



Munich Personal RePEc Archive

**A formula for the optimal taxation in  
Probabilistic Voting Models  
characterized by Single Mindedness**

Canegrati, Emanuele

23 February 2007

Online at <https://mpra.ub.uni-muenchen.de/1896/>

MPRA Paper No. 1896, posted 24 Feb 2007 UTC

# A Formula for the Optimal Taxation in Probabilistic Voting Models characterized by Single Mindedness

Emanuele Canegrati

February 23, 2007

1

## 1 Introduction

This work intends to specify a formula for the optimal taxation in Probabilistic Voting Models with Single Mindedness Theory. The goal is to find an equivalent expression to the Ramsey's rule for a political economy environment where Governments are assumed to be Leviathans rather than benevolents. In a normative approach, one of the most important achievements in deriving an optimal taxation was that a reduction in the compensated demand for a good should be small if the social marginal utility of income for a person  $h$  multiplied by the demand of that good by the same person is large. As a consequence, if good  $k$  is largely demanded by people whose society attributes a large social marginal evaluation, this good should be taxed less. Thus the Ramsey's rule is able to provide useful suggestions not only in terms of efficiency but also in terms of equity. The poor, who traditionally are those who have a large personal marginal utility of income should be taxed less and, obviously, this is a great achievement for the society welfare.

## 2 Framework

Suppose a society where there are  $h = 1, \dots, H$  social groups and where the utility of an individual is given by  $k = 1, \dots, K$  issues plus consumption. A representative individual of the  $h$ -th social group maximises the following utility function:

$$\max U^h(c^h, x_1^h, \dots, x_k^h, \psi_1^h, \dots, \psi_k^h) \quad (1)$$

---

<sup>1</sup>DEFAP - Università Cattolica del Sacro Cuore - Milan; e-mail address: emanuele.canegrati@unicatt.it

where  $\psi^k$  represents the percentage of mindedness devoted to the  $k$  issue. Furthermore, I assume that  $\frac{\partial U^h}{\partial x_k^h} > 0$  and  $\frac{\partial U^h}{\partial \psi_k^h} > 0$ . The budget constraint is given by:

$$c^h = \sum_k \widetilde{p}_k^h x_k^h \quad (2)$$

where  $p_k^h = q_k - \tau_k^h$ , where  $q_k$  is the gross price of the issue (equal for every group) and  $\tau_k^h$  the marginal tax rate on that issue. To solve the individual problem I write the Lagrangian function:

$$L = U^h(c^h, x_1^h, \dots, x_k^h, \psi_1^h, \dots, \psi_k^h) + \mu^h(c^h - \sum_k \widetilde{p}_k^h x_k^h) \quad (3)$$

I write the Focs:

$$\frac{\partial L}{\partial x_1} = \frac{\partial U^h}{\partial x_1^h} = 0$$

⋮

$$\frac{\partial L}{\partial x_k} = \frac{\partial U^h}{\partial x_k^h} = 0$$

$$\frac{\partial L}{\partial \mu^h} = c^h = \sum_k \widetilde{p}_k^h x_k^h$$

Obtain the optimal Marshallian demands:

$$\hat{x}_k = x_k(\widetilde{p}_k, \psi_k^h) \quad (4)$$

Write the IUF:

$$V^h = U^h(x_1(\widetilde{p}_1, \psi_1^h), \dots, x_k(\widetilde{p}_k, \psi_k^h), \psi_1^h, \dots, \psi_k^h) \quad (5)$$

Government maximizes a PVM maximand:

$$\frac{1}{2} + \frac{d}{s} \sum_h n^h s^h (V^h(q^A) - V^h(q^B)) \quad (6)$$

subject to a balanced budget constraint (see Lindbeck and Weibull):

$$\sum_h \sum_i \tau_k^h x_k^h n_h = 0 \quad (7)$$

where in a SMT framework:

$$s^i = s(x_k^i)$$

Solving the problem, we obtain the Focs:

$$\frac{\partial L}{\partial \tau_k} = \sum_h n^h \frac{\partial s^h}{\partial q_k^h} (V^h(q^A) - V^h(q^B)) + n^h s^h \frac{\partial V^h(q^A)}{\partial q_k^h} + \lambda \sum_h n^h (x_k^h + \sum_i \tau_k \frac{\partial x_k^h}{\partial q_k^h}) = 0$$

In a SMT equilibrium must be:

$$q^A = q^B$$

that is the policies chosen by the two candidates must be the same. and thus:

$$V(q^A) - V(q^B) = 0$$

I re-write the Focs:

$$\frac{\partial L}{\partial \tau_k} = \sum_h n^h s^h \frac{\partial V^h(q^A)}{\partial q_k^h} + \lambda \sum_h n^h (x_k^h + \sum_i \tau_k \frac{\partial x_k^h}{\partial q_k^h}) = 0$$

From the Roy's identity:  $\frac{\partial V^h(q^A)}{\partial q_k^h} = (-\mu^h)x_k^h$ , which represents the *social marginal utility of income for person h*, or the social evaluation of the increase in utility of person  $h$  made possible when  $h$  is endowed with an extra unit of numeraire. Substituting we obtain:

$$\frac{\partial L}{\partial \tau_k} = \sum_h n^h s^h (-\mu^h)x_k^h + \lambda \sum_h n^h (x_k^h + \sum_i \tau_k \frac{\partial x_k^h}{\partial q_k^h}) = 0$$

From the Slutsky equation we know that:

$$\frac{\partial x_k^h}{\partial q_k} = S_{ik}^h - x_k^h \frac{\partial x_k^h}{\partial c^h}$$

Rearranging and divide by  $\sum_h x_k^h$

$$\frac{\partial L}{\partial \tau_k} = -\frac{\sum_h s^h \mu^h x_k^h}{\sum_i x_k^h} + \lambda + \lambda \frac{\sum_h \sum_i \tau_k S_{ik}^h}{\sum_i x_k^h} - \lambda \frac{\sum_h \sum_i \tau_k x_k^h \frac{\partial x_k^h}{\partial c^h}}{\sum_h x_k^h} = 0$$

Divide by  $\lambda$  :

$$\frac{\partial L}{\partial \tau_k} = \frac{\sum_h \sum_i \tau_k S_{ik}^h}{\sum_i x_k^h} = \frac{\sum_h s^h \mu^h x_k^h}{\lambda \sum_i x_k^h} - 1 + \frac{\sum_h \sum_i \tau_k x_k^h \frac{\partial x_k^h}{\partial c^h}}{\sum_h x_k^h} \quad (8)$$

The great achievement of the optimal taxation in a PVM with SMT stands in the analysis of the RHS of 8; in particular if we assume that this is negative, we may state that the reduction in the compensated demand for issue  $k$  should be small (in absolute value) as  $\sum_h s^h \mu^h x_k^h$  is large. **But  $\sum_h s^h \mu^h x_k^h$  is larger, the larger is  $s^h$ , and since we have assumed that  $s^h$  is a monotonically increasing function in some issue, this eventually means that the reduction in the compensated demand for issue  $k$  should be lower for those social groups which "consume" that issue more! In other words 8 states that the Government should tax less those social groups who have higher level of density because they get higher levels of consumption for those issues.**

Secondly, the formula also suggests that **groups who gain from a reduction in the taxation for issues where they are more single-minded, may experiment an increase in the taxation for other issues where they are less single-minded at the same time.** Consider now a change in the total welfare resulting from a small change in  $\tau_k$ ; we obtain:

$$\frac{\partial L}{\partial \tau_k} = - \sum_h s^h \frac{\partial V^h(q^A)}{\partial c^h} x_k^h + \lambda \sum_h (x_k^h + \sum_k \tau_k \frac{\partial x_k^h}{\partial \tau_j}) = 0$$

$$\frac{\partial L}{\partial \tau_k} = \lambda \sum_h x_k^h - \sum_h s^h \frac{\partial V^h(q^A)}{\partial c^h} x_k^h + \lambda \theta_k = 0$$

where  $\theta_k = \sum_h \sum_k \tau_k \frac{\partial x_k^h}{\partial \tau_j}$ , which can be thought of as the marginal deadweight loss or indirect change in revenue arising from the tax change. Divide by  $\sum_h x_k^h$  and define  $\widetilde{\theta}_k = \frac{\theta_k}{\sum_h x_k^h}$  as the *normalised marginal deadweight loss*

$$\frac{\partial L}{\partial \tau_k} = \lambda - \frac{\sum_h s^h \frac{\partial V^h(q^A)}{\partial c^h} x_k^h}{\sum_h x_k^h} + \lambda \widetilde{\theta}_k = 0$$

$$\frac{\partial L}{\partial \tau_k} = \lambda(1 + \widetilde{\theta}_k) = \frac{\sum_h s^h \frac{\partial V^h(q^A)}{\partial c^h} x_k^h}{\sum_h x_k^h}$$

$$\widetilde{\theta}_k = \frac{\frac{\sum_h s^h \frac{\partial V^h(q^A)}{\partial c^h} x_k^h}{\sum_h x_k^h}}{\lambda} - 1 \quad (9)$$

Equation 9 shows that the normalized marginal deadweight loss is lower the lower is the level of mindedness of social groups. Thus a society where the presence of single-minded groups is lower is undoubtedly better off.

## References

- [1] Ambrosanio M. F., Bordignon M., Galmarini U. & Panteghini P.: *Lezioni di Teoria delle imposte*, (1997), ETAS Libri
- [2] Brennan G. & Buchanan J.M: *The Power to Tax: Analytical Foundations of a Fiscal Constitution*, (1980) Cambridge: Cambridge University Press
- [3] Ramsey F.P.: *A Contribution to the Theory of Taxation*, (1927) Economic Journal, Vol. 37, No. 1, pp. 47-61