Production Complementarity and Investment Incentives: Does Asset Ownership Matter?

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Abstract

This paper analyzes the role of the allocation of ownership rights in transactions where parties make relationship-specific investments and contracts are incomplete. If there is high mutual dependence in production, the initial allocation of ownership rights is irrelevant. This result contrasts with Grossman and Hart (1986), who, using a similar model, obtain that assets should be owned by the party whose investment is most productive to minimize ex-ante inefficiencies in production. The critical element behind these two different results is that while Grossman and Hart (1986) model uses the Nash bargaining solution treating status quo payoffs as disagreement points, here they are treated as outside options. The model also shows that, when relevant, asset ownership may provide disincentive to invest.

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1 Introduction

There has been contrasting theoretical results on the relationship between asset ownership and investment incentives. The seminal work of Grossman and Hart (1986) (GH, hereafter), followed by Hart and Moore (1990), and Hart (1995), argued that asset ownership boosts incentives to invest. De Meza and Lockwood (1998) and Chiu (1998) (DLC, hereafter), however, have shown that in certain environments, asset ownership may in fact reduce incentives to invest. This paper, in addition to obtaining DLC result in a different setting, presents a case where asset ownership does not affect investment incentives, despite the fact that investments are productive. It also contributes to understanding the link between asset ownership and investment incentives by disentangling the effects of production complementarity and investment productivity.

The model draws on GH where a theory of ownership rights is developed. They consider a relationship between two firms whose productive activities depend on each other. Ex-ante both firms make a relationship-specific investment and, ex-post they make a decision regarding the production process. Due to high transaction costs, ex-ante contracts contingent on the choice variables cannot be written. However, once the ex-ante investments have been made, the ex-post production decisions become contractible. Thus the agents can bargain over the division of surplus before the production decisions are made. Ownership confers residual control rights over the assets. Since none of the variables are ex-ante contractible, the initial contract only specifies the allocation of the residual control rights. Through its effect on the use of the asset in uncontracted states, ownership rights influence agent's bargaining power and the division of ex-post surplus, which in turn affects the parties' incentives to invest in that relationship. If there is a mutual dependence in the production of both firms, integration improves the incentives of the new owner while it weakens the incentives of the acquired firm's ex-owner. This trade-off between the costs and benefits of ownership determines the optimal allocation of control rights, hence ownership.

The main conclusion of GH is that ownership rights should be allocated to minimize ex-ante inefficiencies in production, hence assets should be owned by the agent whose ex-ante investment is the most productive in the relationship. This result is driven by the particular equilibrium of the bargaining game that GH has considered. They considered Nash bargaining where the status quo payoff, which is the payoff received by an agent prior to bargaining, is treated as the disagreement point to the Nash solution. In this paper, the Nash bargaining is replaced with an explicit alternating offers bargaining game where status quo payoffs are treated as outside options.[1]

The main finding of this paper is that if production complementarity and investment productivity are both important in a relationship, redistribution of ownership rights does not have efficiency implications. In GH model the Coase theorem fails to apply (that is ownership matters) because of the existence of transaction costs that are created by the agents' opportunistic behavior during bargaining. In this paper, although the agents cannot implement Pareto efficient outcome through bargaining, the inefficiency cannot be remedied by redistributing ownership rights. This theoretical finding is also supported by empirical evidence. According to Holmstrom and Roberts (1998), the non-integrated organizational structure in steel production from steel scrap (Nucor in U.S., and Co Steel in U.K.) is at odds with the existing property rights theory. The relationship is characterized by significant hold-up risk, yet it is precisely the presence of high degree of mutual dependence that makes cooperation sustainable under non-integration.

The sensitivity of the optimal allocation of ownership rights to the choice of extensive form bargaining game has been argued, earlier by DLC and they have shown that asset ownership does not necessarily boost incentive to invest. In some cases, asset ownership may act as

[1] As it has been previously argued in Binmore, Shaked, and Sutton (1989), and Sutton (1986), a dynamic bargaining game differentiates between a disagreement point and an outside option. When the status quo payoffs are taken as the disagreement point of the Nash solution, the agent's equilibrium payoff, which is called as the *"split-the-difference"* payoff, is the sum of her status quo payoff and half of the difference between the total surplus and both agents' status quo payoffs. This can be an equilibrium of a bargaining game in which the agents receive their status quo payoffs at every period where an agreement has not been reached. However, if the agents do not receive an income flow in the course of the bargaining or there is no exogenous risk of breakdown, then the disagreement payoff, which is the payoff from a perpetual negotiation without an agreement, should be zero. In this case, the agents can obtain their status quo payoff only if they quit the bargaining game unilaterally to implement the status quo. When the status quo payoffs are taken as outside options, they determine the range of validity for the Nash solution. When neither agents' outside option is binding, both receive half of the total surplus, which is called as *"split-the-surplus"* payoff. When only one agent's outside option is binding, she receives her outside option and the opponent claims the residual.

"stick" rather than "carrot" in De Meza and Lockwood (1998) terminology. This paper reinforces the results of DLC on the disincentive effects of asset ownership. When the production complementarity is asymmetric, disowning an asset may increase incentives to invest in the relationship. However, the model in this paper is based on the original model used in GH rather than the model in Hart (1995) (which is used in DLC) which is a special case of the former in some sense. In Hart (1995) the ex-post production decisions are merely a decision on the choice of the trading partner, i.e.: whether to trade with the existing partner or outsider. Hence the right to exercise residual control rights is rather limited.

The paper also unveils the two intertwined factors that affect the optimal distribution of ownership; investment productivity and complementarity in production. Whether an outside option is binding or not depends on both the degree of complementarity between the ex-post productive activities and ex-ante investment levels. If complementarity is mutual and large then, it is more likely that ex-post renegotiation produces a sufficiently large surplus at almost all investment levels and all ownership structures. In this case, neither party's outside option is binding and both parties gets *"split-the-surplus"* payoff. Hence, the distribution of ownership is irrelevant. However, if the relationship is asymmetric, in the sense that my action is significant for you but not vice versa, it is more likely that my outside option will be binding under my ownership. It is optimal to give the assets to me only if my investment is more productive relative to your investment.[2]

The paper is organized as follows. In Section 2, the formal model is introduced. In section 3, the equilibrium to the induced bargaining subgame is derived in the case of non-integration. Section 4 contains the equilibrium of the investment-choice game in the case of non-integration. Section 5 considers integration, in particular the case in which firm 1 owns firm 2 is analyzed. In Section 6, contains the comparative statics with respect to the level of complementarity between the two firm, in

[2]The distinct effects of investment productivity and production complementarity on determination of optimal asset ownership cannot be separated in the De Meza and Lockwood (1998) model but rather their combined affect is represented by assets being productive or unproductive outside relationship. This is because, in their model, the ex-post production decision is reduced down to the choice of trading partner, inside or outside. However, in many situation ex-post production may involve complex design decisions that are not contractible ex-ante.

order to characterize when the equilibrium exist. In Section 7, the two ownership structures are compared and the relationship between asset ownership and investment incentives is discussed. Section 8 contains concluding remarks.

2 The Model

As in the GH model, there are two firms, 1 and 2, that are engaged in a relationship which lasts 2 periods. Each firm is managed by an agent who receives the full return of the firm where she is employed. At the beginning of date 1, the two agents sign a contract that specifies the distribution of ownership rights over each firm's assets. After the contract is signed, each agent makes a relationship-specific investment $a_i \in A_i \subset \mathbb{R}_+$. We assume that the relationship-specific investments require special skills so that the investment a_i in firm i can only be made by agent i. At date 2, the investments become observable to both agents and some further decisions regarding the production process are made. Let $q_i \in Q_i \subset \mathbb{R}_+$ denote ex-post decision of agent $i = 1, 2$. Although a_i is chosen by agent i, the ex-post decision, q_i, is made by the agent who owns firm i.[3] The private benefit to agent i is written as $B_i[a_i, \phi_i(q_1, q_2)]$. The function ϕ_i can be thought of as a monetary payoff from second stage production net of costs. There is a disutility associated with ex-ante investment, which is given by $v_i(a_i)$. All costs and benefits are measured in date 1 dollars. The benefits and costs are the same under any ownership structure. Moreover, ownership does not provide any additional benefit.

None of the variables a_i, q_i and $B_i(\cdot)$ is contractible ex-ante. It is assumed that the non-contractibility of the variables arises either as a result of high transaction costs associated with writing comprehensive contracts, or because of enforcement problems. The ex-ante investment, a_i, is regarded as non-verifiable managerial effort which is non-contractible because of the enforcement problem. The variable q_i is ex-ante non-contractible because it stands for complex production decision and it is difficult to describe ex-ante.

[3] If the firms are separately owned, that is if agent i owns firm i, each agent is an owner-manager who has residual control rights over her firm's physical assets, so agent i chooses a_i and q_i of firm i. If the firms are integrated under i's ownership then agent i owns both firms 1 and 2 and agent j becomes her employee. For example, under 1's ownership, agent 1 chooses a_1, q_1 and q_2 and agent 2 chooses a_2.

Since the decision variables are ex-ante non-contractible, the date 0 contract can only allocate ownership rights between the two agent. Ownership of an asset grants the beholder the right to use it in any way she desires unless these rights are contracted away.[4]

A summary of the sequence of events is as follows. At date 0, a contract is signed. After that, a_1 and a_2 are chosen simultaneously and independently. At date 1, each agent learns the amount invested by her opponent. Before the actual choices of q_i are made they become contractible. If there is no further negotiation, the agent who owns firm i chooses q_i independently. The second stage decision, $q = (q_1, q_2)$, however, becomes contractible at date 1. Thus, a new contract may be negotiated that implements different choices of q_1 and q_2, and specifies how the surplus is divided. Then $B_1(\cdot)$ and $B_2(\cdot)$ are realized and the necessary transfers are made between the two agents according to the new contract.

The following technical assumptions guarantee that the optimization problems have unique solutions and first-order necessary conditions are sufficient. It is assumed that $B_i(\cdot)$ and $v_i(\cdot)$ are twice continuously differentiable and satisfy the following assumptions for all $a_i \in A_i$ and $q_i \in Q_i$.

Assumption 1: $B_i(\cdot)$ is increasing in ϕ_i and a_i. $B_1[\cdot]+B_2[\cdot]$ is strictly concave in its four arguments, (a_1, a_2, q_1, q_2).

Assumption 2: The cost function $v_i(a_i)$ is increasing and convex in a_i.

Assuming that monetary transfers between agents are available, the optimal contract maximizes the total ex-ante net benefits of the two agents,

$$W = B_1[a_1, \phi_1(q_1, q_2)] + B_2[a_2, \phi_2(q_1, q_2)] - v_1(a_1) - v_2(a_2). \quad (1)$$

If a_1 and a_2 were verifiable, and q_1 and q_2 were ex-ante contractible, the first best solution which is obtained by maximizing (1) with respect to a_i and q_i for $i = 1, 2$, could have been implemented. Since it is assumed

[4] Note that in this model, financial returns are not transferable with ownership. Ownership, however, can also be identified with the rights to the residual income stream. Holmstrom and Tirole (1989) argued that the definition of ownership can be a critical element in analyzing the efficiency properties of the initial allocation of ownership rights.

that all date 1 variables are non-contractible as of date 0, the first-best cannot be implemented. There are three cases to consider. In the first case which is called non-integration, the firms are separately owned. In the second and third cases the firms are integrated under the ownership of a single agent, 1 and 2 respectively.[5]

3 Non-integration

The subgame perfect Nash equilibrium of the full game is characterized by a vector of $(a, q) \in A \times Q$ and transfer payments, where $a = (a_1, a_2)$, $q = (q_1, q_2)$, $A = A_1 \times A_2$ and $Q = Q_1 \times Q_2$. Each vector a induces a proper subgame where agent 1 and 2 bargain over the division of total surplus. These subgames are called as the induced bargaining subgames. Below, the equilibrium payoffs in these bargaining subgames are characterized.

In the case where the firms are separately owned, agent i has the right to choose q_i. At date 1, the two agents choose q_1 and q_2 to maximize $B_1 [a_1, \phi_1 (q_1, q_2)]$ and $B_2 [a_2, \phi_2 (q_1, q_2)]$, respectively. It is assumed that there exists a unique Nash equilibrium to the simultaneous q-choice subgame which is

$$\begin{aligned} \hat{q}_1 &= \arg\max_{q_1 \in Q_1} \phi_1(q_1, \hat{q}_2), \\ \hat{q}_2 &= \arg\max_{q_2 \in Q_2} \phi_2(\hat{q}_1, q_2). \end{aligned} \tag{2}$$

In general, the non-cooperative solution $(\hat{q}_1, \hat{q}_2)$ is ex-post inefficient.[6] Therefore the two parties can gain from negotiating a new contract that specifies $(q_1 (a), q_2 (a))$ as the actions to be taken, where

$$(q_1(a), q_2(a)) = \arg\max \{B_1 [a_1, \phi_1 (q_1, q_2)] + B_2 [a_2, \phi_2 (q_1, q_2)]\} \tag{3}$$

is the equilibrium of the cooperative q-choice subgame. The vector of equilibrium actions $(q_1 (a), q_2 (a))$ is unique given that $B_1 (\cdot) + B_2 (\cdot)$ function is concave. The new contract is feasible, since q_1 and q_2 are ex-post contractible. Let $B[a, q(a)]$ denote the value function of problem 3. The division of $B[a, q(a)]$ among the two agents is determined by an alternating offers bargaining game.

[5]Ownership is perceived as a discrete variable which takes the value either 0 or 1 for each agent. Either agent 1 or agent 2 owns the firm. Two agent cannot own the same firm at the same time. Therefore a joint ownership structure is not considered.

[6]The noncooperative choices are efficient when ϕ_i is a function of only q_i or when $\phi_i = \phi_j$, that is the both agents have the same payoff function.

In the GH model the solution to the contract negotiation is characterized by the Nash bargaining solution where the status quo payoffs are treated as disagreement payoffs. In the unique equilibrium outcome of this bargaining game, each agent receives half of the increase in the total surplus which is written as

$$\mathcal{B}_i = \frac{1}{2}\left[B\left[a, q\left(a\right)\right] - B_i(a_i, \hat{\phi}_i) - B_j(a_j, \hat{\phi}_j)\right] \tag{4}$$

where $\hat{\phi}_i = \phi_i\left(\hat{q}_1, \hat{q}_2\right)$, and $B_i(a_i, \hat{\phi}_i)$ is the status quo payoff of agent i under non-integration, $i = 1, 2$. If one considers a bargaining game where the agents do not receive any income flow until they reach an agreement or there is no exogenous risk of breakdown then it is more natural to treat the status quo payoffs as outside options.[7] While a disagreement payoff directly influence the division of the surplus in the equilibrium, an outside option influences the division of the surplus only when it is a credible threat. Outside option constitutes a credible threat only if a player obtains a higher payoff from exercising her outside option than the equilibrium payoff she receives when she continues to bargain. Otherwise, quitting is not a credible threat. In the former case, she should at least receive the value of her outside option in any subgame-perfect equilibrium of the bargaining game. In the latter case, her outside option does not influence the equilibrium of the game.

Lemma 1 *Given the initial ownership structure, and the vector $a = (a_1, a_2)$ of of ex-ante investment levels, the induced bargaining subgame has a unique equilibrium in which the agreement is reached immediately, and firm 1 receives p, given by*

$$p = \begin{cases} B\left[a, q\left(a\right)\right] & \text{if } B_1(a_1, \hat{\phi}_1), B_2(a_2, \hat{\phi}_2) \leq B\left[a, q\left(a\right)\right]/2 \\ B_1(a_1, \hat{\phi}_1) & \text{if } B_1(a_1, \hat{\phi}_1) > B\left[a, q\left(a\right)\right]/2 \\ B\left[a, q\left(a\right)\right] - B_2(a_2, \hat{\phi}_2) & \text{otherwise.} \end{cases}$$

Proof. See Shaked and Sutton (1984). ■

When both outside options are small relative to the *"split-the-surplus"* solution, as in the first case, both agents prefer to continue bargaining than quitting. This would generally be the case when the surplus created

[7] The term outside option does not imply the party's payoff from engaging with a third party. It is the payoff available to the party outside the bargaining game.

by cooperation is large. In the second case agent 1 quits because she receives greater payoff in the status quo than if they split the surplus. In the third case, agent 2 prefers quitting. When agent 1's outside option is binding, agent 2's outside option cannot be binding. This contradicts with the assumption that cooperation generates greater surplus.

Note that, as opposed to GH's *"split-the-difference"* solution, the outside option has no effect on the bargaining outcome if it does not constitute a credible threat. Since the analysis of optimal allocation of ownership rights heavily depends on the outcome of the negotiation, the way the status quo payoff is incorporated into the model is critical.

Given the initial ownership structure and the ex-ante choice of (a_1, a_2), let $\Pi_i(a_1, a_2)$ denote the overall payoff to agent i obtained from the induced bargaining subgame. In the rest of the paper, the game will be analyzed from agent i's perspective, where j denotes the opponent. Let

$$H_i(a_1, a_2) = B[a, q(a)] - B_j(a_j, \hat{\phi}_j) \tag{5}$$

be the agent i's residual payoff after paying the agent j the value of her outside option, and

$$C_i(a_1, a_2) = B[a, q(a)]/2 \tag{6}$$

as agent i's share in the *"split-the-surplus"* solution. Then, using Lemma 1 we obtain agent i's reduced form payoff from the bargaining subgame as

$$\Pi_i(a_1, a_2) = \begin{cases} H_i(a_1, a_2) - v_i(a_i) \text{ if } j\text{'s o. o. is binding,} \\ C_i(a_1, a_2) - v_i(a_i) \text{ if neither o. o. are binding,} \\ B_i(a_i, \hat{\phi}_i) - v_i(a_i) \text{ if } i\text{'s o. o. is binding.} \end{cases} \tag{7}$$

There is a qualitative difference in the way the ex-ante investments affect the payoffs of the parties in this model compared to the GH model. In the GH model, the two agent receives the *"split-the-difference"* payoff in the equilibrium, and as the opponent's action changes agent i responds by maximizing the *"split-the-difference"* payoff. In this model, the opponent's investment first determines the payoff function that agent i is facing. Then it influences the value of this function. Note that the opponent's investment does not affect the value of agent i's outside option because the second period payoff $\hat{\phi}_i$ is independent of ex-ante investment choices of both agents. Fix an a_j, such that agent i's outside option gives her the highest payoff. As the opponent's investment is increased, agent i's response remains constant until the *"split-the-surplus"* payoff becomes equal the value of her outside option. At this point, agent i is indifferent between maximizing the value of her outside option and the *"split-the-surplus"* payoff. As the opponent's investment continues to increase it becomes more profitable for agent i to maximize the *"split-the-surplus"* payoff until the region where the opponent's outside option is binding is reached. From this point on, the agent responds by maximizing the residual payoff. It is worth to note that the status quo payoffs do not affect the agents' payoffs when neither of the firm's outside option is binding because both receive the *"split-the-surplus"* payoff. On the other hand, in a region where the opponent's outside option is binding, the status quo payoff both influences the level of payoff agent i receives and constrains the validity of the payoff function.

In finding the agents' response functions, first the three regions of interest in $\Pi_i(a_1, a_2)$ as a function of a is characterized. However, in doing that I assume that firms are symmetric in order to simplify the calculations.

Lemma 2 *If* $C_i(0,0) > B_i(0, \hat{\phi}_i)$ *and* $\max_{\phi_i} \partial B_i(a_i, \phi_i)/\partial a_i < 2 \min_{\phi_i} \partial B_i(a_i, \phi_i)/\partial a_i$ *for every* $a_i \in A_i$*, then there exists a monotonically increasing function* $\alpha_i : A_j \to A_i$ *such that*

i. *j's outside option is binding if* $a_i \leq \alpha_j^{-1}(a_j)$,

ii. *neither outside option is binding if* $\alpha_j^{-1}(a_j) < a_i \leq \alpha_i(a_j)$,

iii. *i's outside option is binding if* $\alpha_i(a_j) < a_i$.

Proof. See Appendix. ■

The first assumption of the Lemma 2 is automatically satisfied when the firms are symmetric. Otherwise, cooperation generates a smaller total surplus than non-cooperation. The second assumption requires that the marginal benefit from a_i does not change much with ϕ_i. In other words, the marginal private benefit of ex-ante investment must not be very sensitive to the second period payoff.

The α function divides the (a_1, a_2) plane into three regions. In the northwest corner, agent 1's outside option is binding, in the southeast corner, agent 2's outside option is binding and in the between region, neither agent's outside option is binding. On the $45°$ line both agents invest the same amount, $a_1 = a_2$. Given that they are symmetric, the non-cooperative choices of q's will be the same, so will be the value of the status quo payoffs. If agent 1's outside option is binding then agent 2's outside option has to be binding because of symmetry. Both outside option, however, cannot be binding at the same time. Therefore, on the $45°$ line neither outside option is binding. Now consider keeping a_2 at the same level as before but increasing a_1. Since $B_1(\cdot)$ is increasing in a_1, if a_1 is increased enough we reach to a point where agent 1's outside option is just binding. That's why the region where agent 1's outside option is binding should be on the northwest corner. The similar argument applies for agent 2; the region where her outside option is binding should be on the southeast corner. Thus, we have

Claim 3 $\alpha_i(a_j) > a_i$. *The area in which agent i's outside option is binding always lies above* $45°$ *line.*

4 Equilibria to the investment-choice game

The ex-ante investments a_1 and a_2 are chosen simultaneously and independently at date 0 taking into account the outcome of the negotiation between agents 1 and 2. A perfect subgame Nash equilibrium in date 0 investments is a pair $\left(a_1^N, a_2^N\right) \in A_1 \times A_2$ such that

$$\begin{array}{ll} \Pi_1\left(a_1^N, a_2^N\right) \geq \Pi_1\left(a_1, a_2^N\right) & \text{for all } a_1 \in A_1, \\ \Pi_1\left(a_1^N, a_2^N\right) \geq \Pi_1\left(a_1^N, a_2\right) & \text{for all } a_2 \in A_2. \end{array} \tag{8}$$

Define $\rho_i : A_j \rightarrow A_i$ to be agent i's response function, where

$$\rho_i(a_j) = \arg\max_{a_i \in A_i} \Pi_i(a_1, a_2). \tag{9}$$

Before deriving the agents' response functions, some further assumptions are introduced into the model.

Assumption 3: ϕ_i is increasing in q_j. q_1 and q_2 are complementary activities.

Assumption 4: The marginal benefit of a_i is increasing in second period payoff, ϕ_i.

Since $\Pi_i(a_1, a_2)$ depends on the region of choice space considered, it is convenient to separately analyze these regions, find the optimal action in each, and then determine the optimal action which maximizes the overall payoff.

Agent i's best response when her outside option is binding : Let the best response function of agent i in the region where her outside option is binding be defined as

$$\beta_i(a_j) = \max\{\hat{a}_i, \alpha_i(a_j)\}. \tag{10}$$

Here $\hat{a}_i$ is agent i's optimal investment choice when the initial contract is not renegotiated:

$$\hat{a}_i = \arg\max_{a_i \in A_i} \left\{ B_i(a_i, \hat{\phi}_i) - v_i(a_i) \right\}. \tag{11}$$

Let $\overline{a}_j \in A_j$ be defined as, $\alpha_i(\overline{a}_j) = \hat{a}_i$, i.e.: the level of ex-ante investment made by agent j so that agent i's outside option is just binding at its optimum. It is assumed that $\hat{a}_i > \alpha_i(0)$ so that there indeed exists an $\overline{a}_j > 0$. Now $\beta_i(a_j)$ is rewritten as

$$\beta_i(a_j) = \begin{cases} \hat{a}_i & \text{if} \quad a_j \le \overline{a}_j \\ \alpha_i(a_j) & \text{if} \quad a_j > \overline{a}_j. \end{cases} \tag{12}$$

Agent i's best response when neither outside option is binding : Let the best response function of agent i where neither agent's outside option is binding be defined as

$$\eta_i(a_j) = \underset{\alpha_j^{-1}(a_j) < a_i \le \alpha_i(a_j)}{\arg\max} \{C_i(a_1, a_2) - v_i(a_i)\}. \tag{13}$$

In this case, the agents share the total surplus, thus agent i maximizes half of the surplus net of cost of ex-ante investment. We define

$$\delta_i(a_j) = \arg\max_{a_i \in A_i} \{C_i(a_1, a_2) - v_i(a_i)\} \tag{14}$$

as the i's best response to the unconstrained maximization problem. By applying the implicit function theorem it can easily be shown that

Claim 4 *If $\partial q_i(a_i)/\partial a_j > 0$ for $i, j = 1, 2$, then $\delta_i(a_j)$ is increasing in a_j.*

Next, using the definition of $\delta_i(a_j)$ and the second assumption in Lemma 2, it can easily be shown that the best response to the "*split-the-surplus*" payoff is always smaller than the best response to the status quo payoff.

Claim 5 *$\delta_i(a_j) < \hat{a}_i$ for all a_j.*

Essentially, ex-ante investment, a_i, increases the value of the second period payoff, ϕ_i. When the agent receives the status quo payoff she obtains the full benefit of her actions so she has greater incentive to invest than when she receives half of the total surplus.

Next, the critical values of a_j within which $\delta_i(a_j)$ is the relevant response function is defined. Let $a'_j \in A_j$ such that $\delta_i(a'_j) = \alpha_i(a'_j)$, and $a''_j \in A_j$ such that $\delta_i(a''_j) = \alpha_j^{-1}(a''_j)$. That is, a'_j is the level of ex-ante investment of agent j where agent i's outside option is just binding when she maximizes the "*split-the-surplus*" payoff. It is assumed that $\delta_i(0) > \alpha_i(0)$ so that there exists a'_j. The critical value a''_j is the level of ex-ante investment of agent j at which her outside option is just binding when agent i maximizes the *"split-the-surplus"* payoff.

Finally, agent i's best response when neither outside option is binding is written as

$$\eta_i(a_j) = \begin{cases} \alpha_i(a_j) & \text{if} \quad a_j > a'_j, \\ \delta_i(a_j) & \text{if} \quad a'_j \leq a_j \leq a''_j, \\ \alpha_j^{-1}(a_j) & \text{if} \quad a''_j \leq a_j. \end{cases} \tag{15}$$

Agent i's best response when opponent's outside option is binding: Let the best response function of agent i in the region where agent j's outside option binds be defined as

$$\xi_i(a_j) = \arg\max_{\alpha_j^{-1}(a_j) \geq a_i} \{H_i(a_1, a_2) - v_i(a_i)\}. \tag{16}$$

Agent i's outside option does not bind whenever agent j's outside option binds. Agent i claims the residual and chooses a_i to maximize the total surplus net of the cost of ex-ante investment and the payment to the agent j. Let $\epsilon_i(a_j)$ be the maximizer of the unconstrained problem where

$$\epsilon_i(a_j) = \arg\max_{a_i \in A_i} H_i(a_1, a_2) - v_i(a_i). \tag{17}$$

By applying the implicit function theorem it can easily be shown that

Claim 6 *$\epsilon_i(a_j)$ is increasing in a_j.*

Let $\widetilde{a}_j \in A_j$ such that $\alpha_j^{-1}(\widetilde{a}_j) = \epsilon_i(\widetilde{a}_j)$. Here $\tilde{a}_j$ is the level of ex-ante investment of agent j at which her outside option just binds when agent i maximizes the "*split-the-surplus*" payoff. Hence, agent i's response function when agent j's outside option is binding can be written as

$$\xi_i(a_j) = \begin{cases} \alpha_j^{-1}(a_j) & \text{if} \quad a_j > \widetilde{a}_j, \\ \epsilon_i(a_j) & \text{if} \quad a_j < \widetilde{a}_j. \end{cases} \tag{18}$$

Agent i's best response function: Now we evaluate the payoff function $\Pi_i(a_1, a_2)$ at the optimum of each region and compare them to find the best response function of agent i. From claim 5, $\hat{a}_i > \delta_i(a_j)$ for all a_j. Since the function $C_i(a_i, a_j) - v_i(a_i)$ reaches its maximum at $\delta_i(a_j)$, it must be decreasing for all $a_i > \delta_i(a_j)$, so it is for $\hat{a}_i$. This implies that, when agent j invests at $\overline{a}_j$, the value of agent i's *"split-the-surplus"* payoff at its maximum, $C_i(\delta_i(\overline{a}_j), \overline{a}_j) - v_i(\delta_i(\overline{a}_j))$, is higher than the value of her outside option at its maximum, $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$. Since $C_i(\cdot) - v_i(\cdot)$ is increasing in a_j, there exists a level of ex-ante investment, say $\check{a}_j$, which is lower than $\overline{a}_j$ and at $\check{a}_j$ agent i is indifferent between choosing $\hat{a}_i$ or $\delta_i(a_j)$, i.e.: $C_i(\delta_i(\check{a}_j), \check{a}_j) - v_i(\delta_i(\check{a}_j)) = B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$.

We next need to locate $\check{a}_j$. Below it is shown that the jump in agent i's response function occurs at the region where her outside option is not binding.

Claim 7 $a_j' < \check{a}_j$.[8]

Claim 7 says that when agent j's investment is small, agent i can obtain a higher payoff in status quo than the *"split-the-surplus"* payoff by investing at high levels. However, as agent j's investment increases, *"split-the-surplus"* payoff increases because of the complementarity assumption and generates higher payoffs than the status quo payoff. Therefore, for small levels of a_j, agent i continues to choose $\hat{a}_i$ even though her outside option is not binding.

Whether $\check{a}_j$ is greater or smaller than a_j'' depends on the gains from cooperation. Recall a_j'' is the point where agent j's outside option is just

[8]Since $\hat{a}_i$ is greater than $\delta_i(a_j)$ for all a_j by claim 5, $\delta_i(a_j)$ can only be equal to $\alpha_i(a_j)$ when $B_i(a_i, \hat{\phi}_i) - v_i(a_i)$ is increasing. This means that $C_i(\delta_i(a_j'), a_j') - v_i(\delta_i(a_j'))$ is less than $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$. In order to increase the value of $C_i(\cdot) - v_i(\cdot)$ to be equal to $B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$, a_j has to increase. Thus, $a_j' < \check{a}_j$.

binding when agent i chooses $\delta_i(a_j)$. If $\check{a}_j$ is smaller than a_j'', then agent i responds by choosing along $\delta_i(a_j)$ for $a_j \in \left[\check{a}_j, a_j''\right]$. If $\check{a}_j$ is greater than a_j'' and agent i responds with $\delta_i(a_j)$, agent j's outside option becomes binding that implies that agent i does not receive the *"split-the-surplus"* payoff but claims the residual. In fact, she maximizes her payoff if she continues to choose $\hat{a}_i$ for values of $a_j < a_j^*$, where a_j^* is implicitly defined as $C_i(\alpha_j^{-1}(a_j^*), a_j^*) - v_i(\delta_i(a_j^*)) = B_i(\hat{a}_i, \hat{\phi}_i) - v_i(\hat{a}_i)$. For any value $a_j \geq a_j^*$, agent i responds by maximizing the residual, $H_i(\cdot) - v_i(\cdot)$.

Before agent i's response function is presented it is important to note that both $\check{a}_j$ and a_j^* are smaller than $\overline{a}_j$.

Claim 8 *$\check{a}_j < \overline{a}_j$ and $a_j^* < \overline{a}_j$.*

The above analysis is summarized in the following lemma that describes agent i's response function.

Lemma 9 *If $\check{a}_j < a_j''$, then agent i's response function is*

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if} \quad a_j \leq \check{a}_j, \\ \delta_i(a_j) & \text{if} \quad \check{a}_j < a_j \leq a_j'', \\ \alpha_j^{-1}(a_j) & \text{if} \quad a_j'' < a_j \leq \widetilde{a}_j, \\ \epsilon_i(a_j) & \text{if} \quad \widetilde{a}_j < a_j. \end{cases}$$

If $\check{a}_j \geq a_j''$ then agent i's response function is

$$\rho_i(a_j) = \begin{cases} \hat{a}_i & \text{if} \quad a_j \leq a_j^*, \\ \alpha_j^{-1}(a_j) & \text{if} \quad a_j^* < a_j \leq \widetilde{a}_j, \\ \epsilon_i(a_j) & \text{if} \quad \widetilde{a}_j < a_j. \end{cases}$$

Agent i can have two types of response function depending on whether or not she switches from maximizing the status quo payoff to maximizing the *"split-the-surplus"* payoff in the region where the opponent's outside option is binding. If $\check{a}_j < a_j''$, then the jump in the response function occurs in the region where agent j's outside option is not binding. For small a_j, agent i chooses $\hat{a}_i$. At $\check{a}_j$ there is a downward jump in the response function. From this point on, agent i chooses along $\delta_i(a_j)$ until a_j'' is reached. At a_j'', agent j's outside option becomes binding. Agent i responds by choosing along $\alpha_j^{-1}(a_j)$ so that agent j's outside option just binds. After $\widetilde{a}_j$ is reached, agent i responds by choosing $\epsilon_i(a_j)$.

If $\check{a}_j \geq a''_j$, that is, when the jump occurs in the region where agent j's outside option is binding, agent i chooses $\hat{a}_i$ for small a_j. For $a_j \in \left[a^*_j, \widetilde{a}_j\right]$, she responds along $\alpha_j^{-1}(a_j)$ so that agent j's outside option is just binding. For $a_j > \widetilde{a}_j$, she responds along $\epsilon_i(a_j)$. Whether $\check{a}_j$ is smaller or greater than a''_j depends on the B and v functions.

In general, the game will have either a unique pure strategy Nash equilibrium in which both agents maximize the *"split-the-surplus"* payoff (if $\check{a}_j < \delta_i(\check{a}_j)$) or no pure strategy equilibrium (if $\check{a}_j > \delta_i(\check{a}_j)$). The following proposition describes the equilibrium of the investment-choice game.

Proposition 10 *If there exists a Nash equilibrium to the investment-choice game in which neither agent's outside option is binding, then it is the unique equilibrium (in pure strategies).*

Proof. Let $\left(a_1^N, a_2^N\right)$ be the Nash equilibrium in which neither agent's outside option is binding. First I show that regardless of the existence of $\left(a_1^N, a_2^N\right)$, $(\delta_i(\hat{a}_j), \hat{a}_j)$ and $\left(\alpha_j^{-1}(\hat{a}_j), \hat{a}_j\right)$ cannot be equilibria. Since $\overline{a}_j > \check{a}_j$, it is also true by symmetry that $\overline{a}_i > \check{a}_i$. This implies that agent j switches to $\delta_j(a_i)$ at some investment level, a_i, which is below $\overline{a}_i$. Therefore, $\alpha_j^{-1}(a_j)$ never intersects the response function at $\hat{a}_j$. Moreover, $\delta_j(a_i)$ is part of the response function when it is above $\alpha_j^{-1}(a_j)$. Since $\hat{a}_j < \alpha_j^{-1}(\hat{a}_j)$, then $\hat{a}_j < \delta_i(\hat{a}_j)$. Thus, $(\delta_i(\hat{a}_j), \hat{a}_j)$ can never be an equilibrium, either. Given that $\left(a_1^N, a_2^N\right)$ is the Nash equilibrium of the game, it must be true that $\check{a}_i < \delta_i(\check{a}_j) < \delta_i(\hat{a}_j)$ since $\delta_i(a_j)$ is monotonically increasing in a_j. It is also true that $\delta_i(\hat{a}_j) < \epsilon_i(\hat{a}_j)$ which in turn implies that $\hat{a}_j < \epsilon_i(\hat{a}_j)$. Thus, $(\epsilon_i(\hat{a}_j), \hat{a}_j)$ cannot be an equilibrium. ■

The uniqueness of the Nash equilibrium depends on the positive slope of the $\delta_i(\cdot)$ function which arises from the complementarity assumption (Assumption 3). As a_j increases, there is a direct effect on C_i, but also an indirect effect since the second period payoff to both firms, ϕ_i, increases in response to the increase in a_j. The increase in ϕ_i, in return, causes a_i to increase.

Proposition 10 refers to the uniqueness, but not the existence of the pure strategy Nash equilibrium. The existence of the equilibrium will be studied in Section 6.

5 Integration (1's Ownership)

I consider only agent 1's ownership under integration since the case for agent 2's ownership is symmetric. Under agent 1's ownership, agent 1 owns both firms and agent 2 becomes her employee. In our model, this amounts to agent 1 choosing both q_1 and q_2 at date 1. It is, however, still necessary that both agents make the relationship-specific investment at date 0.

Besides having the right to choose both q_1 and q_2 at date 1, agent 1 is also the only agent in the bargaining game who can credibly use her outside option. The residual control rights give her the right to both choose and implement q_1 and q_2. Agent 2 can bribe agent 1 to choose his favorite q but he cannot quit the bargaining game and implement the status quo choices of q_1 and q_2.

At date 1, agent 1 chooses q_1 and q_2 to maximize $B_1\left[a_1, \phi_1\left(q_1, q_2\right)\right]$. It is assumed that there exists a unique equilibrium to the q-choice subgame under 1's ownership. Let

$$\left(\hat{q}_{11}, \hat{q}_{12}\right) = \arg \max_{\substack{q_1 \in Q_1 \\ q_2 \in Q_2}} \phi_1\left(q_1, q_2\right) \tag{19}$$

be the unique Nash equilibrium to this game. In general, the non-cooperative solution $\left(\hat{q}_{11}, \hat{q}_{12}\right)$ is ex-post inefficient.[9] Therefore, the two parties can gain from negotiating a new contract. The rest of the analysis is similar to the case of non-integration. The payoff function for agent 1 is given by

$$\Pi_1\left(a_1, a_2\right) = \begin{cases} C_1\left(a_1, a_2\right) - v_1\left(a_1\right) & \text{if neither o. o. is binding,} \\ B_1\left(a_1, \hat{\phi}_{11}\right) - v_1\left(a_1\right) & \text{if 1's o. o. is binding,} \end{cases} \tag{20}$$

and for agent 2 it is

$$\Pi_2\left(a_1, a_2\right) = \begin{cases} C_2\left(a_1, a_2\right) - v_2\left(a_2\right) & \text{if neither o. o. is binding,} \\ H_2\left(a_1, a_2\right) - v_2\left(a_2\right) & \text{if 1's o. o. is binding.} \end{cases} \tag{21}$$

The assumptions of Lemma 2 are sufficient to prove the existence of $\alpha_{11}\left(a_2\right)$ which divides the space of $\left(a_1, a_2\right)$ into two regions such that, for $a_1 > \alpha_{11}\left(a_2\right)$ agent 1's outside option is binding and $a_1 < \alpha_{11}\left(a_2\right)$ it is not binding. The following lemma describes the agents' response functions under agent 1's ownership.

[9] A variable with subscript ki denotes the choice of agent i under k's ownership.

Lemma 11 *Agent 1's response function is*

$$\rho_{11}(a_2) = \begin{cases} \hat{a}_{11} & \textit{if} \quad a_2 \le \check{a}_{12}, \\ \delta_{11}(a_2) & \textit{if} \quad \check{a}_{12} < a_2. \end{cases}$$

Agent 2's response function is

$$\rho_{12}(a_1) = \begin{cases} \delta_{12}(a_1) & \textit{if} \quad a_2 \le a''_{11}, \\ \alpha_{11}^{-1}(a_1) & \textit{if} \quad a''_{11} < a_2 \le \widetilde{a}_{11}, \\ \epsilon_{12}(a_1) & \textit{if} \quad \widetilde{a}_{11} < a_2. \end{cases}$$

Proof. See Proof of Lemma 9. ■

Under 1's ownership, both agents have a unique response function for any parameter values. This is because the agent 2's outside option is never binding. The jump in the response function always occurs at $\check{a}_{12}$.

As in the case of non-integration the game has either a unique pure strategy Nash equilibrium in which both agents maximize the *"split-the-surplus"* payoff or no equilibrium in pure strategies. The unique Nash equilibrium exists if $\check{a}_{12} < \delta_{11}(\check{a}_{12})$, that is, if the jump in agent 1's response function occurs to the left of 45° line. An argument similar to that used in the proof of proposition 10 shows that if there exists a Nash equilibrium to the investment-choice game in which neither agent's outside option is binding, then it is a unique equilibrium in pure strategies.

6 On the Existence of the Equilibrium

It is apparent that the divergence between the cooperative and non-cooperative equilibria is completely driven by the interdependency in the second period production. The extent to which total surplus can be increased through negotiation depends on the degree of complementarity. Let γ be an index of complementarity where $\gamma \in [0,1]$. When $\gamma = 0$ there is no production complementarity. In that case, the second period payoff function ϕ_i depends solely on q_i. As γ increases the degree of complementarity in the production of the two firms increases. The following assumptions are made:

Assumption 5: $\partial q_i(a)/\partial\gamma > 0$ for $i = 1, 2$. The cooperative choice of ex-post production increases as complementarity increases.

Assumption 6: $\partial B_i(a_i, \hat{\phi}_i)/\partial\gamma < \partial C_i(a_1, a_2)/\partial\gamma$, i.e., as the complementarity between the two firm's production increases the *"split-the-surplus"* payoff increases by more than the non-cooperative payoff.

Assume that $\gamma = 0$. Then the optimal cooperative and non-cooperative choices of q_i are the same and in both cases the value of the second period payoff, $\hat{\phi}_i$ and ϕ_i^c are identical. As a result, regardless of whether or not she cooperates, the payoff to agent i when she claims the residual is the same as the status quo payoff. Thus, $H_i(\cdot) = B_i(a_i, \hat{\phi}_i)$, and they are maximized at the same level of ex-ante investment, $\hat{a}_i = \epsilon_i > \delta_i$. Even though δ_i is independent of the opponent's ex-ante investment level, it is still lower than $\hat{a}_i$ since in the *"split-the-surplus"* solution the agent does not receive the full benefit of her actions. In this non-complementarity case, $\alpha_i(a_j) = \alpha_j^{-1}(a_j) = a_j$, which implies that agent i's outside option is binding in the area above $45°$ line while agent j's is binding below. In other words there is no region in which neither of the agents' outside option is binding. Below the equilibrium to this game is characterized.

Lemma 12 *If there is no complementarity between the two firms' production then* $(\hat{a}_1, \hat{a}_2)$ *is the unique equilibrium of the above game.*[10]

As complementarity is introduced, that is $\gamma > 0$, all the relevant functions and critical points in the response function change. By using the implicit function theorem it is easy to prove that $\hat{a}_i$, $\delta_i(a_j)$, $\alpha_i(a_j)$, and $\epsilon_i(a_j)$ increase as γ increases. This implies that there exists an area in which neither agents' outside option is binding.

For low levels of complementarity it is argued that there is no equilibrium in pure strategies. When γ is zero, a_i'' is smaller than $\check{a}_i$. Thus, for a small degree of complementarity a_i'' is still smaller than $\check{a}_i$ by continuity and the relevant response function is the second response function given in Lemma 9. With this response function, the only possible equilibrium is the one in which neither parties' outside option is binding.

[10] Note that $\overline{a}_i = \hat{a}_i$ by the fact that $\alpha_i(a_j)$ is the $45°$ line and by the definition of $\overline{a}_i$. In claim 8 we have shown that $\check{a}_j < \overline{a}_j$. Thus it follows that, $\check{a}_j < \hat{a}_j$. To show that $\check{a}_i > \delta_i$, consider the opposite, that is $\check{a}_i \leq \delta_i$. Then since $\alpha_i(a_i) = a_i$ in the case of no complementarity, it follows that $B_i(\check{a}_i, \hat{\phi}_i) - v_i(\check{a}_i) = C_i(\check{a}_i, \check{a}_j) - v_1(\check{a}_i)$. By the single crossing property in lemma 2, $C_i(a_i, \check{a}_j) - v_1(a_i) \leq B_i(a_i, \hat{\phi}_i) - v_1(a_i)$ for all $a_i \geq \check{a}_i$. Hence $C_i(\delta_i, \check{a}_j) - v_1(\delta_i) \leq B_i(\delta_i, \widehat{\phi}_i) - v_1(\delta_i) < B_i(\hat{a}_i, \widehat{\phi}_i) - v_i(\hat{a}_i)$, which contradicts with the definition of $\check{a}_i$. Therefore it must be true that $\check{a}_i > \delta_i$. Given that $\check{a}_j$ is in the region where the opponent's outside option is binding, the jump must occur at a_j^*. a_j^* is equal to $\widehat{a}_j$ because of the fact that $H_i(\cdot) = B_i(a_i, \widehat{\phi}_i)$. The best response of agent i is to always play $\widehat{a}_i$. In fact the jump in the response function is fictitious. Because of symmetry $\widehat{a}_i$ intersects $45°$ line at $\widehat{a}_j$, so we have an equilibrium.

Proposition 13 *For low levels of complementarity, there is no equilibrium to the investment-choice game in pure strategies. If the complementarity between the two firm is sufficiently large, then there is a unique Nash equilibrium.*

Proof. In the case where there is no complementarity, $\gamma = 0$, the equilibrium to this game is $(\hat{a}_1, \hat{a}_2)$. Now suppose that the agents are forced to receive the *"split-the-surplus"* payoff. The unique equilibrium of this forced game is (δ_1, δ_2). Let $\mathcal{A}_i$ be the payoff to agent i in this forced equilibrium and $\mathcal{B}_i$ be the payoff to agent i from deviating to a point which enforces outside option. $\mathcal{B}_i$ is greater than $\mathcal{A}_i$ since $B_i(\cdot)$ is increasing in a_i and $\hat{a}_i > \delta_i$. When the complementarity is small, an interior equilibrium, if it exists, has to be close to the equilibrium of the forced division game when there is no complementarity.[11] Let $\mathcal{C}_i$ denote the payoff to agent i in an equilibrium where both agents receive the *"split-the-surplus"* payoff when $\gamma > 0$. Finally let $\mathcal{D}_i$ denote the payoff to agent i from deviating to a point which enforces outside option. We know that $\mathcal{B}_i$ is greater than $\mathcal{A}_i$. $\mathcal{A}_i$ is close to $\mathcal{C}_i$ and $\mathcal{B}_i$ is close to $\mathcal{D}_i$ which implies that $\mathcal{D}_i$ is greater than $\mathcal{C}_i$. This implies that agent i has an incentive to deviate from the (δ_1, δ_2) equilibrium when there is small a complementarity. Therefore (δ_1, δ_2) cannot be an equilibrium. There also cannot be an equilibrium where both agents' outside option are binding. Therefore there is no equilibrium when the firms' production exhibits small complementarity. The second part of the lemma is proved in Proposition 10. ■

The intuition behind proposition 13 is the following. When the agent receives the *"split-the-surplus"* payoff, her incentives are distorted downwards. If we keep the opponent's action fixed, it is profitable for the agent to deviate and choose $\hat{a}_i$ to maximize the status quo payoff. This is true for both agents because of symmetry. There cannot, however, be an equilibrium where both agents' outside options are binding. In fact, the only case when there is an equilibrium where both outside option are binding is when there is no complementarity between the two firms. As complementarity increases, the agents' outside options become non-binding, so the deviations described above do not occur. Then the game has the unique equilibrium where neither of the agents' outside options are binding.

[11] It follows from that the response function has a closed graph.

7 Asset Ownership and Incentives to Invest

When firms are symmetric, the equilibrium to the investment-choice game, provided that it exists, is identical under the two ownership structure. Since the same equilibrium is obtained regardless of the initial distribution of the ownership rights, asset ownership does not affect the incentives to invest. Investments are inefficient, however, this inefficiency cannot be remedied by reallocating the ownership rights.

Now I consider cases where firms are asymmetric. As a benchmark, consider a relationship where ϕ_i only depends on q_i for $i = 1, 2$, which corresponds to the case of $\gamma = 0$ in Section 6. Under non-integration, there is no room for negotiation since the non-cooperative choice of q is identical to cooperative choice. Hence, ex-ante investments are efficient. Under 1's ownership $\hat{q}_1 = q_1^c$ while $\hat{q}_2 \neq q_2^c$. Then, 1's outside option is binding whenever $B_2(a_2, \phi_2^c) > B_1(a_1, \phi_1^c)$.[12] Note that 2's outside option is never binding since she cannot credibly exercise it. Depending on the relative magnitudes of ϕ_2^c and ϕ_1^c, and the marginal productivity of investment of both agents, we can have different kinds of equilibria, including the efficient one.

Next consider an asymmetric case where ϕ_1 depends only on q_1 and ϕ_2 depends on both q_1 and q_2. The cooperative choices of q will differ from non-cooperative choices in all ownership structures, hence there will be room for negotiation. The optimal allocation of ownership is the one that results with less distortion in ex-ante investments. Recall that if an agent becomes a residual claimant in an equilibrium then her investment will be efficient. Hence the ownership should be allocated to make the agent with higher marginal productivity of investment residual claimant in order to minimize the distortion in ex-ante investments. Ownership increases one's bargaining power by giving her residual control rights which in turn allows her to credibly exercise its outside option. But it also makes the level of outside option larger hence makes it more likely to bind. Therefore, similar to the conclusions of DLC, disowning an asset may increase the incentives to invest in a relationship in this model as well.

The distinct roles played by production complementarity $(\partial\phi_i/\partial q_j)$ and marginal productivity of ex-ante investment $(\partial^2 B_i/\partial a_i \partial\phi_i)$ in determining the optimal ownership structure should be stressed. Both

[12] It follows from the fact that $\hat{\phi}_i = \phi_i^c$.

the asset ownership, through its incentive effects, and the production complementarity determine the level of quasi rents in a relationship and whether outside option is binding or not.[13] If production complementarity is not significant then the initial distribution of asset ownership does not change the outcome of ex-post bargaining significantly. However, if production complementarity is important, then ex-post negotiation may improve the surplus created. It is then the initial allocation of ownership that determines the magnitude of the improvement and whether outside option are binding or not. If the degree of complementarity is high and mutual then we have the Coasean result that predicts the irrelevance of the ownership structure. If the degree of complementarity is high but asymmetric then there may be cases in which asset ownership may in fact distort incentives to invest. Hence it is optimal to take away assets from the agent who is significant in the relationship (in the sense of having high marginal productivity).

8 Concluding Remarks

This paper analyzes the role of the initial allocation of ownership rights in transactions where parties make relationship-specific investments and contracts are incomplete. Two ownership structures are compared; non-integration and 1's ownership. In both cases, when firms are symmetric and the degree of complementarity between the two firms' production is high, cooperation generates large surplus. In these cases, the investment-choice game has a unique Nash equilibrium where neither agents' outside option is binding. Since the same equilibrium is obtained regardless of the ownership structure, the distortions in the ex-ante investments are independent of the initial allocation of ownership rights. If, however, the degree of complementarity between the two firms' production is low, then the equilibrium in pure strategies does not exist.

When complementarity between firms is asymmetric, it is found that, as in DLC, there may be cases where taking away the assets from significant partner may boosts her incentives to invest. By removing an asset,

[13] This observation blurs the sharp contrast between property rights theory and transaction cost theory which was sussinctly put forward by Whinston (2001). He states that the likelihood of integration in property rights theory depends on marginal returns to investment while in transaction cost theory it is levels of quasi rents that matter for integration decisions. In my model the likelihood of integration not only depends on marginal returns to investment but also levels of quasi-rents.

bargaining power of the significant player is reduced while the bargaining power of the insignificant one is increased, hence the significant player becomes residual claimant and her ex-ante investment is less distorted.

The main conclusion of the paper, which states that the allocation of initial ownership rights is irrelevant, partially extends the Coase theorem to the relationships in which agents are unable to bargain ex-ante over all aspects of the transaction, due to contractual incompleteness. This irrelevance result also contrasts the findings of earlier studies in property rights theory where the initial allocation ownership rights have efficiency implications. Sustaining cooperation in highly dependent vertical structures without resorting vertical integration is empirically observed.[14] Moreover, this finding can guide privatization policies towards unbundling the vertical structures such as vertical components of public utilities where hold up risk can be significant.

An implicit assumption in this model is regarding the definition of ownership. Following GH, ownership is defined as the power to exercise control. It would be interesting to examine whether the irrelevance result continue to hold if the definition of ownership is broaden to include the rights to the residual income stream. Another assumption in the model is that the relationship lasts only two periods. If, however, the relationship lasts longer and the bargaining takes place concurrently with the production, results may differ. In this case, the status quo payoffs become the income flow accruing to the agents in the course of the bargaining. Then status quo payoffs can be interpreted as the disagreement points. This bargaining game, however, may have many equilibria, some of which are inefficient (see Fernandez and Glazer (1991)).

9 Appendix

Proof of Lemma 2: The lemma is proven for the case of $i = 1$, and it is symmetric for the case $j = 1$. Let $M^* = \max_{\phi_1} \partial B_1(a_1, \phi_1)/\partial a_1$ and $m^* = \min_{\phi_1} \partial B_1(a_1, \phi_1)/\partial a_1$. Then by definition $\partial B_1(a_1, \phi_1^c)/\partial a_1 \leq M^*$ and $\partial B_1(a_1, \hat{\phi}_1)/\partial a_1 \geq m^*$. We have assumed that $\frac{1}{2}M^* < m^*$. One can find a sufficiently small $\delta_1(a_2)$ for each $\phi_1^c(a_1, a_2)$ such that

[14]For instance, Nucor as the producer and David J. Joseph Company as the supplier of scrap as in Holmstrom and Roberts (1998).

$\frac{1}{2}M^* + \delta_1(a_2) < m^*$. Then by substitution we obtain

$$\frac{1}{2}\partial B_1(a_1, \phi_1^c)/\partial a_1 + \delta_1(a_2) \leq \partial B_1(a_1, \hat{\phi}_1)/\partial a_1 \tag{22}$$

that is for every a_2, there exist a $\delta_1(a_2)$ and a unique a_1, such that, where ϕ_1^c is the value of function ϕ evaluated at the cooperative choices.

Next define $D(a_1, a_2) = C_1(a_1, a_2) - B_1(a_1, \hat{\phi}_1)$. By 22, $D(a_1, a_2)$ is a monotonically decreasing function of a_1 and a_2 and $D(0, a_2) > 0$. Rewriting $B_1(a_1, \hat{\phi}_1)$ as $B_1(a_1, \hat{\phi}_1) = B_1(0, \hat{\phi}_1) + \int_0^{a_1}(\partial B_1(a_1, \hat{\phi}_1)/\partial a_1)da_1$ and substituting 22, we obtain $B_1(a_1, \hat{\phi}_1) \geq B_1(0, \hat{\phi}_1) + \int_0^{a_1}(\partial C_1(a_1, a_2)/\partial a_1 + \delta_i(a_j))da_1$. This can be rewritten as $B_1(a_1, \hat{\phi}_1) \geq B_1(0, \hat{\phi}_1) - C_1(0, a_2) + C_1(a_1, a_2) + \delta_i(a_j)a_1$. If $B_1(0, \hat{\phi}_1) - C_1(0, a_2) + \delta_i(a_j)a_1 > 0$ then $B_1(a_1, \hat{\phi}_1) > C_1(a_1, a_2)$. Thus there exists an $a_1^H > (B_1(0, \hat{\phi}_1) - C_1(0, a_2))/\delta$, such that, $D(a_1^H, a_2) < 0$. Using the intermediate value theorem, there exists a point $a_i^* \in [o, a_1^H]$ such that $D(a_1^*, a_2) = 0$. It is unique since $D(a_1, a_2)$ is monotonically decreasing for all $a_i \in A_i$.

Having shown the existence of a unique a_i^* for all a_2, define a function $\alpha_1 : A_2 \to A_1$ such that

$$B_1(\alpha_1(a_2), \hat{\phi}_1) = C_1(\alpha_1(a_2), a_2) \tag{23}$$

By condition 22, $\partial C_1(a_1, a_2)/\partial a_1 - \partial B_1(a_1, \hat{\phi}_1)/\partial a_1 \neq 0$, hence the implicit function theorem can be applied. Differentiating both sides of 23 with respect to a_2 we obtain $\partial \alpha_1(a_2)/\partial a_2 > 0$.

The existence of $\alpha_2(a_1)$ can be shown in a similar manner. Since it is a monotonic function, its inverse, $\overset{-1}{\alpha_2}(a_2)$, is a well defined function. By definition, $D(\alpha_1(a_2), a_2) > 0$ if $a_1 > \alpha_1(a_2)$, hence agent 1 maximizes $B_1(a_1, \hat{\phi}_1) - v_1(a_1)$. For $a_1 < \alpha_1(a_2)$, agent 1's outside option is not binding and for $a_1 > \overset{-1}{\alpha_2}(a_2)$, agent 2's outside option is also not binding. Thus, agent 1 receives $C_1(a_1, a_2) - v_1(a_1)$. For $a_1 \leq \overset{-1}{\alpha_2}(a_2)$, agent 2's outside option binds, therefore agent 1 claims the residual and receives $H_1(a_1, a_2) - v_1(a_1)$.

10 References

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