Marriage and Divorce in a Model of Matching

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Marriage and Divorce in a Model of Matching[1]

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We study the problem of marriage formation and marital distribution in a two-period model of matching, extending the matching with bargaining framework of Crawford and Rochford (1986). We run simulations to find the effects of alimony rate, legal cost of divorce, initial endowments, couple and single productivity parameters on the payoffs and marital status in the society.

Keywords: Matching, bargaining, marriage, divorce.

JEL Codes: C78, J12.

1 Introduction

Being unmarried, married or divorced are decisions faced by every human being during his/her lifetime with far-reaching economic implications both for individuals and the society as a whole. While in most societies, these decisions are taken by individuals, hence determined by individual preferences,

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they are also significantly shaped by social norms that draw the boundaries of the pool of eligible mates, civil laws that govern marital dissolution, and economic environments that frame marriage and divorce possibilities.

'Marriage' that has always lied at the centre of sociology, has also drawn the constant attention of economists in the last fifty years as it involves a choice problem of selecting a mate as well as a wealth allocation problem both within the marriage and at marital dissolution. In fact, these two problems are intertwined and therefore any of them cannot be tackled in isolation.

An early strand of literature that followed the pioneering work of Samuelson (1956) and Becker (1974, 1981) on common preference models of family, omitted the problem of formation of marriages and only studied strategic considerations within the family. Manser and Brown (1980) and McElroy and Horney (1981) modelled family demands as an outcome of a bargaining solution where threat points of agents were taken to be exogenously determined payoffs from divorce. Although these models on marriage commonly missed one important element, namely, the matching market, the implications of its absence was neither long unnoticed nor underestimated. Indeed, Becker (1991) himself emphasized the importance of matching market as a determinant of wealth distribution between men and women. As he pointed out, wealth distribution within marriage not only depends on partners' respective contributions to marriage but also on alternative matches available to each partner in marriage market. The other direction of the interaction between the formation of marriages and marital distribution was later noticed by Lundberg and Pollak (1993), who argued that policies that transfer income from husbands or wives affect not only the marital distribution of surplus within existing marriages but also the number of equilibrium matchings in subsequent marriage markets.

Recently, there have been studies that model the relationship between marital distribution and marital choice in a search-theoretic framework. Agents in this framework draw from a distribution of mates in the marriage market, and thus make their marital decisions. Bougheas and Georgellis (1999) examined the effect of divorce costs on both marriage formation and

dissolution. Aiyagari et. al. (2000) constructed an overlapping generations search model of marriage and divorce, and examined the effects of antipoverty policies (child support and welfare) on decision to marry, divorce and invest on children. Greenwood et. al. (2003) extended the Aiyagari et. al. (2000) by endogenizing the family size. Chiappori and Weiss (2003) studied, in a search theoretic general equilibrium model of marriage, the determination of divorce transfers.

In this paper, we deal with the said problem of marriage formation and marital distribution in a model of 'matching', differing from the existing literature. Individuals' decisions to be unmarried, married, divorced and remarried along with wealth distribution within the marriage are determined and analyzed in a two-period matching model through a recursive bargaining process, which is borrowed from Crawford and Rochford (1986) (hereafter CR).

Given that most individuals consider their immediate network of friends, colleagues and acquaintances as an eligible mate, we consider a prototype of an economy inhabited by two men and two women living for two periods. Each individual is born with an endowment (not necessarily identical) and they have mate-specific emotional utilities reflecting their affection to each possible match. A match between a man and a woman produces a surplus that is assumed to be linear in the total endowments of the couple. If an individual chooses to stay single then he/she produces a surplus combining his/her endowment with a single productivity parameter. Marital distribution is determined via Nash bargaining with the threat point of an individual being given by his/her bargaining payoff from an alternative mate.

Initially no agent is married. At the beginning of each period marriage market opens, allowing agents to change their marital status. In each period, agents reach to men-optimal stable matching and obtain the implied bargaining payoffs using the 'matching with bargaining equilibrium' of CR. The recursive procedure proposed by CR calculates the disagreement points for a given pair of agents as the bargaining equilibrium payoffs that they would receive in a reduced market obtained by setting the match between

the pair in consideration unproductive and letting the match opportunities of all other agents unchanged.[3]

We assume that agents can divorce at the end of period one unilaterally, and in such an event separating partners incur a legal divorce cost and person-specific emotional distress. Moreover, man is obliged to pay to woman a fraction of his period-one marriage surplus as an alimony.[4] Beginning-of-period 2 endowment of each individual is then determined by his/her period-one bargaining payoff net of all types of divorce costs (if any).

Due to the complex nature of the model, the closed-form solution of the problem cannot be obtained. Thus to make comparative statics, we simulate the model for an artificial economy. For each of the model parameters, namely, alimony rate, legal cost of divorce, initial endowments, couple and single productivity parameters, we examine the changes in the payoffs of men, women, and the whole society as well as changes in the frequencies of being i) single in both periods, ii) married only in the first period, iii) married only in the second period, iv) married with distinct mates in the two periods, v) married with the same mate in both periods.

The organization of the paper is as follows: Section 2 introduces the model. Section 3 presents our simulation results and relates them to the

[3]Bargaining equilibrium of CR is criticized for having a drawback that an agent's utility obtained from his or her original marriage cannot affect his or her utility in an alternative marriage. As a remedy, Bennett (1988) proposes a new bargaining equilibrium, which always yields a core matching that is Pareto optimal and maximizes aggregate utility. However, this alternative solution rests upon much stronger behavioral assumptions about players, who must always be consistent in their conjectures and able to solve fixed point problems. While Bennett's bargaining equilibrium is uniquely appealing in the design and implementation of matching algorithms for a market designer who can rationally act on behalf of agents of any degree of rationality, the appropriate choice between the two equilibrium definitions by CR and Bennett in *modelling* the observed marital behavior in a particular society cannot be determined independently from the investigated rationality of the involved agents. In this paper, we prefer to use computationally more tractable equilibrium of CR for our simulations, consciously assuming away the full consistency of agents in the marriage market.

[4]According to McManus and DiPrete (2001), noncustodial parents are ovewhelmingly male and alimony payments almost exclusively flow from men to women.

existing theoretical and empirical literature. Section 4 concludes.

2 Model

We consider a society with two men m_1, m_2 and two women w_1, w_2 living for two periods indexed by $t = 1, 2$. The set of men and women are denoted by M and W, respectively. The set of all possible feasible assignments of women to men are denoted by $\mathcal{M} \equiv \{\mu : W \to M \,|\, \mu \text{ is one-to-one}\}$.

The endowment of agent i in $M \cup W$ at the beginning of period t is $e_i(t) \in \Re_+$. Agent i is born with $e_i(1)$; however $e_i(2)$ which we call an *interim* endowment is acquired from the share of the total marriage surplus at the end of the first period. The emotional utility of agent i derived from a match with agent j in period t is $u_i(j,t) \in \Re$. The emotional utility of agent i from being single is normalized to zero, i.e. $u_i(i,t) = 0$.

The marriage market in period t is defined as the complete list of endowments and all utility possibilities, and denoted by $\Omega(t)$.

We assume that the match between man i and woman j converts total endowments into total utility $E_{ij}(t)$ to be shared (bargained over) through a linear production technology, i.e.

$$(1) \quad E_{ij}(t) = f_{ij}[e_i(t) + e_j(t)]$$

where the linear productivity parameters satisfy $f_{ij} = f_{ji} \in \Re_+$.

The disagreement point from a match between man i and woman j is $\langle d_i(j, \Omega_t), d_j(i, \Omega_t)\rangle$. In our one-to-one matching market with transferrable utility, bargaining problems are solved by applying Nash (1950) solution, which yields to agents equal shares of gains from cooperation. Thus, the payoff to agent i when matched with j is

$$(2) \quad \pi_i(j; \Omega_t) = \max\{b_i e_i(t), u_i(j,t) + \min\{E_{ij}(t), (1/2)[E_{ij}(t) + d_i(j, \Omega_t) - d_j(i, \Omega_t)]\}\}$$

where $b_i \in \Re_+$ is the productivity parameter of the linear technology that converts agent i's endowment into end-of-period utility when he/she is single.

Equation (2) ensures that all the surplus from a marriage is distributed and each agent gets at least as great as when he/she remains single.

Agent i who is matched in period 1 incurs legal divorce cost c, and an emotional distress c_i^u at the end of the period if he/she divorces from his/her mate. In addition, in the event of divorce, man is obliged to pay to woman a fraction, a, of his end-of-period 1 marriage surplus (in endowment terms) as an alimony.

Thus, the period 2 endowment of man i who divorces from woman j at the end of period 1 is

$$(3) \quad e_i(2) = (1-a)[\pi_i(j;\Omega_1) - u_i(j,1)] - c$$

and the period 2 endowment of woman j who divorces from man i at period 1 is

$$(4) \quad e_j(2) = \pi_j(i;\Omega_1) - u_j(i,1) + a[\pi_i(j;\Omega_1) - u_i(j,1)] - c.$$

The period 2 emotional utility of agent i who divorces from agent j at the end of period 1 and considers to be matched with agent k is

$$(5) \quad u_i(k,2) = u_i(k,1) - c_i^u.$$

Agent i feels no emotional distress if he/she does not divorce from his/her mate.

An allocation in the marriage market $\Omega(t)$ consists of an assignment of agents $\mu(.,t)$, and a payoff vector $x(.,t)$. Borrowing from CR, we define a bargaining equilibrium for $\Omega(t)$ as an assignment/payoff pair $(\mu(.,t), x(.,t))$ such that[5]

$$(6) \quad x(m,t) = \pi_m(\mu^{-1}(m,t);\Omega_t) \text{ for all } m \in M, \text{ and}$$

[5]In fact, our two-period model also allows one to define and use a 'dynamic' equilibrium considering the life-time payoffs in individuals' decision problems. In such an equilibrium, a pair of poor agents that have extremely strong affections to each other may rather marry with rich and otherwise 'unattractive' individuals in the first period for the sole purpose of fortune-hunting (wealth-accumulating), and then they may divorce and marry with their 'genuine lovers' in the second period. As we do not believe that the majority of marriages in the modern societies are driven by such far-sighted opportunistic motives, we prefer to use the single-shot equilibrium notion of CR in every period of our model.

$$x(w,t) = \pi_w(\mu(w,t);\Omega_t) \text{ for all } w \in W,$$

(7) there do not exist agents $m \in M$ and $w \in W$ such that

$$\pi_m(w;\Omega_t) > x(m,t) \text{ and } \pi_w(m;\Omega_t) > x(w,t).$$

Condition (6) requires that agents matched under $\mu(,.t)$ obtain payoffs determined by the bargaining solution in (2). Condition (7) simply requires that no pair of agents have strict incentives to block the matching $\mu(.,t)$. Together with the bargaining solution in (2), which checks the incentive of each agent to individually block a given matching, condition (7) defines the stability notion in the usual way.

To determine the disagreement (threat) points, we slightly modify recursive procedure by CR to allow for situations in which agents have emotional utilities. For a given pair $m \in M$ and $w \in W$, we construct the reduced market $\Omega'(t)$ from $\Omega(t)$ by setting the match between m and w as unproductive, i.e.

$$(8) \quad \pi_m(w;\Omega'_t) = b_m e_m(t) \text{ and } \pi_w(m;\Omega'_t) = b_w e_w(t),$$

and letting $\Omega'_t = \Omega_t$ for all agents in $M \cup W$ except for m and w. Let $\langle \mu'(.,t), x'(.,t) \rangle$ be a bargaining equilibrium in Ω'_t. Then

$$(9) \quad d_m(w;\Omega_t) = x'(m,t) - u_m(w,t) \text{ and } d_w(m;\Omega_t) = x'(w,t) - u_w(m,t).$$

By conditions (8) and (9), the threat points of the pair (m,w) in the market Ω_t are taken as their equilibrium payoffs in the reduced market Ω'_t (net of the emotional utilities from their current mates), where they are forced to remain single while all other agents can still exploit all of their opportunities.

Once agents' preferences over matches are fixed at any stage of the recursive procedure by the borrowed wisdom of CR in determining threat points, one can apply Gale and Shapley's (1962) 'Deferred Acceptance Algorithm' (DAA) to find a stable matching that is always known to exist (under some mild assumptions on preferences, such as completeness and transitivity, which are trivially satisfied in our model). Of the two versions of the DAA, we

employ the DAA with men proposing and obtain the men-optimal stable matching in each step of the recursive procedure.[6]

The procedure of finding the bargaining equilibrium in a given period is explicitly described in the Appendix.

3 Simulation Results

Using the GAUSS (version 6.0) we simulated the model for an artificial economy. (The program codes and the simulated data are available from the authors upon request.) We generated 2401 observation values for the model, by drawing each model parameter independently (using uniform random number generators) from a set of seven distinct values.

The parameter values are listed in Table 1. The numbers in curly brackets are the common set of values that the parameters just above them take. Since the primary interest here is to get qualitative effects of model parameters on marital decision and wealth distribution, parameters are picked to yield a rich set of equilibria involving diverse marital decisions. The single and couple productivity parameters take values which are above, below, and equal to, 'one' to allow for all distinct cases of constant returns to scale. The set of couple productivity parameters is a superset of the set of single productivity parameters, which is in line with the recent empirical finding of Zagorsky (2005).[7]

Alimony rate, a, takes values in the unit interval. Legal divorce cost, emo-

[6] By interchanging the names of men and women, the DAA with men proposing simply changes to the DAA with women proposing. In the DAA with men proposing, initially nobody is engaged (assigned) or rejected. In each iteration, an unassigned man proposes to the first acceptable woman on his list that he has not proposed to yet. A woman who receives a proposal that she prefers to her current assignment accepts it and rejects her current assignment. The algorithm stops after any step in which every man has either been rejected by every acceptable women on his list or is engaged to an acceptable woman.

[7] Using US data from the National Longitudinal Survey of Youth (NLSY79), Zagorsky (2005) finds that for respondents who married and stay married, per person net worth was 93 per cent higher than for single respondents.

tional distress parameters, initial endowments of four agents in the marriage market, and emotional utility parameters assume artificial values.

TABLE 1

PARAMETER VALUES

Tastes	$u_{m_1}(w_1,1)$, $u_{m_1}(w_2,1)$, $u_{m_2}(w_1,1)$, $u_{m_2}(w_2,1)$
	$\{0.28, 0.59, 0.94, 1.31, 1.72, 2.15, 2.62\}$
	$u_{w_1}(m_1,1)$, $u_{w_1}(m_2,1)$, $u_{w_2}(m_1,1)$, $u_{w_2}(m_2,1)$
	$\{0.24, 0.50, 0.77, 1.09, 1.44, 1.86, 2.35\}$
	$c^u_{m_1}$, $c^u_{m_2}$, $c^u_{w_1}$, $c^u_{w_2}$
	$\{0.39, 0.49, 0.60, 0.70, 0.81, 0.9, 1.02\}$
Endowments	$e_{m_1}(1)$, $e_{m_2}(1)$, $e_{w_1}(1)$, $e_{w_2}(1)$
	$\{1.04, 2.64, 4.80, 7.51, 10.78, 14.60, 18.98\}$
Productivity parameters	$f_{m_1w_1}$, $f_{m_1w_2}$, $f_{m_2w_1}$, $f_{m_2w_2}$
	$\{0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00\}$
	b_{m_1}, b_{m_2}, b_{w_1}, b_{w_2}
	$\{0.44, 0.66, 0.87, 1.09, 1.30, 1.52, 1.73\}$
Policy variables	a, c
	$\{0.13, 0.25, 0.38, 0.50, 0.63, 0.75, 0.88\}$

For the model parameters $a, c, e_{m_1}(1), e_{w_1}(1), b_{m_1}, b_{w_1}, f_{m_1w_1}$ we shall below analyze the variation in the average values (using 343 observations for each value of the input variables) of the output variables in the list $\langle\, x(M,2)$, $x(W,2)$, $x(MW,2)$, $M(ss)$, $M(sm)$, $M(ms)$, $M(mm\text{-}c)$, $M(mm\text{-}n)$, $W(ss)$, $W(sm)$, $W(ms)$, $W(mm\text{-}c)$, $W(mm\text{-}n)$, $MW(ss)$, $MW(sm)$, $MW(ms)$, $MW(mm\text{-}c), MW(mm\text{-}n)\,\rangle$. Here, $x(M,2), x(W,2), x(MW,2)$ are respectively the total payoffs of men, women, and the whole society, in the second period, while for any group Z in $\{M, W, MW\}$ (men, women, society) the

output variables $Z(ss), Z(sm),\ Z(ms), Z(mm\text{-c}), Z(mm\text{-n})$ respectively denote the frequencies of "being single in both periods, married only in the second period, married only in the first period, married with distinct mates in the two periods, married with the same mate in both periods". We should note that $M(mm\text{-}n)$ and $W(mm\text{-}n)$, and hence $MW(mm\text{-}n)$, always take the same values.

The marital status of the population for the simulated sample is exhibited in Table 2. The percentage of divorced couples, $MW(ms) + MW(mm\text{-}c)$, is 23.28%, while 27% of these couples remarry after divorce. Calculating $MW(mm\text{-}n)/\ [MW(mm\text{-}n) + MW(mm\text{-}c)]$ shows that 89% of the marriages are stable. Moreover, as should be evident from $MW(sm)/[MW(ss)+ MW(sm)]$, approximately 10% out of the first-period singles get married in period two. Strikingly, the characteristics of our sample in terms of marital distribution fit quite well with the latest U.S. demographic data.[8]

TABLE 2

MARITAL STATUS

(Percentage Distribution)

Male	$M(ss)$	$M(sm)$	$M(ms)$	$M(mm\text{-}c)$	$M(mm\text{-}n)$
	23.41	3.00	17.45	5.83	50.31
Female	$W(ss)$	$W(sm)$	$W(ms)$	$W(mm\text{-}c)$	$W(mm\text{-}n)$
	24.32	2.08	16.53	6.75	50.31
Society	$MW(ss)$	$MW(sm)$	$MW(ms)$	$MW(mm\text{-}c)$	$MW(mm\text{-}n)$
	23.87	2.54	16.99	6.29	50.31

Figure 1 through Figure 6 exhibit comparative static results with respect

[8] According to U.S. Census Bureau, the marital status of people 15 years or over for all races as of March 2005 is reported as follows: 17.7% of the population is separated, divorced or widowed; 29% is never married, while 53.2% is married.

to alimony rate. The women's payoff is increasing and men's payoff is decreasing, while the overall payoff is slightly decreasing. The frequency of being single in both periods declines across both genders except for a pike around 0.35. In contrast, the frequency of being married only in the second period is rising except for a slight dip around 0.4. The frequency of being married only in the first period is escalating up to 0.75. However, the frequency of being married to different partners in the two periods is fluctuating with a steep upward trend. The sum of the two, which is the frequency of divorce, is also increasing. Finally, the frequency of being married to the same partner in both periods falls.

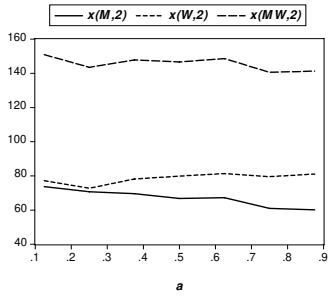


Figure 1

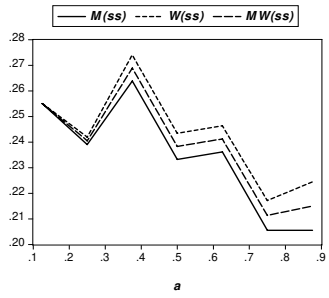
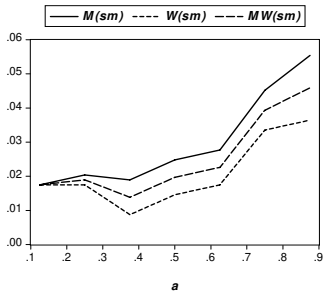


Figure 2



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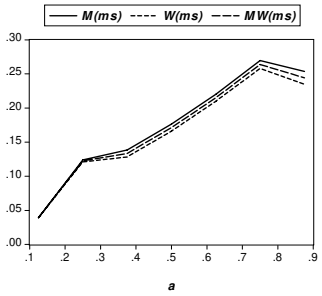

Figure 4

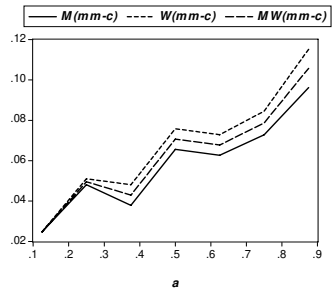

Figure 5

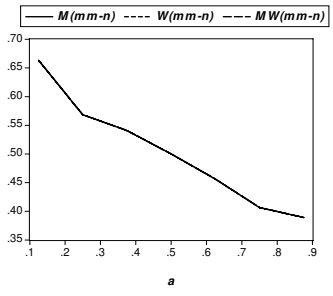

Figure 6

One can also interpret the parameter a as the child support benefit paid to the custodian partner, which happens to be women in our model. Then, our model predicts that child support has positive effects both on the rate of marriage and on the rate of divorce. While the former is in line with, the

latter contrasts with, Aiyagari et. al. (2000). However, in the same context, Greenwood et. al. (2003) finds small effects of child support on the equilibrium number of marriages.

The effects of legal cost of divorce on the model outcomes are presented in Figures 7-12. Since c takes values relatively low with respect to the magnitudes of endowments, its direct effect on the payoffs of separating couples is already expected to be small. The almost horizontal behavior of payoffs in Figure 7 reveals that the indirect effects of c, due to changed marriage opportunities by the changed endowment structure, must also have been negligibly small. The divorce frequency fluctuates on a narrower band with respect to the changes in the legal cost of divorce and reaches a maximum around 0.5, while at this very level, the frequency of continuing marriages attains its minimum.

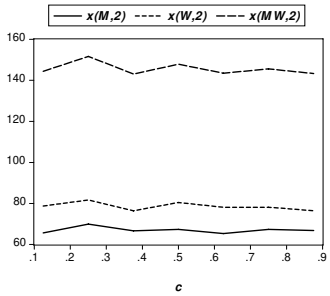

Figure 7

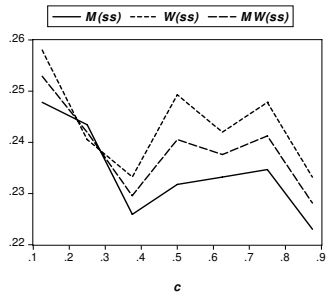

Figure 8

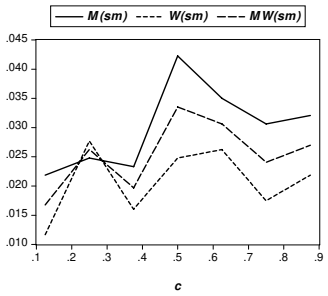

Figure 9

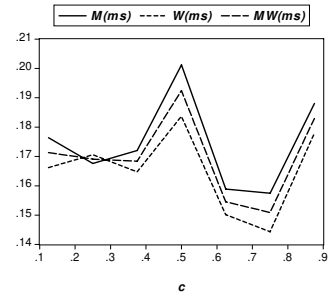

Figure 10

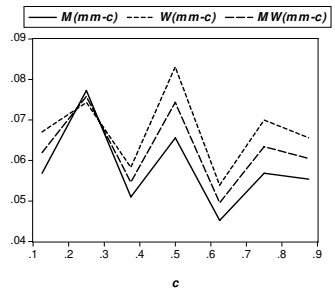

Figure 11

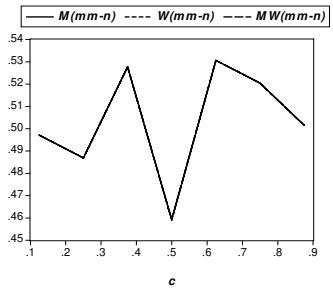

Figure 12

On the effect of divorce cost on the marital decisions, Becker (1976) ar-

gued that when divorce becomes easier, the number of people who are legally married may actually increase, which is partially supported by our findings (Figures 9 and 11). However, empirical studies examining the impact of changing divorce legislation on marital status are somewhat conflicting.[9] While Peters (1986, 1992) find no significant impact of changing divorce laws on divorce rates, Allen (1992) and Friedberg (1998) refute her finding. Recently, Wolfers (2006) reports that the divorce rate rose sharply following the adoption of unilateral divorce laws, but this rise was reversed within about a decade hence the long-run effects are ambigious. Brien et. al. (2006) finds that decrease in divorce cost leads to a slight increase in the rate of marriage.

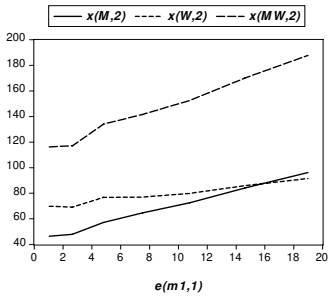

Figure 13

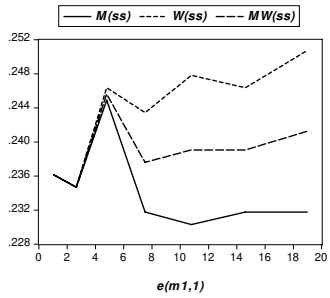

Figure 14

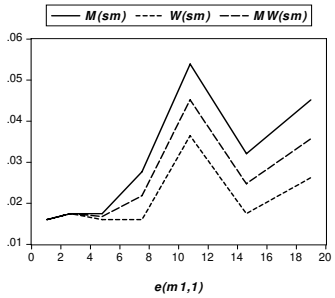

Figure 15

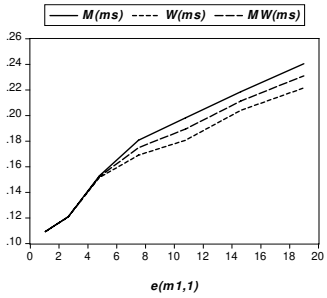

Figure 16

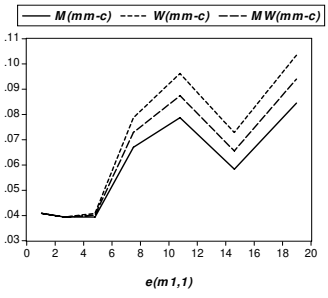

Figure 17

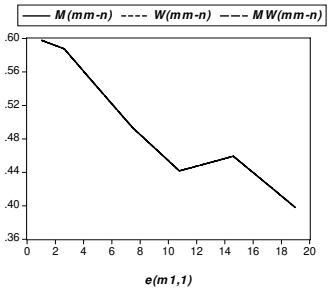

Figure 18

Figure 13 through Figure 18 contain comparative static results with respect to first man's initial endowment (relabeled as $e(m_1, 1)$ in the graphs).

[9] The relaxation of divorce laws across many U.S. states during 1970s and 1980s reduced the legal cost of divorce substantially (see Brien et. al., 2006).

All payoffs are increasing; however, men's aggregate payoff is increasing significantly more. While the frequency of divorcing and becoming single steps up, the frequency of being married to different partners in the two periods fluctuates. Nevertheless, the divorce rate can be shown to be increasing. The frequency of staying single in both periods is on the average is decreasing for men while increasing for women.

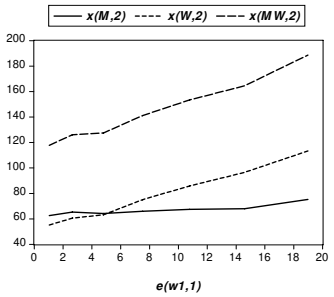

Figure 19

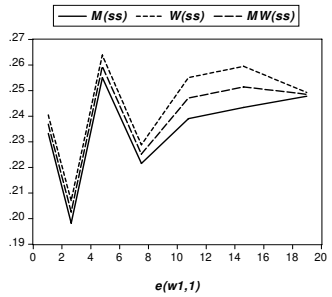

Figure 20

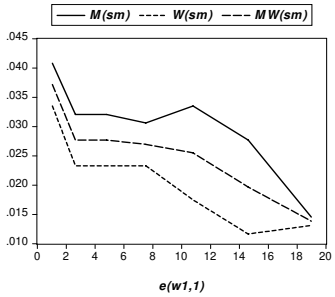

Figure 21

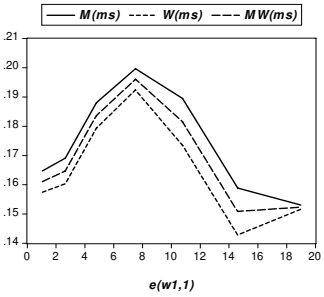

Figure 22

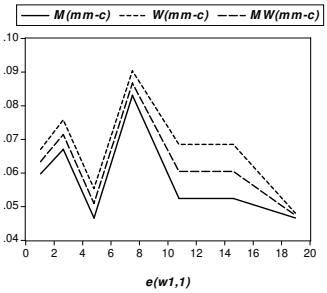

Figure 23

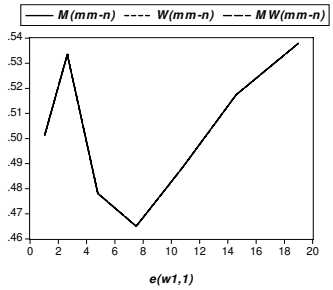

Figure 24

Figures 19-24 exhibit the effects of change in w_1's initial endowment (relabeled as $e(w_1, 1)$ in the graphs). Women's aggregate payoff is significantly rising. By the vertical sum of graphs in Figures 22 and 23, the divorce rate is first increasing and then decreasing in own endowment with a peak around the average wealth level over the simulation grid. The frequency of being single in both periods fluctuates. The frequency of being married only in the second period is decreasing for both genders. The frequency of continuing marriages has a U-shaped graph.

The endowment effects of the two genders on divorce rate are not symmetric since the matching procedure (DAA with men proposing) that we employed to characterize the bargaining equilibrium is not gender-symmetric, either. In a related empirical study, Weiss and Willis (1997) investigated the role of unexpected changes in earning capacity of either spouses in marital dissolution. Unlike our finding, they reported that an unexpected increase in the husband's earning capacity reduces the divorce hazard. However, their finding as to the negative effect of the wife's earning capacity on the divorce hazard is partially replicated by our simulations.

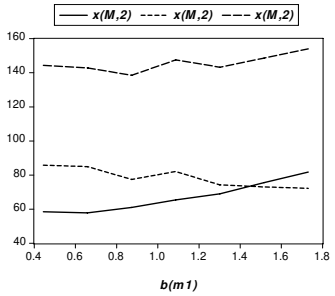

Figure 25

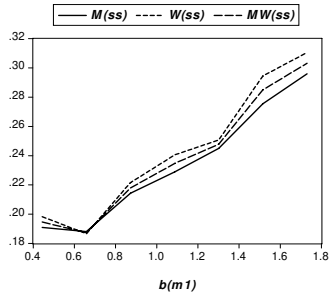

Figure 26

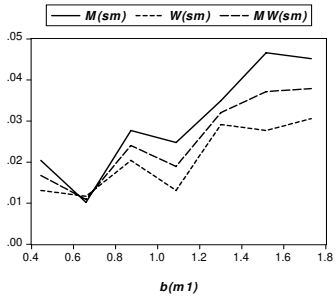

Figure 27

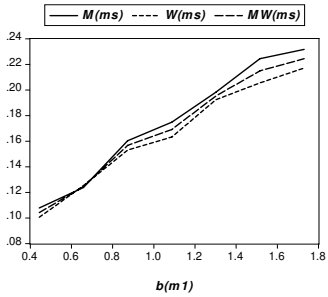

Figure 28

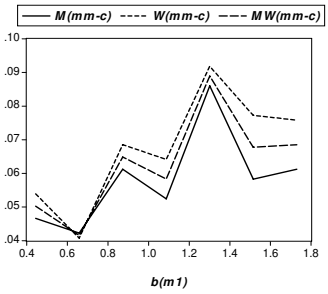

Figure 29

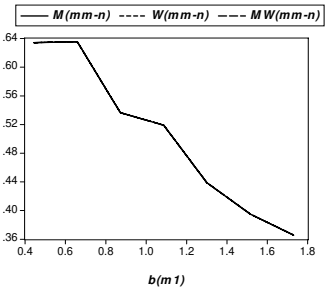

Figure 30

Figures 25-36 show comparative static results with respect to single productivity parameters b_{m_1} and b_{w_1} (relabeled as $b(m_1)$ and $b(w_1)$ in the graphs). Each gender's payoff is increasing in its own productivity parameter, while decreasing in the opposite gender's productivity parameter. The frequency of being single in both periods is almost always increasing for both genders

with respect to both b_{m_1} and b_{w_1}. The frequency of continuing marriage is declining in both gender's single productivity parameter. By the vertical addition of graphs in Figures 28-29, the divorce rate is increasing in man's single productivity parameter. On the other hand, a similar inspection in Figures 34-35 reveals that with respect to woman's single productivity parameter, the divorce rate is falling over the intervals [0.4, 0.65] and [1.05,1.30] and rising elsewhere.

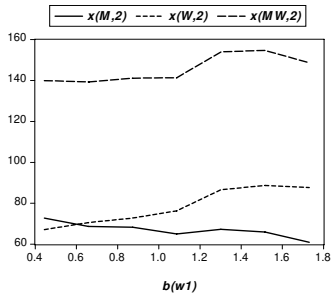

Figure 31

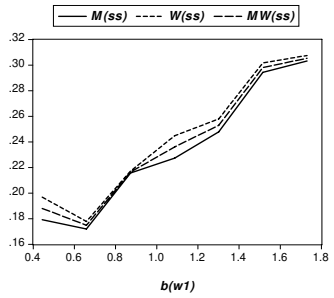

Figure 32

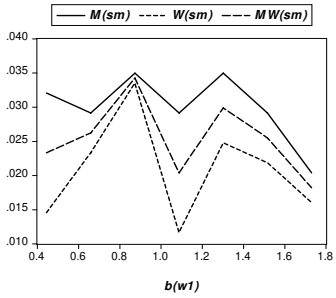

Figure 33

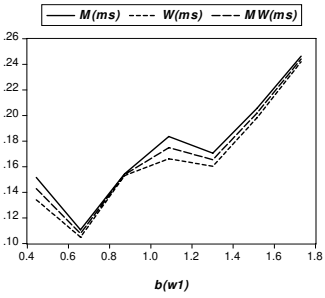

Figure 34

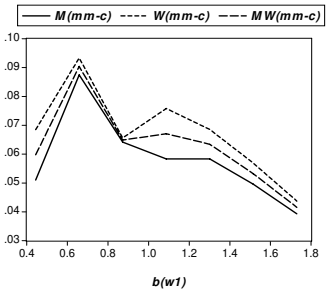

Figure 35

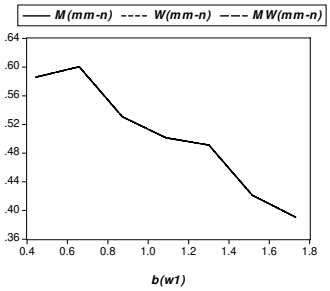

Figure 36

The single productivity parameter b_{w_1} may admit the interpretation of welfare payment collected by an unemployed divorced woman.[10] Then, our restated result that an increase in welfare payment (at high levels of b_{w_1}), as an antipoverty policy tool, to women leads to fewer marriages, is in accordance with Aiyagari et. al. (2000). A similar parallel can be drawn for the

[10] According to Sandefur (1996), 52 per cent of female-headed households were participating in the Aid to Families with Dependent Children Program in 1992.

positive effect of welfare policies on the divorce rate, which we observe at medium and high levels of welfare, i.e. when b_{w_1} lies in either of the intervals [0.4, 0.65] and [1.05,1.30].

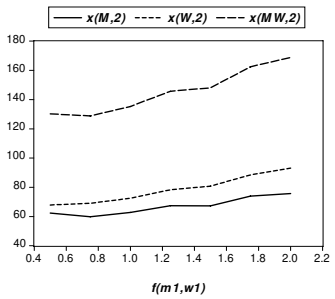

Figure 37

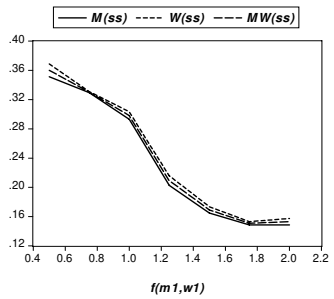

Figure 38

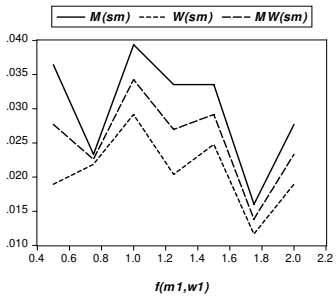

Figure 39

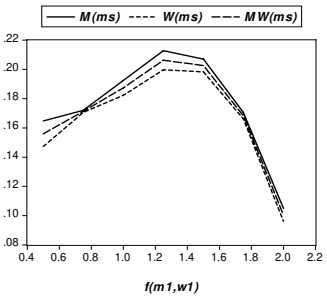

Figure 40

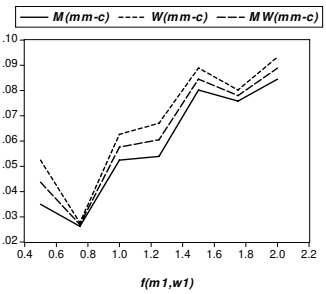

Figure 41

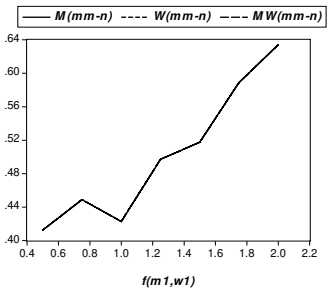

Figure 42

Figure 37 through Figure 42 exhibit the effects of marriage productivity f_{m_1,w_1} (relabeled as $f(m_1, w_1)$ in the graphs). As expected, both genders' payoffs are increasing. The relative benefit from being single decreases, leading to the observed fall in the frequency of remaining single in both periods. The frequency of marrying only in the second period wildly fluctuates over a very narrow band. The frequency of being married only in the first period is slowly rising at low productivity levels and then drops off rapidly, whereas the frequency of changing mates is increasing almost everywhere. Hence, the overall divorce rate can be shown to have an inverted U-shape with a peak around 1.5. The frequency of continuing marriages is increasing for productivity values exceeding one.

Considering the child as the outcome of a '*productive marriage*' (associated with a productivity parameter greater than one), we observe that the stability of marriage is increasing with the value (or the number) of children. In fact, this finding of ours is supported by an experiment of Brien et. al. (2006), which showed that the rate of marriage is increasing in the value of child. A similar result on the stabilizing effect of children on marriage is reported by Weiss and Willis (1997).

4 Conclusion

In the last hundred years, the divorce rates experienced a sharp increase.[11] The positive welfare effects of divorce through facilitating the termination of malfunctioning marriages (e.g. marriages involving domestic violence and child abuse) are on average outweighed by post-separation costs especially borne by women and children.[12] The rise in the divorce rates has consequently attracted the attention of researchers in understanding and modelling marital dissolution, and increased the awareness of policy makers towards taking measures to enhance the stability of marriages as well as to alleviate the post-separation costs. In this respect, our study has analyzed the effects of alimony, child support benefit, welfare programs and divorce laws among such policy measures on the divorce rate and the marriage rate.

The novelty in our paper is to study marital choice and marital distribution in a matching market. The existing literature that handles the same

[11] Stevenson and Wolfers (2007) reports that in the United States over the last 150 years divorce rates have risen with a rate of 3.6 divorces per thousand people and 16.7 per thousand married couples by 2005.

[12] McLanahan and Sandefur (1994) reports that, in 1995, the median income for female headed families with children was about one-third of the median income for married couple with children. Moreover, the rates to drop out of high school, to be idle, to experience teen births and not to take college education are higher for children living in single-parent households than for children from two-parent families. Page and Stevens (2002) finds that in the long-run (six or more years after the most recent divorce) family income falls by 40 to 45% after divorce, and food consumption is reduced by 17%.

problem in a search-theoretic framework emphasizes the random nature of meeting a potential mate. In each period, an agent randomly draws only one from the pool of mates and decides whether to marry or not. In our framework, agents in every period consider the whole set of mates as feasible and decide to whom to propose for marriage.

In reality, the pool of feasible mates for each individual expands over time by controlled as well as uncontrolled entrances of new candidates, i.e. the formation of mate pools is to some extent random in nature. However, given a pool of feasible mates individuals do not determine their prospective partners by random draws. Instead, individuals make pairwise comparisons and form (usually short) preference lists over the pool, according to which they make sequential proposals for marriage. The interaction of the separate decisions of individuals on the opposite sides of the marriage market then determines the equilibrium assignment. In this respect, while both search and matching models have their own uses and strengths, a more complete framework of a marriage market should integrate the two, taking also into consideration the potential benefits of non-cooperative versus cooperative bargaining in simultaneously resolving the marital distribution and determining the marital choice.[13]

Appendix

This Appendix contains the procedure of finding the bargaining equilibrium in a given period.

Marriage Rule: For the agents forming a given pair, calculate the bargaining payoffs using equation (2). Divorce the agents if and only if at least one of the agents is not worse off by remaining single. In that case, assign to both agents in the pair the payoffs from being single.

Step 1: Pick a permutation $\{p_i, p_j, p_k, p_l\}$ of the pairs $p_1 = (m_1, w_1)$, $p_2 = (m_1, w_2)$, $p_3 = (m_2, w_1)$, $p_4 = (m_2, w_2)$.

[13] For a non-cooperative model of marriage, see Lundberg and Pollak (1994, 1996).

Step 2: For the agents forming the pair p_l, set the disagreement points equal to the payoffs from being single and calculate the bargaining payoffs using the Marriage Rule. Next, for the agents entering both p_k and p_l set the disagreement points to their calculated bargaining payoffs in the pair p_l, and for the remaining agents in p_k set the disagreement points to the payoffs from being single. Then, as the outcome of this step, determine the marital status and bargaining payoffs of the pair p_k using the Marriage Rule.

Step 3: Repeat step 2 by interchanging the roles of p_l and p_k, and obtain the marital status and bargaining payoffs of the pair p_l.

Step 4: Using the bargaining allocation obtained for p_k and p_l in steps 2 and 3, respectively, find the men-optimal stable matching for the agents in $p_k \cup p_l$.

Step 5: For the agents in p_j, set the disagreement points to the bargaining payoffs obtained under the stable matching calculated in step 4 if these agents are also in p_k or p_l, and otherwise set them to the payoffs from being single. Then, find the marital status and payoffs for the pair p_j using the Marriage Rule.

Step 6: Repeat steps 2-5 consecutively for the permutations (p_i, p_k, p_j, p_l) and (p_i, p_l, p_j, p_k), and obtain the marital status and payoffs of the pairs p_k and p_l, respectively.

Step 7: Let the pair p_i be unproductive and the agents in p_i have their payoffs from being single.

Step 8: Using the marital status and payoffs assumed or calculated for p_i, p_j, p_k, and p_l in steps 2-7, find the men-optimal stable matching and the implied payoffs.

Step 9: For the agents in p_i, set the disagreement points to their bargaining payoffs calculated in step 8. Next, find the marital status and bargaining payoffs of the pair p_i using the Marriage Rule.

Step 10: Repeat steps 2-9 consecutively for the permutations (p_j, p_i, p_k, p_l), (p_k, p_i, p_j, p_l), (p_l, p_i, p_j, p_k), and obtain the marital status and bargaining payoffs for the pairs p_j, p_k, and p_l, respectively.

Step 11: Using the marital status and bargaining payoffs calculated for the pairs p_i, p_j, p_k, and p_l, determine the men-optimal stable matching (hence the bargaining equilibrium) for the marriage market.

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