Pricing of the Policy Life in Absence of Default Risk and Asset Liability Management

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15 August 2006

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Abstract: The model, by using the option theory, determines the fair value of the insurance life policies in absence of default risk and shows that the fair fixed guaranteed interest rate is less than the risk free interest rate due to the exchange of options between policyholders and shareholders. Furthermore, it shows that the effective liabilities duration is different from the duration of a default free zero coupon bond with the same time of maturity such that the equity value is immunized by using a perfect hedge ratio.

Introduction

The classical contingent claim approach of Black, Scholes(1973) values the insurance life policies as a portfolio of default free bonds and options to value the default risk. If the Insurance Company adopts a strategy to avoid the default risk, this can be achieved, for instance, by investing prevalently in default free bonds, or by transferring the default risk with an operation of securitization, the model brings to value the insurance life policies like a default free bonds. Our approach shows that in this case the classical model misprices the insurance life policies. In effect, there is an exchange of options between shareholders and policyholders that brings to determine a greater value of the insurance life policies such that the fair fixed guaranteed interest rate is less than the risk free interest rate.

The model and its assumptions

The model considers Universal Life with participation to the profit that typically earns a minimum guaranteed rate of return.
We assume that the dynamic of asset portfolio is given by the following stochastic continuous process:

$$d\mathrm{A}_t \,/\, \mathrm{A}_t = \mu\, dt + \sigma_{\mathrm{A}}\, dW_{\mathrm{A}}$$

μ denotes the drift of the process

dW_{A} denotes a standard Wiener process capturing the volatility of asset portfolio

σ_{A} denotes the instantaneous volatility of asset portfolio

At start time t, policyholders pay a premium L_t to be invested in an asset portfolio. For regulatory reasons, the Insurance Company will participate with its own capital for an amount equal to $\lambda\mathrm{L}_t$ to the acquisition of asset portfolio. The starting value invested in the portfolio will be:

$$\mathrm{A}_t = \mathrm{L}_t(1 + \lambda)$$

Where:

$$1 \,/\, (1+\lambda) = \alpha$$

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The final pay off of the insurance life policies maturing at time $T > t$ is given by the following:

$$L_T = L_t\left[1 + \mathrm{Max}\left(\beta \frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right) + \mathrm{Min}\left(\frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right)\right]$$

$$+ \ \mathrm{Max}\left\{L_t\, e^{r^*(T-t)} - L_t\left[1 + \mathrm{Max}\left(\beta \frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right) + \mathrm{Min}\left(\frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right)\right], 0\right\}$$

Where:

β denotes the participation coefficient to the profit
r^* denotes the fixed guaranteed interest rate

Thus, we have:

$$L_T = L_t\left[1 + \mathrm{Max}\left(\beta \frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right) - \mathrm{Max}\left(\frac{\bar{A}_t - A_T}{\bar{A}_t}, 0\right)\right]$$

$$+ \ \mathrm{Max}\left\{L_t\, e^{r^*(T-t)} - L_t\left[1 + \mathrm{Max}\left(\beta \frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right) - \mathrm{Max}\left(\frac{\bar{A}_t - A_T}{\bar{A}_t}, 0\right)\right], 0\right\}$$

$$L_T = L_t + \beta\alpha\, \mathrm{Max}\left[A_T - \bar{A}_t, 0\right] - \alpha\, \mathrm{Max}\left[\bar{A}_t - A_T, 0\right]$$

$$+ \ \alpha\, \mathrm{Max}\left\{\bar{A}_t\, e^{r^*(T-t)} - \left[\bar{A}_t + \mathrm{Max}\left(\beta A_T - \bar{A}_t, 0\right) - \mathrm{Max}\left(\bar{A}_t - A_T, 0\right)\right], 0\right\}$$

At this point, we can compute the fair value of the insurance life policies L_f at time t, it is easy to note that they are the same final pay off of an European option.
Thus, we have:

$$L_f = L_t\, P(t,T) + \alpha\left[\beta C(A_t, \bar{A}_t, T-t) - P(A_t, \bar{A}_t, T-t) + P\left(Q_t, \bar{A}_t\, e^{r^*(T-t)}, T-t\right)\right]$$

Where:

$$Q_t = P(t,T)\bar{A}_t + \beta C(A_t, \bar{A}_t, T-t) - P(A_t, \bar{A}_t, T-t)$$

$P(t,T) = e^{-r(T-t)}$ denotes the price of a default free zero coupon bond and r denotes the continuously compounded internal risk free rate of return. $C(A_t, \bar{A}_t, T-t)$ denotes the value of an European Call option written on the firm's underlying A_t, maturing at time T and with exercise price $\bar{A}_t$. $P(A_t, \bar{A}_t, T-t)$ denotes the value of an European Put option written on the firm's underlying A_t, maturing at time T and with exercise price $\bar{A}_t$.
$P(Q_t, \bar{A}_t\, e^{r^*(T-t)}, T-t)$ denotes the value of an European Put option written on the underlying Q_t, maturing at time T and with exercise price $\bar{A}_t\, e^{r^*(T-t)}$. We can note that the insurance life policies are a portfolio of default free zero coupon bonds with a short position on an European Put option, this reflects the fact that if the insurance life policies are not protected

their value decreases as the value of the asset portfolio decreases. We can note that the Put option is weighted with the weight of the premiums on the asset portfolio; this means that policyholders suffer just the loss on their initial investment. Moreover, there is a long position on an European Put option, this reflects the fact that the value of the insurance life policies is protected against the decrease of the value of asset portfolio. Furthermore, there is a long position on an European Call option weighted by the participation coefficient and the weight of the premiums on the asset portfolio that reflects the opportunity to share the profit when $\alpha A_t > L_t$. This means that shareholders not subsidize policyholders and that policyholders not subsidize shareholders. In fact, policyholders participate just to the profit generated from their initial investment. At this point, we have to observe that the protective Put option is a compound option written on a portfolio of options. Thus, to compute its value can seem a problem, but we have to note that the underlying options mature at same time of the compound option. Therefore, the compound option at time of maturity converges to the final pay off of the portfolio of options. In fact, we have $Q_T = A_T$ when $A_T < \bar{A}_t$ and $Q_T = \bar{A}_t + \beta(A_T - \bar{A}_t)$ when $A_T > \bar{A}_t$. Thus, we can see that the final value of the portfolio of options depends strictly from the value of asset portfolio such that its final value approximately replicates the value of asset portfolio. As we know the value of an European option is determined on the base of expectation on its final pay off. Hence, we can assume that the dynamic of the portfolio of options is given by the following stochastic continuous process:

$$dQ_t \,/\, Q_t \;=\; \mu_t \,\mathrm{dt} \;+\; \sigma_A \, dW_A$$

μ_t denotes the drift of the process that is stochastic because the process reverts to the value of asset portfolio, this is not a problem to get the value of the compound option because the hedging relation permits us to consider the spot-rate like drift.

dW_A denotes a standard Wiener process capturing the volatility of asset portfolio

σ_A denotes the instantaneous volatility of asset portfolio

If we put the following interest rate elasticity measure:

$$\eta_p(t,T) \;=\; -\,[\partial P(t,T) \,/\, \partial r]\;[1 \,/\, P(t,T)]$$

We have:

$$\eta_p(t,T) \;=\; (T - t)$$

$$\sigma_P(t,T) \;=\; \delta_r\,(T - t)$$

δ_r denotes the instantaneous volatility of the continuously compounded internal risk free rate of return

$\sigma_P(t,T)$ denotes the instantaneous volatility of the riskless security $P(t,T)$

Now we assume that the dynamic of the riskless security $P(t,T)$ is given by the following stochastic continuous process:

$$dP(t,T) \,/\, P(t,T) \;=\; \mu_t \; dt \;-\; \sigma_P(t,T)\, dW_r$$

dW_r denotes a standard Wiener process capturing the volatility of interest rate. The drift is determined by the shape of the yield curve and is not constant but stochastic; this is not

according with the absence of arbitrage opportunity but is a good approximation of the reality. Otherwise, we have to accept that the yield curve is the mean of the future expected spot-rate. Heath, Jarrow, Morton(1992) take the observed yield curve as initial condition for the forward rate curve, they assume that the forward rate curve reflects the expectation of the market on the future interest rates such that to avoid arbitrage opportunity it determines the yield curve. In this case the drift of process is given by the spot-rate and the Wiener process captures the volatility of the market expectation.
At this point, we can compute the value of the options by using the *numeraire* $P(t,T)$:

$$P(Q_t, L^*/\alpha, T-t) = P(t,T)(L^*/\alpha) N[-d_2] - Q_t N[-d_1]$$

$$C(A_t, \bar{A}_t, T-t) = A_t N[h_1] - P(t,T) \bar{A}_t N[h_2]$$

$$P(A_t, \bar{A}_t, T-t) = P(t,T) \bar{A}_t N[-h_2] - A_t N[-h_1]$$

Where:

$N[\ldots]$ denotes the cumulative normal distribution

$$d_1 = \frac{\ln\{Q_t / [P(t,T)(L^*/\alpha)]\} + \frac{1}{2} \sigma^2_{(t,T)} (T-t)}{\sigma_{(t,T)} \sqrt{(T-t)}}$$

$$d_2 = \frac{\ln\{Q_t / [P(t,T)(L^*/\alpha)]\} - \frac{1}{2} \sigma^2_{(t,T)} (T-t)}{\sigma_{(t,T)} \sqrt{(T-t)}}$$

$$h_1 = \frac{\ln\{A_t / [P(t,T)\bar{A}_t]\} + \frac{1}{2} \sigma^2_{(t,T)} (T-t)}{\sigma_{(t,T)} \sqrt{(T-t)}}$$

$$h_2 = \frac{\ln\{A_t / [P(t,T)\bar{A}_t]\} - \frac{1}{2} \sigma^2_{(t,T)} (T-t)}{\sigma_{(t,T)} \sqrt{(T-t)}}$$

While:

$$\sigma^2_{(t,T)} = [1/(T-t)] \int_t^T \sigma_A(t)^2 + \sigma_P(t,T)^2 - 2\rho(A,P)\, \sigma_A(t)\, \sigma_P(t,T)\; dt$$

$\rho(A,P)$ represents the correlation between the asset portfolio A_t and the riskless security $P(t,T)$

If we assume that $\sigma_A(t)$ is deterministic we get:

$$\sigma^2_{(t,T)} = \sigma_A^2 + \tfrac{1}{3} \sigma_P(t,T)^2 - \rho(A,P)\, \sigma_A\, \sigma_P(t,T)$$

We can note that the drift of the process of asset portfolio doesn't appear in the value of options, this is due to the fact that the hedging relation permits us to consider the spot-rate like drift. Thus, the expected value of asset portfolio, that can be get by using a Monte Carlo simulation, doesn't change the fair value of liabilities.
Now we can put:

$$\beta C(A_t, \bar{A}_t, T-t) = C(A_t, \bar{A}_t, T-t) - (1-\beta)\, C(A_t, \bar{A}_t, T-t)$$

The parity Put-Call gives us:

$$C(A_t, \bar{A}_t, T-t) - P(A_t, \bar{A}_t, T-t) = A_t - \bar{A}_t\, P(t,T)$$

If we insert it in the fair value of insurance life policies we have:

$$L_f = \alpha A_t + \alpha \left[P(Q_t, \bar{A}_t\, e^{r^*(T-t)}, T-t) - (1-\beta)\, C(A_t, \bar{A}_t, T-t) \right]$$

The policyholders participate to the value of asset portfolio for the amount α , and they have a Put option to protect the guaranteed value. In fact, if we put $\bar{A}_t\, e^{r^*(T-t)} = L^*/\alpha$, where L^* denotes the guaranteed value of the insurance life policies, the Put option goes in-the-money when $\alpha Q_t < L^*$. We can note that policyholders have a short position on a Call option that reflects the opportunity given to the shareholders to participate to the profit generated from the initial premium of the policyholders.
We can note that the fair value of the insurance life policies L_f is equal to the premium L_t when:

$$P(Q_t, \bar{A}_t\, e^{r^*(T-t)}, T-t) = (1-\beta)\, C(A_t, \bar{A}_t, T-t)$$

We can get an explicit formula for the participation coefficient:

$$\beta = \frac{C(A_t, \bar{A}_t, T-t) - P(Q_t, \bar{A}_t\, e^{r^*(T-t)}, T-t)}{C(A_t, \bar{A}_t, T-t)}$$

We can note that by varying the participation coefficient and the fixed guaranteed interest rate we can get the equilibrium. From a deep examination of the formula we can see that when the value of asset portfolio is nil, we have $L_f = \alpha\, \bar{A}_t\, e^{(r^* - r)(T-t)}$. If we don't consider the participation to the profit, from this value the fair value of the insurance life policies begins to increase as the value of asset portfolio increases. Thus, if $r^* = r$ we can't find a solution because the fair value of the insurance life policies is always greater than the initial premiums. Hence, we can find a solution just for:

$$r^* < r$$

We can note that as the participation to the profit increases the fixed guaranteed interest rate decreases.
As we can see from the equilibrium condition the value of λ doesn't influence the fair value of the insurance life policies. This is due to the fact that there isn't default risk. Thus, the initial value of equity E_t doesn't influence the fair value of the insurance life policies, but the fair value of the insurance life policies influences the fair value of equity E_f such that we have:

$$E_f = A_t - L_f$$

We can see that:

$$L_f = L_t \quad \Rightarrow \quad E_f = E_t$$

To achieve our aim we have to compute the elasticity measure of asset portfolio and liabilities. Thus, we put the following interest rate elasticity measure:

$$\eta_L = -(\partial L_f / \partial r)(1 / L_f)$$

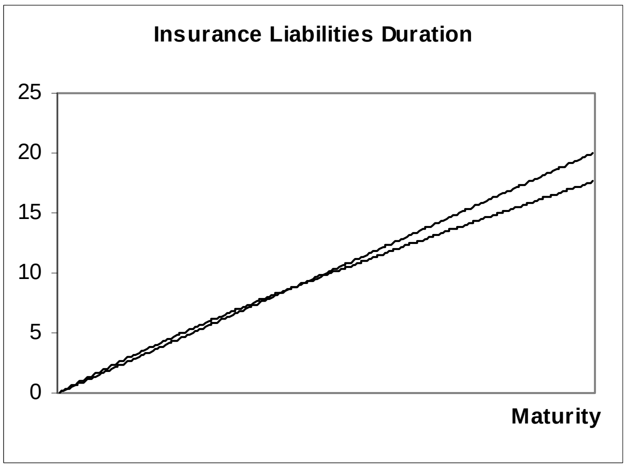
$$\eta_A = -(\partial A_t / \partial r)(1 / A_t)$$

If we assume that the asset portfolio is invested in the riskless security $P(t,T)$, where $Þ(A,P)$ is its weight on the asset portfolio, we have:

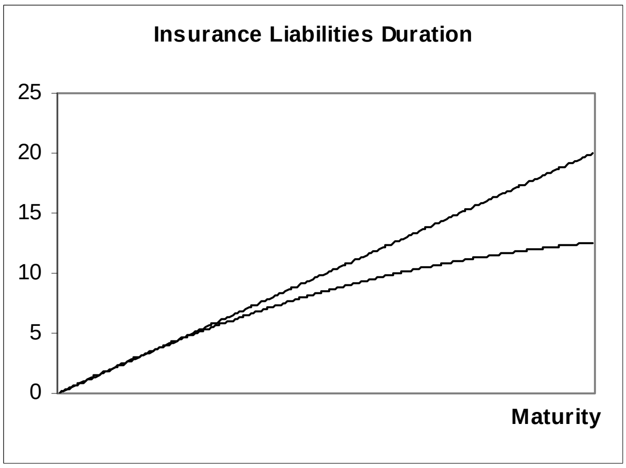
$$\eta_A = Þ(A,P)\,\eta_p(t,T)$$

$$\eta_L = [L^* P(t,T) / L_f]\,\eta_p(t,T) + [\alpha\, A_t \eta_A / L_f]\Big\{1 - (1-\beta)N[h_1] - \{\beta N[h_1] + N[-h_1]\}N[-d_1]\Big\}$$
$$- [\alpha\, \bar{A}_t P(t,T)\eta_p(t,T) / L_f]\Big\{e^{r^*(T-t)}N[d_2] + N[-d_1] - \{\beta N[h_2] + N[-h_2]\}N[-d_1] - N[h_2](1-\beta)\Big\}$$

The first term denotes the interest rate elasticity measure of a default free zero coupon bonds portfolio, the second and the third term measure the impact of the options on the insurance liabilities duration. We can note that for a high value of the elasticity of asset portfolio the options increase the insurance liabilities duration. Otherwise, they reduce the effective duration of the insurance liabilities. For rational values of parameters, we have the following prospect:



This result is due to our assumption that the Insurance Company invests prevalently in default free zero coupon bonds. In fact, we get this prospect by assuming that the weight of the bonds on the asset portfolio is equal to the weight of the debt on the asset portfolio, and that the participation coefficient to the profit is $\beta = 85\%$. Instead, if we don't consider the participation to the profit, we have the following prospect:



We can note that the participation to the profit increases the insurance liabilities duration. In fact to a maturity of twenty years corresponds a duration of twelve years. One of our assumptions is that fair equity value is equal to the difference between the market value of asset portfolio and the fair value of liabilities. As such, its effective elasticity is directly affected by both the elasticity of asset portfolio, the elasticity of liabilities and the leverage effect. Thus, we have:

$$\eta_A = (E_f / A_t)\, \eta_E + (L_f / A_t)\, \eta_L$$

$$\eta_E = (A_t / E_f)\, \eta_A - (L_f / E_f)\, \eta_L$$

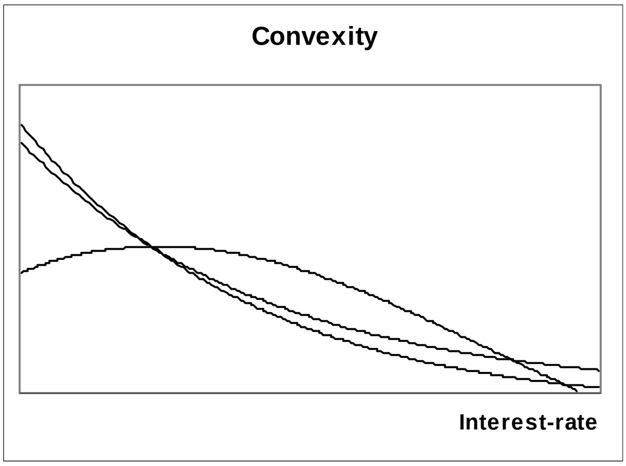
From this equation we can see that the elasticity of equity is nil when:

$$A_t\, \eta_A = L_f\, \eta_L$$

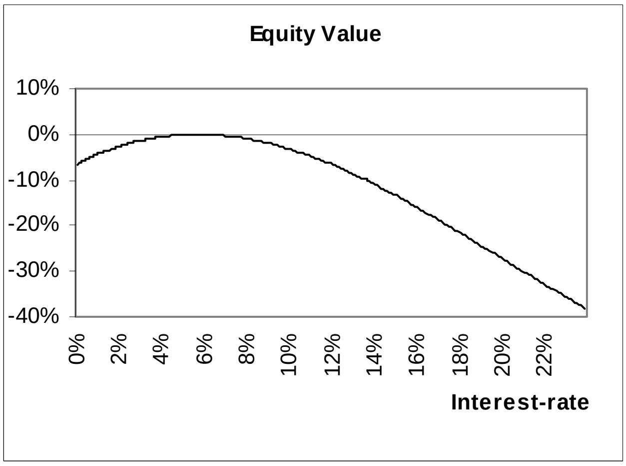
Where we equal the relative variation of the market value of asset portfolio and the fair value of liabilities. Thus, the value of Þ(A,P)* that makes nil the elasticity of equity and equals the convexity of asset portfolio and liabilities around the present value of the interest rate is:

$$\text{Þ}(A,P)^* = \frac{\alpha\, P(t,T)\Big\{ \{\beta N[h_2] + N[-h_2]\} N[-d_1] + N[h_2](1-\beta) + e^{r^*(T-t)} N[-d_2] - N[-d_1] \Big\}}{1 - \alpha \Big\{ 1 - (1-\beta) N[h_1] - \{N[h_1] + N[-h_1]\} N[-d_1] \Big\}}$$

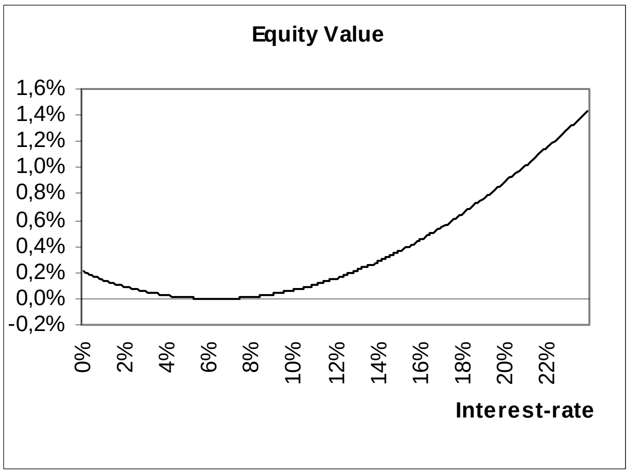
As we can see by the following figures the equity value is approximately immunized in a perfect way. In fact, the convexity of asset portfolio and liabilities is approximately equal around the present value of the interest rate.



More specifically, the behaviour of equity value resembles to a short position on a Collar written on the interest rate.



If we don't consider the participation to the profit, the behaviour of equity value resembles to a long position on a Collar written on the interest rate.



We have to note that for a big movement of the interest rate and the values of parameters, we have to rebalance the hedge ratio. This result is due to the impact of the derivatives of the cumulative normal distributions. In fact, the convexity of liabilities is stronger than the convexity of asset portfolio. If Þ(A,P) > Þ(A,P)* the behaviour of equity value resembles to a short position on interest rate and if Þ(A,P) < Þ(A,P)* the behaviour of equity value resembles to a long position on interest rate. At this point, we have to note that the equity value is immunized with respect to the fair value of the insurance life policies. Nevertheless, if the insurance life policies are not traded the only way for the policyholders to get back the loan is to exercise the surrender options. Thus, if the fair value of the insurance life policies is less than the refund value we get a loss on the equity value. However, we can consider the surrender options in the pricing of the insurance life policies by using the option theory. We can note that the surrender option is an American Put option written on the insurance life policies value. We have to note that the financial approach values the American Put option under the assumption of absence of arbitrage opportunity. As such, its value is greater than its pay off, this doesn't permit us to find a solution for the price of the insurance life policies. In fact, the initial fair value of the insurance life policies is always greater than the initial premiums. However, we have to note that the surrender options may expire without being exercised. Hence, we can weigh the American Put options with the probability that they will be exercised that can depend from the market conditions and the mortality tables. In fact, except for the start time, there isn't an arbitrage opportunity but just an incentive to exercise the surrender options. Thus, it is rational to expect that the market value of the surrender options is less than their pay off because many of them will expire without being exercised either if they are in-the-money. Thus, we have the following:

$$B_f = L_f + f_x P_A(L_f, B^*, T - t)$$

Where f_x denotes the probability that the surrender options will be exercised. B_f denotes the fair value of the insurance life policies with the surrender option and $P_A(L_f, B^*, T - t)$ denotes the value of an American Put option written on the underlying L_f, maturing at time T and with exercise price B^* that represents the refund value of the insurance life policies. If the interest rate increases such that the American Put option goes deeper in-the-money there is an incentive to exercise the surrender options. In Giandomenico(2006), we have:

$$P_A(L_f, B^*, T - t) = B^* N[b_1] - L_f N[b_2]$$

Where:

$$b_1 = \frac{\ln(B^*/L_f) + \frac{1}{2}\sigma^2_{(t,T)}(T - t)}{\sigma_{(t,T)}\sqrt{(T - t)}}$$

$$b_2 = \frac{\ln(B^*/L_f) - \frac{1}{2}\sigma^2_{(t,T)}(T - t)}{\sigma_{(t,T)}\sqrt{(T - t)}}$$

While:

$$\sigma^2_{(t,T)} = [1 / (T - t)] \int_t^T \sigma_L(t)^2 + \sigma_P(t,T)^2 - 2\rho(L,P)\, \sigma_L(t)\, \sigma_P(t,T)\; dt$$

$\rho(L,P)$ represents the correlation between the liabilities L_f and the riskless security $P(t,T)$

$\sigma_L(t)$ denotes the instantaneous volatility of the liabilities L_f

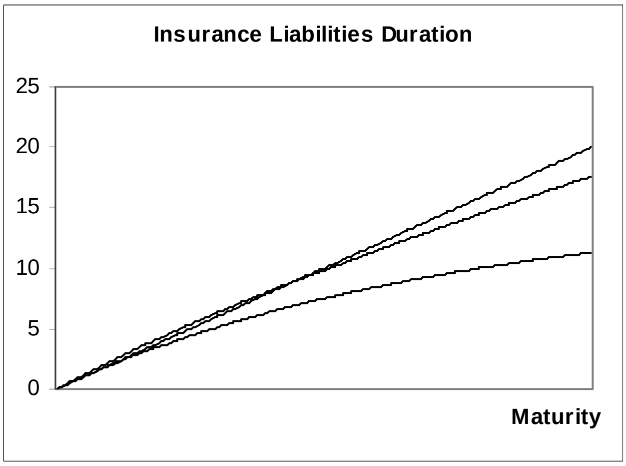
Thus, we have the following elasticity measure of the insurance liabilities:

$$\eta_B = - \frac{(\partial L_f / \partial r)\,(1 - f_x N[b_2])}{B_f}$$

We can note that the surrender options reduce the effective duration of the insurance liabilities. This means:

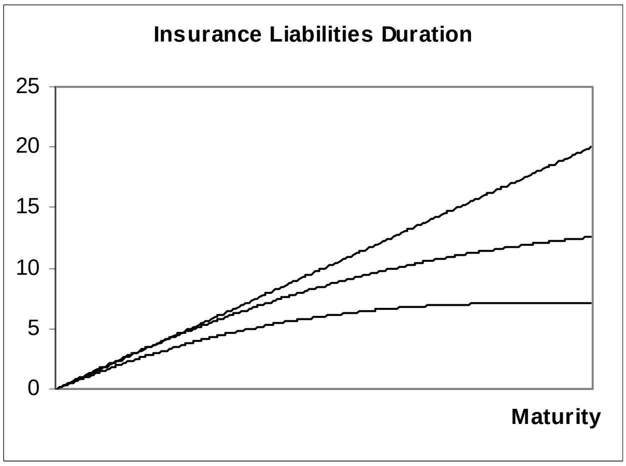
long is quite short

Thus, we have the following prospect:

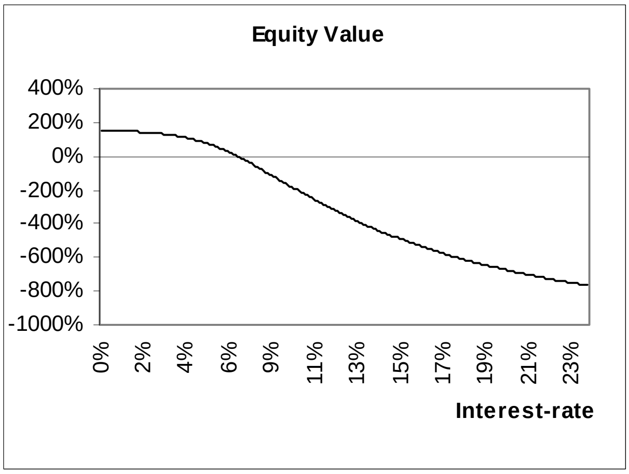


We can note that to a maturity of twenty years corresponds a duration of eleven years. If we assume that the weight of the bonds on the asset portfolio is less than the weight of the debt on the asset portfolio the surrender options reduce even more the insurance liabilities duration. At the same time, if we decrease the participation coefficient to the profit the insurance liabilities duration decreases even more.

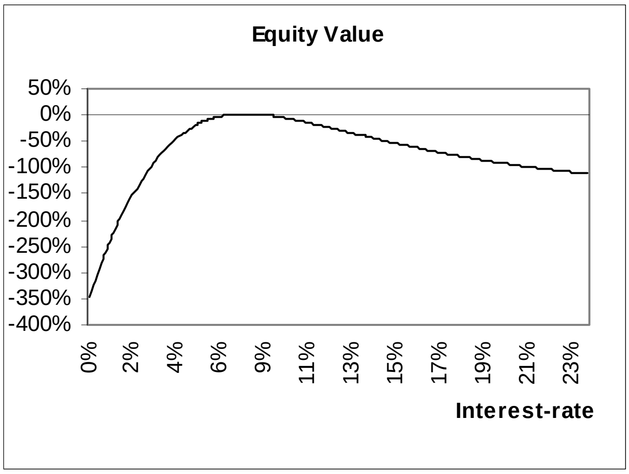
In fact, if we don't consider the participation to the profit, we have the following prospect:



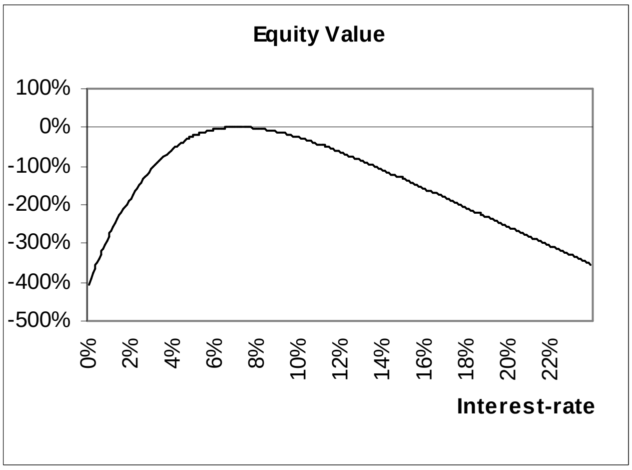
We can note that to a maturity of twenty years corresponds a duration of seven years. Furthermore, in the case of mortality issue the cash-flows can occur sooner than expected, this brings us to think that it reduces the effective liabilities duration. In fact, it is like to sell an American Put option to the policyholders. Insurers always insist upon the long maturity of their liabilities but the message covered here is something different. As result, the behaviour of equity value resembles to a short position on interest rates. Thus, we have the following prospect:



We can note that the Insurance Companies can be insolvent either if they invest prevalently in default free bonds. Thus, they have to hedge from the interest rate risk exposure to avoid the default risk. All this explains the problems of the insurance industry in the eighties when interest rates increased for the inflation. In fact, policyholders exercised the surrender option because they were attracted from alternative investments that offered greater rate of return. As result, the Insurance Companies got loss on the equity value given by the interest rates and the surrender options. To avoid this result and the past mistakes Insurance Companies have to reduce the duration of asset portfolio and to put a penalty on the insurance life policies in the case policyholders exercise the surrender options. If the Insurance Companies reduce the weight of the bonds on the asset portfolio, again the behaviour of equity value resembles to a short position on a Collar written on the interest rate but we have a greater loss on the equity value with respect to a movement of the interest rate. This result is due to the impact of the surrender options that increase the convexity of liabilities. Thus, we have the following prospect:



Instead, if the Insurance Companies reduce the maturity of asset portfolio the equity value is immunized with respect to small movement of the interest rate because the convexity of liabilities is stronger than the convexity of asset portfolio. As such, the behaviour of equity value resembles to a short position on a Straddle written on the interest rate. Thus, we have the following prospect:



We can note that in this case the loss on the equity value with respect to a movement of the interest rate is greater, this is due to the greater spread between the convexity of asset portfolio and liabilities. Note that we can immunize the equity value in a perfect way by taking a long position on a Straddle written on the interest rate. Otherwise, given our assumption that the dynamic of the interest rate follows a stochastic continuous process, we can keep the immunization in a dynamic way by selling and purchasing the default free bonds in the asset portfolio. Instead, if we assume that the interest rate can have a jump, the efficacy of immunization will depend negatively by the width of the jump. This solution is based on the hypothesis that there is a parallel shift of the yield curve with respect to a movement of the interest rates. Instead, if we assume that the interest rates are independent to immunize the equity value we can take a long position on a Call option written on the interest rate for each maturity such that the value of asset portfolio doesn't decrease if interest rates increase. We definitely conclude by noting that the contract of a insurance life policy determines the fixed guaranteed interest rate for all future cash-flows. Thus, the Insurance Companies are exposed to the risk that they can’t invest the future cash-flows at the same risk free interest rate. A way to avoid this problem in the pricing of the insurance life policies is to compute them on the base of the present value of the future cash-flows. In fact, the future cash-flows are credits of the Insurance Company. This approach is based on the hypothesis that the Insurance Companies discount in the financial market their credits, operations of this kind take place in London, where the Reinsurance Companies take either the risk that the insurance life policies underwritten will decrease. Instead, if the Insurance Companies don’t discount their credits, they are exposed to the risk that they can’t invest the future cash-flows at the same risk free interest rate. Hence, we can assume that Insurance Companies live in a world where the risk free interest rate for each maturity is determined on the base of the forward rate.

Conclusion

By using a contingent claim approach we have developed a model for Insurance Companies subjects to securitization. A fair value of insurance liabilities was derived with its duration measure. Hedging strategy was also examined and the equity value was immunized by using a perfect hedge ratio. This is a good approach for little Insurance Companies that desire to operate no more as brokerage firms but as investment firms for wider the profit without risking the default. The idea to invest prevalently in default free bonds deserves a careful inspection.

I thank Prof. Franco Nardini, University of Bologna, Prof. Eric Briys, HEC School of Management, My Family and some other anonymous for the contributions.

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