Sequential Spatial Competition in Vertically Related industries with Different Product Varieties

Beladi, Hamid and Chakrabarti, Avik and Marjit, Sugata

Department of Economics, College of Business, University of Texas at San Antonio, One UTSA Circle, San Antonio, Texas 78249-0633., Department of Economics, 816 Bolton Hall, College of Letters and Science, University of Wisconsin-Milwaukee, P.O. Box 413, Wisconsin 53201., Centre for Studies in Social Sciences, Calcutta 700-094, West Bengal, India.

2009

Online at https://mpra.ub.uni-muenchen.de/19278/
MPRA Paper No. 19278, posted 13 Dec 2009 13:21 UTC
Sequential Spatial Competition in Vertically Related Industries with Different Product Varieties

Hamid Beladi a
Avik Chakrabarti b
and
Sugata Marjit c

We demonstrate the sensitivity of the location of downstream firms, engaged in sequential spatial competition, to the vertical structure of an industry where no downstream firm can produce all varieties demanded.

JEL Classification Code: L13, L42, D43, R32
Keywords: Product-differentiation, Price-discrimination, Spatial-competition, Firm-location, Merger.

2009

a Corresponding author; Professor, Department of Economics, College of Business, University of Texas at San Antonio, One UTSA Circle, San Antonio, Texas 78249-0633.
Tel: (210) 458-7038; Fax: (210) 458-7040; Email: hamid.beladi@utsa.edu
b Associate Professor, Department of Economics, 816 Bolton Hall, College of Letters and Science, University of Wisconsin-Milwaukee, P.O. Box 413, Wisconsin 53201.
Tel.: (414)229-4680; Fax: (414)229-3860; E-mail: chakra@uwm.edu
c Professor, Centre for Studies in Social Sciences, Calcutta 700-094, West Bengal, India.
Tel.: 852-2788-7745; Fax: 852-2788-8806; Email: smarjit@hotmail.com
1. Introduction

In this paper, we extend Braid (2008) to capture the sensitivity of equilibrium locations of downstream firms, selling different varieties of a product, to the vertical structure of an industry when spatial moves are sequential. Braid (2008) showed that the equilibrium locations of two firms are partially centralized to the socially optimal extent if there is spatial price discrimination, and if each firm has two out of three products, or else one variety of a differentiated product with some consumers indifferent between varieties. In what follows, we demonstrate the effect of sequential moves by downstream firms engaged in spatial competition in a vertically related industry where no firm can produce all the varieties that consumers demand.

2. Model and Propositions

Consider a vertically related industry with one upstream firm ($M$) producing an intermediate good and selling this good to 2 downstream firms ($R_i; i=1,2$) who transform (on a one-to-one basis) the intermediate good into differentiated final goods and sell to consumers distributed uniformly with unit density on a uni-dimensional (linear) market interval with support [0, 1]. The downstream firms $R_1$ and $R_2$ are located at $x$ and $y$, respectively, on this interval. $R_1$ sells products $U$ and $W$, and $R_2$ sells products $V$ and $W$: a fraction $c$ of consumers want to buy product $U$; a fraction $c$ of consumers want to buy product $V$; and a fraction $b$ of consumers want to buy product $W$. Transportation costs are $td$, where $t$ is a constant and $d$ is the distance shipped. There is a maximum reservation price ($k$) consumers are willing to pay which is sufficiently large so that it comes into play only for product varieties where there is no competition between the two firms. The game is played with perfect monitoring i.e. all past actions become
common knowledge at the end of each stage. The following figure summarizes the sequence in which the game is played.

<table>
<thead>
<tr>
<th>M offers contract</th>
<th>R₁ chooses location</th>
<th>R₂ chooses location</th>
<th>Spatial Price Discrimination</th>
<th>Final Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

Figure 1. Sequence of Events

But for the sequential choice of location by $R₁$ and $R₂$, the structure of the game is identical to that in Beladi et al. (2008). As usual, the solutions are obtained by backward induction. For expositional convenience, let $R = \frac{b}{4(b + c)}$ and $\Delta = (5b^2 + 16c^2 + 20bc)$.

Absent the possibility of a merger, $M$ charges a uniform wholesale price $w_j$ and a fixed fee $F_j$ that extracts all of the profits from each downstream firm.

The profits of $R₂$, from selling varieties V and W, are

(1) $\Pi₂ = \left( c(k - w_2) - \frac{ct}{2} \left[ y^2 + (1 - y)^2 \right] \right) + \int_0^1 b\left[ t(x + y - 2z) - w_2 \right] dz + \int_0^1 b\left[ t(x - y) - w_2 \right] dz - F_2$

The first order condition for profit maximization yields $R₂$’s reaction function

(2) $y(x) = \frac{1}{2} \left( \frac{4(b + c) - bw_2 + 2bx}{3b + 4c} \right)$

The profits of $R₁$, from selling varieties U and W, are

(3) $\Pi₁ = \left( c(k - w₁) - \frac{ct}{2} \left[ x^2 + (1 - x)^2 \right] \right) + \int_0^1 b\left[ t(y(x) - x) - w₁ \right] dz + \int_0^1 b\left[ t(x + y(x) - 2z) - w₁ \right] dz - F₁$
Solving the first order conditions for $R_1$’s profit-maximization and plugging this solution into (2), we obtain

$$x = \left(\frac{1}{2} - R\right) - \frac{b}{4t(b + c)\Delta} \left[2(b + c)(btw_2 + w_1(6b + 8c)) - b^2 t(3b + 4c)\right]$$

$$y = \left(\frac{1}{2} + R\right) + \frac{b}{4t(b + c)\Delta} \left[b^2 t - 4(b + c)(tw_2(b + 2c) + w_1b)\right]$$

This leads to our first proposition.

**Proposition I.** Absent the possibility of any merger, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically related industry, are

$$\left[\left(\frac{1}{2} - R\right) - \frac{b}{4t(b + c)\Delta} \left(2(b + c)(btw_2 + w_1(6b + 8c)) - b^2 t(3b + 4c)\right)\right] \text{ for the leader and}$$

$$\left[\left(\frac{1}{2} + R\right) + \frac{b}{4t(b + c)\Delta} \left(b^2 t - 4(b + c)(tw_2(b + 2c) + w_1b)\right)\right] \text{ for the follower.}$$

If there is a merger between $M$ and $R_1$, $M$ sets $(w_1, F_1) = (0,0)$ and charges a wholesale price $w_2 > 0$ and a fixed fee $F_2$ that extracts all of the profits from $R_2$. The profits of $R_1$, from selling varieties $U$ and $W$, are

$$\Pi_1 = \left(ck - \frac{ct}{2}\right)\left[x^2 + (1 - x)^2\right] + \int_0^x b[t(y - x)]dz + \int_x^{xy} \frac{b[z]}{x}dz$$

Solving the first order conditions for $R_1$’s profit-maximization and plugging this solution into (2), we obtain

$$x = \left(\frac{1}{2} - R\right) - \frac{b^2}{4(b + c)\Delta} \left[2(b + c)w_2 - (3b + 4c)\right]$$

$$y = \left(\frac{1}{2} + R\right) + \frac{b}{4(b + c)\Delta} \left[b^2 - 4(b + c)(b + 2c)w_2\right]$$
This leads to our next two propositions.

**Proposition 2.** If the downstream leader merges upstream, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically related industry, are

\[
\left( \frac{1}{2} - R \right) - \frac{b^2}{4(b + c)\Delta} \left( 2(b + c)w_2 - (3b + 4c) \right) \]

for the integrated firm and

\[
\left[ \frac{1}{2} + R \right] + \frac{b}{4(b + c)\Delta} \left( b^2 - 4(b + c)(b + 2c)w_2 \right) \]

for the un-integrated firm.

**Proposition 3.** A merger between the downstream leader and the upstream monopolist induces the integrated firm to move further away from the socially optimal location by

\[
\left( \frac{3b + 4c}{\Delta t} \right)w_i
\]

and the un-integrated firm to move further away from the socially optimal location by

\[
\left( \frac{b^2}{\Delta t} \right)w_i
\].

If there is a merger between \( M \) and \( R_2 \), \( M \) sets \((w_2, F_2) = (0,0)\) and charges a wholesale price \( w_i > 0 \) and a fixed fee \( F_1 \) that extracts all of the profits from \( R_1 \). The profits of \( R_2 \), from selling varieties \( V \) and \( W \), are
The first order condition for profit maximization yields $R_2$’s reaction function

$$y(x) = \frac{2(b + c) + bx}{3b + 4c}$$

Solving the first order conditions for maximizing $R_1$’s profit (3) and plugging this solution into (10), we obtain

$$x = \left(\frac{1}{2} - R\right) - \frac{b(3b + 4c)}{4(b + c)\Delta t}\left[4(b + c)w_1 - bt\right]$$

$$y = \left(\frac{1}{2} + R\right) + \frac{b^2}{4(b + c)\Delta t}\left[b^2 - 4(b + c)w_1\right]$$

This leads to our final propositions.

**Proposition 4.** If the downstream follower merges upstream, the Nash equilibrium locations of the two downstream firms, engaged in spatial competition in a vertically
related industry, are \[
\left[ \frac{1}{2} - R \right] - \frac{b(3b + 4c)}{4(b + c)\Delta t}(4(b + c)w_1 - bt) \] for the un-integrated firm
and \[
\left[ \frac{1}{2} + R \right] + \frac{b^2}{4(b + c)\Delta t}(b^2 - 4(b + c)w_1) \] for the integrated firm.

Proposition 5. A merger between the downstream follower and the upstream monopolist induces the integrated firm to move further away from the socially optimal location by \[
\left( \frac{(b + 2c)bw_2}{(5b^2 + 16c^2 + 20bc)} \right) \] and the un-integrated firm to move further away from the socially optimal location by \[
\left( \frac{b^2w_2}{2(5b^2 + 16c^2 + 20bc)} \right).
\]

It is worth noting (ref. figures 2 and 3) that when the downstream firms are allowed to move sequentially, in contrast\(^3\) with Beladi et al. (2008), the merged firm (notwithstanding the order of its move) locates farther away from the social optimum than does the firm outside the merger. Also, notwithstanding the vertical structure of the industry, the equilibrium locations of the sequentially moving downstream firms no longer gravitate toward Braid’s (2008) partially social optimum \[\frac{1}{2} \pm R\] even if the upstream monopoly is broken up.

3. Conclusion

In this paper we have taken a step forward along the path of efforts that continue to capture the implications of the vertical structure\(^4\) of an industry for the location of downstream firms that move sequentially. We have shown that, with sequentially moving downstream firms that can not produce all the varieties that consumers demand, the
merged firm (notwithstanding the order of its move) will locate farther away from the social optimum than will the firm outside the merger.\textsuperscript{5} A couple of interesting extensions, we are currently working on, include a) allowing incomplete information when each downstream firm’s unit cost is unknown\textsuperscript{6} to its rival and b) endogenizing the sequence of moves by adding a prior stage where each firm chooses when to pick its location.

References


Endnotes

\textsuperscript{1} If firms cannot price discriminate at each location between the different types of consumers who find its own variety desirable, then it might be possible to assume mixed price strategies, but unlike Dasgupta and Maskin (1986), who have a single mixed-strategy Nash equilibrium in mill prices for any given set of firm locations, there would be a different mixed-strategy Nash equilibrium in delivered prices at each consumer location for any given set of firm locations. We maintain that there is spatial price discrimination for product W of the sort first examined in Hoover (1937) and Lerner and Singer (1937), in which there is a Nash equilibrium in delivered price schedules.

\textsuperscript{2} Beladi \textit{et al.} (2008) showed that in a vertically related industry, with an upstream monopoly, the downstream firms will not choose the socially optimal location. A vertical merger will exacerbate the distance of the downstream firms from the socially optimal location: the firm outside the merger will deviate more than the firm that is part of the merger when the firms move simultaneously.

\textsuperscript{3} Analogous to Beladi \textit{et al.} (2008), notwithstanding the vertical structure of the market, the downstream firms gravitate toward the social optimum if \( b = 0 \) (i.e. when, in the absence of any demand for W, the downstream firms are reduced to spatial-price-discriminating monopolists choosing uniform delivered prices). This replicates the equilibrium one would expect, à la Hotelling (1929), in the market for a homogeneous product where the (mill) prices of the two firms are exogenous and equal, and consumers pay travel costs.

\textsuperscript{4} See Lafontaine and Slade (2007) for an insightful review of the contributions relevant to the related literature.

\textsuperscript{5} Our results are likely to have important implications for a firm’s inter-temporal choice of entry mode. See, for instance, Haller (2009), Lahiri (2009), Kurata \textit{et al.} (2009) and Raff \textit{et al.} (2009).

\textsuperscript{6} See Arozamena and Weinschelbaum (2009) for an elegant model that compares across equilibria resulting from sequential and simultaneous moves when firms’ unit costs are private information.