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Depression as a Nash Equilibrium Consisting of Strategies of Choosing a Pareto Inefficient Transition Path

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Abstract

This paper shows that a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path is selected by households even without frictions as a result of the revealed government failure in supervision of financial markets. The Pareto inefficiency causes the generation of many of the phenomena observed in a depression (e.g., a persistently large amount of unutilized resources), and it is not necessary to use “animal spirits” to explain the generation of a depression. The revealed government failure in the supervision of financial markets and the resulting increased policy-induced uncertainty makes non-cooperative and risk-averse households behave more myopically, resulting in a Nash equilibrium of a Pareto inefficient path. When the failure of financial supervision is revealed, the household rate of time preference shifts upwards when the expected variance of steady-state consumption increases and/or its expected value shifts downwards.

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1. INTRODUCTION

During a depression, the unemployment rate rises and the capital utilization rate falls more sharply and persistently than during a recession. The generation mechanism of unemployment and idle capital (i.e., unutilized resources) is usually attributed either to friction on quantity adjustments (e.g., prolonged searching and matching processes of employment) or friction on price adjustments (i.e., sticky prices and wages). During the Great Depression in the 1930s, unutilized resources were so huge that Keynes and his followers argued that the friction on quantity adjustments was not sufficient to explain the magnitude and persistence of unutilized resources and that friction on price adjustments was more important. Since then, the generation mechanism of unutilized resources has remained a puzzle, as has the shock that generates depression. Whether friction on quantity adjustments or on price adjustments is more important has been disputed, but the outcome of that discussion has been inconclusive.

Frictions on quantity adjustments of employment are usually explained by the search and matching models initiated by Mortensen and Pissarides (e.g., Pissarides, 1985; Mortensen and Pissarides, 1994). However, Shimer (2005) argues that the standard version of search and matching models fails to account for the observed volatility of unemployment and vacancies. Shimer (2004), Farmer and Hollenhorst (2005), Hall (2005), Kennan (2006), Hall and Milgrom (2008), and Gertler and Trigari (2009) suggest the necessity of modifying the mechanism of wage formation in these models (e.g., introducing wage rigidity) to solve this shortcoming, because the wage-setting mechanism in the standard version of search and matching models (i.e., the Nash bargaining solution) is increasingly regarded as unsatisfactory (see also Hornstein et al., 2005; Yashiv, 2007).

Frictions on price adjustments cause rigidities in price movements. The rigidities hinder markets from quickly clearing, and unutilized resources such as unemployment are temporarily generated. However, friction on price adjustments has been criticized for its inability to explain the persistent nature of inflation, and skepticism about its economic importance remains. Mankiw (2001) argues that the so-called new Keynesian Phillips curve is ultimately a failure and is not consistent with the standard stylized facts about the dynamic effects of monetary policy (see also, e.g., Fuhrer and Moore 1995; Galí and Gertler 1999). The hybrid new Keynesian Phillips curve of Galí and Gertler (1999) solves this problem partly by incorporating lagged inflation into the models, but this solution raises another serious problem—why would rational agents behave partly in a backward-looking manner? Fuhrer (2006) concludes that the success of the hybrid new Keynesian Phillips curve is merely superficial, because the persistent nature of inflation is attributed mainly to lagged inflation. These arguments imply that stickiness of prices and wages is not very important economically.

Although friction on both quantity adjustments and price adjustments may well explain small-scale temporary phenomena, both appear insufficient as the mechanism generating the large-scale persistent phenomena that are observed in a depression. This insufficiency suggests that some unknown mechanism that amplifies the effects of the frictions exists. The main purpose of this paper is to search for such a mechanism. The presence of a persistently large amount of unutilized resources

implies that the economy is not persistently Pareto efficient. Pareto inefficiency usually may not be left as it is for a long period, but a Nash equilibrium can conceptually coexist with Pareto inefficiency. If a Nash equilibrium that consists of strategies generating Pareto inefficient payoffs is selected, unutilized resources as large and persistent as those observed in a depression may exist. This paper shows that a depression at such a Nash equilibrium—that is, a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (hereafter called a “Nash equilibrium of a Pareto inefficient path”)—is generated even in a frictionless economy if—and probably only if—the rate of time preference shifts upwards. If the frictions are combined with this Nash equilibrium of a Pareto inefficient path, the effects of the frictions will be substantially amplified by the Pareto inefficiency of the transition path. An essential reason for the generation of this path is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household’s strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path.

The Nash equilibrium of a Pareto inefficient path should not be confused with a Pareto inferior Nash equilibrium or a Nash equilibrium that is Pareto inefficient. They are conceptually quite different, although the Nash equilibrium of a Pareto inefficient path discussed in this paper is also a Pareto inferior Nash equilibrium and a Nash equilibrium that is Pareto inefficient. Multiple equilibria due to, for example, increasing returns, an externality or a complementarity in a macro-economic framework are usually Pareto ranked equilibria and include a Pareto inferior equilibrium (e.g., Morris and Shin, 2001). Such a Pareto inferior equilibrium usually indicates lower production and consumption than Pareto superior equilibria, suggesting a depression. However, if consumption is immediately adjusted completely when the economy is switched from a Pareto superior equilibrium to the inferior one, unutilized resources will not be generated as a result of the switch; therefore, merely showing the possibility of multiple Pareto ranked equilibria is not sufficient to explain the depression generation mechanism. A mechanism that generates huge and persistent unutilized resources during the transition path to the new equilibrium should be also presented, and the Nash equilibrium of a Pareto inefficient path fully explains this mechanism.

If households are cooperative, they will always proceed on Pareto efficient paths because they will coordinate with each other to perfectly utilize all resources. Conversely, if they do not coordinate with each other, they may strategically not utilize all resources; that is, they may select a Nash equilibrium of a Pareto inefficient path. Such a possibility cannot be denied *a priori*, because a Nash equilibrium can coexist with Pareto inefficiency. In fact, households are intrinsically not cooperative—they act independently of one another. Suppose that an upward shift of the time preference rate occurs. All households will be knocked off the Pareto efficient path on which they have proceeded until the shift occurred. At that moment, each household must decide on a direction in which to proceed. Because they are no longer on a Pareto efficient path, households choose a path strategically on the basis of the expected utility calculated considering other households’ choices; that is, each household behaves non-cooperatively in its own interest considering other households’ strategies. This

situation can be described by a non-cooperative mixed strategy game. In this paper, I show that there is a Nash equilibrium of a Pareto inefficient path in this game.

The important question remains, however: What causes the upward time preference shock? In other words, what ultimately causes a depression? Keynes (1936) suggests animal spirits as a driving force of economic fluctuations. Keynes's (1936) definition of animal spirits is vague but probably indicates that some psychological factors (e.g., moods such as pessimism or optimism) overwhelm optimal actions from the point of view of rationally expected outcomes. These psychological factors drive households and firms to take actions even though the actions do not maximize their expected utilities and profits. The animal spirits argument implies that depression is a result of irrationality. This paper argues, however, that irrationality is not necessary to explain the generation of a depression.

The rate of time preference has been naturally supposed and actually observed to be time-variable since the era of Böhm-Bawerk (1889) and Fisher (1930). This paper presents an endogenous time preference model, in which the rate of time preference is inversely proportionate to the expected steady-state consumption. Hence, the model is consistent with many observations that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991) and thus escapes from the drawback of Uzawa's (1968) well-known endogenous time preference model. The model in this paper indicates that a shock to the expected steady-state consumption changes the rate of time preference. Nevertheless, steady-state consumption is intrinsically smoothed by the Ramsey-Euler equation (e.g., Brock and Mirman, 1972; Mirman and Zilcha, 1977); that is, it is nearly deterministic. In addition, the expected probability distribution of natural science technologies and knowledge is usually not substantially time-variable. Thus, the number of shocks that changes the expected steady-state consumption is limited, but policy-induced shocks are among the few such shocks. Such shocks make TFP (total factor productivity) become a stochastic process with an absorbing state, which results in substantially random steady-state consumption. Government policies in financial markets are particularly important. It has long been argued that financial development affects the level and growth of an economy (e.g., Levine, 1997; Wachtel, 2003; Do and Levchenko, 2007). In addition, there is a significant imperfection in financial markets—asymmetric information between financial institutions and investors (e.g., Gertler, 1988; Mishkin, 1991)—that needs to be reduced by government supervision. This paper argues that the revealed failure in government supervision of financial markets is the origin of the shock to the expected steady-state consumption.

The paper is organized as follows. Section 2 shows that a Nash equilibrium of a Pareto inefficient path is rationally generated when the time preference rates of risk-averse and non-cooperative households shift upwards. In Section 3, an endogenous time preference model is constructed, in which the rate of time preference is inversely proportionate to steady-state consumption. Section 4 shows that the probability distribution of steady-state consumption is affected by policy-induced elements in TFP, particularly its financial element, and that the revealed failure of government supervision of financial markets originates the shock to steady-state consumption. In Section 5, the mechanism of depression is summarized, and policies to prevent and recover from depression are suggested. Finally, I offer concluding remarks in Section 6.

2. NASH EQUILIBRIUM OF A PARETO INEFFICIENT PATH

2.1 *Model with non-cooperative households*

2.1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path (other possible shocks are discussed in Section 2.5). Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower production and consumption than before the shock, which is consistent with the phenomena actually observed in a depression.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference. Because time preference is naturally time-variable, models of endogenous time preference have been presented, the most familiar of which is Uzawa's (1968) model. In Section 3, the endogeneity of time preference is examined in detail and an endogenous time preference model is presented as the mechanism of generation of the shock.

2.1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical and infinitely living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E \int_0^{\infty} \exp(-\theta t) u(c_t) dt, \quad (1)$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - c_t, \quad (2)$$

where y_t , c_t , and k_t are production, consumption, and capital per capita in period t respectively; A is technology; u is the utility function; $y_t = f(A, k_t)$ is the production function; $\theta (> 0)$ is the rate of time preference; and E is the expectation operator. y_t , c_t , and k_t are monotonously continuous and differentiable in t , and u and f are monotonously continuous functions of c_t and k_t , respectively. All households initially have an identical amount of financial assets equal to k_t , and all households gain the identical amount of income $y_t = f(A, k_t)$ in each period. It is assumed that $\frac{du(c_t)}{dc_t} > 0$

and $\frac{d^2u(c_t)}{dc_t^2} < 0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion (CRRA) utility function

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \quad \text{if } \gamma \neq 1$$

$$u(c_t) = \ln(c_t) \quad \text{if } \gamma = 1,$$

where $0 < \gamma < \infty$. In addition, $\frac{\partial f(A, k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$. Technology A and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure 1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line $\frac{dc_t}{dt} = 0$ moves to the left (from the solid line to the dashed line in Fig 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point Z . After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point Z to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to Z may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward time preference shift. The first is a jump option " \mathcal{J} ", in which a household's consumption jumps to Z and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option " \mathcal{NJ} ", in which a household's

consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure 1. The household that chose the NJ option reaches the posterior steady state in period s (≥ 0). The difference in consumption between the two options in each period t is b_t (≥ 0). Thus, b_0 indicates the difference between Z and the prior steady state. b_t diminishes continuously and becomes zero in period s . The NJ path of consumption (c_t) after the shock is monotonously continuous and differentiable in t and $\frac{dc_t}{dt} < 0$ if $0 \leq t < s$. In addition,

$$\begin{aligned} \bar{c} < c_t < \hat{c}_t & \quad \text{if } 0 \leq t < s \\ c_t = \bar{c} & \quad \text{if } 0 \leq s \leq t, \end{aligned}$$

where \hat{c}_t is consumption when proceeding on the posterior Pareto efficient saddle path and \bar{c} is consumption in the posterior steady state. Therefore,

$$\begin{aligned} b_t = \hat{c}_t - c_t > 0 & \quad \text{if } 0 \leq t < s \\ b_t = 0 & \quad \text{if } 0 \leq s \leq t. \end{aligned}$$

It is also assumed that, when a household chooses the option that is different from the option the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption (b_t) before period s between the household and the other households is reflected in consumption after period s . That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period s . The exact functional form of the addition (or subtraction) is shown in Section 2.1.4.

2.1.3 Firms

Unutilized products (b_t) are eliminated quickly in each period by firms, because holding b_t for a long period is a cost to firms. Elimination of b_t is done by discarding the goods or preemptively suspending production, leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital that generates b_t . Disposing of the excess capital is rational for firms, because the excess capital is an unnecessary cost for firms, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the NJ path (that is, if all households choose the NJ option), firms dispose all of the remaining excess capital that generates b_t and adjust their capital to the posterior steady-state level in period s , corresponding to households' reaching the posterior steady state. Thus, if the economy proceeds on the NJ path, capital k_t is

$$\bar{k} < k_t \leq \hat{k}_t \quad \text{if } 0 \leq t < s$$

$$k_t = \bar{k} \quad \text{if } 0 \leq s \leq t,$$

where \hat{k}_t is capital per capita when proceeding on the posterior Pareto efficient saddle path and \bar{k} is capital per capita in the posterior steady state.

The real interest rate i_t is

$$i_t = \frac{\partial f(A, k_t)}{\partial k_t}. \quad (3)$$

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the **NJ** path,

$$\begin{aligned} \tilde{\theta} &\leq i_t < \theta & \text{if } 0 \leq t < s \\ i_t &= \theta & \text{if } 0 \leq s \leq t, \end{aligned} \quad (4)$$

where $\tilde{\theta}$ is the rate of time preference before the shock and θ is the rate of time preference after the shock. i_t is monotonously continuous and differentiable in t if $0 \leq t < s$.

2.1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of **J** or **NJ**. Let **Jalone** indicate that the household chooses the **J** option but the other households choose the **NJ** option, **NJalone** indicate that the household chooses the **NJ** option but the other households choose the **J** option, **Jtogether** indicate that all households choose the **J** option, and **NJtogether** indicate that all households choose the **NJ** option. Let p ($0 \leq p \leq 1$) be the subjective probability of the household that the other households choose the **J** option (e.g., $p=0$ indicates that all the other households choose option **NJ**). With p , the expected utility of the household when it chooses option **J** is,

$$E(J) = pE(Jtogether) + (1-p)E(Jalone), \quad (5)$$

and when it chooses option **NJ** is

$$E(NJ) = pE(NJalone) + (1-p)E(NJtogether), \quad (6)$$

where $E(Jalone)$, $E(NJalone)$, $E(Jtogether)$, and $E(NJtogether)$ are the expected utilities of the household when choosing **Jalone**, **NJalone**, **Jtogether**, and **NJtogether**, respectively. With the properties of **J** and **NJ** shown in Sections 2.1.2 and 2.1.3,

$$E(J) = pE\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t)dt\right]$$

$$+(1-p)E\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c} - \bar{a})dt\right], \quad (7)$$

and

$$E(NJ) = pE\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t + a_t)dt\right] \\ + (1-p)E\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c})dt\right], \quad (8)$$

where

$$\bar{a} = \theta \int_0^s b_r \exp \int_r^s i_q dq dr, \quad (9)$$

and

$$a_t = i_t \int_0^s b_r \exp \int_r^s i_q dq dr, \quad (10)$$

and the shock occurred in the period $t = 0$. Figure 2 shows the paths of *Jalone* and *NJalone*. Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, then in the case of *Jalone* the economy almost proceeds on the *NJ* path, and in the case of *NJalone* it almost proceeds on the *J* path. If the other households choose the *NJ* option (*Jalone* or *NJtogether*), consumption after s is constant as \bar{c} and capital is adjusted to \bar{k} by firms in the period s . In addition, a_t and i_t are constant after s such that a_t equals \bar{a} and i_s equals θ , because the economy is at the posterior steady state. Nevertheless, during the transition period before s , the value of i_t changes from the value of the prior time preference rate to that of the posterior. If the other households choose option *J* (*NJalone* or *Jtogether*), however, consumption after s is \hat{c}_t and capital is not adjusted to \bar{k} by firms in the period s and remains at \hat{k}_t .

As mentioned in Section 2.1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period s . This is described by a_t and \bar{a} in equations (7) and (8), and equations (9) and (10) indicate that the accumulated difference in financial assets due to b_t increases by compound interest between the period r to s . That is, if the household takes the *NJalone* path, it accumulates more financial assets than each of the other *J* households, and instead of immediately consuming these extra accumulated financial assets after period s , the household consumes the returns on them in every subsequent period. If the household takes the *Jalone* path, however, its consumption after s is $\bar{c} - \bar{a}$, as shown in equation (7). \bar{a} is subtracted because the income of each household $y_t = f(A, k_t)$, including the *Jalone* household, decreases equally by b_t . Each of the other *NJ* households decreases consumption by b_t at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; k_t) are kept equal

to \hat{k}_t . The *Jalone* household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other *NJ* households, which results in the subtraction of \bar{a} after period s .

2.2 Pareto inefficient transition path ¹

2.2.1 Rational Pareto inefficient path

2.2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option *J* is chosen as the path after the shock because it gives a higher expected utility than option *NJ*. Because there is no possibility of *Jalone* and *NJalone* if households are cooperative, then $E(J) = E(Jtogether)$ and $E(NJ) = E(NJtogether)$. Therefore, $E(J) - E(NJ) = E\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt\right] - E\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt\right]$
 $= E\left\{\int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\hat{c}_t) - u(\bar{c})] dt\right\} > 0$ since $c_t < c_t + b_t$ and $\bar{c} < \hat{c}_t$.

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small γ is examined.

Lemma 1: If $\gamma (0 < \gamma < \infty)$ is sufficiently small, then $E(Jalone) - E(NJtogether) > 0$.

Proof: $\lim_{\gamma \rightarrow 0} [E(Jalone) - E(NJtogether)]$

$$\begin{aligned} &= E \int_0^s \exp(-\theta t) \lim_{\gamma \rightarrow 0} [u(c_t + b_t) - u(c_t)] dt + E \int_s^\infty \exp(-\theta t) \lim_{\gamma \rightarrow 0} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &= E \int_0^s \exp(-\theta t) b_t dt - E \int_s^\infty \exp(-\theta t) \bar{a} dt \\ &= E \int_0^s \exp(-\theta t) b_t dt - E \theta \left[\int_0^s \left(b_r \exp \int_r^s i_q dq \right) dr \right] \int_s^\infty \exp(-\theta t) dt \\ &= E \int_0^s \exp(-\theta t) b_t dt - E \exp(-\theta s) \int_0^s \left(b_r \exp \int_r^s i_q dq \right) dr \\ &= E \exp(-\theta s) \int_0^s b_t \left\{ \exp[\theta(s-t)] - \exp \int_t^s i_q dq \right\} dt > 0, \end{aligned}$$

because, if $0 \leq t < s$, then $i_t < \theta$ and $\exp[\theta(s-t)] > \exp \int_t^s i_q dq$. Therefore, because $\exp[\theta(s-t)] > \exp \int_t^s i_q dq$, $E(Jalone) - E(NJtogether) > 0$ for sufficiently small γ . ■

Second, the opposite special case (i.e., a utility function with a sufficiently large γ) is examined.

Lemma 2: If $\gamma (0 < \gamma < \infty)$ is sufficiently large and if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}}$, then

¹ The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004b).

$$E(\text{Jalone}) - E(\text{NJtogether}) < 0.$$

Proof: Because $0 < b_t$, then for any period $t (< s)$, $\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(c_t + b_t) - u(c_t)] =$

$$\lim_{\gamma \rightarrow \infty} \left[\left(\frac{c_t + b_t}{\bar{c}} \right)^{1-\gamma} - \left(\frac{c_t}{\bar{c}} \right)^{1-\gamma} \right] = 0. \text{ On the other hand, because } 0 < \bar{a}, \text{ then for any period}$$

$$t (< s), \text{ if } 0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1, \lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [u(\bar{c} - \bar{a}) - u(\bar{c})] = \lim_{\gamma \rightarrow \infty} \left[\left(1 - \frac{\bar{a}}{\bar{c}} \right)^{1-\gamma} - 1 \right] = \infty. \text{ Hence,}$$

$$\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} [E(\text{Jalone}) - E(\text{NJtogether})] = \lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_0^s \exp(-\theta t) \lim_{\gamma \rightarrow \infty} [u(c_t + b_t) - u(c_t)] dt +$$

$$\lim_{\gamma \rightarrow \infty} \frac{1-\gamma}{\bar{c}^{1-\gamma}} \int_s^\infty \exp(-\theta t) \lim_{\gamma \rightarrow \infty} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt = 0 + \infty > 0. \text{ Because } \frac{1-\gamma}{\bar{c}^{1-\gamma}} < 0 \text{ for any}$$

$\gamma (1 < \gamma < \infty)$, then if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$, $E(\text{Jalone}) - E(\text{NJtogether}) < 0$ for sufficiently large $\gamma (< \infty)$. ■

The condition $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ indicates that path **NJ** from c_0 to \bar{c} deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state \bar{c} not too late. Because steady states are irrelevant to the degree of risk aversion (γ), both c_0 and \bar{c} are irrelevant to γ .

By Lemmas 1 and 2, it is proved that $E(\text{Jalone}) - E(\text{NJtogether}) < 0$ is possible.

Lemma 3: If $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$, then there is a $\gamma^* (0 < \gamma^* < \infty)$ such that if $\gamma^* < \gamma < \infty$,

$$E(\text{Jalone}) - E(\text{NJtogether}) < 0.$$

Proof: If $\gamma (> 0)$ is sufficiently small, then $E(\text{Jalone}) - E(\text{NJtogether}) > 0$ by Lemma 1, and if $\gamma (< \infty)$ is sufficiently large and if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$, then

$E(\text{Jalone}) - E(\text{NJtogether}) < 0$ by Lemma 2. Hence, if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$, there is a certain

$\gamma^* (0 < \gamma^* < \infty)$ such that, if $\gamma^* < \gamma < \infty$, then $E(\text{Jalone}) - E(\text{NJtogether}) < 0$. ■

However, $E(\text{Jtogether}) - E(\text{NJalone}) > 0$ because both **Jtogether** and **NJalone** indicate that all the other households choose option **J**; thus, the values of i_t and k_t are same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these i_t and k_t , deviating alone from the Pareto efficient path (**NJalone**) gives a lower expected utility than **Jtogether** to the **NJ** household. Opposite to **Jtogether** and **NJalone**, both **Jalone** and **NJtogether** indicate that all the other households choose option **NJ** and i_t and k_t are not those of the Pareto efficient path. Hence, the sign of $E(\text{Jalone}) - E(\text{NJtogether})$ varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property $E(Jtogether) - E(NJalone) > 0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E(J) - E(NJ) < 0$, is shown.

Proposition 1: If $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, then there is a $p^* (0 \leq p^* \leq 1)$ such that if $p = p^*$, $E(J) - E(NJ) = 0$, and if $p < p^*$, $E(J) - E(NJ) < 0$.

Proof: By Lemma 3, if $\gamma^* < \gamma < \infty$, then $E(Jalone) - E(NJtogether) < 0$ and $E(Jtogether) - E(NJalone) > 0$. Here, $E(J) - E(NJ) = p[E(Jtogether) - E(NJalone)] + (1-p)[E(Jalone) - E(NJtogether)]$ by equations (5) and (6). Thus, if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, $\lim_{p \rightarrow 0} E(J) - E(NJ) = E(Jalone) - E(NJtogether) < 0$ and $\lim_{p \rightarrow 1} E(J) - E(NJ) = E(Jtogether) - E(NJalone) > 0$. Hence, by the intermediate value theorem, there is $p^* (0 \leq p^* \leq 1)$ such that if $p = p^*$, $E(J) - E(NJ) = 0$ and if $p < p^*$, $E(J) - E(NJ) < 0$. ■

Proposition 1 indicates that, if $0 < \lim_{\gamma \rightarrow \infty} \frac{\bar{a}}{\bar{c}} < 1$, $\gamma^* < \gamma < \infty$, and $p < p^*$, then the choice of option **NJ** gives the higher expected utility than that of option **J** to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, and a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose any Pareto inefficient transition path in a frictionless economy.

2.2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, $\gamma^* < \gamma < \infty$ may appear rather strict. If γ^* is very large, option **NJ** will be rarely chosen. However, if path **NJ** is such that consumption is reduced sharply after the shock, option **NJ** gives the higher expected utility than option **J** even though γ^* is very small. For example, for any $\gamma (0 < \gamma < \infty)$,

$$\begin{aligned} & \lim_{s \rightarrow 0} \frac{1}{s} [E(Jalone) - E(NJtogether)] \\ &= \lim_{s \rightarrow 0} \frac{1}{s} \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \lim_{s \rightarrow 0} \frac{1}{s} \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &= u(c_0 + b_0) - u(c_0) - \frac{1}{\theta} \lim_{s \rightarrow 0} \frac{u(\bar{c}) - u(\bar{c} - s\theta b_0)}{s} = u(c_0 + b_0) - u(c_0) - b_0 \frac{du(\bar{c})}{d\bar{c}} \\ &= \frac{(c_0 + b_0)^{1-\gamma} - c_0^{1-\gamma}}{1-\gamma} - b_0 \bar{c}^{-\gamma} = \bar{c}^{-\gamma} \left\{ \bar{c}^\gamma \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] - b_0 \right\} < 0, \end{aligned}$$

because $\lim_{\gamma \rightarrow 1} \bar{c}^\gamma \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \bar{c} [\ln(c_0 + b_0) - \ln(c_0)] = \bar{c} \ln \left(1 + \frac{b_0}{c_0} \right) < b_0$ and

$$\lim_{\gamma \rightarrow \infty} \bar{c}^\gamma \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \lim_{\gamma \rightarrow \infty} \bar{c}^\gamma c_0^{1-\gamma} \left[\frac{\left(1 + \frac{b_0}{c_0}\right)^{1-\gamma} - 1}{1-\gamma} \right] = 0 \quad \text{due to } \bar{c} < c_0. \text{ That is, for}$$

each combination of path \mathbf{NJ} and γ , there is $s^*(>0)$ such that, if $s < s^*$, then $E(\text{Jalone}) - E(\text{NJtogether}) < 0$.

Consider an example in which path \mathbf{NJ} is such that b_t is constant as $b_t = \bar{b}$ before s (Figure 3); thus $E \int_0^s b_t = s\bar{b}$. In this \mathbf{NJ} path, consumption is reduced more sharply than it is in the case shown in Figure 2. In this case, because $\bar{a} > E\theta \int_0^s b_t = \theta s\bar{b}$, $0 < \gamma$, and $c_s < c_t$ for $t < s$, then $E \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt < E \int_0^s \exp(-\theta t) dt [u(c_s + \bar{b}) - u(c_s)] = E \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)]$, and in addition, $E \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt = E \int_s^\infty \exp(-\theta t) dt [u(\bar{c} - \bar{a}) - u(\bar{c})] = E \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \bar{a}) - u(\bar{c})] < E \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s\bar{b}) - u(\bar{c})]$. Hence,

$$\begin{aligned} & E(\text{Jalone}) - E(\text{NJtogether}) \\ &= E \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + E \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &< E \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)] + E \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s\bar{b}) - u(\bar{c})] \\ &= E \frac{1 - \exp(-\theta s)}{\theta} \left\{ [u(c_s + \bar{b}) - u(c_s)] - \frac{\exp(-\theta s)}{1 - \exp(-\theta s)} [u(\bar{c}) - u(\bar{c} - \theta s\bar{b})] \right\}. \end{aligned}$$

As γ becomes larger, the ratio $\frac{u(c_s + \bar{b}) - u(c_s)}{u(\bar{c}) - u(\bar{c} - \theta s\bar{b})}$ becomes smaller; thus, larger values of s can satisfy $E(\text{Jalone}) - E(\text{NJtogether}) < 0$. For example, suppose that $\bar{c} = 10$, $c_s = 10.2$, $\bar{b} = 0.3$, and $\theta = 0.05$. If $\gamma = 1$, then $s^* = 1.5$ at the minimum, and if $\gamma = 5$, then $s^* = 6.8$ at the minimum. This result implies that, if option \mathbf{NJ} is such that consumption is reduced relatively sharply after the shock (e.g., $b_t = \bar{b}$) and $p < p^*$, option \mathbf{NJ} will usually be chosen. It is not a special case observed only if γ is very large, but it will normally be generated when the value of γ is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a depression, consumption usually declines sharply after the shock, which suggests that households have chosen the \mathbf{NJ} option.

2.3 Nash equilibrium

2.3.1 A Nash equilibrium consisting of NJ strategies

A household strategically determines whether to choose the J or NJ option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are $H (\in N)$ identical households in the economy where H is sufficiently large (as assumed in Section 2.1). Let q_η ($0 \leq q_\eta \leq 1$) be the probability that a household $\eta (\in H)$ chooses option J . The average utility of the other households almost equals that of all households because H is sufficiently large. Hence, the average expected utilities of the other households that choose the J and NJ options are $E(Jtogether)$ and $E(NJtogether)$, respectively. Hence, the payoff matrix of the H -dimensional symmetric mixed strategy game can be described as shown in Table 1.

Table 1: The payoff matrix

		Any other household	
		J	NJ
A household	J	$E(Jtogether), E(Jtogether)$	$E(Jalone), E(NJtogether)$
	NJ	$E(NJalone), E(Jtogether)$	$E(NJtogether), E(NJtogether)$

Each identical household determines its behavior on the basis of this payoff matrix. In this mixed strategy game, strategy profiles

$$(q_1, q_2, \dots, q_H) = \{(1, 1, \dots, 1), (p^*, p^*, \dots, p^*), (0, 0, \dots, 0)\} \quad (12)$$

are Nash equilibria for the following reason. By Proposition 1, the best response of a household η is J (i.e., $q_\eta = 1$) if $p > p^*$, indifferent between J and NJ (i.e., any $q_\eta \in [0, 1]$) if $p = p^*$, and NJ (i.e., $q_\eta = 0$) if $p < p^*$. Because all households are identical, the best-response correspondence of each household is identical such that $q_\eta = \{1\}$ if $p > p^*$, $[0, 1]$ if $p = p^*$, and $\{0\}$ if $p < p^*$ for any household $\eta \in H$. Hence, the mixed strategy profiles $(1, 1, \dots, 1)$, (p^*, p^*, \dots, p^*) , and $(0, 0, \dots, 0)$ are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution $(1, 1, \dots, 1)$; i.e., ***Jtogether*** is a pure strategy Nash equilibrium, but a Pareto inefficient transition path $(0, 0, \dots, 0)$; i.e., ***NJtogether*** is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium (p^*, p^*, \dots, p^*) .

2.3.2 Selection of equilibrium

Determining which Nash equilibrium, either ***NJtogether*** $(0, 0, \dots, 0)$ or

Jtogether $(1,1,\dots,1)$, is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose very low p and select the **NJtogether** $(0,0,\dots,0)$ equilibrium. Because

$$\begin{aligned} E(\text{Jalone}) - E(\text{NJalone}) &= E \left\{ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\hat{c}_t + a_t)] dt \right\} \\ &< E \left\{ \int_0^s \exp(-\theta t) [u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t) [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \right\} \\ &= E(\text{Jalone}) - E(\text{NJtogether}) < 0, \end{aligned} \quad (13)$$

by Lemma 3, then **Jalone** is the worst choice in the sense of the amount of payoff, followed by **NJtogether**, and **NJalone**, and **Jtogether** is the best. The outcome of choosing option **J** is more dispersed than that of option **NJ**. If households have the risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option (**NJ**), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Since all households are identical and know inequality (13), all households will equally suppose that they all prefer the less dispersed **NJ** option; therefore, all of them will suppose a very low p , particularly $p=0$, and select the **NJtogether** $(0,0,\dots,0)$ equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and animal spirits (e.g., pessimism or optimism) are unnecessary to explain the selection.

2.4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products (b_t). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the **J** path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, b_t will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates b_t will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, b_t is generated and eliminated, but in the next period, another, new, b_t is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium.

Because of the liquidation of firms, many employees are dismissed and capital is discarded. Note that a Nash equilibrium of a Pareto inefficient path can be selected irrespective of frictions (as discussed in Section 2.2), and if the economy is frictionless, the liquidation will not raise the unemployment rate. However, if there are frictions on

quantity and price adjustments, the unemployment rate will rise substantially as a result of the liquidation. Unemployment “naturally” exists even on a Pareto efficient path because of frictions. The extra unemployment resulting from the liquidation caused by unutilized products is in addition to this natural unemployment. In this sense, unutilized products amplify the generation of unutilized resources. The extra unemployment and discarded capital can be huge and persistently generated during the Pareto inefficient transition path, matching the observed magnitudes of unutilized resources in depressions.

2.5 Time preference shock as the exceptional shock

Not all shocks result in a Nash equilibrium of a Pareto inefficient path. If anything, this type of shock is limited because it needs to force consumption to fluctuate very jaggedly to maintain Pareto efficiency. A Pareto inefficient path is preferred, because these substantially jagged fluctuations can be averted. An upward time preference shock is one such shock, as shown in Figure 1. Other examples are rare, because shocks that do not change the steady state (e.g., monetary shocks) are not relevant. One other example is a technology regression, which would move the vertical line $\frac{dc_t}{dt} = 0$ to the left in Figure 1 and necessitate a jagged consumption path to keep Pareto efficiency. In this sense, technology and time preference shocks have similar effects on economic fluctuations. However, a technology regression also simultaneously moves the curve $\frac{dk_t}{dt} = 0$ downwards, and accordingly, the Pareto efficient saddle path also moves downwards. Therefore, the jagged consumption is smoothed out to some extent. As a result, the substantially jagged consumption that can generate a depression would require a large-scale, sudden, and sharp regression in technology, which does not seem very likely. An upward time preference shock, however, only moves the vertical line $\frac{dc_t}{dt} = 0$ to the left, which suggests that most depressions are generated by upwards time preference shocks.

In some macro-economic models with multiple equilibria, however, changing equilibria may necessitate substantially jagged consumptions to keep Pareto optimality. There are many types of multiple equilibria models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). Examining the properties, validity, and plausibility of each of these many and diverse models is beyond the scope of this paper.

3. ENDOGENOUS TIME PREFERENCE

The results in Section 2 raise the question: what force drives households to shift their rates of time preference upwards? Keynes’s (1936) argument suggests that an upward time preference shift is caused by a change in households’ moods. Indeed, preferences may change stochastically by fluctuating moods. However, it is not compelling to accept the idea of animal spirits *ad hoc* because it implies irrationality. Before arbitrarily assuming irrationality, we should search for all possibilities of

mechanisms by which an upward time preference shift is endogenously generated as a consequence of rational agents' rational behavior.

3.1 *Endogenous time preference models*

3.1.1 Uzawa's (1968) endogenous time preference model

The most well-known endogenous time preference model is that of Uzawa (1968). It has been applied to many analyses (e.g., Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990). However, Uzawa's model has not necessarily been regarded as a realistic expression of endogeneity of time preference because it has a serious drawback in that impatience increases as income, consumption and utility increase. The basic structure of Uzawa's model is

$$\begin{aligned}\theta_t &= \theta^*[u(c_t)], \\ 0 &< \frac{d\theta_t}{du(c_t)},\end{aligned}\tag{14}$$

in which the rate of time preference θ_t in period t is time-variable and an increasing function of present utility $u(c_t)$. The problem is that $0 < \frac{d\theta_t}{du(c_t)}$ is necessary for the

model to be stable. This property is quite controversial and difficult to accept *a priori*, because many empirical studies have indicated that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991); thus, many economists are critical of Uzawa's model. Epstein (1987), however, argues the plausibility of increasing impatience and offers some counter-arguments. However, his view is in the minority and most economists support arguments in favor of the decreasing rate of time preference such that $\frac{d\theta_t}{du(c_t)} < 0$. Hence, although Uzawa's

model attracted attention from economists such as Epstein and Hynes (1983), Lucas and Stokey (1984), and Obstfeld (1990), analysis of the endogeneity of time preference has progressed very little. Although Uzawa's model may be flawed, that does not necessarily mean that the conjecture that the rate of time preference is influenced by future income, consumption, and utility is fallacious, just that an appropriate model in which the rate of time preference is negatively correlated with income, consumption, and utility has not been presented.

3.1.2 Size effect on impatience

The problem of $0 < \frac{d\theta_t}{du(c_t)}$ in Uzawa's model arises because distant future

levels of consumption have little influence on factors that form the rate of time preference; that is, it is formed only with the information on present consumption, and it must be revised every period in accordance with consumption growth. However, there is no *a priori* reason why information on distant future activities should be far less important than the information on the present and near future activities. Fisher (1930) argued that

[O]ur first step, then, is to show how a person's impatience depends on

the *size* of his income, assuming the other three conditions to remain constant; for, evidently, it is possible that two incomes may have the same time shape, composition and risk, and yet differ in size, one being, say, twice the other in every period of time.

In general, it may be said that, other things being equal, the smaller the income, the higher the preference for the present over the future income. It is true of course that a permanently small income implies a keen appreciation of wants as well as of immediate wants. ... But it increases the want for immediate income *even more* than it increases the want for future income.” (p. 72)

According to Fisher’s (1930) view, a force that influences time preference is a psychological response derived from the perception of the “size of the entire income or utility stream.” This view indicates that it is necessary to probe how people perceive the size of the entire income or utility stream.

Little effort has been directed towards probing the nature of the size of utility or income stream on time preference, although a large number of psychological experiments have been made with regard to anomalies of the expected utility model with a constant rate of time preference (e.g., Frederick et al., 2002). Turning to research in economics, analyses using endogenous time preference models so far have merely introduced the *a priori* assumption of endogeneity of time preference without explaining its reasoning in detail. Hence, even now, Fisher’s (1930) insights are very useful for the examination of the size effect. An important point in Fisher’s above quote is that the size of the infinite utility stream is perceived as “permanently” high or low. The size difference among the utility streams may be perceived as the permanent continuing difference of utilities among different utility streams. Anticipation of the permanently higher utility may enhance an emotional sense of well-being because people feel they have a long-lasting secure situation, which will generate a positive psychological response and make people more patient. If that is true, distant future utilities should be taken into account equally with the present utility. Otherwise, it is impossible to distinguish whether the difference of utilities continues permanently.

From this point of view, the specification that only the present utility influences the formation of time preference, as is the case of Uzawa’s model, is inadequate as the specification of the size of utility stream. Instead, a simple measure of the size where entire utilities from the present to distant future are summed with equal weight will be more appropriate as the measure of the size of a utility stream.²

3.2 Model of time preference³

3.2.1 The model

Because no strategic situation is supposed in this section unlike in Section 2, the usual representative household is assumed for simplicity, and the representative household solves the maximization problem indicated in equations (1) and (2). Taking

² Das (2003) shows another stable endogenous time preference model with decreasing impatience. Her model is stable, although the rate of time preference is decreasing because endogenous impatience is almost constant. In this sense, the situation her model describes is very special.

³ The idea of this type of endogenous time preference model was originally presented by Harashima (2004a).

the arguments in Section 3.1 into account, the “size” of the infinite utility stream can be defined as follows.

Definition 1: The size of the utility stream W for a given technology A is

$$W = \lim_{T \rightarrow \infty} E \int_0^T w(t) u(c_t) dt, \quad (15)$$

where

$$\begin{aligned} w(t) &= \frac{1}{T} & \text{if } 0 \leq t \leq T \\ w(t) &= 0 & \text{otherwise.} \end{aligned}$$

The variable $w(t)$ indicates weights and has the same value in any period. Thus, the weights for evaluation of future utilities are distributed evenly over time, as argued in Section 3.1.

To this point in my argument, technology A has been assumed to be constant, but if A is time-variable (A_t) and grows at a constant rate and the economy is on a balanced growth path such that A_t , y_t , k_t , and c_t grow at the same rate, then the definition of W needs to be modified because any stream of c_t and $u(c_t)$ grows to infinity, and it is impossible to distinguish the sizes of the utility stream by simply summing up c_t with $T \rightarrow \infty$ as shown in Definition 1. Because balanced growth is possible only when technological progress is Harrod neutral, I assume a Harrod neutral production function such that

$$y_t = \omega A_t^\alpha k_t^{1-\alpha}, \quad (16)$$

where $\alpha (0 < \alpha < 1)$ and $\omega (0 < \omega)$ are constants. To distinguish the sizes of utility stream, the following value is set as the standard stream of utility,

$$u(\tilde{c} e^{\psi t}),$$

where $\tilde{c} (0 < \tilde{c})$ is a constant and $\psi (0 < \psi)$ is a constant rate of growth. Streams of utility are compared with this standard stream. Because the utility function is CRRA as shown in Section 2, a stream of utility in comparison with the standard stream of utility is

$$\frac{u(c_t)}{u(\tilde{c} e^{\psi t})} = \frac{c_t^{1-\gamma}}{(\tilde{c} e^{\psi t})^{1-\gamma}} = \frac{1-\gamma}{\tilde{c}^{1-\gamma}} u\left(\frac{c_t}{e^{\psi t}}\right).$$

By using this ratio, a stream of utility can be distinguished from the standard stream of utility. That is, the size of a utility stream W for a given stream of technology A_t that grows at the same rate ψ as y_t , k_t , and c_t can be alternatively defined as

$$W = \lim_{T \rightarrow \infty} E \int_0^T w(t) u \left(\frac{c_t}{e^{\psi t}} \right) dt. \quad (17)$$

Clearly, if $\psi = 0$, then the size (W) degenerates into the one shown in Definition 1.

If there is a steady state such that

$$\lim_{t \rightarrow \infty} E [u(c_t)] = E [u(c^*)], \quad (18)$$

or for the case of expected balanced growth

$$\lim_{t \rightarrow \infty} E \left[u \left(\frac{c_t}{e^{\psi t}} \right) \right] = E [u(c^*)], \quad (19)$$

where c^* is a constant and indicates steady-state consumption, then

$$W = E [u(c^*)] \quad (20)$$

for the following reason. Because $\lim_{t \rightarrow \infty} E [u(c_t)] = E [u(c^*)]$ (or $\lim_{t \rightarrow \infty} E \left[u \left(\frac{c_t}{e^{\psi t}} \right) \right] = E [u(c^*)]$), then

$$\begin{aligned} & \lim_{T \rightarrow \infty} \int_0^T w(t) \{E [u(c^*)] - E [u(c_t)]\} dt = E [u(c^*)] - W \\ \text{(or } & \lim_{T \rightarrow \infty} \int_0^T w(t) \left\{ E [u(c^*)] - E \left[u \left(\frac{c_t}{e^{\psi t}} \right) \right] \right\} dt = E [u(c^*)] - W \text{)}. \end{aligned}$$

In addition,

$$\begin{aligned} & \lim_{T \rightarrow \infty} \int_0^T w(t) \{E [u(c^*)] - E [u(c_t)]\} dt = 0 \\ \text{(or } & \lim_{T \rightarrow \infty} \int_0^T w(t) \left\{ E [u(c^*)] - E \left[u \left(\frac{c_t}{e^{\psi t}} \right) \right] \right\} dt = 0 \text{)}. \end{aligned}$$

Hence, $W = E [u(c^*)]$; that is, the rate of time preference is determined by steady-state consumption (c^*).

The model of time preference in this paper is constructed on the basis of this measure of W . An essential property that must be incorporated into the model is that the rate of time preference is sensitive to, and a function of, W such that

$$\theta = \theta^{**}(W),$$

where $\theta^{**}(W)$ is monotonously continuous and continuously differentiable. Because W is a sum of utilities, this property simply reflects the core idea of endogenous time

preference. However, this property is new in the sense that the rate of time preference is sensitive not only to the present utility but also the entire stream of utility, that is, the size of utility stream represented by the utility for steady-state consumption. This property is intuitively acceptable because it is likely that people set their principles or parameters for their behaviors considering the final consequences (i.e., the steady state; see, e.g., Barsky and Sims, 2009).

Another essential property that must be incorporated into the model is

$$\frac{d\theta}{dW} < 0 .$$

Because $W = E[u(c^*)]$ and $0 < \frac{du(c_t)}{dc_t}$, the rate of time preference is inversely proportionate to c^* . This property is consistent with the findings in many empirical studies, which have shown that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991).

In summary, the basic structure of the model is:

$$\begin{aligned} \theta &= \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\}, \\ \frac{d\theta}{dW} &= \frac{d\theta}{dE[u(c^*)]} < 0 . \end{aligned} \quad (21)$$

This model is deceptively similar to Uzawa's endogenous time preference model (14), simply replacing c_t with c^* and $0 < \frac{d\theta_t}{du(c_t)}$ with $\frac{d\theta}{dE[u(c^*)]} < 0$. However, the two models are completely different because of the opposite characteristics between $0 < \frac{d\theta_t}{du(c_t)}$ and $\frac{d\theta}{dE[u(c^*)]} < 0$.

3.2.2 Nature of the model

The model (21) can be regarded as successful only if it exhibits stability. In Uzawa's model, the economy becomes unstable if $0 < \frac{d\theta_t}{du(c_t)}$ is replaced with

$\frac{d\theta_t}{du(c_t)} < 0$. In this section, I examine the stability of the model.

3.2.2.1 Equilibrium rate of time preference

In Ramsey-type models, such as equations (1) and (2), if a constant rate of time preference is given, the value of marginal product of capital (i.e., the value of the real interest rate) converges to that of the given rate of time preference as the economy approaches the steady state. Hence, when a rate of time preference is specified at a certain value, the corresponding expected steady-state consumption is uniquely determined. Given fixed values of other exogenous parameters, any predetermined rate of time preference has unique values of expected consumption and utility at steady

state. There is a one-to-one correspondence between the expected utilities at steady state and the rates of time preference; therefore, the expected utility at steady state can be expressed as a function of the rate of time preference. Let c_x^* be a set of steady-state consumptions, given a set of time preference rates (θ_x) and other fixed exogenous parameters. The function $\theta \rightarrow W$ argued above can be described as

$$g(\theta) = E[u(c^*)](=W),$$

where $c^* \in c_x^*$ and $\theta \in \theta_x$. On the other hand, the rate of time preference is a continuous function of steady-state consumption as shown in the model (21) such that $\theta = \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\}$. The reverse function is

$$h(\theta) = E[u(c^*)](=W).$$

The equilibrium rate of time preference is determined by the point of intersection of the two functions, $g(\theta)$ and $h(\theta)$, as shown in Figure 4. Figure 5 shows a special but conventionally assumed $h(\theta)$, in which θ is not sensitive to W , and the rate of time preference is constant permanently. There exists a point of intersection because both $g(\theta)$ and $h(\theta)$ are monotonously continuous for $\theta > 0$. $h(\theta)$ is monotonously continuous because $\theta^{**}(W)$ is monotonously continuous. $g(\theta)$ is monotonously continuous because, as a result of utility maximization, $c^* = f(k^*)$ and $\theta = \frac{df(k^*)}{dk^*}$, where k^* is capital input per capita at steady state such that $k^* = \lim_{t \rightarrow \infty} (k_t)$. Because $f(k^*)$ and $\frac{df(k^*)}{dk^*}$ are monotonously continuous for $k^* > 0$, c^* is a monotonously continuous function of θ for $\theta > 0$. Here, because u is monotonously continuous, then $E_0[u(c^*)] = g(\theta)$ is also monotonously continuous for $\theta > 0$.

The function $g(\theta) = E[u(c^*)] = W$ is a decreasing function of θ because the higher rate of time preference results in the lower steady state consumption. The function $h(\theta) = E[u(c^*)] = W$ is also a decreasing function of θ because $\frac{d\theta}{dW} < 0$. Thus, both $g(\theta)$ and $h(\theta)$ are decreasing, but the slope of $h(\theta)$ is steeper than that of $g(\theta)$ as shown in Figure 4. This is true because $g(\theta) = W$ is the consequence of the Ramsey-type model indicated in equations (1) and (2); thus, if $\theta \rightarrow \infty$, then $g(\theta) = W \rightarrow 0$ because $\theta = i_t \rightarrow \infty$ and $k_t \rightarrow 0$, and if $\theta \rightarrow 0$, then $g(\theta) = W \rightarrow \infty$ because $\theta = i_t \rightarrow 0$ and $k_t \rightarrow \infty$. On the other hand, the function $h(\theta) = W$ indicates the endogeneity of time preference, and because the rate of time preference is usually neither zero nor infinity, then even if $h(\theta) = W \rightarrow 0$, $\theta < \infty$, and $h(\theta) = W \rightarrow \infty$, $0 < \theta$. Hence, the locus $h(\theta) = W$ cuts the locus $g(\theta) = W$ downwards from the top, as shown in Figure 4. Because the locus $h(\theta) = W$ is more vertical than $g(\theta) = W$, a permanently constant rate of time preference, as shown in

Figure 5, has probably been used as an approximation of the locus $h(\theta)=W$ for simplicity.

3.2.2.2 Stability of the model

The rate of time preference is constant unless a shock that changes the expectation of c^* occurs. This is self-evident by $W = E[u(c^*)]$. W does not depend on t but on the expectation of c^* ; thus, the same rate of time preference and steady state continue until such a shock hits the economy. Therefore, the endogeneity of time preference matters only when such a shock occurs. This constancy is the key for the stability of the model (21). Once the rate of time preference corresponding to the intersection is determined, it is constant and the economy converges at a unique steady state unless a shock that changes the expectation of c^* occurs. This shock is exogenous to the model, and the economy does not explode endogenously but stabilizes at the steady state. Hence, the property $\frac{d\theta}{dW} < 0$ in model (21), which is consistent with empirical findings, does not cause instability.

Model (21) therefore is acceptable as a model of endogenous time preference, which indicates that, because the rate of time preference is endogenously determined, irrationality is not necessary for determination of the time preference rate. Nevertheless, a shock on the rate of time preference is initiated by a shock on the expectation of c^* ; thus, even though animal spirits are directly irrelevant to determination of the time preference rate, they may be relevant to the generation of shock on the expectation of c^* . This possibility is examined in Section 4.

3.3 Uncertainty and time preference

An important feature of the model (21) is that a shock on uncertainty makes the rate of time preference shift, where uncertainty means the stochastic nature of the steady-state consumption (c^*). This is not a new idea. Fisher (1930) argued that uncertainty, or risk, must naturally have an influence on the rate of time preference, and higher uncertainty tends to raise the rate of time preference. This feature is particularly important for examining the mechanism of depression, because it has been reported that uncertainty increases in a depression (e.g., Romer, 1990).

The uncertainty about c^* can be described by the stochastic dominance of the distribution of c^* in a second-degree sense or a Rothschild-Stiglitz sense. Given $F(c^*)$, a subjective cumulative distribution function of c^* ($0 \leq a < c^* < b$),

$$W = E[u(c^*)] = \int_a^b u(c^*) dF(c^*). \quad (22)$$

Consider two steady-state consumptions c_1^* and c_2^* . Because $u(c^*)$ is increasing and concave in c^* , then $E_0[u(c_2^*)] \leq E_0[u(c_1^*)]$ if $F(c_1^*)$ second degree stochastically dominates $F(c_2^*)$, with strict inequality for a set of values of c^* with positive probability. If $F(c_1^*)$ stochastically dominates $F(c_2^*)$ in the Rothschild-Stiglitz sense, then $E[u(c_2^*)] \leq E[u(c_1^*)]$ and the mean of c^* is preserved as well.

Suppose that a shock on the distribution of c^* occurs, which preserves the mean but makes the uncertainty increase for any θ . Because utility $u(c^*)$ is increasing and concave, this increase in uncertainty indicates a shift of the locus $g(\theta)=W$ downwards to the bold dashed line shown in Figure 4, because $W = E[u(c^*)]$ becomes smaller for any θ . Hence, if the uncertainty about c^* increases from $F(c_1^*)$ to $F(c_2^*)$ in the Rothschild-Stiglitz sense, $W = E[u(c^*)]$ decreases. Even though the mean of c^* is not preserved, if the uncertainty about c^* increases from $F(c_1^*)$ to $F(c_2^*)$ in the second-degree sense, $W = E[u(c^*)]$ also decreases. If the mean of c^* decreases simultaneously, the locus $g(\theta)=W$ shifts further downwards to the thin dashed line in Figure 4. Therefore, the equilibrium rate of time preference increases; that is, increased uncertainty makes households more myopic. The effect of uncertainty in the model (21) is thus consistent with Fisher's (1930) argument.⁴

4. GOVERNMENT FAILURE

Animal spirits may influence the generation of shocks on the expectation of c^* , but the arbitrary assumption of animal spirits is not compelling. In this section, I explore a mechanism that generates a shock on the expectation of c^* without the need to invoke animal spirits.

4.1 Policy-induced stochastic processes

4.1.1 A stochastic process with an absorbing state

Because it is not present consumption (c_t) but steady-state consumption (c^*) that matters, the factor that generates a shock on the expectation of c^* should have persistent effects on consumption. Thereby, the factor should be one of the deep parameters (e.g., TFP and preferences) that can change the steady state. In addition, since it has been reported that uncertainty increases in a depression (e.g., Romer, 1990), the factor should make c^* be expected to be random with a constant probability distribution. For the endogenous variable c^* to be expected to be random, exogenous random variables are required because, without exogenous random variables, endogenous variables are constant at steady state. Nevertheless, exogenous variables that make c^* be expected to be substantially random with a constant probability distribution are not easily found among the deep parameters. If the exogenous stochastic variable is a stationary process with a known constant steady-state probability distribution, c^* is expected to be smoothed by the stochastic Ramsey-Euler equation and to become nearly deterministic (e.g., Brock and Mirman, 1972; Mirman and Zilcha, 1977). On the other hand, if it is a random walk, it does not have a constant probability distribution.

Hence, for c^* to be expected to be substantially random with a constant probability distribution, a special process of the exogenous stochastic variable is required. The following jump process with an absorbing state (V_t) is such a process. For an unknown future period \bar{t} ($0 < \bar{t}$),

⁴ Harashima (2004a) shows that the rate of time preference and uncertainty in Japan simultaneously rose at the end of 1990s just before Japan entered a severe and persistent economic slump.

$$V_t = \begin{cases} \text{random}(\text{non-absorbing state}) & \text{if } 0 \leq t < \bar{t} \\ \text{deterministic}(\text{absorbing state}) & \text{if } \bar{t} \leq t \end{cases}, \quad (23)$$

where there are finite $M(\in \mathbb{N})$ deterministic states after the period \bar{t} . Which of the states becomes the absorbing state of V_t after \bar{t} is unknown until \bar{t} , but the probability distribution of the absorbing state is known for any t before \bar{t} . Let state $m(\in M)$ take the value v_m and its probability density function be $\tau(v_m)$. Then, the present expected value of V_t at steady state is $E\left(\lim_{t \rightarrow \infty} V_t\right) = \sum_{m=1}^M v_m \tau(v_m)$. If the value of each state is time-variable as $v_{m,t}$ but converges at each constant value if $t \rightarrow \infty$, then the present expected value of V_t at steady state is $E\left(\lim_{t \rightarrow \infty} V_t\right) = \sum_{m=1}^M \lim_{t \rightarrow \infty} v_{m,t} \tau\left(\lim_{t \rightarrow \infty} v_{m,t}\right)$ and its probability density function is $\tau\left(\lim_{t \rightarrow \infty} v_{m,t}\right)$. An important feature of the process V_t is that c^* is not expected to be smoothed by the stochastic Ramsey-Euler equation because it is only after \bar{t} that one of the deterministic paths ($v_{m,t}$) that is chosen as the absorbing state is known, and consumption proceeds solely in accordance with this unique deterministic path. Therefore c^* is expected to be random with a constant probability distribution depending on randomly distributed deterministic paths $v_{m,t}$ after \bar{t} .

4.1.2 Policy-induced elements

An important feature of this V_t -type process is that the unique future deterministic path is decided in the future. This feature is often observed in a government's policy decisions, which often take time to make. Once the government has made a decision, the path is deterministic, but before the decision, the path is uncertain. Governments sometimes postpone decisions because they are difficult (e.g., tax hike decisions). As a result, before the policy is decided, households have uncertainty with a constant probability distribution of the deterministic path. Hence, the necessity of a V_t -type process for the exogenous variable that makes c^* be expected to be substantially random with a constant probability distribution suggests that the exogenous variable is policy related.

A V_t -type process implies that, even though the exogenous variable is a stationary process, if it has break points in its process then c^* can be expected to be substantially random. The factors that break a stationary process require exogenous mechanisms. Some structural changes in the mechanism of forming TFP or preferences will be necessary. Nevertheless, the mechanisms of forming TFP and preferences do not usually change. One of the few possibilities for change is that the mechanism is policy related because policies are changed at the discretion of governments, and stationary processes will occasionally break if they are related to policies. Therefore, the necessary properties of the exogenous variable, whether it takes a V_t -type process or not, suggest that the exogenous variable is policy related. The policy-induced element in TFP is particularly important, because production is substantially affected

by the TFP level.⁵

4.2 A policy-induced financial element in TFP

4.2.1 Financial elements in TFP

An important element in TFP is natural science technologies and knowledge. They are usually assumed to be stochastic, primarily because of the random nature of scientific discoveries and inventions. However, that randomness implies a random walk that has no constant probability distribution and, more importantly, no steady state. Therefore, scientific technology and knowledge will not be the element in TFP that changes the expected distribution of c^* .

Elements in TFP are not limited, however, to natural science technologies and knowledge. In the production function $y_t = \omega A_t^\alpha k_t^{1-\alpha}$ (equation (16)), A_t usually indicates natural science technologies and knowledge, but TFP is not A_t but ωA_t^α . If ω contains a policy-induced element, TFP is affected by the policy. Financial elements are included in this group of policy-induced elements. Economic development is proportionate to the level of financial development (e.g., Wachtel, 2003; Do and Levchenko, 2007), and wide differences of financial development have existed between developed and developing economies. Many studies have concluded that the causality is from financial development to economic activities (e.g., Levine, 1997; La Porta et al., 1998; Levine et al., 2000). In addition, the importance of financial development as a driving force of economic growth has been repeatedly emphasized (e.g., Levine, 1997; Levine et al., 2000; Temple, 2000; Easterly and Levine, 2003). Financial development reduces friction in markets, especially in capital accumulation and technological innovation (e.g., Levine, 1997), and financial systems play a critical role in allocation of resources, which is crucial for innovative activities (e.g., Schumpeter, 1912/1934; Shaw, 1973). These facts and arguments indicate that the financial element in TFP is an important determinant of the parameter ω and has significant effects on TFP. An important feature of the financial element is that it is closely related to government policies and thus has a V_t -type process, because there is an important imperfection in financial markets—there is asymmetric information between borrowers (firms) and lenders (investors)—and it needs to be eliminated by government.

4.2.2 Asymmetric information

The problem of imperfection in financial markets has long been studied (e.g., Gertler, 1988; Mishkin, 1991). Lenders usually have less information than borrowers. Under this asymmetric information, lenders may lend their money to less appropriate and lower quality borrowers, which indicates that resources including technologies are not optimally allocated in the economy.⁶ If there is no asymmetric information, the optimal allocation of resources in the economy will be achieved by rational activities of investors, but if there is asymmetric information, the allocation will be distorted.

⁵ The policies on TFP related to c^* are usually policies on economic structure and do not include discretionary macro-economic (fiscal and monetary) policies.

⁶ Not all technologies are embodied in a unit of capital, and each capital embodies only a portion of technologies. The adequate allocation of technologies over capital is important for maximizing production efficiency.

Non-optimal allocation of resources decreases the economy's overall efficiency, and TFP becomes lower in the long run if asymmetric information is left as it is.

Financial intermediaries mitigate the asymmetric information. Because financial intermediaries join in activities between firms and investors, the asymmetric information is separated into two parts: between firms and financial intermediaries, and between financial intermediaries and investors. The former will be reduced substantially by activities of financial intermediaries that monitor and investigate information on firms. Nevertheless, the latter is usually not easily minimized because of the principal-agent problem between investors and financial intermediaries. A financial intermediary (the agent) has an incentive to divert its behaviors from what an investor (the principal) wishes if there is asymmetric information and the investor does not know whether the contract has been satisfied. As a result, markets are distorted.

To reduce the principal-agent problem, investors must sufficiently monitor financial intermediaries. Investors, however, including individual small depositors of banks, cannot sufficiently monitor the intermediaries because such monitoring requires very complex processes, special skills, and a great deal of technical knowledge. More importantly, it is necessary to access perfect information on financial intermediaries and firms. If signals in financial markets contain and transmit perfect information on financial intermediaries and firms, investors may sufficiently monitor financial intermediaries, but many empirical studies have shown that the information is not perfect. For example, DeYoung et al. (2001) show that supervisors' assessments of banks contain some information that is not incorporated into prices of subordinated debts in markets. Other studies have also shown that signals from financial markets do not contain and transmit information perfectly (e.g., see Berger et al., 2000; Curry et al., 2003; Furlong and Williams, 2006). Such imperfect market signals suggest that some information—in particular, bad information—is deliberately hidden from markets.

4.2.3 The financial supervision authority

The market's inability to solve the problem of asymmetric information justifies the government's intervention to eliminate the distortion. On behalf of investors, the financial supervision authority eliminates the asymmetric information. As argued in Section 4.2.2, the problem of asymmetric information is separated into two parts. With addition of a financial supervision authority, the problem is further divided: asymmetric information between firms and financial intermediaries, between financial intermediaries and the authority, and between the authority and investors. The first two parts can be solved by financial intermediaries and the authority, respectively. The last part is not necessarily easily solved, however, because investors cannot fully monitor the authority's activities. They have to trust the authority. Hence, self-regulation is quite important for the authority.

It is very difficult to be perfect, and the supervision may occasionally fail. Such failure is more likely to occur and be more severe after regulations have been substantially changed, for example, after deregulation. In such cases, the financial supervision authority has to innovate to adapt to the new regulations. Because the authority is a monopoly, its failure is not a single negligible error among many authorities,⁷ and once the supervision fails, its negative effects will spread widely

⁷ In some economies, the authority is separated across a few branches in the government, depending on

through financial markets. In addition, there is also a principal-agent problem between the authority and investors. The authority has an incentive to hide its failure from investors, and if the authority deliberately hides its failure, investors cannot easily know of the failure.

If asymmetric information is unchecked because of the failure of supervision, financial intermediaries will obtain extra profits thanks to the asymmetric information. The negative effect of non-optimal allocation of resources will be recognized only by less-informed investors and households far later. Faced with the extra profits of financial intermediaries, less-informed investors and households may wrongly guess that technology is unexpectedly progressing more than it actually is. The less-informed households will then undertake activities on the basis of this incorrect guess—activities that would be considered to be irrational if perfect information were available—and this may make the economy spuriously appear to be in a boom in the short run.

4.2.4 Revelation of the failure of supervision

Even if an authority deliberately hides its failure, it is impossible to hide it forever. Because there is a gap between the distorted expectation by less-informed households and actual economic activities, the failure will eventually be revealed, perhaps by accident. When the failure is revealed, the trust in the financial supervision authority will immediately be lost, and the expectation of future policy will change suddenly and sharply. Because the financial element in TFP is a policy-induced element and has a V_t -type process, the expected probability distribution of the financial element in TFP at steady state will also immediately change.

The arguments in Section 4.2.2 indicate that the present financial element in TFP will not change suddenly and sharply, because the already allocated resources cannot change suddenly and sharply. Nevertheless, unlike the present financial element in TFP, the expected probability distribution of the financial element in TFP at steady state can change suddenly and sharply with the revelation of the failure of supervision. In addition, the failure of supervision implies that the expected distributions of the financial element in TFP and c^* were wrongly formed before the revelation of the failure; thus, the revisions of the expected distributions of the financial element in TFP and c^* resulting from the revelation will be more substantial than usual. As a result, the rate of time preference is immediately raised and a Nash equilibrium of a Pareto inefficient path will be immediately selected even though the present TFP is almost unchanged.

5. THE MECHANISM OF DEPRESSION

5.1 *Essence of the mechanism of depression*

The mechanism of depression shown in this paper takes the following steps:

- 1) There is asymmetric information between financial intermediaries and investors.
- 2) The financial supervision authority fails to sufficiently eliminate the

the type of financial intermediary, but each branch is a monopoly authority for each of type of intermediary.

asymmetric information.

- 3) Financial intermediaries exploit extra profits as a result of the asymmetric information and the failed supervision, and the economy may spuriously look like a boom in the short run to less-informed investors and households.
- 4) The exploitation distorts the allocation of resources and lowers TFP in the long run.
- 5) The failure of supervision is revealed.
- 6) The revealed government failure immediately lowers the expected financial element in TFP at steady state and increases uncertainty about it.
- 7) The lowered expected financial element in TFP at steady state immediately lowers the expected steady-state consumption, and the increased uncertainty about the financial element in TFP at steady state immediately increases the uncertainty about steady-state consumption.
- 8) The lowered expected steady-state consumption and increased uncertainty about it immediately raise the time preference rate of households.
- 9) The effect of the upward time preference shift is immediately transmitted to the real economy through the link $i_t = \theta$ (equation 4).
- 10) The steady state shifts to one with lower production and consumption.
- 11) A Nash equilibrium of a Pareto inefficient path is immediately rationally selected by non-cooperative and risk-averse households, even in a frictionless economy.
- 12) If there are the frictions, their effects are amplified by the Pareto inefficiency, and huge amounts of unutilized resources (e.g., unemployment and idle capital) are persistently generated.
- 13) Production and consumption steadily decline to the level at the posterior steady state.

This multi-stage complex mechanism has the following distinguishing features:

- The necessity of not only market but government failure
- TFP as a catalyst
- Uncertainty as an intermediate
- The endogeneity of time preference
- A Nash equilibrium of a Pareto inefficient path selected by non-cooperative households

The government failure to eliminate the market failure originates the depression by affecting the financial element in TFP. Thus, depression is closely related to TFP. However, this does not mean that a depression is caused by a technology shock, i.e., by a shift of the present TFP, but instead by a change of the expected TFP at steady state. In that sense, TFP does not play the leading role but rather is a catalyst for depression. During the process, an increase of uncertainty about c^* is generated as an intermediate, which increases the rate of time preference. The increase in the time preference rate as the discount factor may trigger a stock market crash, which has accompanied depressions (e.g., Barro and Ursúa, 2009). The turbulences in the financial markets are then transmitted to the real variables through the link $i_t = \theta$, to satisfy which at steady state, i_t must be adjusted (see equation (4)). Then, a Nash equilibrium of a Pareto

inefficient path is selected by non-cooperative households.

The above-mentioned features are equally important, but the most essential element in the mechanism of depression is the endogeneity of time preference. A Nash equilibrium of a Pareto inefficient path is probably generated only if the rate of time preference shifts upwards. With the endogeneity, turbulences in financial markets are immediately transmitted to the real economy through the link $i_t = \theta$. The key is not i_t but θ . In short, the mechanism shown in this paper indicates that a depression as a Nash equilibrium of a Pareto inefficient path is generated because the revealed incompetence of government and the resulting increased policy-induced uncertainty make non-cooperative and risk-averse households behave more myopically.

5.2 Policies for preventing and recovering

What should or should not be done to prevent and recover from depressions? The following discussion offers answers suggested by the mechanism of depression shown in this paper.

5.2.1 Policies for preventing a depression

Because the origin of depression is the failure of government supervision, the key to prevent depression is for the financial supervision authority to perfectly implement its delegated task. If the supervision is perfectly prudential and effective and sufficient information is provided to investors, markets approach perfect information and thereby few depressions will be generated. Therefore, it is important for the authority to self-regulate. Note, however, that the purpose of the supervision is not to tightly control financial intermediaries but to eliminate the asymmetric information, and the task delegated to the authority is to disseminate sufficient information on firms and financial intermediaries to investors. The government is not the manager of financial intermediaries but the referee of markets in which the financial intermediaries, firms, and investors participate. Conversely, financial intermediaries can freely do their business in principle if they give the authority perfect information about themselves and the firms they intermediate. Except for regulations with regard to providing perfect information to the authority, only activities that confuse investors and induce *ex post* irrational behaviors should be regulated.

5.2.2 Policies for recovering from a depression

5.2.2.1 Fiscal policy

Because the economy is on a Pareto inefficient path during a depression, government intervention is justified. Fiscal policies to fill the gap between the Pareto efficient and inefficient paths can improve the utilities of households without affecting capital formation. Fiscal policies to utilize unutilized resources (b_t ; Section 2) increase the utilities of households but do not necessarily reduce capital, because b_t is neither consumed nor invested but simply discarded or preemptively not produced. Therefore, fiscal policies will ease the problems caused by the Pareto inefficiency without harming the future economy. If utilization of b_t is done through government borrowings, government debt will increase. However, the increased debts are irrelevant to the formation of capital; thus, future production and consumption are also not affected.

Even though fiscal policies will help, however, they do not have the power to change the steady state. Even if a large amount of fiscal stimulus is injected into the economy, the economy eventually will converge on the posterior lower production steady state corresponding to the raised rate of time preference. Therefore, it will not be possible to make the economy come back to the prior higher production steady state only by relying on fiscal policies.

5.2.2.2 Monetary policy

Monetary policies are taken according to the instrument rule of the central bank and are basically taken in the same manner even during depression. Because the Pareto inefficiency widens output gaps, the interest rate as the instrument of the central bank needs to be lowered by the rule. The lower interest rate will temporarily increase production. However, as is the case with fiscal policies, monetary policies do not have the power to change the steady state and cannot make the economy come back to the prior higher production steady state.

In addition, lowering the interest rate according to the rule may not be sufficient. To prevent a deep deflation, the interest rate may have to be lowered more substantially and rapidly than is usually required by the rule. Harashima (2004c, 2007, 2008) shows that, if an upward time preference shift of households widens the gap between the time preference rates of households and the government, inflation decreases, and in some cases, deflation is generated. If a deep deflation sets in and the real interest rate is forced to exceed the marginal productivity by the zero bound on nominal interest rates, markets cannot be cleared, and the situation will be significantly exacerbated (e.g., the Great Depression). A deep deflation can be prevented if the central bank lowers the interest rate more substantially and rapidly than usually required by the rule.

5.2.2.3 Supervision reform

Supervision reform is necessary because the expectation of the financial element in TFP should be improved and policy-induced uncertainty should be reduced. Only with this reform can the economy return to the prior higher production steady state, because fiscal and monetary policies cannot shift the rate of time preference downwards. As shown in Section 4, policy-induced uncertainty is reduced after the policy is decided. If the reform decision is delayed, then uncertainty and the rate of time preference remain high for a longer period, and the costs of the depression will be exacerbated. In this context, the decision should be made as soon as possible. If the reform, however, overlooks the asymmetric information problem, the expectation of the financial element in TFP at steady state will remain low, and the rate of time preference will not fully return to the previous low rate even after the policy-induced uncertainty is eliminated. Therefore, reforms must never overlook the problem of asymmetric information.

Putting most financial intermediaries under tight governmental control or, alternatively, replacing their role in financial allocation in the economy by governments, may be effective as an emergency measure to make the economy recover rapidly from depression, because this measure will significantly reduce policy-induced uncertainty if the government is politically stable. The famous rapid recovery of the German economy from the Great Depression in the mid-1930s (e.g., Temin, 1989) may

be partly attributed to this effect. Recapitalization of failed financial intermediaries by using public money may also be needed. The purpose of the recapitalization is not to rescue the failed financial intermediaries but to protect small investors, smooth the difficulties of the reform, and reduce policy-induced uncertainty. Recapitalization, as well as many other government interventions, nevertheless can generate the problem of moral hazard. Thus, these measures should be as temporary as possible and should be accompanied by various measures to keep the moral hazard problem to a minimum.

6. CONCLUDING REMARKS

During a depression, unemployment substantially increases and capital utilization rate significantly falls. Frictions on quantity and price adjustments appear to be insufficient to explain the generation mechanism of these large-scale persisting phenomena. This insufficiency suggests that some unknown mechanism amplifies the effects of the frictions. In this paper, I argue that the Nash equilibrium of a Pareto inefficient path is that mechanism. Such a Nash equilibrium is generated because households are risk averse and non-cooperative. If the Nash equilibrium is generated, additional huge extra unutilized resources (e.g., unemployment and idle capital) will be persistently generated. The paper shows that a depression as a Nash equilibrium of a Pareto inefficient path is generated even in a frictionless economy if, and probably only if, the rate of time preference shifts upwards. The situation when the rate of time preference shifts upwards is described by a non-cooperative mixed strategy game. In this game, a strategy profile consisting of strategies of choosing a Pareto inefficient transition path of consumption to the posterior steady state is a Nash equilibrium.

Nevertheless, why does the rate of time preference shift upwards? Keynes (1936) implied that animal spirits—moods like pessimism or optimism—overwhelm rationality and are an important factor in a depression. This paper argues that animal spirits are not necessary and presents an endogenous time preference model, in which the rate of time preference is determined not by the present but by the expected steady-state consumption. The model is consistent with many empirical observations that the rate of time preference is negatively correlated with permanent income (e.g., Lawrance, 1991). A shock that changes the expected steady-state consumption shifts the rate of time preference, and a source of the shock is policy-induced shocks to the financial element in TFP. A failure of government supervision of financial markets originates the shock to the expected steady-state consumption, resulting in an upward time preference shift. With the endogeneity of time preference, the turbulences in financial markets are transmitted to the real economy through the link $i_t = \theta$. It is not the behavior of i_t but that of θ that induces a depression.

In summary, a depression is generated because the revealed incompetence of government and increased policy-induced uncertainty make non-cooperative and risk-averse households behave more myopically, resulting in a Nash equilibrium of a Pareto inefficient path.

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Figure 1: A time preference shock

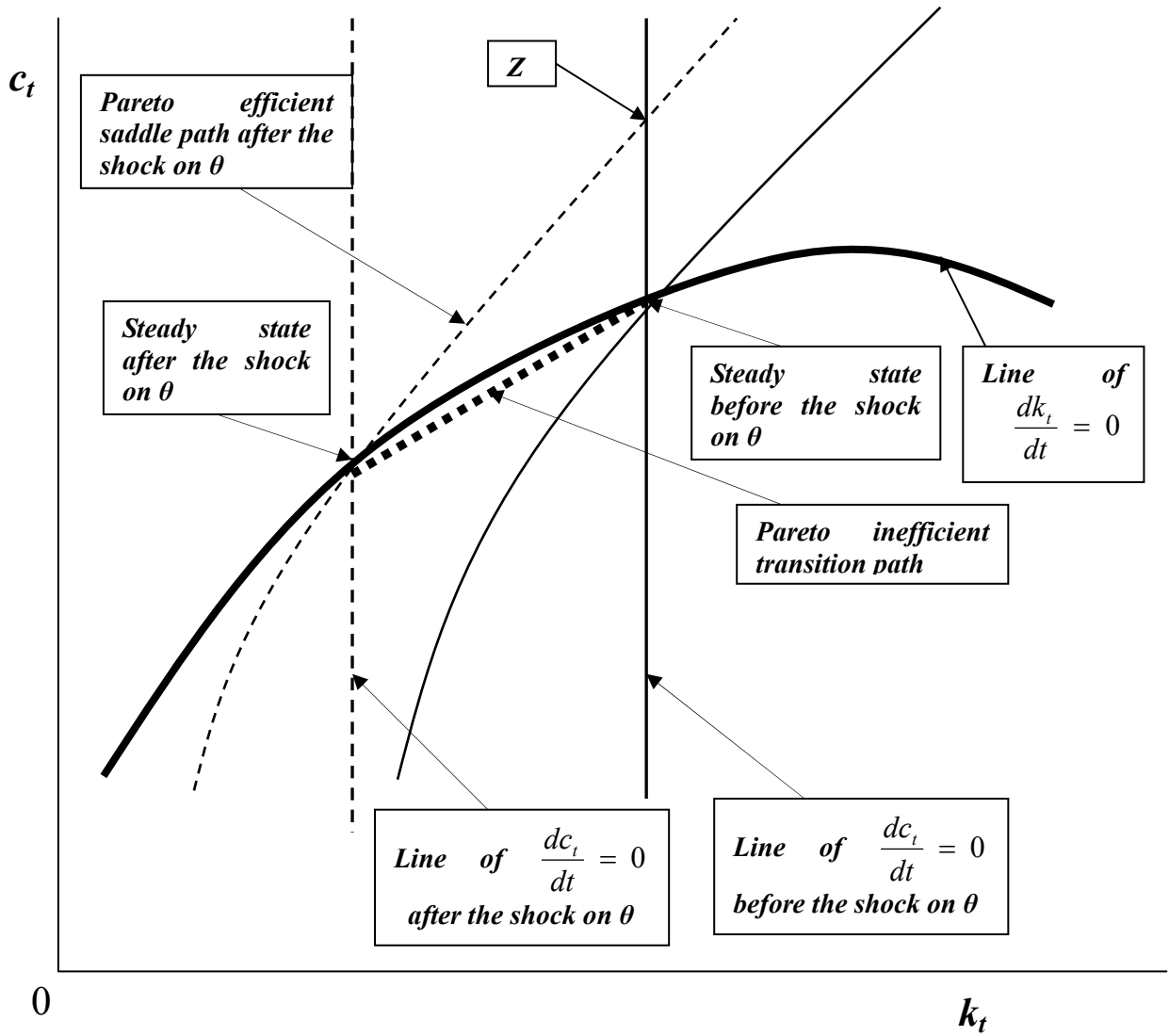


Figure 2: The paths of *Jalone* and *NJalone*

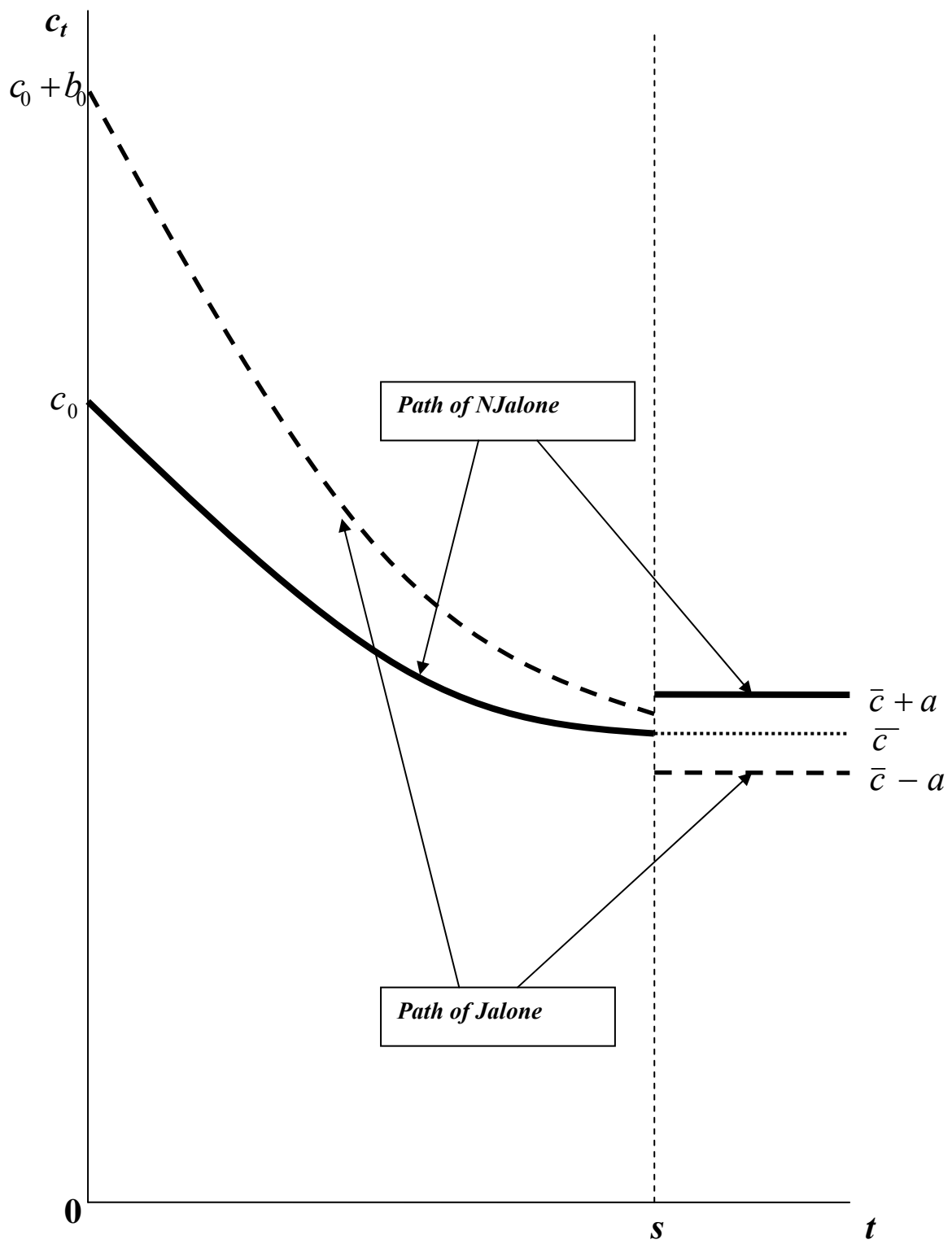


Figure 3: A Pareto inefficient transition path

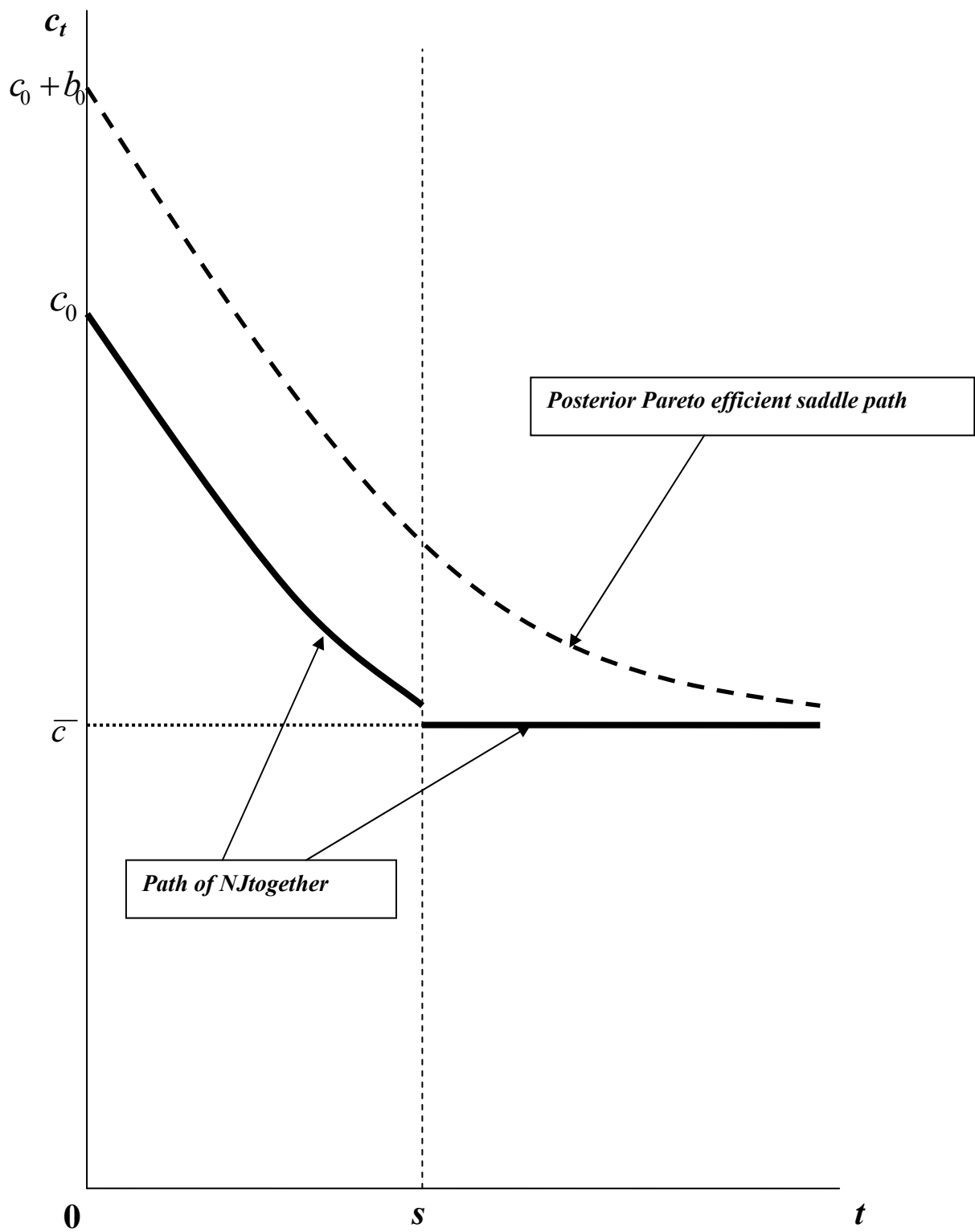


Figure 4: Endogenous time preference

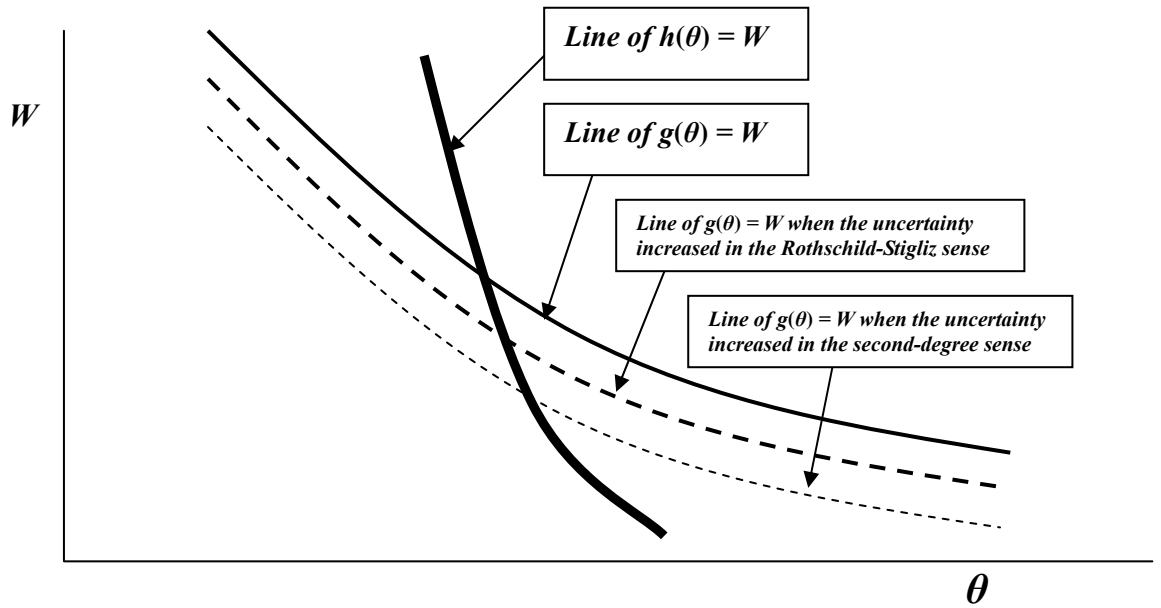


Figure 5: Permanently constant time preference

