The Illusion of Irrationality

Kontek, Krzysztof

Artal Investments

6 December 2009

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Krzysztof Kontek
Artal Investments sp. z o.o.
ul. Chrościckiego 93/105
02-414 Warsaw, Poland
e-mail: kontek@artal.com.pl
kkontek2000@yahoo.com
tel.:+48 504 16 17 50

Abstract

This short paper shows that the Allais Paradox and the Common Ratio Effect – regarded as classic examples of the violation of the Expected Utility Theory Axioms – may be easily explained by assuming that changes in wealth (i.e. gains and losses) are perceived in relative terms. The preference reversal observed in experiments is therefore predictable and the choices shall consequently be assumed to be rational. By contrast, the assumption that wealth changes are perceived in absolute terms leads to the conclusion that the choices violate the axioms underlying Expected Utility Theory, and are therefore irrational. This state of affairs is called the illusion of irrationality.

JEL classification: C91, D03, D81, D87

Keywords: Expected Utility Theory, Relative Utility Function, Allais Paradox, Common Ratio Effect, Prospect Theory, Markowitz's hypothesis

1. Introduction

1.1. The concept of expected utility was first introduced by Bernoulli and Cramer as early as the first half of the 18th century, although they did not consider the question of rationality when presenting their solutions. Much later, von Neumann and Morgenstern (1944) derived an identical solution from axioms which assumed that people made rational decisions. Their Expected Utility Theory has four axioms: completeness, transitivity, independence and continuity. If all these axioms are satisfied, then the individual is said to be rational and his or her preferences can be represented by a utility function.

1.2. Expected Utility Theory has dominated mainstream economic thought since the middle of the twentieth century. Since NM's work, however, an increasing number of psychological experiments have pointed to the irrationality of human decision making, where "irrationality" means any deviation from the axioms underlying Expected Utility Theory. Most of the arguments against the Expected Utility approach have concerned the independence axiom with the Allais Paradox and the Common Ratio Effect being the main examples of inconsistency of choices. The concept of probability weighting has allowed these experimental results to be explained and has led to the development of several theories, including Prospect Theory (Kahneman, Tversky, 1979) and its Cumulative version (Tversky, Kahneman, 1992).

1.3. Kontek (2009) argues that lottery experiments can be explained using the relative utility function without recourse to the probability weighting function. This solution, whose shape strongly resembles the utility function hypothesized by Markowitz (1952), is based on the assumption that people perceive wealth changes (i.e. gains and losses) in relative rather than absolute terms as assumed by Prospect Theory[1]. These arguments, however, only apply to simple lotteries. This paper goes further and analyzes examples which are regarded to violate the EUT axioms. It will be shown that both paradoxes can be easily explained by using the concept of the relative utility function. The preference reversal observed in the experiments is therefore only irrational if it is assumed that wealth changes are perceived in absolute terms. This state of affairs is called the illusion of irrationality.

2. Common Ratio Effect and Allais Paradox.

2.1. The Common Ratio Effect will be considered first. This was presented by Prospect Theory as follows:

Problem 1: Choose between

A: 4,000 with a probability of 0.80 or B: 3,000 with a probability of 1.00

0 with a probability of 0.20

Problem 2: Choose between

C: 4,000 with a probability of 0.20 or D: 3,000 with a probability of 0.25

0 with a probability of 0.80 0 with a probability of 0.75

Experimental results consistently reveal that most people choose option B in Problem 1 and option C in Problem 2. Expected Utility Theory, per contra, predicts that people would choose either A and C or B and D, as the probabilities of winning the main prize in the second pair of choices differ by a common ratio factor of 4 compared with the first pair. The graphical solution to this problem using the relative utility function is presented in Fig.1.

The shape of both the red and blue curves are the same as they are rescaled relative utility

[1] This observation, which results from Weber's Law is not new and was noticed by Kahneman and Tversky (1984) as well: *The topical organization of mental accounts leads people to evaluate gains and losses in* <u>*relative*</u> *rather than in absolute terms* (emphasis added). This was, however, not implemented in PT and CPT.

functions[2]. The difference is that the range of the red curve is [0, 4000] as this corresponds with options A and C, which have a maximum outcome of 4,000, whereas the range of the blue curve is [0, 3000] as this corresponds with options B and D, which have a maximum outcome of 3,000.

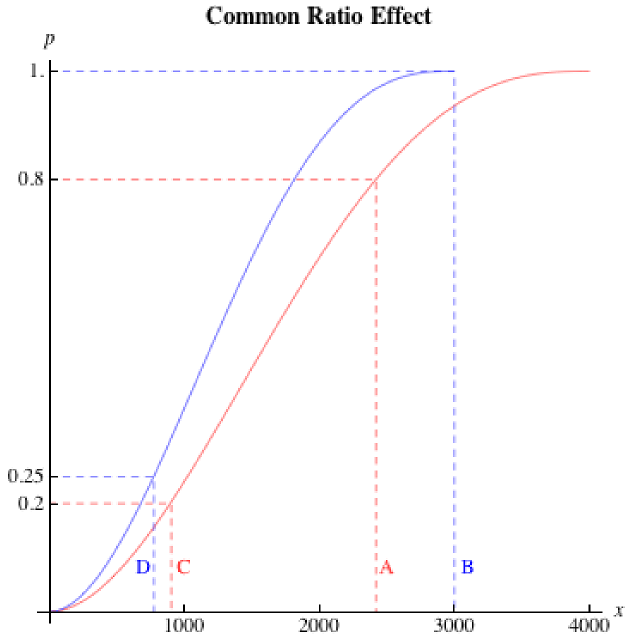

Fig. 1. The Common Ration Effect explained using the relative utility function.

The certainty equivalents of each option can be easily found on the *x*-axis as presented in the figure. There is obviously no inconsistency in choices between options having the same maximum outcome (e.g. between options A and C or B and D). The preference reversal only appear when options having different maximum outcomes are compared. For greater probabilities, option B prevails over option A, but for lower probabilities, C is a better option than D. As presented this effect is easily predicted and explained by assuming that people perceive wealth changes in relative terms.

2.2. The second classical problem considered here is the Allais Paradox, which is presented by Prospect Theory as follows:

Problem 3: Choose between

A: 2,500 with a probability of 0.33	B: 2,400 with a probability of 1.00
2,400 with a probability of 0.66	
0 with a probability of 0.01	

Problem 4: Choose between

C: 2,500 with a probability of 0.33	D: 2,400 with a probability of 0.34
0 with a probability of 0.67	0 with a probability of 0.66

[2]The parts for gains are presented.

This problem is similar to the Common Ratio Effect except that here both pairs differ by an outcome of 2,400 with a probability of 0.66. Once again, the experiments show a similar pattern in that most people choose option B in Problem 3 and option C in Problem 4. The graphical solution to this problem using the relative utility function is presented in the Fig. 2.

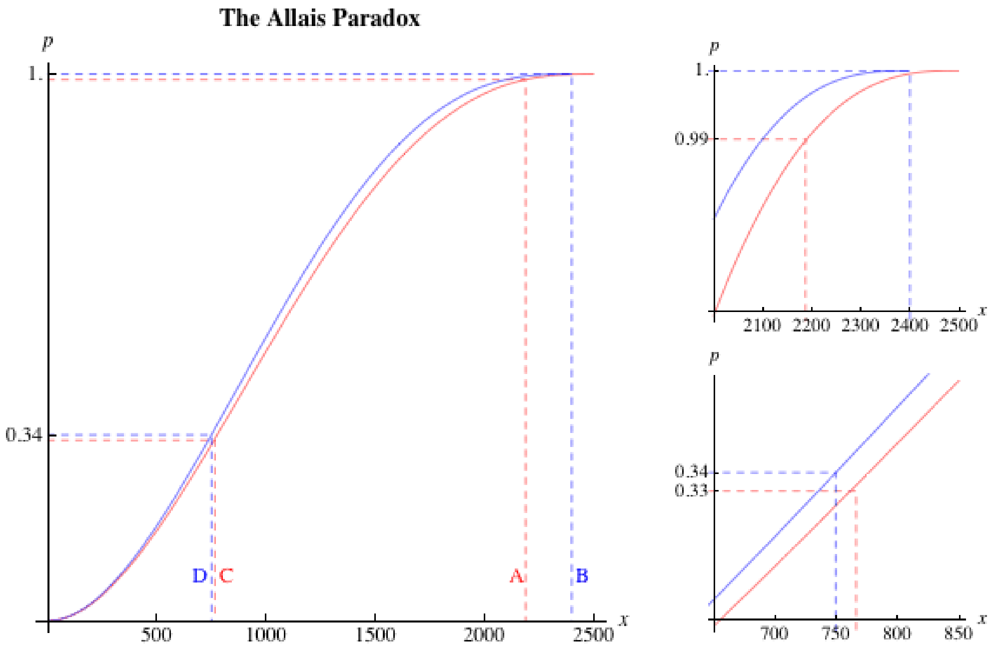

Fig. 2. The Allais Paradox explained using the relative utility function. The diagrams on the right show the details of the main diagram.

The red and blue curves have the same shape, as was the case with the Common Ratio Effect. Similarly, they differ in that the range of the red curve is [0, 2500] as this corresponds with options A and C, which have a maximum outcome of 2,500, whereas the range of the blue curve is [0, 2400] as this corresponds with options B and D, which have a maximum outcome of 2,400. The certainty equivalents of each option are found as presented in the figure. In order to simplify the analysis, the two option A outcomes can be combined to create (2500, 0.99), which is a better outcome. Even though its certainty equivalent is lower than that of the certain outcome of 2400. The preference reversal appears when comparing options having different maximum outcomes. For greater probabilities option B prevails over A, but for lower probabilities option C is better than D. It follows that this reversal is wholly predictable and can be explained using the relative utility approach.

3. Conclusions

Expected Utility Theory, based as it is on the assumption that wealth changes are perceived in absolute terms, can neither predict nor explain the preference reversals observed in the Common Ratio and Allais paradoxes. This leads to the conclusion that people are irrational. However, once it is assumed that wealth changes are perceived in relative terms, the preference reversals are fully predictable and there is no reason to regard people as irrational. As the perception of wealth changes in relative terms was stated even by Kahneman and Tversky, people's behavior should be regarded as rational and the relative utility function should be used for its description. It follows that the stated irrationality is merely a result of using an improper model. This is what we call the illusion of irrationality.

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