Asset Liability Management for Banks

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Abstract: The model, by using a contingent claim approach, determines the fair value of the banks liabilities accounting for the protection and the surrender possibility. Furthermore, it determines the implied duration of banks liabilities so to show that the surrender possibility will reduce the effective duration of banks liabilities. Implications for the immunization are also treated.

Introduction

The asset liability management has its origins in the duration analysis proposed by Macaulay(1938) and Redington(1952). In recent years it becomes a tool of integrated analysis of assets and liabilities so to value not only the interest rate risk but the liquidity risk, solvency risk, firm strategies and asset allocation as well. Indeed, the new banks rules developed in Basilea II focused on the solvency risk so to impose a required amount of equity value on the base of the risk associated to the investments of asset portfolio. Bank industry responded to the convergence by developing internal model based essentially on the Value-at-Risk (VaR), parametric (GARCH, EGARCH) and simulated (Monte Carlo), extended to Conditional Value-at-Risk (CVaR) and Copula approaches. Other banks extended the analysis to the cash flows by using a stress testing to generate different scenarios. In this case it is possible to analyse how can evolve the cash flows so to study a strategy to hedge the risk exposure. The banks with greater equity value that means to have the possibility to invest in riskier assets focused on the portfolio insurance. The basic idea is to construct a synthetic Put option on the value of asset portfolio by taking a long position on the risky assets and on the default free bonds such that their weight will be rebalanced dynamically so to replicate the value of a portfolio of risky assets with a protective Put option. This approach has the advantage that the banks don't pay the premiums for the options but the replication isn't perfect due to the possibility of a jump of the risky assets. The duration analysis of banks liabilities is still an open question due to the fact that the deposits don't have a time to maturity known *ex-ante* because it is possible to know it just *ex-post*. Indeed, the duration of deposits is not only a matter of maturity but it is affected by the contractual geometry and the possibility of surrender as we will show in the next section. In fact, by using a contingent claim approach, it is possible to get the implied duration of banks liabilities in a closed form. Straightforward, it is possible to immunize the equity value of a bank by equalling the duration of bonds portfolio in the assets to the implied duration of the bank liabilities as suggested by Macaulay(1938) and Redington(1952).

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The model and its assumptions

The banks liabilities differ from other liabilities because usually they are securitized. Hence, we have to value them in absence of default risk. Furthermore, the majority of asset portfolio is composed by default free bonds and mortgages. Moreover, in the case of solvency problems the deposits are protected from the central bank or other banks.
In the model we assume that the dynamic of asset portfolio is given by the following stochastic continuous process:

$$dA_t \,/\, A_t = \mu\, dt + \sigma_A\, dW_A$$

μ denotes the drift of the process

dW_A denotes a standard Wiener process capturing the volatility of asset portfolio

σ_A denotes the instantaneous volatility of asset portfolio

At start time t the clients make deposits L_t to be invested in an asset portfolio. For regulatory reasons, the bank will participate with its own capital for an amount equal to λL_t to the acquisition of asset portfolio. The starting value invested in the portfolio will be:

$$A_t = L_t(1 + \lambda)$$

Where:

$$1 \,/\, (1+\lambda) = \alpha$$

The final pay off of the deposits maturing at time $T > t$ is given by the following:

$$L_T = L_t\left[1 + \mathrm{Min}\left(\frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right)\right]$$

$$+ \mathrm{Max}\left\{L_t\, e^{r^*(T-t)} - L_t\left[1 + \mathrm{Min}\left(\frac{A_T - \bar{A}_t}{\bar{A}_t}, 0\right)\right], 0\right\}$$

r^* denotes the fixed guaranteed interest rate

Thus, we have:

$$L_T = L_t\left[1 - \mathrm{Max}\left(\frac{\bar{A}_t - A_T}{\bar{A}_t}, 0\right)\right]$$

$$+ \mathrm{Max}\left\{L_t\, e^{r^*(T-t)} - L_t\left[1 - \mathrm{Max}\left(\frac{\bar{A}_t - A_T}{\bar{A}_t}, 0\right)\right], 0\right\}$$

$$L_T = L_t - \alpha\, \mathrm{Max}\left[\bar{A}_t - A_T, 0\right]$$

$$+ \alpha\, \mathrm{Max}\left\{\bar{A}_t\, e^{r^*(T-t)} - \left[\bar{A}_t - \mathrm{Max}\left(\bar{A}_t - A_T, 0\right)\right], 0\right\}$$

At this point, we can compute the fair value of the deposits L_f at time t . It is easy to note that they are the same final pay off of an European option. Thus, we have:

$$L_f = L_t P(t,T) + \alpha \left[P(Q_t , \bar{A}_t e^{r^*(T - t)} , T - t) - P(A_t , \bar{A}_t , T - t) \right]$$

Where:

$$Q_t = P(t,T)\bar{A}_t - P(A_t , \bar{A}_t , T - t)$$

$P(t,T)$ denotes the price of a default-free zero coupon bond such that $P(T,T) = 1$.
$P(A_t , \bar{A}_t , T - t)$ denotes the value of an European Put option written on the firm's underlying A_t , maturing at time T and with exercise price $\bar{A}_t$.
$P(Q_t , \bar{A}_t e^{r^*(T - t)} , T - t)$ denotes the value of an European Put option written on the underlying Q_t , maturing at time T and with exercise price $\bar{A}_t e^{r^*(T - t)}$.
We can note that the deposits are a portfolio of default-free zero coupon bonds with a short position on an European Put option, this reflects the fact that if the deposits are not protected their value decreases as the value of the asset portfolio decreases. We can note that the Put option is weighted with the weight of the deposits on the asset portfolio; this means that clients suffer just the loss on their initial investment. Moreover, there is a long position on an European Put option, this reflects the fact that the value of the deposits is protected against the decrease of the value of asset portfolio. At this point, we have to observe that the protective Put option is a compound option written on an option. Thus, to compute its value can seem a problem, but we have to note that the underlying option matures at same time of the compound option. Therefore, the compound option at time of maturity converges to the final pay off of the option. In fact, we have $Q_T = A_T$ when $A_T < \bar{A}_t$. Thus, we can see that the final value of Q_t depends strictly from the value of asset portfolio. As we know the value of an European option is determined on the base of expectation on its final pay off.
The parity Put-Call gives us:

$$C(A_t , \bar{A}_t , T - t) - P(A_t , \bar{A}_t , T - t) = A_t - \bar{A}_t P(t,T)$$

If we insert it in the fair value of the deposits we have:

$$L_f = \alpha A_t + \alpha \left[P(Q_t , \bar{A}_t e^{r^*(T - t)}, T - t) - C(A_t , \bar{A}_t , T - t)\right]$$

The clients participate to the value of asset portfolio for the amount α . They have a Put option to protect the guaranteed value. In fact, if we put $\bar{A}_t e^{r^*(T - t)} = L^*/\alpha$, where L^* denotes the guaranteed value of the deposits, the Put option goes in-the-money when $\alpha Q_t < L^*$. We can note that the clients have a short position on a Call option that reflects the opportunity given to the shareholders to take the profit generated from the deposits. Now we assume that the dynamic of the default free zero coupon bond $P(t,T)$ is given by the following stochastic continuous process:

$$dP(t,T) / P(t,T) = r_t \, dt - \sigma_P(t,T) \, dW_r$$

r_t is the spot rate and denotes the drift of the process

dW_r denotes a standard Wiener process capturing the volatility of the market expectation

$\sigma_P(t,T)$ denotes the instantaneous volatility of the default free zero coupon bond

Heath-Jarrow-Morton(1992) take the observed yield curve as initial condition for the forward rate curve, they assume that the forward rate curve reflects the expectation of the market on the future interest-rates such that to avoid arbitrage opportunity it determines the yield curve. They assume that the yield curve is the mean of the future expected spot rate.
If we put the following interest rate elasticity measure:

$$\eta_p(t,T) = -[\partial P(t,T) / \partial r]\,[1 / P(t,T)]$$

We have:

$$\eta_p(t,T) = (T-t)$$

Thus, we have:

$$\sigma_P(t,T) = \delta_r\, \eta_p(t,T)$$

Where:

δ_r denotes the instantaneous volatility of the market expectation

At this point, we can compute the value of the options by using the *numeraire* $P(t,T)$:

$$P(Q_t, L^*/\alpha, T-t) = P(t,T)\,(L^*/\alpha)\,N[-d_2] - Q_t\,N[-d_1]$$

$$C(A_t, \bar{A}_t, T-t) = A_t\,N[h_1] - P(t,T)\,\bar{A}_t\,N[h_2]$$

$$P(A_t, \bar{A}_t, T-t) = P(t,T)\,\bar{A}_t\,N[-h_2] - A_t\,N[-h_1]$$

Where:

N[…] denotes the cumulative normal distribution

$$d_1 = \frac{\ln\{Q_t / [P(t,T)(L^*/\alpha)]\} + \tfrac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln\{Q_t / [P(t,T)(L^*/\alpha)]\} - \tfrac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

$$h_1 = \frac{\ln\{A_t / [P(t,T)\bar{A}_t]\} + \tfrac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

$$h_2 = \frac{\ln\{A_t / [P(t,T)\bar{A}_t]\} - \tfrac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

While:

$$\sigma^2_{(t,T)} = [1/(T-t)] \int_t^T \sigma_A(t)^2 + \sigma_P(t,T)^2 - 2\rho(A,P)\,\sigma_A(t)\,\sigma_P(t,T)\ dt$$

$\rho(A,P)$ represents the correlation between the asset portfolio A_t and the riskless security $P(t,T)$

If we assume that $\sigma_A(t)$ is deterministic we get:

$$\sigma^2_{(t,T)} = \sigma_A{}^2 + ⅓\,\sigma_P(t,T)^2 - \rho(A,P)\,\sigma_A\,\sigma_P(t,T)$$

Now we assume that we have the following portfolio of default free zero coupon bonds in the assets:

$$V_t = \lambda\, P_1(t,\tau) + (1-\lambda)\, P_2(t,T)$$

Where:

$$\lambda = þ(A,P_1) / þ(A,V)$$

$$1-\lambda = þ(A,P_2) / þ(A,V)$$

$$þ(A,V) = þ(A,P_1) + þ(A,P_2) \leq 1$$

$þ(A,V)$ denotes the weight of the riskless bonds portfolio V_t on the total value of assets A_t
$þ(A,P_1)$ denotes the weight of the riskless security $P_1(t,\tau)$ on the total value of assets A_t
$þ(A,P_2)$ denotes the weight of the riskless security $P_2(t,T)$ on the total value of assets A_t

The dynamic of the default free zero coupon bonds portfolio V_t is given by the following stochastic continuous process:

$$dV_t / V_t = r_t\, dt - \sigma_V\, dW_r$$

Where:

$$\sigma_V{}^2 = \lambda^2 \sigma^2_p(t,\tau) + (1-\lambda)^2 \sigma^2_p(t,T) + 2\,\rho(P_1P_2)\lambda\,\sigma_p(t,\tau)\,(1-\lambda)\,\sigma_p(t,T)$$

$\rho(P_1P_2) = 1$ that is equal to assume a parallel shift of the yield curve with respect to a movement of the market expectation
It represents the correlation between the riskless security $P_1(t,\tau)$ and the riskless security $P_2(t,T)$.

Thus, we have:

$$\eta_V = \lambda\,\eta_p(t,\tau) + (1-\lambda)\,\eta_p(t,T)$$

$$\sigma_V = \eta_V \delta_r$$

To achieve our aim we have to compute the elasticity measure of asset portfolio and liabilities. Hence, we put the following interest rate elasticity measure:

$$\eta_L = -(\partial L_f / \partial r)(1 / L_f)$$

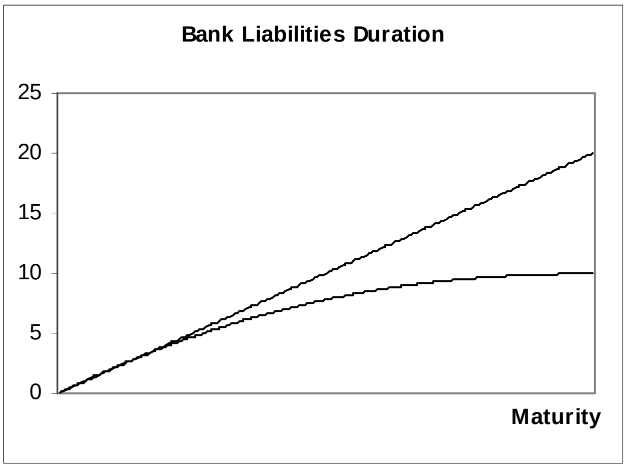
$$\eta_A = -(\partial A_t / \partial r)(1 / A_t)$$

Thus, we have:

$$\eta_A = þ(A,V)\,\eta_V$$

$$\eta_L = [L^*P(t,T) / L_f]\,\eta_p(t,T) + [\alpha\, A_t \eta_A / L_f]\Big\{1 - N[h_1] - N[-h_1]N[-d_1]\Big\}$$
$$- [\alpha\, \bar{A}_t P(t,T)\eta_p(t,T) / L_f]\Big\{e^{r^*(T-t)}N[d_2] + N[-d_1] - N[-h_2]N[-d_1] - N[h_2]\Big\}$$

The first term denotes the interest-rate elasticity measure of a default-free zero coupon bond, the second and the third term measure the impact of the options on the bank liabilities duration. If we assume that the weight of the portfolio of bonds on the assets is equal to the weight of bank liabilities on the assets and that the duration of the bonds portfolio in the asset is equal to the time to maturity, for rational value of parameters, we have the following prospect:



We can note that the options reduce the effective duration of bank liabilities. In fact, to a maturity of twenty years corresponds a duration of ten years. At this point, we have to note that if the fair value of the bank liabilities is less that the account value of the deposits we get a loss on the equity value. Indeed, there are surrender options for the clients. The surrender options are American Put options written on the fair value of bank liabilities with strike price equal to the account value of the deposits. We have to note that the surrender options may expire without being exercised. Hence, we can weigh the American Put options with the probability that they will be exercised. Thus, we have the following:

$$B_f = L_f + f_x\, P_A(L_f, L_t, T - t)$$

Where f_x denotes the probability that the surrender options will be exercised. B_f denotes the fair value of the bank liabilities with the surrender option and $P_A(L_f, L_t, T-t)$ denotes the value of an American Put option written on the underlying L_f, maturing at time T and with exercise price L_t that represents the account value of the deposits. If the interest rate increases such that the American Put option goes deeper in-the-money there is an incentive to exercise the surrender options. In Giandomenico(2006), we have:

$$P_A(L_f, B^*, T-t) = L_t N[b_1] - L_f N[b_2]$$

Where:

$$b_1 = \frac{\ln(L_t / L_f) + \frac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

$$b_2 = \frac{\ln(L_t / L_f) - \frac{1}{2}\sigma^2_{(t,T)}(T-t)}{\sigma_{(t,T)}\sqrt{(T-t)}}$$

While:

$$\sigma^2_{(t,T)} = [1/(T-t)] \int_t^T \sigma_L(t)^2 + \sigma_P(t,T)^2 - 2\rho(L,P)\,\sigma_L(t)\,\sigma_P(t,T)\; dt$$

$\rho(L,P)$ represents the correlation between the liabilities L_f and the riskless security $P(t,T)$

$\sigma_L(t)$ denotes the instantaneous volatility of the liabilities L_f

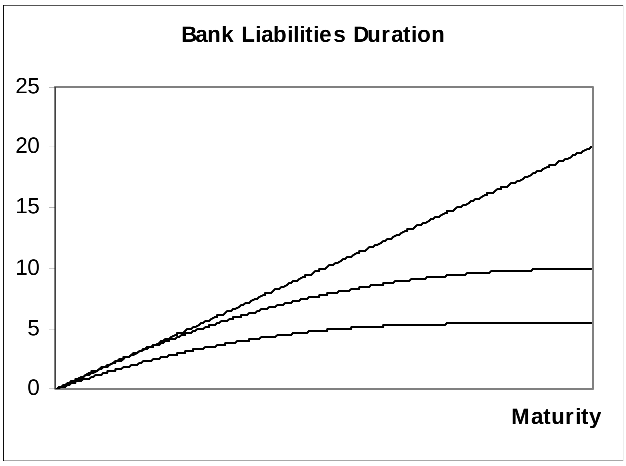
Now we have to observe that the mark to market of the bank liabilities is equal to the account value of the deposits. Thus, we can calibrate the probability of surrender for each time to maturity such that:

$$B_f = L_t$$

From the solution of this formula we can get the implied probability of surrender for each time to maturity and for a fixed guaranteed interest rate. This is a good tool to monitor the liquidity risk and to get the implied duration of bank liabilities. Moreover, if the interest rate increases the fair value of bank liabilities decreases. Thus, we can get the fair fixed guaranteed interest rate given the probability of surrender. Otherwise, we will accept a greater probability of surrender, while, if the interest rate decreases we have the opposite.
At this point, we can compute the implied liabilities duration of a bank by solving the following equation for $B_f = L_t$:

$$\eta_B = -\frac{(\partial L_f / \partial r)(1 - f_x N[b_2])}{B_f}$$

We can note that the surrender options reduce the effective duration of the bank liabilities. In fact, if the time of maturity increases the implied probability of surrender will increase so to reduce the effective implied duration of bank liabilities. Thus, we have the following prospect:



We can note that to a maturity greater than ten years corresponds an implied duration of five years. This means:

long is quite short

Now it is possible to immunize the equity value by equalling the duration of bonds portfolio and mortgages in the assets to the implied duration of bank liabilities such that:

$$\eta_B = \eta_V$$

What we get is an immunization with respect to the cash flows not with respect to the market value because the account value of the deposits doesn't change with respect to the movement of interest rate. We suggest constructing a portfolio of bonds with equals cash flows such that the duration of the bonds portfolio is equal to the implied duration of the bank liabilities.

Conclusion

By using a contingent claim approach we have developed a model for the fair value of bank liabilities accounting for the protection and the surrender possibility. The implied duration of bank liabilities has been showed to be very short. Hence, banks must be very careful to invest prevalently in financial instruments with long maturity such as mortgages. Otherwise, they have to rebalance their duration by investing in bonds with short maturity.

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