Revealing the naked truth behind price determinacy, infinite-horizon rational expectations, and inflation targeting

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ABSTRACT

By presenting two examples where the non-exploding criterion fails miserably, we demonstrate that that criterion does not universally apply. Therefore, by normal academic standards and burdens of proof, the previous price-determinacy literature has the burden to prove that the non-explosive criterion does apply to their models. However, that literature has not met and probably cannot meet that burden. Instead of using the non-explosive criterion, this paper looks at an economy with an arbitrarily large, but finite horizon and concludes that inflation targeting leads to price indeterminacy even with a Taylor-like feedback rule for setting the nominal interest rate.

Revealing the Naked Truth Behind Price Determinacy, Infinite-Horizon Rational Expectations, and Inflation Targeting

> *Many years ago there lived an emperor who … cared much about his clothes. One day he heard [about two tailors who] could make the finest suit of clothes from the most beautiful cloth... [which] was invisible to anyone who was either [ignorant] or not fit for his position. …[The] emperor first sent two of his trusted men to see it..., neither[of whom] would admit that they could not see the cloth and so praised it... . The emperor then allowed himself to be dressed in the clothes for a procession through town, never admitting that he was too unfit and [ignorant] to see what he was wearing. ...[All] the townspeople wildly praised the magnificent clothes of the emperor, afraid to admit that they could not see them, until [a child yelled, "But he is naked!"]* Wikopeida's Synopsis of Hans Christian Andersen's fairy tale, *The Emporer's New Clothes*, [brackets added][1]

The economic profession's acceptance of the non-explosive criterion for solving infinite-horizon expectational difference equations is so widespread that McCallum (1999, p. 622) notes that the profession usually applies the non-explosive criterion without even referencing that it is using the non-explosive criterion. For me to challenge such a widely accepted criterion and its use by the price-determinacy literature is for me to risk being labeled ignorant and unfit to be an economist. Not only do I take that risk, but I also risk alienating many economists by blowing the whistle on the price-determinacy literature. In this paper, I appeal to the whole economic profession to police that literature, to force that literature to adhere to normal academic standards and normal distributions of burdens of proof, to question that literature's abandonment of price theory in lieu of the non-exploding criterion. I appeal to the whole economic profession

[1] (http://en.wikipedia.org/wiki/The_Emperor's_New_Clothes, accessed on August 7, 2006 (***The Emperor's New Clothes*** is a Danish fairy tale written by Hans Christian Andersen and first published in 1837, as part of *Eventyr, Fortalte for Born* (*Fairy Tales, Told for Children)*

to clean its own laundry with respect to this literature. The stakes are high as the non-explosive criterion is the major obstacle preventing the profession from realizing that pegging the nominal interest rate while targeting inflation is naked in the sense that it leads to price indeterminacy even with a Taylor-like feedback rule.

One reason why the economic profession as a whole needs to step in to police the price-determinacy literature is because that literature has abandoned price theory. This abandonment of price theory is clearest in the price-determinacy literature's models concerning inflation targeting where the only equation that contains the current inflation rate is one that reflects the mechanism whereby the inflation rate <u>affects</u> the nominal interest rate the central bank pegs. Clearly, there is no basis in price theory for such a mechanism to be the basis for the determination of a price level.

In an ideal world, sound logic would be sufficient to reveal the nakedness of the non-explosive criterion, the price-determinacy literature, and inflation targeting. However, by diverging from normal academic standards and normal distributions of burdens of proof, the previous price-determinacy literature has made it nearly impossible for anyone outside that literature to be critical about that literature.

The price-determinacy literature applies the non-explosive criterion as though it was accepted fact, without referencing the non-explosive criterion, without referencing its foundations, without referencing the assumptions upon which it is based. Without such references, readers outside the literature find it difficult to determine why the literature thinks the non-explosive criterion applies to its models. This violation of normal academic standards is made even worse when referees reject any critiques of the price-determinacy literature's use of the non-explosive criterion by stating that some unreferenced literature exists that justifies the use of the non-explosive criterion. Because busy editors do not have time to consider authors' rebuttals to referees' comments showing that the literature cited by the referees does not apply, this shifting of the burden of proof has created an almost impenetrable barrier to sound logical challenges to this literature. The situation is made even worse when some authors refuse to respond to requests to explain why they use the non-explosive criterion and what justifies their use of the criterion. Later in this paper, we will talk more about these violations of normal academic standards.

This paper reveals the "nakedness" of the non-exploding criterion in two examples that show that the criterion cannot be universally applied. By applying the non-explosive criterion to a central bank using the money supply as its monetary instrument as it targets the price level, we reach the absurd conclusion that the central bank should increase the money supply whenever the price level exceeds its target and decrease the money supply whenever the price level falls below its target. If we apply the non-explosive criterion to the design of a cruise control, we reach a similarly absurd conclusion that that the cruise control should speed up the vehicle when the vehicle is going too fast and slow down the vehicle when the vehicle is going too slow.

The purpose of the non-explosive criterion is to eliminate speculative bubbles. I agree with the desirability of eliminating speculative bubbles for most economic applications. However, the problem with the criterion is that it eliminates fundamentally-caused bubbles as well as speculative bubbles. The price-determinacy literature has taken this absurdity to the extreme by designing policy rules that make endogenous certain fundamental variables such as the interest rate to "cause" bubbles to occur for all but one solution, which that literature then claims by the non-exploding criterion to be the sole legitimate solution.

In addition to poor academic standards in the papers that have been published, several other contributing factors have led to the widespread acceptance of the non-explosive criterion. First, a disconnect has occurred between the early literature emphasizing speculative bubbles (e.g., Flood and Garber, 1980) and later literature such as Blanchard and Fisher (1992) which operationally defined bubbles without distinguishing between speculative and fundamental variables. Another contributing factor is that the non-explosive criterion is applied to expectational difference equations regardless of the economic theoretical basis for those equations. Still another contributing factor is the cloak of infinity, which hides the mechanisms that exist or do not exist by which the price level could be determined.

The foundational literature to the non-exploding criterion includes Blanchard and Kahn (1980) and Sargent (1979). We use "B&K" to refer to Blanchard and Kahn (1980). The B&K approach is to look at the roots of the characteristic equation of the expectational difference equation and determine whether they are inside or outside the unit circle. According to B&K (p. 1307), their condition "in effect rules out exponential growth of the" time-t expectation of the exogenous and endogenous variables. Usually, economists apply the B&K condition to the change in price so that the inflation rate cannot grow exponentially. Sargent's (1979) approach is to solve the expectational difference equation forward and assume the solution is bounded. Both the B&K condition and Sargent's bounded assumption do rule out speculative bubbles since speculative bubbles do cause prices to explode. However, an explosive price is a necessary condition of a speculative bubble, not a sufficient condition.

The non-explosive conditions of B&K and Sargent (1979) are mathematical conditions. Therefore, the non-explosive conditions of B&K and Sargent (1979) can be applied to expectational difference equations without understanding the economic underpinnings of those equations. In the next four sections, this paper demonstrates by example that such blind application of the non-explosive criterion fails.

There are two approaches for the economic profession to deal with the failure of the blind application of the non-explosive criterion. One approach is to make qualifications to the use of the non-explosive criterion. The other approach is to abandon it entirely. This paper argues for the latter. However, if the price-determinacy literature chooses to take the first approach, then under normal academic standards, that literature needs to justify its use of the non-explosive criterion; that literature has the burden of proof to show that the non-explosive criterion applies to their models. So far, that literature has failed to meet this burden of proof.

Sections II, III, IV, and V show that the non-explosive criterion does not universally apply. Section VI discusses the history of the literature behind the non-explosive criterion, interjecting notes about where this literature has gone astray. Section VII proposes an alternative to the non-exploding criterion and applies that alternative approach to a central bank using the nominal interest rate as its instrument as it either targets the price level or targets the inflation rate. This reveals that inflation targeting leads to price indeterminacy even with a Taylor-like feedback rule. Section VIII further advances the case that the profession needs to pressure the price-determinacy literature to conform to normal academic standards and distributions of burdens of proof. Section IX summarizes this paper's findings and reflects on its broader implications including the price indeterminacy of inflation targeting.

II. A Naked Difference Equation

In this section, we apply the non-explosive criterion to a naked difference equation. By "naked", we mean without a complete description of the model underlying the difference equation. The purpose of this exercise is to show that the non-explosive criterion cannot be blindly applied, that the non-explosive criterion does not universally apply.

Let P_t be the price level at time t and P_t^* be the central bank's targeted price level at time t. For some constant $\tau \geq -1$, we assume the following difference equation holds:

$$\frac{P_{t+1}}{P_{t+1}^*} = \left(\frac{P_t}{P_t^*} \right)^{1+\tau} \qquad (1)$$

At this point, (1) is "naked" in the sense that we have not yet described a model underlying (1). For now, we will transform the "naked" equation (1) into a linear difference equation and "solve" it using the non-explosive criterion.

Taking the natural logarithm of both sides of (1) gives:

$$\ln\left(\frac{P_{t+1}}{P_{t+1}^*} \right) = (1+\tau) \cdot \ln\left(\frac{P_t}{P_t^*} \right)$$

Following Woodford (2003), define $\hat{P}_t \equiv \ln\left(\frac{P_t}{P_t^*} \right)$. We then can rewrite the above as:

$$\hat{P}_{t+1} = (1+\tau)\cdot \hat{P}_t \qquad (2)$$

We can solve (2) backwards to conclude that:

$$\hat{P}_t = (1+\tau)^t \cdot \hat{P}_0 \qquad (3)$$

Equation (3) is a simplified version of the equations upon which much of the previous price-determinacy literature is based. If we follow the logic of Woodford (2003), we would conclude that if τ>0, then the unique bounded solution to (3) is where $\hat{P}_t = 0$ for all t, which means that, in this example, the price level will always equal its targeted level.[2] This is because $\lim_{t\to\infty}\hat{P}_t = \hat{P}_0 \cdot \lim_{t\to\infty}(1+\tau)^t = sign(\hat{P}_0)\cdot\infty$, which means that the price sequence of $\hat{P}_t$ will be unbounded whenever $\hat{P}_0 \neq 0$. Hence, if and only if $\hat{P}_t = 0$ will the sequence { $\hat{P}_t$ } be bounded.

This assumption of boundedness follows Sargent (1979). On the other hand, the B&K conditions do allow unbounded prices but rule out exponential growth in the inflation rate or to the percentage change in the inflation rate depending on how the B&K conditions are applied. When $P_t^* = (1+\pi^*)P_{t-1}^*$ where π^* is the constant implicit targeted inflation rate, the appendix shows that the actual inflation rate will explode if and only if $\hat{P}_t = 0$ when $\tau > 0$. Therefore, regardless if we use Sargent's (1979) bounded assumption or the B&K conditions, applying the non-explosive criterion when τ>0 rules out all solutions to (3) except the solution where $\hat{P}_t = 0$. On the other hand, when $-1<\tau\leq 0$ there is an uncountably infinite number of non-explosive solutions to (3).

[2] When uncertainty exists, the unique solution will normally differ from being on track all the time. However, the literature is confused on this because of lognormalized approximations. As such I don't want us to be distracted in this paper by that confusion.

The price-determinacy literature uses this analysis to justify designing policy feedback rules where $\tau>0$, which causes all solutions to explode except one. By the non-explosive criterion, that literature then declares that unique non-explosive solution to be the only legitimate solution, which is its basis for claiming that the price level is determined.

The next section discusses a model that involves the central bank implementing monetary policy through the nominal interest rate, which is in the spirit of the price-determinacy literature and which leads to the expectational difference equation (1). However, section IV presents another model that also leads to equation (1) where the central bank uses the money supply to implement monetary policy. Applying the non-explosive model to section IV's model leads to absurd results akin to an endorsement of a cruise control that speeds up when the vehicle is going too fast and that slows down when the vehicle is going too slow. This shows by example that the non-explosive criterion does not universally apply.

III. An Interest-Rate-Pegging Model

This section presents a model very similar to the models used by the price-determinacy literature. Our presentation of the model and analysis is an integration of Woodford (2003) and Carlstrom and Fruest (2001), which allows us to discuss the solutions in general without the need for dealing with steady states or with a loglinear approximation. The price-determinacy literature is based on the Fisher-Euler equation which states that

$$\frac{U'(c_t)}{P_t} = R_t \beta E_t \left[\frac{U'(c_{t+1})}{P_{t+1}} \right] \qquad (4)$$

where R_t is the gross nominal interest rate from time t to time t+1, which equals one plus the nominal interest rate. The Fisher-Euler equation (4) states that the marginal utility per "buck" today equals today's gross nominal interest rate times the expected marginal utility per "buck" tomorrow.[3]

For the simplified model of this section, we also assume (i) a representative agent, (ii) future output is known in advance, and (iii) all output is consumed so that c_t=Y_t. The Fisher-Euler equation then becomes:

$$\frac{U'(Y_t)}{P_t} = R_t \beta E_t \left[\frac{U'(Y_{t+1})}{P_{t+1}} \right] \qquad (5)$$

Next, suppose the central bank sets the gross nominal interest rate according to the following feedback rule:

$$\hat{R}_t = \frac{U'(Y_t) P_{t+1}^*}{\beta U'(Y_{t+1}) P_t^*} \left(\frac{P_t}{P_t^*} \right)^{\tau} \qquad (6)$$

This is known as "current price-level targeting".[4] The parameter τ represents the sensitivity of the feedback rule to when the current price level differs from its target.

Substituting (6) into (5) and simplifying gives:

$$\left(\frac{P_t}{P_t^*} \right)^{1+\tau} = E_t \left[\frac{P_{t+1}}{P_{t+1}^*} \right] \qquad (7)$$

[3] Carlstrom and Fuerst (2001) argue that (4) only applies for what they call CWID timing. However, Eagle (2007) uses a cash-in-advance (CIA) timing constraint that has money only being held within the period but not from one period to the other, which then does result in (4). Also, Woodford (2003) uses (4) except he also assumes utility shocks.

[4] Eagle (2007) argues that current price-level targeting does not make sense as a feedback rule since when the central bank sets the R_t, it does not know P_t. We will ignore this issue in this paper as the price-determinacy literature, primarily Woodford (2003), usually does analyze current price-level targeting.

By assuming the representative consumer has perfect foresight, (7) becomes (1), which then through logarithms can be transformed into (3) as done in section II.

The price determinacy literature such as Woodford (2003) uses this result to conclude that the central bank should set the nominal interest rate according to feedback rule (6) with $\tau>0$. In this section's simplified model, this causes all solutions to (3) to be explosive except the $\hat{P}_t = 0$ solution. By the non-explosive criterion, the price-determinacy literature rules out all the explosive solutions and concludes that the price level is determined since there is a unique non-explosive solution.

According to Carlstrom (2005) and McCallum (1999), the justification for using the non-explosive criterion is to rule out speculative bubbles. However, if the explosive behavior in the nominal interest rate is "causing" this explosive behavior in prices, then that explosive price level is due to a fundamental variable, not speculation.

Before we delve more deeply into the causal mechanisms in this model, the next section presents another model that also leads to equations (1) and (3), where the same logic of applying the non-explosive criterion leads to a totally absurd result.

IV. A Money-Supply-Setting Model

This section presents another model that leads to equations (1) and (3), where the central bank uses the money supply as its monetary instrument as it pursues a price-level target. Applying the non-explosive criterion in the same manner as the previous section leads to the totally absurd result that the central bank should follow a policy of increasing the monetary growth rate whenever the price level exceeds its target and decreasing the monetary growth rate whenever the price level falls short of its target.

Once again assume a representative consumer where all the output is consumed, and future output is known. Also, assume the cash-in-advance (CIA) constraint $M_t \geq P_t c_t$ where M_t is the money held at the beginning of the period by the representative consumer. As long as the nominal interest rate is positive, the CIA constraint will be binding so that

$$M_t = P_t Y_t \tag{8}$$

Assume that the central bank does not know what the current money supply is, but is able to perfectly control the growth rate of the money supply. The central bank sets this growth rate using the following rule:

$$\frac{M_{t+1}}{M_t} = \frac{P_{t+1}^* Y_{t+1}}{P_t^* Y_t}\left(\frac{P_t}{P_t^*}\right)^{\tau} \tag{9}$$

If $P_t = P_t^*$, then the central bank will increase the money supply by exactly the amount necessary to increase the price level according to the targeted price change. The τ represents the sensitivity of the central bank's money supply to when the current price level differs from its targeted level. By (8), $M_{t+1} = P_{t+1}Y_{t+1}$. Substituting this and (8) into (9), we get $\frac{P_{t+1}}{P_t} = \frac{P_{t+1}^*}{P_t^*}\left(\frac{P_t}{P_t^*}\right)^{\tau}$, which can be rewritten as equation (1) in section II.

What should the central bank use for the value of τ in equation (8)? From section II, we know that if $\tau<0$, there will be an uncountably infinite number of solutions, but that if $\tau>0$, then there will be a unique bounded solution. If we apply the non-explosive criterion to this example as the price-determinacy literature applies that criterion, we would conclude that in order for the price level to be determined, the central bank should follow feedback rule (8) with τ>0. If τ>0, all solutions would explode except the one where the price level is forever equal to its target. However, τ>0 in this model means that the central bank will increase (decrease) the growth rate in the money supply when the price level exceeds (is less than) its target. This conclusion is absurd; it is the opposite of what would be considered as the common-sense strategy for central banks. I therefore conclude that this is one situation where the non-explosive criterion fails to apply.

V. Applying the Non-Explosive Criterion to the Design of a Cruise Control

Let us now look at a non-economic example when the non-exploding criterion leads to absurd results. Consider a vehicle traveling in space forever along a particular trajectory, but we do not know the initial speed of the vehicle. Assume a cruise control that is set by the following equation:

$$\frac{s_{t+1}}{s_t} = \left(\frac{s_t}{s^*}\right)^{\tau} \tag{10}$$

where s_t is the speed of the vehicle at time t and s^* is the vehicle's targeted speed, which we assume to be constant.

We can rewrite (10) as $\frac{s_{t+1}}{s^*} = \left(\frac{s_t}{s^*}\right)^{1+\tau}$. Taking the logarithms of both sides and defining $\hat{s}_t \equiv \ln\left(\frac{s_t}{s^*}\right)$ gives $\hat{s}_{t+1} = (1+\tau)\hat{s}_t$. This shows all the possible solutions of (10). One solution is where $\hat{s}_t = 0$ for all t, which means that the speed always equals its targeted speed. If τ<0, then there will be an uncountably infinity of solutions to (10) and each of these solutions will be bounded since $\hat{s}_{t+1} < \hat{s}_t$. On the other hand, if τ>0 then any solution where $\hat{s}_t \neq 0$ will explode with the speed going to infinity or going to zero (which will cause $\hat{s}_t$ to go to negative infinity).

If we applied the non-explosive criterion to this cruise control when $\tau > 0$, we would eliminate all the possible solutions except where the actual speed equals the projected speed for all t. If we really believed the non-explosive criterion could be applied to this example, we would then conclude that we could guarantee that the actual speed will equal the targeted speed by choosing a cruise control with $\tau>0$. However, that would imply that the vehicle would speed up when the vehicle is going too fast and slow down when it goes too slow. This is the opposite of how a cruise control normally works. Again, the non-exploding criterion has led us to an absurd conclusion.

The reason that a backwards cruise control violates the non-explosive criterion should be obvious. If the speed exceeds the targeted speed and $\tau > 0$, the cruise control will cause the speed to increase, which will cause the excess speed to increase, which will in turn lead an even greater acceleration by the cruise control. In other word, the speed will escalate or explode whenever the speed exceeds its target.

On the other hand, if the speed falls short of the targeted speed, the backwards cruise control responds by decelerating the vehicle, which will mean that the gap between the actual and targeted speeds is even greater. This greater speed gap then causes the backwards cruise control to decelerate more with the percentage change in speed approaching -100%, which would be considered a speed explosion in the negative sense.

The cruise control is very analogous to the money-supply example of the previous section. If $\tau>0$, then if the price level exceeds its targeted level, then the central bank would increase the money supply further causing the price level to increase further. This process escalates with an increasing rate of money supply increases and an increasing rate of inflation, i.e., both the money supply and inflation explode. For the central bank to respond in this fashion would be analogous to the cruise control accelerating when the vehicle goes too fast. Granted, in both cases the price level or the vehicle's speed explodes. However, in neither case is the explosion caused by a speculative bubble. In the case of the cruise control, the explosion of the vehicle's speed is caused by a fundamental variable –the cruise control. In the money supply example, the explosion in the price level occurs because of a fundamental variable –the money supply.

Because many economists learned about the difference between speculative bubbles and fundamental variables in the context of Blanchard and Fisher's (1992) operational definition of their "fundamental solution" and their "bubble", many of these economists may think that anything that is endogenous is a speculative variable; they may think that fundamental variables must be exogenous.

But would we not consider the acceleration of a cruise control a fundamental variable even though it is made endogenous by the cruise control? Would we not consider the money supply a fundamental variable even though it is made endogenous by a central bank policy rule? Would we not consider the nominal interest rate a fundamental variable even though it is made endogenous by a central bank policy rule?

Under the philosophy of determinism, everything is endogenous. Therefore, if we said that any fundamental variable must be exogenous and that any endogenous variable is speculative, we would conclude that in the real world, there are no fundamental variables and that all variables are speculative. Such a conclusion is absurd. Clearly, whether a variable is endogenous or exogenous has no relevance to whether the variable is a fundamental variable or a speculative variable. In conclusion, we should not use our desire to rule out speculative bubbles as justification to rule out explosive results caused by explosive endogenous fundamental variables.

While the current literature has failed to emphasize the difference between speculative bubbles and fundamental variables, the early rational-expectations literature did clearly make that distinction. The next section reviews the literature to see how that literature developed and has gone astray.

VI. History of Infinite-Horizon Rational Expectations Techniques

When the Rational Expectations Revolution took place in the 1970s, a problem plaguing early rational expectations models having infinite horizons were speculative bubbles. These speculative bubbles are self-fulfilling price effects, which exist not for any fundamental reason but because they are expected to exist. These speculative bubbles mean an infinite number of solutions exist with these rational expectations models. As Flood and Garber (1980, p. 746) stated, "A bubble can arise when the actual market price depends positively on its own expected rate of change, as normally occurs in asset markets. … In such conditions, the arbitrary, self-fulfilling expectation of price changes may drive actual price changes independently of market fundamentals; we refer to such a situation as a price bubble." Please note that Flood and Garber's view of a speculative bubble was independent of market fundamentals. Other early literature similarly also emphasized the independence of speculative bubbles from market fundamentals (See Obstfelt and Rogoff, 1983; and McCallum, 1981).

In the late 1970s and the early 1980s, several economists argued that these speculative bubbles were nuisances, distracting from what could really be learned from rational expectations models. McCallum (1999) summarizes this point of view by saying, "… many dynamic models with rational expectations (RE) feature a multiplicity of paths that satisfy all the conditions for intertemporal equilibrium. …in many applications the analyst is not specifically concerned with this multiplicity … and wishes to focus on one particular path that is presumed to be of economic relevance, e.g., if bubbles were absent."

I do agree with the literature that the speculative bubbles are distractions and that for most economic analysis we should rule out these speculative bubbles. However, the non-explosive criterion has also ruled out solutions caused by fundamental factors. My critique of the non-explosive criterion is similar to the following argument McCallum stated against the non-explosive criterion, which he refers to as the stability criterion: "…one important objective of dynamic economic analysis … will often be to determine the conditions under which a system will be dynamically stable and unstable. … To the extent, then, that this objective of analysis is important, the stability criterion is inherently unsuitable." By requiring any legitimate solution to be stable or non-explosive as required by the non-explosive criterion, we are unable to consider the possibility of explosive solutions, even if the explosive solution is caused by fundamental factors.

My critique applies not only to the non-explosive criterion but to any criterion where an analyst need only work with the expectational difference equations without understanding the economics behind the difference equations. Flood and Garber (1980) stated that, "An explicit definition of market fundamentals depends on a particular model's structure; indeed, the very notion of a bubble can make no sense in the absence of a precise model detailing a market's operation. Without such a model, it is impossible both to define market fundamentals and to isolate the trajectory characteristic of a bubble." In this paper, I argue that there is more truth in Flood and Garber's (1980) statement than most economists realize. I argue that a criterion such as the non-exploding criterion cannot be applied blindly to expectational difference equations. Instead, to eliminate speculative bubbles, we need to work carefully with the specific model to rule out speculative bubbles without also ruling out situations caused by fundamental factors.

Part of the confusion leading to the use of the nonexplosive criterion stems from an operational definition of bubbles by Blanchard and Fisher (B&F) in 1992. They start with the following expectational difference equation:

$$y_t = aE_t[y_{t+1}] + cx_t \tag{11}$$

where y_t is the value of some variable y at time t and x_t is the value of a particular exogenous variable at time t. The parameters a and c are constants, with $|a| < 1$. The expectations operator given the public's information set at time t is represented by $E_t[\cdot]$. Using the law of iterated expectations and solving (11) recursively, B&F get

$$y_t = c\sum_{k=0}^{T-t-1} a^k E_t[x_{t+k}] + a^{T-t} E_t[y_T] \tag{12}$$

By taking the limit as T goes to infinity, B&F get

$$y_t = \lim_{T\to\infty} c\sum_{k=0}^{T-t-1} a^k E_t[x_{t+k}] + \lim_{T\to\infty} a^{T-t} E_t[y_T]$$

B&F then define their "fundamental solution" to be where $\lim_{T\to\infty} c\sum_{k=0}^{T-t-1} a^k E_t[x_{t+k}]$ is well defined and where $\lim_{T\to\infty} a^{T-t} E_t[y_T] = 0$. Thus, their "fundamental solution" is:

$$y_t = y_t^* \equiv c\sum_{k=0}^{\infty} a^k E_t[x_{t+k}] \tag{13}$$

By B&F calling (13) the "fundamental solution", they have biased the profession to think that if only fundamental variables affect y_t, then (13) must be the solution. However, as we have seen in previous sections, explosive endogenous fundamental variables can also create explosive price behavior. Since equation (11) only assumes exogenous variables, (13) does not apply to situations where there are endogenous fundamental variables.

B&F then writes the more general solution to (11) as

$$y_t = y_t^* + b_t \tag{14}$$

for some value *b* that would be a function of time. Since y_t^* should solve (11),

$$y_t^* = aE_t[y_{t+1}^*] + cx_t \tag{15}$$

Substituting (14) into (11) for both y_t and y_{t+1} gives:

$$y_t^* + b_t = aE_t[y_{t+1}^* + b_{t+1}] + cx_t$$

Substituting (15) into the above for y_t^* and simplifying gives $b_t = aE_t[b_{t+1}]$, which is itself an expectational difference equation, which we can solve backwards to get that:

$$E_t[b_{t+k}] = \frac{b_t}{a^k} \tag{16}$$

What B&F show by deriving (16) is that when $|a| < 1$, any solution to (11) other than y_t^* will be explosive. While I consider B&F's labeling of y_t^* as "the fundamental solution" to be misleading, I do agree with their analysis and conclusion that if and only if $b_t = 0$ will a solution to (11) be non-explosive.

B&F then go on to operationally define b_t as a bubble. Unfortunately, B&F and much of the subsequent literature has failed to distinguish B&F's bubble from a speculative bubble. This may have been true because B&F's "bubble" b_t has the same exponential behavior as does Flood and Garber's (1980) speculative bubble.

VII. An Alternative Approach to Price Determinacy

As noted at the beginning of this paper, one of the contributing factors to the wide acceptance of the non-explosive criterion has been the cloak of infinity, which has hidden the true economic meaning of the expectational difference equations. In this section, we utilize an alternative to the non-explosive criterion that avoids logical errors involving infinity by assuming an arbitrarily large, but finite horizon to the economy. Let T represent the last period of the economy. We assume T is known in advance.

Let us continue to assume a representative consumer, all output is consumed, and future output is known for all periods. However, we will no longer assume that future prices are known (although in some situations, we will conclude that they are known).

With a finite horizon, the Fisher-Euler equation (5) applies for t=0,1,…,T-1. It does not apply for period T since no period T+1 exists. Therefore, no new loans or bonds are issued at time T and no interest rate exists at time T. Similarly, the central bank sets the gross nominal interest rate according to feedback rule (6) for t=0,1,…,T-1. Substituting (6) into (5) results in (7) which applies for t=0,l,…,T-1.

Since there is no interest rate at time T, the central bank will have no choice at time T but to use the money supply as its instrument to conduct monetary policy. At time T, assume the cash-in-advance constraint $M_T \leq P_T c_T$ holds for the representative consumer in the same manner as in section IV. Since there is no tomorrow at time T, consumers will spend all their money. Since all output is consumed and we have a representative agent, this cash-in-advance constraint says that $M_T = P_T Y_T$. Therefore, as it pursues its price-level target P_T^*, the central bank would set $M_T = \frac{P_T^*}{Y_T}$, which implies that $P_T = P_T^*$.

Since $P_T = P_T^*$, (7) implies that $P_{T-1} = P_{T-1}^*$. By backwards recursion, we conclude that $P_t = P_t^*$ for t=1,2,…,T. In conclusion, the price level is technically determined for this finite-horizon economy. This is true for any arbitrarily large T, so that the price level is technically determined as T goes to infinity.

A strong assumption we made is that the central bank at time T will set the money supply so it will meet its price-level target and that the public perfectly trusts the central bank to do so. This means the public has perfect confidence that the central bank in the last period will set the money supply at time T to meets its price-level target. The next subsection looks at less-than-perfect confidence in the central bank, which leads to a conclusion that is similar to the conclusions made by the previous price-determinacy literature.

The Central-Bank Credibility Issue:

In this subsection, we assume that the public is not 100% confident that the central bank will perfectly set the money supply at time T so that $P_T = P_T^*$. This may be caused by a number of factors, including (i) the central bank has less-than-perfect control over the money supply, (ii) uncertainty with regard to velocity, or (iii) the central bank may change its mind between time t and time T concerning its price-level target at time T.[5]

Assume for t<T, the public's expectation of the price level at time T is unbiased in the sense that $E_t\left[\frac{1}{P_T}\right] = \frac{1}{P_T^*}$, which implies that

$$E_t\left[\frac{P_T^*}{P_T}\right] = 1. \tag{17}$$

Using backwards recursion, (7) and (17) imply that $P_t = P_t^*$ for t=1,2,…,T-1.

While the prices continue to be determined at their targeted levels, the uncertainty the public has concerning P_T affects their confidence about the current price being what it should be. We use confidence intervals to help us study this issue of confidence.

Define $\tilde{P}_t$ to be the hypothetical value of P_t that would exist if the public knew at time t the actual value of all future prices including P_T. We will refer to $\tilde{P}_t$ as the "hypothetical true price at time t." Under this hypothetical situation, (7) becomes:

$$\left(\frac{P_t^*}{\tilde{P}_t}\right)^{1+\tau} = \frac{P_{t+1}^*}{\tilde{P}_{t+1}} \tag{18}$$

[5] For tractability reasons we assume that the announced price level target P_T^* does not change over time.

Taking the reciprocal of (18), taking the natural logarithm of both sides, and solving it backwards gives:

$$\tilde{\hat{P}}_t = \frac{\hat{P}_T}{(1+\tau)^{T-t}} \tag{19}$$

where $\tilde{\hat{P}}_t \equiv \ln\left(\frac{\tilde{P}_t}{P_t^*}\right)$. We refer to $\tilde{\hat{P}}_t$ as the "hypothetical true value for $\hat{P}_t$."

(Remember that $\hat{P}_t \equiv \ln\left(\frac{P_t}{P_t^*}\right)$). Let $[\hat{P}_T]_{\alpha,t}$ represent the public's confidence interval at time t of $\hat{P}_T$ for a certain defined level of confidence (e.g. α=95%).

Substituting this confidence interval into (19), we conclude that

$$\left[\tilde{\hat{P}}_t\right]_{\alpha,t} = \frac{\left[\hat{P}_T\right]_{\alpha,t}}{(1+\tau)^{T-t}}. \tag{20}$$

where $\left[\tilde{\hat{P}}_t\right]_{\alpha,t}$ is the confidence interval the public has for the hypothetical true value of $\hat{P}_t$. Equation (20) states the relationship between the confidence interval for the true hypothetical price at time t and the confidence interval of the price level at time *T*.

Consider the case where τ=0. Then, by (20) the size of $[\tilde{\hat{P}}_t]_{\alpha,t}$ will equal the size of $[\hat{P}_T]_{\alpha,t}$. The size of the public's confidence interval $[\hat{P}_T]_{\alpha,t}$ will likely increase as *T* increases but *t* remains constant. If $[\hat{P}_T]_{\alpha,t}$ goes to infinity in size as *T* goes to infinity, then so would $\left[\tilde{\hat{P}}_t\right]_{\alpha,t}$. Such an infinite confidence interval for $\tilde{\hat{P}}_t$ means the public would have no confidence in the current price level being close to its true level. We interpret this result as being consistent with the problem of price indeterminacy as discussed in the previous price-determinacy literature.

On the other hand, if τ>0 then the term $(1+\tau)^{T-t}$ will increase as T goes to infinity. The increase in $(1+\tau)^{T-t}$ may be sufficient to offset the increase in the size of $[\hat{P}_T]_{\alpha,t}$. In fact, the larger is τ the more likely the increase in $(1+\tau)^{T-t}$ will offset the increase in the size of $[\hat{P}_T]_{\alpha,t}$. This conclusion I think is similar to the issue addressed by the previous price determinacy literature. In other words, while for a finite-horizon economy, prices are technically determined for any arbitrarily large horizon T, the confidence the public has in the central bank meeting the price target it has announced at time t for time T could go to zero (meaning the size of the confidence interval for P_T goes to infinity) as T goes to infinity. By following the feedback rule (6) for some τ in (0,1), the public's confidence in the true value of P_t can be kept high; the larger is τ, the greater will be the public's confidence in $\hat{P}_t$.

In summary, with respect to price-level targeting, applying the finite-horizon approach to price determinacy resulted in the same policy implications as did the previous literature using the non-explosive criterion. However, most monetary economists give more attention to inflation targeting (IT) than price-level targeting. As the next subsection shows, applying the finite-horizon approach to IT gives substantially different results than has the previous literature.

Inflation Targeting:

In this subsection we look at inflation targeting (IT). We start with a brief review of how the previous literature used the non-explosive criterion to claim that prices are determined under IT as long as the central bank follows a feedback rule such as the Taylor's (1993) rule. We then look at IT in a finite economy. We find that in a finite-horizon economy, no matter how large is the horizon, prices are indeterminate under IT.

Assume again an economy where the Fisher-Euler equation (5) applies, P_{-1}=1.0, $\pi_{-1} = \pi^*_{-1}$, and the representative consumer has perfect foresight of future prices and output. Where $\pi_t \equiv \frac{P_t}{P_{t-1}}$, the Fisher-Euler equation (5) then implies that

$$\pi_{t+1} = R_t \beta \frac{U'(Y_{t+1})}{U'(Y_t)} \tag{21}$$

Assume the central bank follows a feedback rule for setting the nominal interest rate so that

$$\hat{R}_t = \frac{U'(Y_t)\pi^*_{t+1}}{\beta U'(Y_{t+1})}\left(\frac{\pi_t}{\pi^*_t}\right)^{\tau} \tag{22}$$

Equation (22) reflects "current inflation targeting".[6] Substituting (22) into (21) gives

$$\frac{\pi_{t+1}}{\pi^*_{t+1}} = \left(\frac{\pi_t}{\pi^*_t}\right)^{\tau} \tag{23}$$

Taking the natural log of both sides we get

$$\hat{\pi}_{t+1} = \tau\, \hat{\pi}_t \tag{24}$$

[6] Eagle (2007) argues that current inflation targeting does not make sense as a feedback rule since at the time the central bank sets R_t, it does not know P_t and hence does not know π_t. In this paper, we will ignore this issue as the price-determinacy literature usually does analyze current inflation targeting.

where $\hat{\pi}_t \equiv \ln\left(\frac{\pi_t}{\pi_t^*}\right)$. If $\tau>1$, then a unique bounded solution exists for $\hat{\pi}_t$, which is where $\hat{\pi}_t = 0$ or $\pi_t = \pi_t^*$ for t=0,1,…,∞. The previous literature has used the non-exploding criterion and followed this line of logic to conclude that prices are determined under IT. Since $\pi_t = \pi_t^*$ for t=0,1,…,∞ and P_{-1} is given, all prices can be determined recursively since $P_t = \pi_t P_{t-1}$.

Because this paper rejects the non-explosive criterion, this section takes a different approach to the issue of price determinacy. We now look at an economy with a finite horizon to see if prices are determined under IT. We find that the price level is indeterminate under IT, regardless how large is the finite horizon.

We first need to determine how this economy will work at the end of the horizon. We must be careful so that how the central bank sets R_{T-1} is consistent with how the central bank sets M_T. We assume the central bank will set M_T so that $\pi_T = \pi_T^*$. To do so under the CIA constraint of this paper, the central bank would set $M_T = \pi_T^* P_{T-1} Y_T$.

For t=0,1,…,T-2, we assume the central bank sets the gross nominal interest rate following feedback rule (22), which implies through (21) that (23) holds for t=0,1,…,T-2. Note that if we also assumed (22) and hence (23) applied for t=T-1, then if π_{T-2} differed from π_{T-2}^*, then (23) would conflict with our assumption that the central bank will set the money supply so that $\pi_T = \pi_T^*$. Instead, assume at time t=T-1, the central bank pegs the gross nominal interest rate to equal

$$\hat{R}_{T-1} = \frac{U'(Y_{T-1})\pi_T^*}{\beta U'(Y_T)} \tag{25}$$

By substituting (25) into (21) evaluated at t=T-1, we then get that $\pi_T = \pi_T^*$.

For this finite economy, inflation targeting leads to price indeterminacy because the number of unknowns exceeds the number of equations. There are T+1 unknowns in the system. These unknowns are π_0, π_1, …, π_T. There are T equations. These equations are (23) for t=0,1,…,T-2 and $\pi_T = \pi_T^*$. Since there are more unknowns than equations, this system is unable to determine all the unknowns. This is true regardless how large, but finite, T is. For simple examples verifying this indeterminacy, see Eagle (2007). The reasons that the previous literature has not realized that this difference between unknowns and equations is that for an infinite-time economy, it makes no sense to compare an infinite number of unknowns to an infinite number of equations, and because economists have relied on the flawed non-exploding criterion.

Eagle (2007) discusses this price-indeterminacy of IT further. He finds that even in an infinite horizon economy, there is no mechanism to determine the current price level other than the mechanism by which the central bank looks at the current inflation rate (and hence price level) when it pegs the nominal interest rate. He argues that it is absurd to think that a mechanism by which the current inflation rate affects the nominal interest rate is the mechanism by which that current inflation rate and hence current price level are determined.

VIII. Should Normal Academic Standards Apply To Economics?

This section further discusses the reasons that I have asked the whole economic profession to step in and police the price-determinacy literature. My charge is that the price-determinacy literature usually uses the non-explosive criterion as though it is a well accepted, undeniable mathematical fact, without identifying the non-explosive criterion, without referencing the foundational literature for the criterion, and without stating the assumptions underlying the criterion. Dittmer and Gavin (2005) are typical of how the price-determinacy literature uses the non-explosive criterion. On p. 340, they state, "For a unique solution to this system, we need one eigenvalue inside the unit circle and the other outside." They do not identify that they are applying the non-explosive criterion. They do not reference Blanchard and Kahn (1980), which is from where this condition comes. They do not state the assumptions that they are making when they apply this condition, which is that they are implicitly assuming that their solution cannot explode. They make no mention of speculative bubbles, even though ruling out speculative bubbles was the reason why Blanchard and Kahn (1980) developed their condition. Without such references, it is difficult for someone from outside the literature to read this literature and understand the logical basis for their arguments.

Woodford (2003) provides another example of the referencing or lack thereof within the price-determinacy literature. On p. 81, he states, "…this equation can be solved forward (as discussed further in Appendix A) to obtain a unique bounded solution…" Nowhere, does Woodford cite that the basis for his assuming his solution is bounded is the non-explosive criterion, Sargent's (1979) precedent, or Blanchard and Kahn's (1980) condition. As with Dittmar and Gavin (2005), Woodford makes no mention of speculative bubbles. Instead on p. 632 in his Appendix A, Woodford discusses some inverse function theorem that results in a unique bounded solution. Also, on p. 79, Woodford states, "In the case of tight enough bounds on the variations that are considered in these variables, it suffices to take into account only the bounded solutions to a system consisting of log-linear approximations …" Given that the basis for assuming a bounded solution in the price-determinacy literature is the non-exploding criterion, these statements by Woodford mke it difficult for readers to learn the true basis of that bounded assumption. This situation has been made even more aggravating when Woodford has refused to respond to my repeated requests for the reason and justification for his assuming his solution is bounded.[7] Is this how the field of economics is supposed to operate? Is our theory held together by economists ignoring questions and critiques? If so maybe, economics deserves its label as the "dismal science."

The lack of referencing concerning the non-explosive criterion seems to be epidemic in rational-expectations literature. McCallum (1999) conducted an informal review of that literature and found it often does not identify the non-explosive criterion (or what he called the saddle-point or stability criterion).

These instances where references are lacking, I consider to be violations of normal academic standards. Without these proper references, it is difficult for the reader of this literature to understand why this literature is assuming the solutions must be bounded or why the roots must be inside and outside the unit circle as Blanchard and Kahn (1980) state even though this literature does not reference Blachard and Kahn (1980). Fortunately, I did eventually find someone in the literature who took the time to answer my questions and provided me with the history of the non-explosive criterion. Also, I learned from several papers by Bennett McCallum who has followed normal academic standards of referencing.

[7] Over the last several years, I have repeatedly asked him by email, by phone, and by formal letter to explain why he assumes his solution is bounded. He has not responded to my requests for this information.

However, even after I have learned the basis for the non-explosive criterion and have challenged it showing examples where it fails,[8] referees have rejected my challenges by stating that some unreferenced literature exists that does justify the use of the non-explosive criterion. Explaining a rejection from *Econometrica*, Levin (2004) wrote, "You may not be aware that there is a large literature in economics that attacks the issue of the boundedness of solutions to infinite horizon problems generally with the use of examples and models from economics... . In the case of single person optimization problems, this falls under the category of transversality conditions. Infinite horizon existence results generally follow from carefully relating a finite horizon truncated economy to the infinite horizon economy. In some cases, boundedness follows naturally from economic assumptions. In other cases (most obviously models of growth) they do not." Dr. Levine did not provide references for me to find this literature, but with *Econometrica*, I consider myself fortunate to have received any feedback at all.

To explain why s/he was recommending rejection of my critique of Woodford (2003), an anonymous referee from *Economic Inquiry* wrote, "While a complete presentation of the conditions necessary for this to be true is not provided by Woodford, there is a well defined literature which explains under what circumstances the solution is bounded. As a result, the paper should not be published." The referee goes on to say that, "While Woodford does not provide details necessary to show the solution is bounded, there are well known arguments which show under what conditions the solution is bounded." In essence, the referee was saying that Woodford did not have the burden of proof to reference this literature, but that I as a critic did have the burden to prove that no literature existed that justified Woodford's bounded assumption.

[8] See Eagle and Murf (2005) and Eagle (2005c).

In this case, the referee did cite some literature including Brock and Mirman (1972), Lucas (1978), and Calin, Chen, Cosiman and Himonas (2005). S/he also stated "a comprehensive proof of this solution to such models can be found in Altug and Labadie (1994, Chapter 5)." However, close inspection of this "well-defined literature" shows that none of it applies to the models used by the price-determinacy literature. The "well-defined literature" included several models that did not include money: Brock and Mirman (1972), Lucas (1978), and Calin, Chen, Cosmano and Himaonas (2005). Clearly, the presumption should be that results involving non-monetary economies do not apply to monetary economies unless someone proves that the results apply even in a monetary economy. The price-determinacy literature has not met that burden of proof and probably will be unable to do so as we will explain later.

The only model of a monetary economy cited by the referee was Altug and Labadie's (1995) Chapter 5 model. However, Altug and Labadie's Chapter 5 model includes a cash-in-advance constraint and assumes upper bounds to the growth rate of the money supply. I am aware of no model in the price-determinacy literature involving a central bank pegging the nominal interest rate that assumes an upper bound to growth rate of the money supply as such a constraint would also constrain the central bank's feedback rule for setting the nominal interest rate. In particular; Woodford's (2003) Chapter 2 model definitely does not assume any upper bound to the growth rate in the money supply. As a result, Altug and Labadie's Chapter 5 model does not apply because it has different assumptions from models of the price-determinacy literature.

As a profession, we need to ask ourselves, "Who has the burden of proof in discussions involving logic?" Should it be the burden of proof for challengers of analyses to prove that no literature exists that supports the original analysis or should it be the burden of proof for the original authors of the analyses to prove that that the analysis is correct and to cite any literature upon which that analysis relies? The normal academic burden of proof lies with the original authors of the analyses for obvious reasons. Otherwise challengers of an analysis would have to review and reference the "almost infinite" economic literature in order to show that no literature supports the original author's analysis. Such a burden of proof would discourage or make nearly impossible critical thinking of analyses economists read. To reduce the justification for others labeling economics "the dismal science," economists need to encourage critical thinking, not discourage it.

I did write up a rebuttal showing that the "well-defined literature" cited by the anonymous *Economic Inquiry* referee did not apply to Woodford's model. However, the editor of *Economic Inquiry* merely considered my rebuttal to be considered as a resubmission and that a resubmission was not appropriate. Thus, by diverging from normal academic standards and burdens of proofs, the price determinacy literature has succeeded in putting down my logical critique of Woodford's analysis. As a critic of this literature, I am now in the awkward position of having to appeal to academics outside the price-determinacy literature. I first appeal to the broad economic profession to police the price-determinacy literature, to clean its own laundry with respect to that literature. I am in the bizarre situation of writing a profession paper where I have to rely on the report of an anonymous referee for the references that supposedly justify the use of the non-explosive criterion by the price-determinacy literature. Even though under normal academic standards I should not have such a burden of proof, I nevertheless have shown that the “well-defined literature” cited by that anonymous referee does not apply to the models in the price-determinacy literature. Where does this leave the price-determinacy literature? There is only one answer according normal academic standards. The answer is “naked,” as naked as an emperor parading down the street without any clothes.

Since the price-determinacy literature has not followed normal academic standards of referencing, a question needs answering, “Is the price-determinacy literature justified in its use of the non-exploding criterion?” I ask that the economic profession demand that the price-determinacy literature meet its burden of proof to provide this justification, rather than expecting critics of that literature to go on wild goose chases to find this justification.

If the price-determinacy literature was given this burden of proof (which by normal academic standards it already has, but has not met), I believe that they would be unable to find any such justification. As Cochrane (2006, p. 2) states, “…nothing in economics rules out explosive nominal paths… . (Transversality conditions rule out real explosions, but not nominal ones.) Therefore, nothing in economics allows us to insist on the unique ‘locally bounded’ equilibrium." Remember that the referee for *Economic Inquiry* had cited several papers that he thought did justify the assumption of boundedness. However, the models of those papers include no money; they only resulted in the conclusion that real prices or the real pricing kernel was bounded; they did not conclude that nominal prices were bounded.

While all economists should understand the difference between a nominal cash flow and a real cash flow, many economists may not understand the difference between a nominal price and a real price. Eagle and Domian (2005) make the distinction between the nominal pricing kernel and the real pricing kernel in a pure-exchange, one-good Arrow-Debrue economy. In that model, the real pricing kernel is the units of the consumption good at time 0 per unit of the consumption good at time t; the nominal pricing kernel is the units of money at time 0 per unit of the consumption good at time t. In models involving many goods, the real price of a good X is the units of the numeraire consumption good per unit of good X; the nominal price of a good X is the units of fiat money per unit of good X. Clearly, true nominal prices can only occur in models which include fiat money.

If the price-determinacy literature had followed normal academic standards and had referenced the non-explosive criterion and referenced its justification for using the criterion, it is likely that we would have realized much sooner that there was no justification for applying the non-explosive criterion to the price-determinacy literature.

IX. Summary and Reflections

This paper has argued that the profession's use of the non-explosive criterion is naked, especially as applied by the price-determinacy literature. This paper shows that the non-explosive criterion does not universally apply. The criterion fails miserably when we apply it to the design of a cruise control where the resulting control would accelerate the vehicle when it goes too fast and decelerate it when it goes too slow. Similarly, the criterion fails when we apply it to the design of a feedback rule for the central bank to use when it sets the money supply as it pursues a price-level target. In this latter application, the non-explosive criterion led to the absurd conclusion that the central bank should increase the growth rate in the money supply when the price level exceeds its target and the opposite when the price level falls short of its target.

Because the non-explosive criterion does not universally apply, those who use the non-explosive criterion should have the burden of proof to show that the criterion applies to their models. However, the price-determinacy literature has fallen drastically short in meeting its burden of proof. Also, based on information we have received from referees and others familiar with that literature, it appears that the price-determinacy literature will be unable to meet that burden of proof because there is no basis in economics to assume that nominal prices must be bounded (other than Altug and Labadie (1994) where they assumed an upper limit to the money supply).

The profession's reliance on the non-explosive criterion is the major obstacle preventing the profession from realizing that inflation targeting leads to price indeterminacy even with a Taylor-like feedback rule for pegging the interest rate. Therefore, the ramifications of this paper are likely to be significant. Inflation targeting which is the current fad in central banking, will likely fall in popularity due to papers such as Eagle (2007) and Eagle and Domian (2005). However, not all is bad with inflation targeting; the emphasis on central bank transparency I think has been a positive step. As a result, central banks may begin to look at price-level targeting as a preferred direction over inflation targeting. However, this paper by no means is an endorsement of price-level targeting even though we do believe that price-level targeting does lead to price determinacy when the central bank follows a McCallum-Woodford feedback rule. Price determinacy is a necessary condition for good monetary policy, but it is not a sufficient condition. Another very important goal to strive for in monetary policy is

Pareto efficiency. Eagle and Domian (2005) argue that nominal-income targeting brings the economy much closer to Pareto efficiency than does price-level targeting.

Appendix

This appendix shows that when $P_t^* = (1+\pi^*)P_{t-1}^*$ where π^* is the constant implicit targeted inflation rate and equation (3) holds, the actual inflation rate and the percentage change in the inflation rate will explode whenever $\hat{P}_0$ differs from zero. By applying (3) to both $\hat{P}_{t+1}$ and $\hat{P}_t$, we conclude that

$$\hat{P}_{t-1} - \hat{P}_t = \hat{P}_0 \tau (1+\tau)^{t-1} \qquad \text{(A1)}$$

Since $\frac{P_{t+1}^*}{P_t^*} = 1+\pi^*$, $\frac{P_{t+1}}{P_t} = \frac{P_{t+1}}{P_{t+1}^*}\frac{P_t^*}{P_t}(1+\pi^*)$. Taking logarithms of both sides gives:

$$\ln\left(\frac{P_{t+1}}{P_t}\right) = \hat{P}_{t+1} - \hat{P}_t + \ln(1+\pi^*)$$

Substituting (A1) into the above gives:

$$\ln\left(\frac{P_{t+1}}{P_t}\right) = \hat{P}_0 \tau (1+\tau)^{t-1} + \ln(1+\pi^*) \qquad \text{(A2)}$$

Since P_{t+1}/P_t equals one plus the inflation rate, (A2) shows that the inflation rate will explode (i.e., increase more than exponentially) whenever $\tau > 0$ and $\hat{P}_0$ differs from zero.

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