A two-sector OLG economy: economic growth and demographic behaviour

Fanti, Luciano and Gori, Luca

University of Pisa

26 November 2009

A two-sector OLG economy: economic growth and demographic behaviour

Luciano Fanti[*] and Luca Gori[**]

Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy

Preliminary draft – Please do not quote

Abstract We analyse an overlapping generations economy with two sectors of production: a capital-intensive commodity sector and a labour-intensive services sector. First, we consider an economy with exogenous population and study the effects of a change in the individual preference for old-aged services that causes a reallocation of labour between sectors on per capita income. Then, we compare the results with the standard Diamond (1965) style one-sector economy. Second, we endogenise fertility founding that a reallocation of labour in favour of the services sector causes an additional beneficial effect on per capita income with respect to the model with exogenous population. Third, we further introduce endogenous lifetime through public health investments, showing that multiple regimes of development may exist. In this context, the a rise in the preference for old-aged services may help escaping from poverty.

Keywords Fertility, Life expectancy; OLG model; Public health expenditure; Services market

JEL Classification I18; O41

[*] *E-mail address*: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 22 16 384.

[**] Corresponding author. *E-mail address*: luca.gori@ec.unipi.it; tel.: +39 050 22 16 212; fax: +39 050 22 16 384.

1. Introduction

This paper analyses two prominent problems on the political and economic debate of many countries: (*i*) population ageing, and (*ii*) the increasing demand for both health care provision and services for old age people. The productive structure of an economy can be broadly shared into two categories: capital-intensive commodities and labour-intensive services. A specific feature of the demand of the two categories is worth to be noted: the demand is different depending on the consumers' age and, in particular, the demand for services is an increasing function of it. The need of older people to use more services that the younger emerges from both anecdotic and statistical sources: (*i*) for instance, The Economist argued that: "Older people are likely to spend more on medical care and domestic services, such as those of gardeners and cleaners. Younger people are likely to spend more on new products, such as mobile phones or computers (The price of age, The Economist, 21 December, 2000); (*ii*) budget surveys tend to reveal a larger service expenditure for the elderly. For instance, from the ONS Family Spending 2000–01 for the United Kingdom and from the CBS Budgetonderzoek for The Netherlands may be seen that the elderly spend 1.5 times of their total expenditures on services more than younger people.[1] Therefore, to capture this feature of consumptions in this model we assume, for simplicity, that only the old-aged spend on services.[2]

In this paper the number of children is endogenously chosen by individuals, while the rate of longevity depends on the public health expenditure.

[1] Both instances are drawn by van Groezen et al. (2005, p. 648; 2007a, p. 736).

[2] This assumption captures the feature of the need of higher demand for services by the older people, standing in the middle between the assumption made by van Groezen et al. (2007b), where young- and old-aged consume both goods and services, and the assumption made by van Groezen et al. (2005, 2007a), where young people only consumes goods and old people only consume services. It must be noted, however, that while a similarity as regards the assumption of a two-sector economy exists between the present model and theirs, van Groezen et al. (2007a; 2007b) focused on pension problems, while van Groezen et al (2005) built on a model with exogenous demographic variables and analysed the effect of a rise in the rate of longevity on both the production structure of the economy and economic growth.

Most articles that investigated the mutual relationship between economic and demographic outcomes only considered either a static partial equilibrium context (e.g., Apps and Rees, 2004) or, in a general equilibrium context accounted for neither the existence of aggregate consumption of goods and services, nor the consequences that a two-sector economy may have on economic and demographic performances (e.g. Chakraborty, 2004). In addition to the above mentioned assumption of a two-sector economy, and different from most papers that neglect either endogenous fertility (e.g. Chakraborty, 2004) or the production side of the economy (e.g. Blackburn and Cipriani, 2002), another distinguishing feature of the present paper is that the components of demographic behaviour (i.e. fertility and longevity) are endogenously determined in a dynamic general equilibrium context.

A crucial key ingredient for a change in the sector composition is represented by the change in individual savings. In fact, the saving rate directly affects the demand for services by augmenting the income of the elderly, and by determining capital and wages it affects the capital intensity in the commodity sector, the return on savings – and thus both the income of the elderly and demand for services – and the price of the service. Therefore, to the extent that changes in fertility and longevity affect individual savings, the sector composition changes as well. But the effect of the demographic variable through savings is only a part of the story: indeed the dependency ratio also causes a direct effect on the demand for old-aged services, and thus population ageing in itself shifts the sector composition towards a larger share for the production of services.

The focus of the paper grounds on studying the effect of the demand for old-aged services within a neoclassical overlapping generations (OLG) growth model (Diamond, 1965) modified to account for two production sectors (i.e. the commodity and the service sectors), where an endogenous relationship between fertility, life expectancy, health care provision and output emerges. The importance of an endogenous relationship between economic and demographic variables stands in the fact that a higher level of per worker GDP induced by a certain change in economic or policy parameters, does not necessarily imply a higher level of per capita GDP when both fertility and longevity are endogenous. For example, a rise in the per worker stock of capital (and thus in the per worker GDP) may be associated with a higher longevity, but the final effect on the level of per capita GDP is a priori uncertain. This argument holds even in a one-sector economy, but it is even more relevant in a two-sector context because, for instance, a rise in life expectancy may be associated with a change in the sector composition so that san increasing longevity has an even more uncertain effect on the neoclassical economic growth.

The structure of the paper may be divided into three parts. The first part consider the effect on economic growth occurring when the basic Diamond's (one-sector) model is extended with two productive sectors and shows that per worker GDP (which is the same of per capita GDP in such a context because both fertility and longevity are exogenously given) increases monotonically with the preference for the demand of old-aged services.

The second part extends the two sector model by considering fertility as a choice variable determined by individuals, in line with the approach of the home economics (Becker, 1960; Eckstein and Wolpin; 1985; Galor and Weil, 1996). We found that the positive effect of the preference for old-aged services on economic growth – as shown in the first part – is reinforced when fertility is endogenous. This because a rise in the preference for the demand for services increases the wage rate and this in turn raises the individual demand for children. As a consequence, the dependency ratio is reduced and thus the per capita GDP is increased.

The third part of the paper also accounts for the endogenisation of mortality rates assuming that the probability of surviving from the first period of life to the next depends upon health capital that is augmented through public investment financed by a wage tax.[3] While the provision of health care services on the one side competes resources away (through the taxation) from consumption, savings and children, on the other side it prolongs life expectancy that leads in turn to higher capital accumulation, higher demand of services and a larger number of heads, so that the effect of health

[3] The assumption that the probability of surviving depends on a certain measure of public health care services follows Chakraborty (2004).

policies on economic growth may appear, a priori, uncertain. We show – with respect to the previous models with exogenous longevity – that the role played by the preference for old-aged services on economic growth is enriched by two new effects: since a rise in the preference for services tends increase life expectancy (through the rise in wages and thus in the provision of public health care services), then (*i*) the per worker GDP increases because of the positive effect of longevity on capital accumulation, but (*ii*) the per capita GDP shrinks owing to the increased number of living old age people in the economy. The overall effect, however, is positive and thus per capita GDP increases.

Moreover, when lifetime is endogenous, multiple steady states may exist, generating the so-called poverty trap: unless a high-mortality society starts out with a high enough capital stock (above a certain threshold level), it is unable to escape the vicious cycle of poverty and ill-health.[4] We show that the higher the preference for old age services, the more likely high-mortality societies will escape from poverty.

The remainder of the paper is organised as follows. In Section 2 we present the model with exogenous fertility and exogenous longevity. In Section 3 takes into endogenous fertility decision of individuals. Section 4 considers an economy where both fertility and longevity are endogenous. Section 5 concludes.

2. The model with exogenous population

Consider a general equilibrium overlapping generations (OLG) closed economy populated by identical individuals and identical firms. Young population evolves according to $N_{t+1} = nN_t$, where

[4] The multiple steady state scenario exists, however, even in the one-sector model when the output elasticity of capital is relatively high and threshold effects exists in the distribution of public health investments across population (see Fanti and Gori, 2009).

n is the exogenous number of children of individuals, and two sectors of production exist: the commodity sector and the services sector (see van Groezen et al., 2007b).

2.1. Firms

The commodity sector. At time t firms produce a homogeneous good (Y_t) combining capital (K_t) and the number of young people employed in the commodity sector ($L_{Y,t}$) through the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_{Y,t}^{1-\alpha}$, where $A > 0$ is a scale parameter and $0 < \alpha < 1$ the output elasticity of capital. Firms maximise profits and perfect competition guarantees that factor inputs are paid their marginal products, that is

$$r_t = \alpha A \left(\frac{k_t}{l_{Y,t}} \right)^{\alpha - 1} - 1, \tag{1}$$

$$w_t = (1 - \alpha) A \left(\frac{k_t}{l_{Y,t}} \right)^{\alpha}, \tag{2}$$

where $k_t := K_t / N_t$ is the per worker stock of capital and $l_{Y,t} := L_{Y,t} / N_t$ is the fraction of young people in the economically active population employed in the commodity sector.[5]

The services sector. Production takes place only with labour. The technology is linear and given by $D = L_{D,t}$ where $L_{D,t}$ is the number of people employed in that sector. Since labour is homogenous and perfectly mobile, the price of services in terms of commodities (p_t) equals the wage in the commodity sector, that is $p_t = w_t$.

2.2. Individuals

[5] The price of final output is normalised to unity and capital totally depreciates at the end of each period.

Agents of each generation live for two periods: youth (working period) and old age (retirement period). Each young supplies inelastically his endowment of one unit of time in each period and receives a unitary wage income at the competitive rate w. Therefore, the budget constraint of an individual born at t reads as:

$$c_{1,t} + s_t = w_t \,, \tag{3}$$

i.e. wage income is divided into young-aged consumption for commodities, $c_{1,t}$, and savings, s_t.

Moreover, we assume that at the end of youth survival is uncertain and the probability of surviving from work time to retirement time is $0 < \pi < 1$.

Old individuals are retired and live uniquely with the amount of resources saved when young plus the interest accrued from time t to time $t+1$ at the rate r_{t+1}. The existence of a perfect annuity market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint of an old retired agent started working at t can be expressed as

$$c_{2,t+1} + p_{t+1} d_{2,t+1} = \frac{1 + r_{t+1}}{\pi} s_t \,, \tag{4}$$

where $c_{2,t+1}$ is old-aged consumption for commodities and $d_{2,t+1}$ is the number of services used when old.

The representative individual entering the working period at t must choose consumption for commodities when young and old and the number of both commodities and services when old in order to maximise the following logarithmic utility function:

$$U_t = \ln\left(c_{1,t}\right) + \pi \beta \left[(1-\lambda)\ln\left(c_{2,t+1}\right) + \lambda \ln\left(d_{2,t+1}\right)\right], \tag{5}$$

subject to Eqs. (3) and (4), where β is the subjective discount factor and λ captures the relative importance of consuming services when old. The constrained maximisation of Eq. (5) gives the following demand functions:

$$c_{1,t} = \frac{w_t}{1+\pi\beta}, \tag{6}$$

$$c_{2,t+1} = \frac{\beta(1-\lambda)(1+r_{t+1})w_t}{1+\pi\beta}, \tag{7}$$

$$d_{2,t+1} = \frac{1+r_{t+1}}{w_{t+1}}\frac{\beta\lambda w_t}{1+\pi\beta}, \tag{8}$$

while the saving rate is

$$s_t = \frac{\pi\beta w_t}{1+\pi\beta}, \tag{9}$$

2.3. Equilibrium

The equilibrium in the services market is given by $L_{D,t} = \pi N_{t-1} d_{2,t}$, or

$$l_{D,t} = \frac{\pi d_{2,t}}{n}, \tag{10}$$

The equilibrium in the labour market is given by $N_t = L_{Y,t} + L_{D,t}$, or

$$1 = l_{Y,t} + l_{D,t}, \tag{11}$$

where $l_{D,t} = L_{D,t}/N_t$ is the fraction of young people in the economically active population employed in the services sector.

The equilibrium in the commodity sector is $AK_t^{\alpha} L_{Y,t}^{1-\alpha} = N_t(c_{1,t} + s_t) + \pi N_{t-1} c_{2,t}$, or $AK_t^{\alpha} L_{Y,t}^{1-\alpha} = N_t w_t + \pi N_{t-1} c_{2,t}$, that is

$$Ak_t^{\alpha} l_{Y,t}^{1-\alpha} = w_t + \frac{\pi c_{2,t}}{n}, \tag{12}$$

Market-clearing in goods and capital market leads to the usual equilibrium condition $N_{t+1}k_{t+1} = s_t N_t$, or

$$n k_{t+1} = s_t . \tag{13}$$

Exploiting Eqs. (13) and (9) we get:

$$k_{t+1} = \frac{\pi \beta w_t}{(1+\pi \beta)n} . \tag{14}$$

Substituting out for w_t from Eq. (2), the dynamic path of capital accumulation is:

$$k_{t+1} = \frac{\pi \beta (1-\alpha) A}{(1+\pi \beta)n} \left(\frac{k_t}{l_Y} \right)^{\alpha} . \tag{15}$$

Now, using Eqs. (1), (2) and (11) together with the one period backward Eqs. (8) and (14) we find that

$$l_D = \frac{\lambda \alpha}{1-\alpha(1-\lambda)}, \tag{16}$$

so that

$$l_Y = \frac{1-\alpha}{1-\alpha(1-\lambda)} . \tag{17}$$

The reallocation of labour between sectors depends on both the preference for old-aged services and the output elasticity of capital in the commodity sector. In particular, a rise in either λ or α reduces (increases) the fraction of young population employed in the commodity (services) sector, that is $\frac{\partial l_Y}{\partial \lambda} = \frac{-\alpha(1-\alpha)}{[1-\alpha(1-\lambda)]^2} < 0$ and $\frac{\partial l_Y}{\partial \alpha} = \frac{-\lambda}{[1-\alpha(1-\lambda)]^2} < 0$ ($\frac{\partial l_D}{\partial \lambda} = \frac{\alpha(1-\alpha)}{[1-\alpha(1-\lambda)]^2} > 0$ and $\frac{\partial l_D}{\partial \alpha} = \frac{\lambda}{[1-\alpha(1-\lambda)]^2} > 0$). Moreover, the higher the preference for old-aged services, the lower the intensity of labour reallocation in favour of the services sector, that is $\frac{\partial^2 l_D}{\partial \lambda^2} = \frac{-2\alpha^2(1-\alpha)}{[1-\alpha(1-\lambda)]^3} < 0$.

Steady state implies $k_{t+1} = k_t = k$. Therefore, combining Eqs. (15) and (17) the steady state stock of capital is

$$k = \left[\frac{\pi\beta(1-\alpha)A}{(1+\pi\beta)n}\right]^{\frac{1}{1-\alpha}} \cdot \left[l_Y(\lambda)\right]^{\frac{-\alpha}{1-\alpha}}. \tag{18}$$

From Eq. (18) it can easily be seen that a rise in λ increases the steady state stock of capital, that is $\frac{\partial k}{\partial \lambda} > 0$, since $\frac{\partial k}{\partial l_Y} < 0$ and $\frac{\partial l_Y}{\partial \lambda} < 0$. A rise in λ reduces the fraction of young people employed in the commodity sector and this increases the wage rate. A rise in the wage rate increase savings and this in turn increases the stock of capital in per worker terms.

In this two-sector economy the per capita income at time t is obtained as $\frac{Y_t + w_t D_t}{N_t + \pi N_{t-1}}$, or $\frac{N_t(y_t + w_t d_t)}{N_t + \pi N_{t-1}}$ where y_t and d_t represent the per worker production in the commodity and services sectors, respectively. The latter equation can be rearranged to obtain:

$$q_t := (y_t + w_t d_t) \cdot F, \tag{19}$$

where $F := \frac{n}{n+\pi} = \frac{1}{1+\frac{\pi}{n}}$ with $\frac{\pi}{n}$ being the dependency ratio.

At the steady state $y = Ak^{\alpha} l_Y^{1-\alpha}$ and $d = 1 - l_Y$. Therefore, the effects of a rise in λ on per capita income are the following:

$$\frac{dq}{d\lambda} = \underbrace{\left(\overbrace{\frac{\partial y}{\partial k}}^{+} \cdot \overbrace{\frac{\partial k}{\partial l_Y}}^{-} + \overbrace{\frac{\partial y}{\partial l_Y}}^{+}\right)\overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-}}_{+/-} + \underbrace{w \cdot \overbrace{\frac{\partial d}{\partial l_Y}}^{-} \cdot \overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-}}_{+} + \underbrace{d \cdot \left(\overbrace{\frac{\partial w}{\partial k}}^{+} \cdot \overbrace{\frac{\partial k}{\partial l_Y}}^{-} + \overbrace{\frac{\partial w}{\partial l_Y}}^{-}\right)\overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-}}_{+}, \tag{20}$$

From Eq. (20) we see that a rise in λ has a threefold effect on per capita income. First, it reduces is the fraction of young people in the economically active population employed in the commodity sector. Moreover, a reduction in l_Y increases the per worker stock of capital. Then both forces act positively on wages and then on per capita income. Second, it increases the fraction of young people in the economically active population employed in the services sector, and this increases per worker production in that sector, and through this channel per capita income. Third, an ambiguous effect exist on per worker production in the commodity sector. In fact, a rise λ reduces l_Y, and this tends to lower y. However, the reduction in l_Y also increases k, and this in turn produces a positive effect on y. As can easily be shown when the output elasticity of capital is $\alpha < 1/2$ ($\alpha > 1/2$), the negative (positive) effect dominates causing a reduction (increase) in the production of commodities. Therefore, if $\alpha < 1/2$ ($\alpha > 1/2$) a rise λ causes a counterbalancing negative force due to lower production in the commodity sector (a strengthening positive force due to higher production in the commodity sector) on per capita income.

Therefore, the following remarks holds:

Remark 1. *The wage rate increases with λ because the labour input in the commodity sectors is reduced.*

Remark 2. *A rise in λ reduces (increases) output per worker in the commodity sector y if $\alpha < 1/2$ ($\alpha > 1/2$).*

Remark 3. *A rise in λ increases monotonically output per worker in the services sector d.*

In the following proposition, however, we show the overall effect of a rise in λ on per capita income is always positive, that is irrespective of whether the output elasticity of capital is relatively high or low a reallocation of labour in favour of the services sector is income improving, and this in turn implies that the per capita income in a two-sector economy is always higher than that would be obtained in the standard Diamond's (1965) one-sector economy.

Proposition 1. *The per capita income a two-sector economy is always higher than that in a one-sector economy.*

Proof. The steady-state per capita income in this two-sector economy may be expressed as

$$q = F \cdot A \cdot [k(\lambda)]^{\alpha} \cdot G(\lambda), \tag{21}$$

where $G(\lambda) := \dfrac{1-\alpha+\alpha l_Y(\lambda)}{[l_Y(\lambda)]^{\alpha}}$.

Since

$$\frac{\partial q}{\partial \lambda} = F \cdot A \cdot \left\{ G(\lambda) \cdot \overbrace{\frac{\partial k}{\partial \lambda}}^{+} + k^{\alpha} \cdot \overbrace{\frac{\partial G}{\partial l_Y}}^{-} \cdot \overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-} \right\} > 0, \tag{22}$$

then Proposition 1 follows. **Q.E.D.**

3. Endogenous fertility

In this section we consider fertility to be endogenously determined by individuals as a rational comparison between benefits and costs of children. Agents of each generation are identical and live for three periods: childhood (during which individuals do not make economic decisions), youth (working and child bearing time) and old age (retirement time). The representative individual, therefore, that seeks to maximise the lifetime utility function with respect to consumption for commodities and fertility when young and consumption for commodities and services when old:

$$U_t = \ln(c_{1,t}) + \phi \ln(n_t) + \pi\beta[(1-\lambda)\ln(c_{2,t+1}) + \lambda \ln(d_{2,t+1})], \tag{23}$$

subject to

$$c_{1,t} + s_t + e n_t = w_t, \tag{24}$$

and Eq. (4), where ϕ is the relative taste for children, e is the fixed cost of raising a child and n_t is the number of children at time t. The demand functions, therefore, are the following:

$$c_{1,t} = \frac{w_t(1-\tau)}{1+\pi\beta+\phi}, \tag{25}$$

$$c_{2,t+1} = \frac{\beta(1-\lambda)(1+r_{t+1})w_t(1-\tau)}{1+\pi\beta+\phi}, \tag{26}$$

$$d_{2,t+1} = \frac{1+r_{t+1}}{w_{t+1}} \frac{\beta\lambda w_t(1-\tau)}{1+\pi\beta+\phi}, \tag{27}$$

$$n_t = \frac{\phi w_t(1-\tau)}{(1+\pi\beta+\phi)e}, \tag{28}$$

while the saving rate is

$$s_t = \frac{\pi_t \beta w_t(1-\tau)}{1+\pi\beta+\phi}. \tag{29}$$

The equilibrium in the services market is given by

$$l_{D,t} = \frac{\pi d_{2,t}}{n_{t-1}}, \tag{30}$$

The equilibrium in the labour market is determined by Eq. (11), while the equilibrium in the commodity sector is

$$Ak_t^{\alpha} l_{Y,t}^{\ 1-\alpha} = w_t - en_t + \frac{\pi c_{2,t}}{n_{t-1}}, \tag{31}$$

Market-clearing in goods and capital market implies

$$n_t k_{t+1} = s_t. \tag{32}$$

Exploiting Eq. (32) together with Eqs. (28) and (29) we get:

$$k_{t+1} = k = \frac{\pi\beta e}{\phi}. \tag{33}$$

Therefore,

Remark 4. *When fertility is endogenously determined by individuals and the cost of children is fixed, the per worker equilibrium stock of capital is constant and independent of* l_Y.

Now, using Eqs. (1), (2) and (11) together with the one period backward Eqs. (33), (28) and (27) we find that l_D and l_Y are constant and determined by Eqs. (16) and (17), respectively. Therefore, Remark 1 and 3 hold, while

Remark 5. *A rise in λ reduces monotonically the output per worker in the commodity sector y.*

Since $y = Ak^{\alpha} l_Y^{1-\alpha}$ and k is independent of λ, a rise in the preference for old-aged services, increases labour in that sector while reducing labour in the commodity sector. Hence, output per worker in the commodity sector shrinks. Moreover,

Remark 6. *A rise in λ reduces l_Y and in turn increases the wage rate. Since n is a monotonic increasing function of the wage, a reallocation of the labour input in favour of the services sector increases the steady-state fertility rate.*

Different from the world with exogenous population, in an endogenous fertility setting the effects of λ on per capita income are different. From Remark 6, the preference for old-aged services affects fertility and, through this channel, the per capita income.

The steady-state per capita income expressed in terms of commodities is given by

$$q = A \cdot k^{\alpha} \cdot F(\lambda) \cdot G(\lambda), \tag{34}$$

where $F(\lambda) := \dfrac{n(\lambda)}{n(\lambda)+\pi}$. Therefore,

$$\frac{\partial q}{\partial \lambda} = A \cdot k^{\alpha} \cdot \left[G(\lambda) \cdot \overbrace{\frac{\partial F}{\partial \lambda}}^{+} + F(\lambda) \cdot \overbrace{\frac{\partial G}{\partial \lambda}}^{+} \right] > 0, \tag{35}$$

where $\frac{\partial F}{\partial \lambda} = \overbrace{\frac{\partial F}{\partial n}}^{+} \cdot \overbrace{\frac{\partial n}{\partial w}}^{+} \cdot \overbrace{\frac{\partial w}{\partial l_Y}}^{-} \cdot \overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-} > 0$ and $\frac{\partial G}{\partial \lambda} = \overbrace{\frac{\partial G}{\partial l_Y}}^{-} \cdot \overbrace{\frac{\partial l_Y}{\partial \lambda}}^{-} > 0$. The effect of $G(\lambda)$ on per capita income is the same as in the model with exogenous fertility. However, an additional effect exists that passes through the function $F(\lambda)$: a rise in the preference for old-aged services reduces the labour input in the commodity sector and this in turn increases the wage rate. Higher wages mean higher fertility. A higher fertility rate gives rise to a positive additional effect of λ on per capita income. Therefore, the following propositions hold:

Proposition 2. *The per worker income* $y + wd$ *in a two-sector endogenous fertility economy increases monotonically with* λ.

Proof. Since k is constant and independent of λ, the proof is obvious from the sign of the second addendum in square brackets in (35). **Q.E.D.**

Proposition 3. *When fertility is endogenous, a rise in* λ *causes a positive fertility effect on per capita income in addition to the positive monotonic effect on per worker income.*

Proof. The proof is given by the sign Eq. (35).

4. Endogenous fertility and endogenous longevity

The government now invests in public health (e.g. hospitals, vaccination programmes, scientific research and so on) and finances health capital through a constant wage income tax $0 < \tau < 1$ (see Chakraborty, 2004). Therefore, at time t the public health budget (in per worker terms) reads as

$$h_t = \tau w_t , \tag{36}$$

where the left-hand side is the health expenditure and the right-hand side the tax receipt.

The budget constraint of an individual born at t becomes:

$$c_{1,t} + s_t + en_t = w_t(1-\tau), \tag{37}$$

i.e. wage income – net of contributions paid to finance health expenditure – is divided into consumption for commodities when young, $c_{1,t}$, savings, s_t and the fixed cost e of raising n_t children.

Following Chakraborty (2004), we assume that at the end of youth survival is uncertain and the probability of surviving from work time to retirement time, π_t, is an increasing – though bounded – function of the public health measure h_t. Different from Chakraborty (2004), however, we follow Blackburn and Cipriani (2002) and Blackburn and Issa (2002) and specialise the relationship between public health investments and life expectancy with the following logistic-typed function

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}, \tag{38}$$

where $\delta, \Delta > 0$, $0 < \pi_1 \leq 1$, $0 \leq \pi_0 < \pi_1$, $\pi(0) = \pi_0 \geq 0$, $\pi'_h(h) = \dfrac{\delta \Delta h^{\delta-1}(\pi_1 - \pi_0)}{\left(1 + \Delta h^{\delta}\right)^2} > 0$,

$\lim_{h\to\infty} \pi(h) = \pi_1 \leq 1$ and $\lim_{h\to 0} \pi'_h(h) = \pi_1 - \pi_0 < \infty$, $\pi''_{hh}(h) < 0$ if $\delta \leq 1$ and $\pi''_{hh}(h) \gtrless 0$ for any $h \lessgtr h_T := \left[\dfrac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Eq. (5) allows us to capture various aspects of the evolution of the length of life of the typical agent through the retirement age. First, we define π_0 as being the exogenous "natural" rate of longevity of people in a country: it represents the fraction of time lived by individuals at the end of youth irrespective of whether the government invests in public health or not, and may affected by both economic and non-economic factors, e.g. the lifestyle of people, education, economic growth and the standards of living, the degree of culture and civilisation, weather and climate changes, ethnical and civil wars, endemic diseases and so on. Thus, we may expect π_0 to be higher in developed rather than developing or under-developed nations, and the more individuals naturally live longer, the smaller the reduction in adult mortality due to a rise in public health expenditure. Second, the parameter π_1 captures the intensity of the efficiency of public health investments on the rate of longevity. A rise in π_1 may be interpreted as exogenous medical advances due, for instance, to scientific research, vaccination programmes and so on. Third, we may think, realistically, that health investments have a more intense effect in reducing adult mortality often when a certain threshold level of public health expenditure has been reached, while becoming scarcely effective when longevity is close to its saturating value (e.g., the functional relationship between health investment and longevity may be S-shaped). The parameters δ and Δ both allow to capture such an idea and determine both the turning point of $\pi_h'(h)$ and speed of convergence from the natural length of life π_0 to the saturating value π_1. In particular, given the value of Δ, the parameter δ represents the degree of proportionality in the distribution of public health investments across population as an inducement to higher life expectancy, other things being unchanged. In other words, it measures how an additional unit of public health capital is transformed into higher longevity through the public health technology. If $\delta \leq 1$, $\pi(h)$ is concave for any h and, hence, no threshold effects of public health investments on longevity exist, i.e. the distribution of an additional unit of health capital is relatively widespread and smooth across population, so that longevity increases monotonically with decreasing returns from the starting point π_0 to the saturating value π_1 as h rises, and the more δ is close to zero the more efficiently and rapidly an additional unit of health capital is transformed into higher longevity when h is relatively low, while reaching the saturating value π_1 more slowly as h becomes larger. Figure 1 illustrates in a stylised way the evolution of the rate of longevity as a function of public health capital when $\delta \leq 1$: the solid (dashed) [dotted] line refers to the case $\delta = 1$ ($\delta < 1$) [$\delta \to 0$]. As can readily be seen the lower (higher) is δ the more (less) efficient is an additional unit of public health capital expenditure as an inducement to higher life expectancy until (once) a certain level of h is reached.

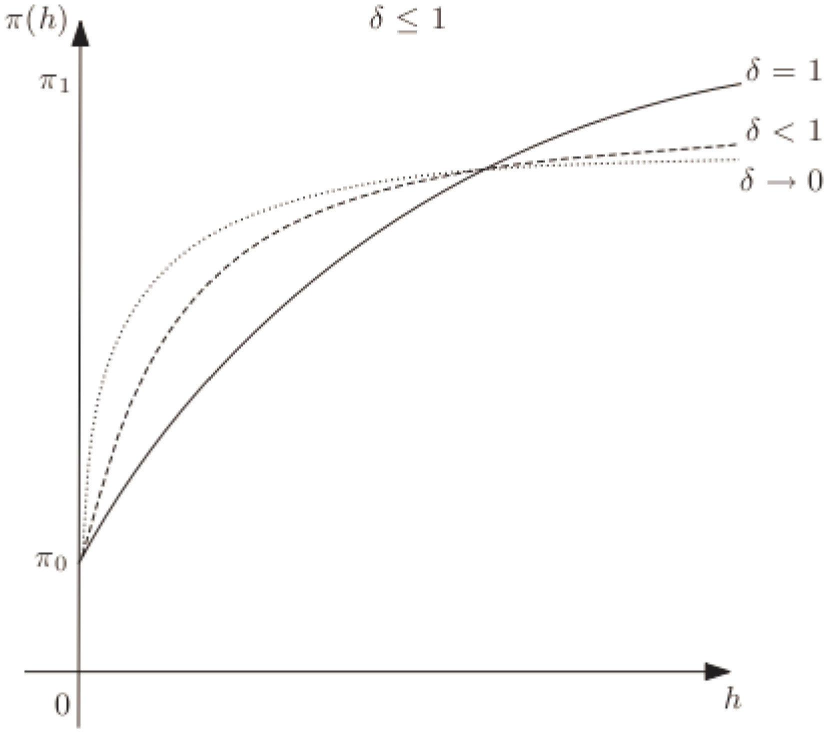

Figure 1. Longevity and public health capital when $\delta \leq 1$.

In contrast, when $\delta > 1$ the longevity function is S-shaped and, hence, threshold effects exist in the way in which public health investments are distributed across population and then transformed into higher longevity. In particular, longevity increases more (less) than proportionally when $h < h_T$ ($h > h_T$), i.e. it grows with increasing (decreasing) returns until (once) the turning point h_T is reached. However, a rise in δ shifts the longevity function to the right while also increasing the speed of convergence from π_0 to π_1, as clearly shown in Figure 2 below, where the solid (dashed) [dotted] line refers to δ_{low} (δ_{high}) [$\delta \to +\infty$]. The distribution of an additional unit of health capital is not smooth across population in that case and, in particular, an increasing amount of public health expenditure is required to trigger beneficial effects on longevity, and the higher δ is the more slowly an additional unit of health investments is transformed into higher life expectancy when h is relatively low, while reaching the saturating value π_1 more efficiently and rapidly as h becomes larger.

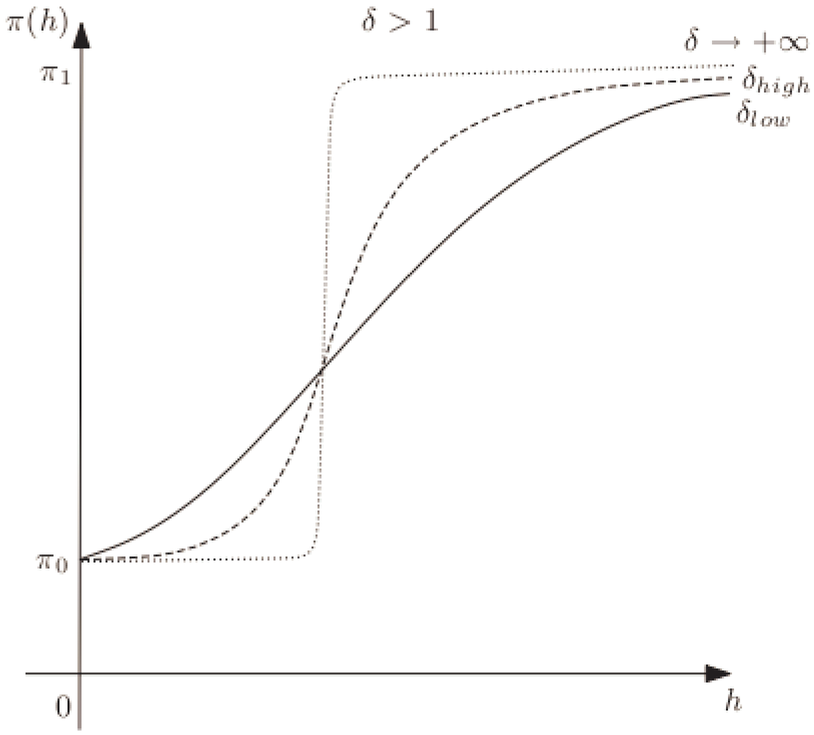

Figure 2. Longevity and public health capital when $\delta > 1$.

This means that increasing public health expenditure is not effective as an inducement to higher longevity until a threshold value of health capital is accumulated (and this value is larger the larger is δ), because, for instance, a certain degree of knowledge to enable health investments to be distributed and transformed efficiently into a higher longevity has not yet been reached. Beyond that threshold, however, a "sudden" effect exists that efficiently allows such a transformation. As an example, we may think about the existence of threshold effects in the accumulation of knowledge required for new medical advances and discoveries in the treatment of diseases (e.g. vaccines) to be effective and efficient: the public health expenditure to finance new research projects may be high and apparently useless up to a certain degree of knowledge is achieved. Once that threshold is reached, however, there exists a sudden effect that allows to trigger and bring to light the beneficial effects of the new discoveries, to make them efficient, usable and operative across population and eventually transformed into a higher life expectancy.

The representative individual entering the working period at t must choose consumption for commodities when young and old, the number of services when old and the number of children in order to maximise

$$U_t = \ln\left(c_{1,t}\right) + \phi \ln\left(n_t\right) + \pi_t \beta\left[\left(1-\lambda\right)\ln\left(c_{2,t+1}\right) + \lambda \ln\left(d_{2,t+1}\right)\right], \tag{39}$$

Eq. (4) and (37). The demand functions therefore are the following:

$$c_{1,t} = \frac{w_t(1-\tau)}{1+\pi_t\beta+\phi}, \tag{40}$$

$$c_{2,t+1} = \frac{\beta(1-\lambda)(1+r_{t+1})w_t(1-\tau)}{1+\pi_t\beta+\phi}, \tag{41}$$

$$d_{2,t+1} = \frac{1+r_{t+1}}{w_{t+1}} \frac{\beta\lambda w_t(1-\tau)}{1+\pi_t\beta+\phi}, \tag{42}$$

$$n_t = \frac{\phi w_t(1-\tau)}{(1+\pi_t\beta+\phi)e}, \tag{43}$$

while the saving rate is

$$s_t = \frac{\pi_t\beta w_t(1-\tau)}{1+\pi_t\beta+\phi}, \tag{44}$$

with π_t being determined by Eq. (38) upon substitution of h_t from Eq. (36).

The equilibrium in the services market is given by $L_{D,t} = \pi_{t-1}N_{t-1}d_{2,t}$, or

$$l_{D,t} = \frac{\pi_{t-1}d_{2,t}}{n_{t-1}}, \tag{45}$$

The equilibrium in the labour market is given by $N_t = L_{Y,t} + L_{D,t}$, or

$$1 = l_{Y,t} + l_{D,t}. \tag{46}$$

The equilibrium in the commodity sector is $AK_t^{\alpha}L_{Y,t}^{1-\alpha} = N_t w_t(1-\tau) - N_{t+1}e + \pi_{t-1}N_{t-1}c_{2,t}$, that is

$$Ak_t^{\alpha}l_{Y,t}^{1-\alpha} = w_t(1-\tau) - en_t + \frac{\pi_{t-1}c_{2,t}}{n_{t-1}}, \tag{47}$$

Market-clearing in goods and capital market leads to

$$n_t k_{t+1} = s_t \,. \tag{48}$$

Therefore, upon substitution for n_t and s_t from (43) and (44) into (48) we get

$$k_{t+1} = \frac{\pi_t(k_t, l_Y)\beta e}{\phi} \,. \tag{49}$$

Since l_D and l_Y are determined by (16) and (17), respectively, using (49), (38), (36) and (2) the dynamic path of capital accumulation is:

$$k_{t+1} = \frac{\beta e\left[\pi_0 + \pi_1 B k_t^{\alpha\delta} M(\lambda)\right]}{\phi\left[1 + B k_t^{\alpha\delta} M(\lambda)\right]}, \tag{50}$$

where $B := \Delta[\tau(1-\alpha)A]^{\delta}$ and $M(\lambda) := \left[\frac{1-\alpha(1-\lambda)}{1-\alpha}\right]^{\alpha\delta}$ are two positive constant used to simplify notation.

Different from the model of the previous sections, when longevity is endogenous and determined by public health investments the dynamic path of capital accumulation Eq. (50) may give rise to multiple development regimes, and changes in the allocation of labour between sectors through a change in the preference for old-aged services affects both the transitional dynamics and steady states of the economy. In the next section, therefore, we study how the steady state outcomes (i.e. macroeconomic and demographic variables) in both low and high income countries are affected by a rise in λ.

4.1. Dynamics

Analysis of Eq. (50) gives the following proposition:

Proposition 4. *Let either (i) $\delta \le 1$ or (ii) $\delta > 1$ and $0 < \alpha < \frac{1}{\delta}$ hold. Then the dynamic system described by Eq. (50) possesses only one positive asymptotically stable steady state $\{\bar{k}\}$. Let $\delta > 1$*

and $\frac{1}{\delta}<\alpha<1$ *hold. Then the dynamic system described by Eq. (50) admits either one positive asymptotically stable steady state* $\{\bar{k}\}$ *or three positive steady states* $\{\bar{k}_1,\bar{k}_2,\bar{k}_3\}$, *with* $\bar{k}_3>\bar{k}_2>\bar{k}_1$ *(only the first and the third being asymptotically stable).*

Proof. Let first the following lemma be established.

Lemma 1. *Define the right-hand side of (50) as* $J(k)$. *Then, we have: (1.i)* $J(0)=\frac{\beta e\pi_0}{\phi}$, *(1.ii)* $J_k'(k)>0$ *for any* $k>0$, *(1.iii)* $\lim_{k\to+\infty}J_k'(k)=0$, *(1.iv) if* $\delta\le 1$, *then* $J_{kk}''(k)<0$ *for any* $k>0$, *(1.v) if* $\delta>1$ *and* $0<\alpha<\frac{1}{\delta}$, *then* $J_{kk}''(k)<0$ *for any* $k>0$, *(1.vi) if* $\delta>1$ *and* $\frac{1}{\delta}<\alpha<1$, *then* $J_{kk}''(k)\underset{<}{\overset{>}{-}}0$ *if and only if* $k\underset{>}{\overset{<}{-}}k_f:=\left[\frac{\alpha\delta-1}{BM(\lambda)(1+\alpha\delta)}\right]^{\frac{1}{\alpha\delta}}$.

From Eq. (50), property (1.*i*) is straightforward. Differentiating the right-hand side of (50) with respect to k gives

$$J_k'(k)=\frac{\alpha\delta\beta eB(\pi_1-\pi_0)k^{\alpha\delta}M(\lambda)}{\phi k\left[1+Bk^{\alpha\delta}M(\lambda)\right]^2}>0, \tag{51}$$

which proves (1.*ii*). Moreover,

$$\begin{aligned}\lim_{k\to+\infty}J_k'(k)&=\lim_{k\to+\infty}\frac{\alpha\delta\beta eB(\pi_1-\pi_0)k^{\alpha\delta}M(\lambda)}{\phi k\left[1+Bk^{\alpha\delta}M(\lambda)\right]^2}\\&=\frac{\alpha\delta\beta eB(\pi_1-\pi_0)M(\lambda)}{\phi}\cdot\lim_{k\to+\infty}\frac{1}{k^{1+\alpha\delta}\left\{\frac{1}{k^{2\alpha\delta}}+\frac{BM(\lambda)}{k^{\alpha\delta}}+B^2[M(\lambda)]^2\right\}^2}=0,\end{aligned}$$

which proves (1.*iii*).

Now, differentiating (51) with respect to k gives

$$J''_{kk}(k)=\frac{\alpha\delta\beta eBM(\lambda)(\pi_1-\pi_0)k^{\alpha\delta}\left[\alpha\delta-1-BM(\lambda)(1+\alpha\delta)k^{\alpha\delta}\right]}{\phi k^2\left[1+Bk^{\alpha\delta}M(\lambda)\right]^3}. \tag{52}$$

From (52), it is straightforward to see that if $\delta\le 1$, then $J''_{kk}(k)<0$ for any $k>0$. This proves (1.*iv*). If $\delta>1$ and $0<\alpha<\frac{1}{\delta}$, then $J''_{kk}(k)<0$ for any $k>0$. This proves (1.*v*). If $\delta>1$ and $\frac{1}{\delta}<\alpha<1$, then $J''_{kk}(k)\underset{<}{\overset{>}{-}}0$ if and only if $k\underset{>}{\overset{<}{-}}k_f:=\left[\frac{\alpha\delta-1}{BM(\lambda)(1+\alpha\delta)}\right]^{\frac{1}{\alpha\delta}}$. This proves (1.*vi*).

Proposition 4 therefore follows. In fact, by property (1.*i*), zero can never be an equilibrium of Eq. (50) and the phase map $J(k)$ starts on the vertical axis at the point $J(0)=\frac{\beta e\pi_0}{\phi}$. Moreover, by (1.*ii*) and (1.*iii*), $J(k)$ is a monotonic increasing function of k and eventually falls below the 45° line, so that at least one positive stable steady state exists for any $k>0$.

If $\delta\le 1$, then (1.*iv*) holds and $J(k)$ is a concave function for any $k>0$. Hence, one and only one positive asymptotically stable steady state exists in that case.

If $\delta>1$ and $0<\alpha<\frac{1}{\delta}$, then (1.*v*) holds and $J(k)$ is a concave function for any $k>0$. Hence, one and only one positive asymptotically stable steady state exists in that case.

If $\delta>1$ and $\frac{1}{\delta}<\alpha<1$, then (1.*vi*) holds and a unique positive inflection point k_f exists for any $k>0$, so that $J(k)$ is a convex (concave) function for any $0<k<k_f$ ($k>k_f$). Since at least one positive stable steady state exists, then for any $k>0$ the phase map $J(k)$ may intersect the 45° line from below *at most* once before falling below it. Therefore, either one positive asymptotically stable steady exists or *at most* one positive steady state separates the lowest asymptotically stable steady state from the highest asymptotically stable one, and, thus, the number of equilibria is three. **Q.E.D.**

Proposition 4 shows that when both threshold effects in the distribution of health investments across population exist and the output elasticity of capital is high enough two development regimes are possible in this stylised two-sector economy. The high regime is characterised low mortality and high capital accumulation, per capita income, wages and fertility. The low regime, instead, is characterised by high mortality and low capital accumulation, per capita income, wages and fertility.

4.2. Discussions

A change in the preference for the number of old-aged service changes the allocation of the fraction of workers employed in both the commodity and services sector. In particular, as noted in the previous sections a rise in λ increases in the fraction of workers employed in the services sector while reducing those employed in the commodity sector, i.e. l_D increases and l_Y shrinks. A reduction in l_Y causes in turn a rise in wages that directly promotes savings and capital accumulation. Higher wages, however, means both higher fertility and longevity. In particular, the increased life expectancy of people causes an impetus to increase further the saving rate and this produces a positive indirect effect on capital accumulation. Definitively, the shape of the function $J(k)$ raises as a consequence of a redistribution of labour in favour of the services sector, i.e. capital accumulation in both regimes of development increases. In particular, the following result holds:

Result 1. *A rise in the preference for old-aged services increases both the lowest and highest stable steady states, reduces the intermediate unstable steady state, while shrinking the size of the basin of attraction towards the poverty trap. Moreover, a large enough increase in* λ *may cause the loss of the lowest stable steady state, thus allowing poorer economies to escape permanently from poverty and, hence, converge towards the high regime of development..*

Figure 3 below illustrates in a stylised way Result 1, showing the beneficial effect of λ on capital accumulation.

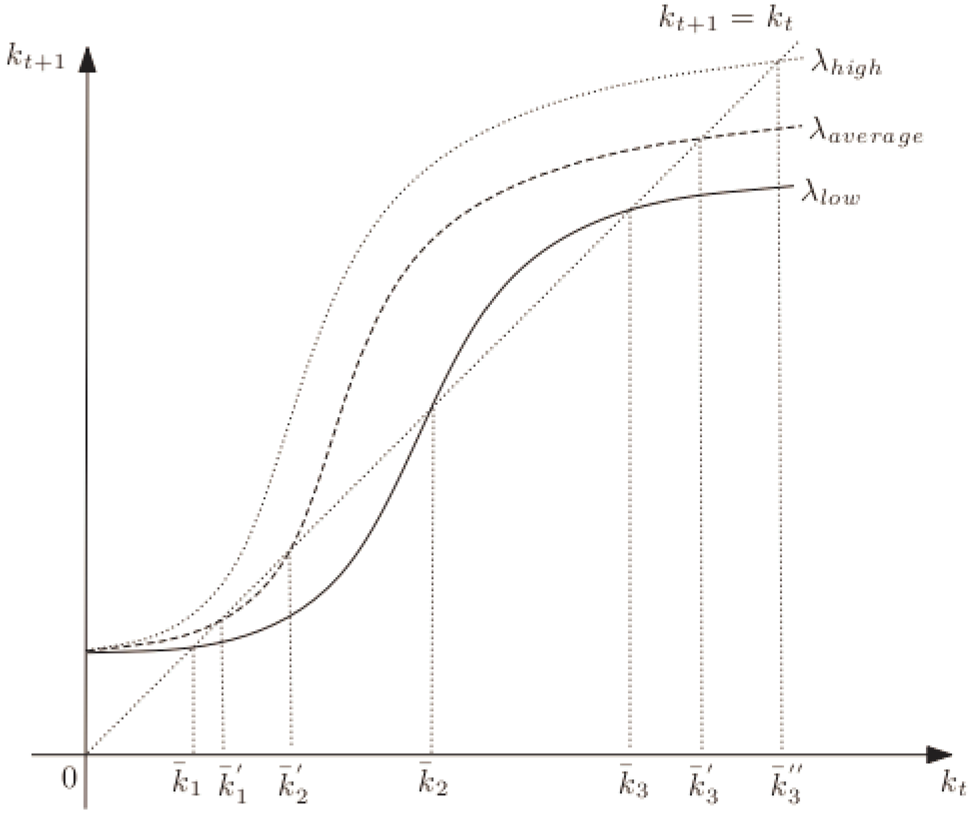

Figure 3. Multiple steady states in a two-sector economy.

As can be seen from Figure 3, when λ is high enough the low equilibrium vanishes and all economies converge towards the high regime of development (see the dotted line in Figure 3).

4.3. Steady-state: macroeconomic and demographic outcomes

Tables 1 and 2 below show the behaviour of macroeconomic and demographic variables when λ varies for two different values of the output elasticity of capital in the commodity sector, i.e.,

$\alpha = 0.33$ (Table 1) and $\alpha = 0.37$ (Table 2).[6] We take the following parameter set (chosen only for illustrative purposes): $A = 13$, $\beta = 0.60$, $\phi = 0.30$, $\pi_0 = 0.10$, $\pi_1 = 0.85$, $\delta = 10$, $\Delta = 1$, $\tau = 0.10$ and $e = 2$.

Table 1. Long-run outcomes of the economy when $\alpha = 0.33$ and λ varies.

Low regime

λ	0	0.10
$\bar{k}_1$	0.455	0.472
$w(\bar{k}_1)$	6.71	6.91
$\pi(\bar{k}_1)$	0.113	0.118
$n(\bar{k}_1)$	0.662	0.680
$q(\bar{k}_1)$ (per capita income)	8.556	8.652
l_D	0	0.046
l_Y	1	0.953

High regime

λ	0	0.10	0.30	0.50	0.70	0.90	0.99
$\bar{k}_3$	3.152	3.194	3.25	3.29	3.31	3.33	3.34
$w(\bar{k}_3)$	12.72	12.98	13.45	13.87	14.26	14.62	14.78
$\pi(\bar{k}_3)$	0.788	0.798	0.813	0.822	0.829	0.833	0.835

[6] We used values of the output elasticity of capital around 1/3 as it is usually assumed in literature (for estimates of α in different countries see, e.g., Jones, 2003).

$n(\bar{k}_3)$	0.968	0.985	1.01	1.04	1.07	1.09	1.10
$q(\bar{k}_3)$ (per capita income)	10.47	10.53	10.67	10.82	10.98	11.14	11.22
l_D	0	0.046	0.128	0.197	0.256	0.307	0.327
l_Y	1	0.953	0.871	0.802	0.743	0.692	0.672

Table 2. Long-run outcomes of the economy when $\alpha = 0.37$ and λ varies.

Low regime

λ	0	0.10	0.30	0.50
$\bar{k}_1$	0.415	0.42	0.433	0.457
$w(\bar{k}_1)$	5.91	6.069	6.381	6.744
$\pi(\bar{k}_1)$	0.103	0.105	0.108	0.114
$n(\bar{k}_1)$	0.586	0.601	0.631	0.665
$q(\bar{k}_1)$ (per capita income)	7.98	8.03	8.16	8.36
l_D	0	0.055	0.149	0.226
l_Y	1	0.944	0.85	0.773

High regime

λ	0	0.10	0.30	0.50	0.70	0.90	0.99
$\bar{k}_3$	3.096	3.168	3.253	3.30	3.329	3.348	3.354
$w(\bar{k}_3)$	12.44	12.81	13.45	14.01	14.51	14.98	15.18

$\pi(\bar{k}_3)$	0.774	0.792	0.813	0.825	0.832	0.837	0.838
$n(\bar{k}_3)$	0.951	0.974	1.015	1.053	1.089	1.122	1.136
$q(\bar{k}_3)$ (per capita income)	10.89	10.99	11.20	11.42	11.65	11.88	11.98
l_D	0	0.055	0.149	0.226	0.291	0.345	0.367
l_Y	1	0.944	0.85	0.773	0.708	0.654	0.632

As can be seen from Tables 1 and 2, a rise in λ causes a rise in capital accumulation, wages, life expectancy, fertility and per capita income in both regimes. Moreover, when $\alpha = 0.33$ values of λ beyond 0.10 are able to allow low-income economies to escape permanently from poverty and converge towards the high regime where per capita income, fertility and longevity are higher. The beneficial effect of λ on capital accumulation, however, are slightly reduced when the output elasticity of capital increases (see the case $\alpha = 0.37$ in Table 2), other things being unchanged. In fact, in that case the threshold value of λ beyond which the poverty trap is lost is 0.50.

5. Conclusions

We analysed an overlapping generations economy with two sectors of production: the commodity sector and the services sector. We assumed that these services are consumed only by the old age people. We studied how a reallocation of labour in favour of the labour-intensive services sector affects both macroeconomic and demographic variables. A crucial key ingredient for a change in the sector composition is represented by the change in individual savings.

The paper is divided into three parts. The first part analyses the effect of a reallocation of labour in favour in the services sector on economic growth and compares the results with that obtained in the basic Diamond's (one-sector) model. We found that the per capita GDP increases monotonically with the individual preference for old-aged services.

The second part extends the two sector model by considering endogenous fertility. In that case, the positive effect of the preference for old-aged services on the (neoclassical) economic growth is reinforced due to the increased fertility rate.

The third part of the paper also takes into account endogenous longevity through public investments. With respect to the previous models with exogenous longevity multiple regimes of development may exist. In this context, a rise in the preference for old age services may help escaping from poverty.

References

Apps, P., Rees, R., 2004. Fertility, taxation and family policy. Scandinavian Journal of Economics 106, 745–763.

Becker, G.S., 1960. An economic analysis of fertility. In Demographic and Economic Change in Developing Countries. National Bureau Committee for Economic Research (Eds.), Princeton, NJ, Princeton University Press.

Blackburn, K., Cipriani, G.P., 2002. A model of longevity, fertility and growth. Journal of Economic Dynamics and Control 26, 187–204.

Blackburn, K. and Issa, H., 2002. Endogenous life expectancy in a simple model of growth. Centre for Growth and Business Cycle Research, School of Economic Studies, University of Manchester, Working Paper no. 13, available at http://www.ses.man.ac.uk/cgbcr/discussi.htm.

Chakraborty, S., 2004. Endogenous lifetime and economic growth. Journal of Economic Theory 116, 119–137.

Diamond, P.A., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150.

Eckstein, Z., Wolpin, K.I., 1985. Endogenous fertility and optimal population size. Journal of Public Economics 27, 93–106.

Fanti, L., Gori, L., 2009. Endogenous fertility, endogenous lifetime and economic growth: the role of health and child policies. Discussion Paper no. 91, Department of Economics, University of Pisa.

Galor, O., Weil, D.N., 1996. The gender gap, fertility, and growth. American Economic Review 86, 374–387.

Groezen, B. van, Meijdam, L., Verbon, H.A.A., 2005. Serving the old: ageing and economic growth Oxford Economic Papers 57, 647–663.

Groezen, B. van, Meijdam, L., Verbon, H.A.A., 2007a. Increased pension savings: blessing or curse? Social security reform in a two-sector growth model. Economica 74, 736–755.

Groezen, B. van, Meijdam, L., Verbon, H.A.A., 2007b. The case for pay-as-you-go pensions in a service economy. Scottish Journal of Political Economy 54, 151–165.

Jones, C.I., 2003. Growth, capital shares, and a new perspective on production functions. U.C. Berkeley Working Paper, available at http://elsa.berkeley.edu/~chad.