Altruism, Lifetime Uncertainty and Optimal Public Pension Contribution Rate

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***Abstract*—Assuming that individuals are altruistic, this paper employs an overlapping generations model with lifetime uncertainty to study the partially funded public pension in China. By comparing the market economy equilibrium with the social optimum allocation, we find the optimal firm contribution rate. Our simulation results show that this rate increases when the life expectancy rises, while decreases when the population growth rate falls. It decreases in the joint case of risen life expectancy and fallen population growth rate because it is much more sensitive to the latter than to the former. The result has some policy implications.**

Keywords-altruism; lifetime uncertainty; public pension; contribution rat

I. INTRODUCTION

The Chinese State Council Document 38 of 2005, "Decision on Improving Basic Pension System for Enterprise Employees", was issued in December, 2005. It introduces a new public pension system in the urban area: The government establishes an individual account for each employee and a social pool for all employees and retirees. Each firm contributes 20% of its payroll to the social pool, while each employee contributes 8% of her wage to her individual account. The social pool fund is used to pay the current retirees as pay-as-you-go (PAYG) pension benefits, while the accumulation in the individual account is used to pay the individual herself when she retires as fully funded pension benefits. Each retiree receives funded pension benefits from her individual account and PAYG pension benefits from the social pool. Such a public pension system is called a partially funded system.

It is worth studying the contribution rates in the following environment: The people are altruistic through bequests or gifts. The population growth rate has fallen because of the family planning (one-child) policy, and the life expectancy has risen because of improved living and medical conditions.

Sheshinski and Weiss [1] examine the annuity aspect of social security within the framework of an overlapping generations (OLG) model, in which the duration of life is assumed to be uncertain. Abel [2] solves the consumption and portfolio decision problem of a consumer who lives for either one period or two periods and who can hold his wealth in the form of riskless bonds and actuarially fair annuities. Abel [3] demonstrates the effect of a lump-sum tax on debt neutrality in an OLG model with lifetime uncertainty. There is not any production sector in these models. Fuster [4] studies how the lack of an annuities market affects savings behavior and intergenerational transfers in a dynastic OLG economy. It is found that the answer to this question depends crucially on altruism. Zhang *et al*. [5] examine how mortality decline affects long-run steady state growth by assuming actuarially fair annuity markets in an OLG model with uncertain lifetime and social security.

Samuelson [6] studies the optimum social security in a life-cycle growth model. He adjusts the capital-labor ratio to the modified golden rule level to maximize the social welfare by controlling the social security taxes. The approach to find the optimal social security is to equate the interest rate in the decentralized economy to the growth rate of economy. Blanchard and Fischer [7] elaborate the principle of social optimum. Social planner maximizes social welfare by rationally allocating social resources. This approach can be used to derive the optimal pension contribution rate in China's partially funded public pension system.

Based on the literature, we employ an OLG model with altruism and lifetime uncertainty to investigate the public pension system in China's urban area. We introduce production sector into Abel's [2] model, and replace the fully funded public pension system with a partially funded one. Instead of only individuals make pension contributions in the literature; both individuals and firms make contributions in this model. Applying the approach used by Samuelson [6], we seek the optimal contribution rates, and obtain a result that is helpful to firms.

II. MARKET ECONOMY

The generation born at the beginning of period t is called generation t. The population grows at the rate of $n=N_t/N_{t-1}-1$, where N_t is the population of generation t. Each individual survives to the end of her working period certainly, but survives in retirement period with probability $p \in [0,1]$.

A. Individuals

In the working period, each individual earns wage by supplying inelastically one unit of labor and makes pension contributions. She gets bequests or gifts from her parent, consumes part of her incomes and saves the rest. If she dies at the beginning of her retirement period, her savings with accrued interest and individual account benefits are inherited equally by her children as bequests. If she survives in the retirement period, she distributes her savings with accrued interest, individual account benefits and social pool benefits between her consumption and the gifts to her children.

Each individual derives utility from her working-period consumption c_{1t}, possible retirement-period consumption c_{2t+1}, bequests b_{t+1}^D or gifts b_{t+1}^S. The utility is described by an additively separable logarithmic function. Thus, the utility maximization problem is:

$$\mathrm{Max}\, U_t = \ln c_{1t} + \theta p \ln c_{2t+1} + \delta(1-p)\ln b_{t+1}^D + \delta p \ln b_{t+1}^S, \quad (1)$$

$$s.t. \quad c_{1t} = \left[(1-p)b_t^D + p b_t^S\right]/(1+n) + (1-\tau)w_t - s_t, \quad (2)$$

$$b_{t+1}^D = (1+r_{t+1})s_t + B_{t+1}, \quad (3)$$

$$c_{2t+1} + b_{t+1}^S = (1+r_{t+1})s_t + B_{t+1} + P_{t+1}, \quad (4)$$

where $\theta \in (0,1)$ denotes the individual discount rate, $\delta \in (0,1)$ the altruism intensity, w_t the wage, τ the individual contribution rate, s_t the savings, r_{t+1} the interest rate, B_{t+1} the individual account benefits, and P_{t+1} the social pool benefits. Suppose that $\theta > \delta$ since the limitation of altruism. The first-order conditions for the utility maximization problem are:

$$\delta(1+r_{t+1})\left[(1-p)/b_{t+1}^D + p/b_{t+1}^S\right] = 1/c_{1t}, \quad (5)$$

$$\theta b_{t+1}^S = \delta c_{2t+1}. \quad (6)$$

Equation (5) describes the tradeoff between the marginal utility of working-period consumption and that of bequests or gifts. Equation (6) describes the tradeoff between the marginal utility of gifts and that of retirement-period consumption.

B. Firms

Firms produce homogenous commodity in competitive markets. The production is described by Cobb-Douglas function $Y_t = AK_t^\alpha N_t^{1-\alpha}$ or $y_t = Ak_t^\alpha$, where Y_t is the output in period t, K_t the capital stock, $\alpha \in (0,1)$ the capital share of income, A the productivity, $k_t = K_t/N_t$ the capital-labor ratio, and y_t the output-labor ratio.

Firms make pension contributions at the rate of $\eta \in (0,1)$ on their payroll. According to the product distribution, one can get $AK_t^\alpha N_t^{1-\alpha} = r_t K_t + (1+\eta)w_t N_t$. The first-order conditions for the profit maximization are:

$$r_t = \alpha A k_t^{\alpha-1}, \quad (7)$$

$$w_t = (1-\alpha)Ak_t^\alpha/(1+\eta). \quad (8)$$

C. The Government

The social pool fund is paid to the retirees in the current period as PAYG pension benefits: $pN_{t-1}P_t = \eta w_t N_t$, or

$$P_t = (1+n)\eta w_t / p. \quad (9)$$

The accumulation in the individual account is used to pay the individual when she retires in the next period as funded pension benefits:

$$B_{t+1} = (1+r_{t+1})\tau w_t. \quad (10)$$

D. The Capital Market

The savings and the individual pension contributions in period t generate the capital stock in period t+1 (See Blanchard and Fischer [7] or Barro and Sala-i-Martin [8] for details):

$$s_t + \tau w_t = (1+n)k_{t+1}. \quad (11)$$

E. Dynamic Equilibrium System

Substituting (2)-(4) and (7)-(11) into (5)-(6) and arranging gives a dynamic equilibrium system described by the following difference equation:

$$\left[k_t + Ak_t^\alpha - (1+n)k_{t+1} - \frac{p\theta}{\theta+\delta}\left(k_t + \alpha Ak_t^\alpha + \frac{\eta}{p}\cdot\frac{1-\alpha}{1+\eta}Ak_t^\alpha\right)\right]\cdot$$
$$\left[(1-p)\frac{\delta}{\theta+\delta}\left(k_{t+1} + \alpha Ak_{t+1}^\alpha + \frac{\eta}{p}\cdot\frac{1-\alpha}{1+\eta}Ak_{t+1}^\alpha\right) + p\left(k_{t+1} + \alpha Ak_{t+1}^\alpha\right)\right]\cdot$$
$$= k_{t+1}\frac{1+n}{\theta+\delta}\left(k_{t+1} + \alpha Ak_{t+1}^\alpha + \frac{\eta}{p}\cdot\frac{1-\alpha}{1+\eta}Ak_{t+1}^\alpha\right) \quad (12)$$

The individual contribution rate has no effect on the capital-labor ratio because the mandatory savings (individual pension contributions) crowd out the voluntary savings by one-for-one. However, the firm contribution rate has effect on the capital-labor ratio because it appears in the dynamic system.

III. Social Optimum

By controlling the firm contribution rate, one can adjust the capital-labor ratio to the modified golden rule level to maximize social welfare. The social welfare is the sum of the lifetime utilities of all current and future generations (Blanchard and Fischer [7] and Groezen *et al*. [9] also use an analogous social welfare function):

$$W = \theta p \ln c_{20} + \delta(1-p)\ln b_0^D + \delta p \ln b_0^S + \sum_{i=0}^{\infty}\rho^i\left[\ln c_{1i} + \theta p \ln c_{2i+1} + \delta(1-p)\ln b_{i+1}^D + \delta p \ln b_{i+1}^S\right], \quad (13)$$

where $\rho \in (0,1)$ is the social discount rate, which indicates how much the social planner weights different generations in the social welfare calculations.

The resource constraint is

$$k_i + Ak_i^\alpha = (1+n)k_{i+1} + c_{1i} + p\,c_{2i}/(1+n). \quad (14)$$

The initial condition, k_0, is given. The social planner maximizes the social welfare subject to the resource constraint and initial condition. The first-order conditions for the social welfare maximization problem are:

$$\theta(1+n)c_1^* = \rho c_2^*, \quad (15)$$

$$k^* = \left[(1+n-\rho)/(\rho\alpha A)\right]^{\frac{1}{\alpha-1}}, \quad (16)$$

where the superscript * denotes the optimal steady state values of variables. The capital-labor ratio satisfying (16) is at the modified golden rule level, where the social welfare achieves the maximum.

In order to maximize the social welfare of the market economy in the steady state, we control the policy variable to adjust the capital-labor ratio of the market economy to the modified golden rule level, namely, $k=k^*$. Substituting (16) into (12) and arranging gives

$$p\theta\delta(1-p)\varphi^2 + p\frac{1+n}{\rho}\left(n-\frac{1+n-\rho}{\rho\alpha}\right) + \left[p^2\theta\frac{1+n}{\rho}+(1+n)+\delta(1-p)\left(n-\frac{1+n-\rho}{\rho\alpha}\right)\right]\varphi = 0 \quad (17)$$

where $\varphi=\frac{1}{\theta+\delta}\left(\frac{1+n}{\rho}+\frac{\eta^*}{p}\frac{1-\alpha}{1+\eta^*}\cdot\frac{1+n-\rho}{\rho\alpha}\right)$. Hence, the optimal firm contribution rate η^* is an implicit function of θ, δ, ρ, α, p and n.

IV. OPTIMAL FIRM CONTRIBUTION RATE

The effects of the life expectancy and population growth rate on the optimal firm contribution rate can be obtained by partially differentiating η^* with respective to p and n. It is shown that the signs of the derivatives depend on the values of the parameters. Hence, we estimate the parameter values and check the effects by simulating.

A. *Estimation of Parameter Values*

Analogous to Pecchenino and Pollard [10], we assume that the individual discount rate per year is 0.985, the altruism intensity per year 0.965, and a period length 26 years. Hence, in a period, $\theta=0.985^{26}$, and $\delta=0.965^{26}$.

The capital share of income, α, is usually to be estimated as 0.3 in developed countries. The labor in China is comparatively cheaper, and thus the labor share of income is lower, while the capital share of income is higher than that in developed countries. Hence, we assume that α in China could be 0.35.

According to UN Secretariat [11], the life expectancy of Chinese people in 2000-2005 is 72.0 years old. Since the life-span from birth to the end of working-period is 52 years and that from birth to the end of retirement-period is 78 years, by virtue of the concept of life expectancy, one can get that $(1-p)\times52+p\times78=72.0$, which gives $p\approx76.92\%$.

There are several calibers for population statistics in China. Since the public pension system in urban area is different from that in rural area, and only the former is studied in this paper, so the caliber of "Urban Population" is selected. The population growth rate during 1978-2004 is computed to be $n\approx2.148$ according to the "Population and Its Composition" in China Statistical Yearbook, which has reflected immigration from countryside.

The social discount rate is estimated in accordance with the stipulated firm contribution rate, 20%, which is the optimal firm contribution rate adopted by the government. Substituting the above relative parameter values into (17) and calculating repeatedly until the equation holds, we get $\rho\approx0.7473$. These estimated values are baseline parameter values.

B. *Risen Life Expectancy*

According to UN Secretariat [11], the life expectancy of Chinese people in 2005-2010 is 73.0 years old, and that in 2010-2015 is 74.0 years old. Hence, the survival probabilities in retirement period are 80.77% and 84.62%, respectively. When the survival probability is 80.77%, substituting it and the baseline values of θ, δ, α, ρ and n into (17) gives $\eta^*\approx20.60\%$. When the survival probability is 84.62%, analogous simulation gives the result shown in Table Ⅰ. The rise in the life expectancy leads to the increase in the optimal firm contribution rate.

C. *Fallen Population Growth Rate*

Since the current term, 2005-2015, is more concerned, we look into the population growth rates during the periods including the current term, such as period 1984-2010 and period 1989-2015. It is necessary to predict the urban population in each year from 2007 to 2015. The sample is the "Urban Population" in China Statistical Yearbook in each year from 1978 to 2006. Using the TREND function in Excel gives the result shown in Appendix A. The value of $R^2\approx0.9725$ implies a very good fit. Computing the population growth rates gives that the rate during 1984-2010 is 1.500, and that during 1989-2015 is 1.272. Simulating with the population growth rates and the baseline values of θ, δ, α, ρ and p gives the result shown in Table Ⅱ. The fall in the population growth rate induces the decrease in the optimal firm contribution rate.

D. *Risen Life Expectancy and Fallen Population Growth Rate*

Simulating the four combinations of the above survival probabilities and population growth rates gives the result shown in Table Ⅲ. The optimal firm contribution rate falls under the joint case of risen life expectancy and fallen population growth rate. This is because that the elasticity of η^* with respect to n is much higher than to p. It implies that the optimal firm contribution rate is much more sensitive to the population growth rate than to life expectancy.

TABLE I. η^* UNDER RISEN LIFE EXPECTANCY

p	76.92%	80.77%	84.62%
η^*	20.00%	20.60%	21.11%

TABLE II. η^* UNDER FALLEN POPULATION GROWTH RATE

n	2.148	1.500	1.272
η^*	20.00%	14.19%	11.23%

TABLE III. η^* UNDER RISEN LIFE EXPECTANCY AND FALLEN POPULATION GROWTH RATE AND THE ELASTICITY

p	n	η^*	elasticity of η^* with respect to p	elasticity of η^* with respect to n
80.77%	1.500	14.58%	–	–
80.77%	1.272	11.51%	–	138%
84.62%	1.500	14.87%	43%	–
84.62%	1.272	11.71%	35%	140%

V. CONCLUSIONS

Supposing that individuals are altruistic, this paper employs an OLG model with lifetime uncertainty to investigate the urban public pension in China. By controlling the policy variable and adjusting the capital-labor ratio in market economy to the modified golden rule level to maximize the social welfare, we find the optimal firm contribution rate. It is shown to be dependent on the individual discount rate, altruism intensity, social discount rate, capital share of income, life expectancy and population growth rate. Simulations show that the rise in the life expectancy leads to the increase in the optimal firm contribution rate, whereas the fall in the population growth rate induces the decrease in the optimal firm contribution rate. Because the optimal firm contribution rate is much more sensitive to the population growth rate than to life expectancy, it decreases under the joint case of risen life expectancy and fallen population growth rate.

This result is advantageous for firms to reduce the heavy burden regarding social insurance. The social insurance in China includes public pension, medical, unemployment, work-related injury, and maternity insurances. The firm contribution rates for the five lines of insurance are 20%, 6%, 2%, 1%, and 1%, respectively. The total contribution rate composed of social insurance and housing security has amounted to 40% of firm's payroll. The heavy burden can be reduced if the firm pension contribution rate could be decreased.

APPENDIX A. PREDICTED URBAN POPULATION IN EACH YEAR FROM 2007 TO 2015

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015
Population (10000 persons)	55794	57208	58621	60035	61448	62862	64276	65689	67103

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