

V. Conclusions

Applying the overlapping-generations model with lifetime uncertainty, we have examined China's partially funded public pension program that combines a social pool with individual accounts to seek the optimal individual and firm contribution rates. It is shown that the individual contribution rate does not affect the capital-labor ratio, but the firm contribution rate does. The optimal firm contribution rate depends on the capital share of income, social discount factor, survival probability, and population growth rate.

Using the data in China Statistical Yearbook – 2005 and World Population Prospects and the anticipation of China Population Society and China Family Planning Association, we simulate cases in which life expectancy is rising and/or the population growth rate is falling, especially the case in the 2030s when the share of elderly people in China reaches its peak. The simulation gives the following results. First, the optimal firm contribution rate rises with life expectancy. Second, surprisingly, the optimal firm contribution rate falls with the population growth rate. Third, when the effect of falling population growth rate dominates that of rising life expectancy, the optimal firm contribution rate should be cut. Fourth, the optimal firm contribution rate is much more sensitive to the population growth rate than to life expectancy. Finally, if life expectancy rises to 74.7-75.5 years old in the 2030s and the population growth rate falls to zero by the year of 2040, then the optimal firm contribution rate should be in the interval of (12.16%, 17.05%), which is the optimal interval to cope with China's population aging peak in the 2030s.

Having knowledge about the optimal interval of firm contribution rate is helpful for the government and private entities to design their public and private pension programs, respectively. For example, the firm contribution rate has been so high in China that there has been widespread evasion of firms' pension contributions. The social insurance in China comprise public pension, medical, unemployment, work-related injury, and maternity insurance. The firm contribution rates for these five lines of insurance are 20%, 6%, 2%, 1%, and 1%, respectively. Thus, the social insurance contribution rate burdened by enterprise is 30% of payroll. In addition to the social insurance, the enterprises have to participate in a housing security, where the firm contribution rate is about 10%. Consequently, a firm's total contribution rate can reach or even exceed 40%. If the enterprises reported and made contributions strictly based on reality, many of them would go bankrupt. Given the potentially high contribution rate, particularly for pension contribution, some companies may attempt to refuse to participate or, more commonly, understate their number of employees and payroll.

If the firm contribution rate can be cut from 20% to the interval of (12.16%, 17.05%), numerous companies can lighten their heavy pension burden and market will observe fewer market conduct cases. More importantly, China's public pension program will become financially healthier.

There are some shortcomings and limitations in the model. Altruism in real life is not taken into account. The survival probability is assumed and estimated simply. The arbitrariness in the choice of period length is also a limitation of the model. The estimations for the capital share of income and population growth rate are simplified as well. The above simplifications in the model might lead to a significant deviation from reality. For further research, any extension to counteract any one of the above shortcomings would be valuable.

Appendix A

The assumption that there is a unique, stable and nonoscillatory steady state equilibrium is equivalent to $dk_{t+1}/dk_t \in (0,1)$ around the steady state (k). In order to find the stability condition, we differentiate equation (11) with respect to k_{t+1} and k_t. Evaluating the derivatives around the steady state gives:

$$idk_{t+1} + jdk_t = 0\,, \tag{A1}$$

where:

$$i = (1+n)\left(1+\alpha^2 Ak^{\alpha-1} + \frac{\eta}{p}\frac{1-\alpha}{1+\eta}\alpha Ak^{\alpha-1}\right) + p(1+n)\left(1+\alpha Ak^{\alpha-1}\right)$$
$$+ p\alpha(1-\alpha)Ak^{\alpha-2}\left[\frac{1-\alpha}{1+\eta}Ak^{\alpha} - (1+n)k + (1-p)k\left(1+\alpha Ak^{\alpha-1}\right)\right],$$

$$j = -p\left(1+\alpha Ak^{\alpha-1}\right)\left[\frac{1-\alpha}{1+\eta}\alpha Ak^{\alpha-1} + (1-p)\left(1+\alpha^2 Ak^{\alpha-1}\right)\right] < 0 .$$

Therefore, the stability condition is $0 < \frac{dk_{t+1}}{dk_t} = -\frac{j}{i} < 1$, or:

$$i + j > 0 . \tag{A2}$$

Appendix B

The Lagrange function for the social welfare maximization problem is:

$$\begin{aligned} L = \cdots & \\ & + \rho^{t-1}\left(\ln c_{1t-1} + p\ln c_{2t}\right) + \lambda_{t-1}\left[k_{t-1} + Ak_{t-1}^{\alpha} - (1+n)k_t - c_{1t-1} - \frac{pc_{2t-1}}{1+n}\right] \\ & + \rho^{t}\left(\ln c_{1t} + p\ln c_{2t+1}\right) + \lambda_{t}\left[k_{t} + Ak_{t}^{\alpha} - (1+n)k_{t+1} - c_{1t} - \frac{pc_{2t}}{1+n}\right] \\ & + \rho^{t+1}\left(\ln c_{1t+1} + p\ln c_{2t+2}\right) + \lambda_{t+1}\left[k_{t+1} + Ak_{t+1}^{\alpha} - (1+n)k_{t+2} - c_{1t+1} - \frac{pc_{2t+1}}{1+n}\right] \\ & + \cdots \end{aligned}$$

where λ_t is the Lagrange multiplier for the resource constraint in period *t*. Differentiating *L* with respect to c_{1t}, c_{2t} and k_{t+1} gives:

$$\rho^t / c_{1t} - \lambda_t = 0 , \tag{A3}$$

$$\rho^{t-1} p / c_{2t} - \lambda_t p / (1+n) = 0 , \tag{A4}$$

$$-\lambda_t (1+n) + \lambda_{t+1}\left(1 + \alpha Ak_{t+1}^{\alpha-1}\right) = 0 . \tag{A5}$$

Arranging equations (A3) through (A5) at the optimal steady state (k^*, c_1^*, c_2^*) yields equations (14) and (15).

Appendix C

Table A-1: Computed $(1+l_j)$ From 1990 to 2004

Year j	Urban Population (10,000 persons)	$(1+l_j)$
1978	17,245	
1980	19,140	
1985	25,094	
1989	29,540	
1990	30,195	1.0222
1991	31,203	1.0334
1992	32,175	1.0312
1993	33,173	1.0310
1994	34,169	1.0300
1995	35,174	1.0294
1996	37,304	1.0606
1997	39,449	1.0575
1998	41,608	1.0547
1999	43,748	1.0514
2000	45,906	1.0493
2001	48,064	1.0470
2002	50,212	1.0447
2003	52,376	1.0431
2004	54,283	1.0364

Columns 1 and 2 are cited from China Statistical Yearbook – 2005. Column 3 is computed based on Column 2 data.

Table A-2: Predicted $(1+l_j)$ From 2005 to 2030

Year j	$(1+l_j)$ Predicted by Line (17)	$(1+l_j)$ Predicted by Line (18)
2005	1.048	1.035
2006	1.047	1.034
2007	1.045	1.033
2008	1.044	1.032
2009	1.043	1.031
2010	1.041	1.030
2011	1.040	1.029
2012	1.039	1.028
2013	1.037	1.027
2014	1.036	1.026
2015	1.034	1.025
2016	1.033	1.024
2017	1.032	1.023
2018	1.030	1.022
2019	1.029	1.021
2020	1.028	1.020
2021	1.026	1.019
2022	1.025	1.018
2023	1.023	1.017
2024	1.022	1.016
2025	1.021	1.015
2026	1.019	1.014
2027	1.018	1.013
2028	1.017	1.012
2029	1.015	1.011
2030	1.014	1.010

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