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# **Optimal Monetary Policy and Expectation Driven Business Cycles**

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## **Abstract**

We explore the optimal response of central bank when a news shock hits the economy, that is, agents' optimistic expectation of an improvement in technology does not realize. Ramsey optimal policy and simple policy rules are studied in a two-sector model with price rigidities in each of non-durable and durable sector. We find that a simple policy rule reacting to the inflation rates in both non-durable and durable sector with appropriate weights can mimic the performance of the Ramsey policy closely. Another interesting result is that monetary policy plays an important role in generating expectation driven business cycles.

## **1. Introduction**

The objectives of this paper are: (1) to study the effect of monetary policy on the expectation driven business cycles (Pigou cycles); (2) to find the Ramsey optimal monetary policy for the economy in our model hit by a news shock and compare this Ramsey optimal policy with several simple monetary policy rules. We are particularly interested in studying if central bank can mimic the Ramsey optimal policy by targeting only several macroeconomic variables.

Beaudry and Portier (2004) formalized Pigou (1926)'s idea and defined Pigou cycles as: (i) agents receive signals or news indicating that technology will improve in near future. An optimistic forecast of future technological improvement leads to a boom defined as an increase in aggregate output, employment, investment and consumption, and (ii) the realization that a forecast is too optimistic leads to a recession defined as a fall in all the same aggregate variables. The economy is said to be hit by a news shock. They also illustrate that standard one-sector and two-sector equilibrium models used in the macroeconomic literature can not produce Pigou cycles. Of course, their largest contribution is to find a particular multi-sector model in which Pigou cycles can arise. Their finding is that expectation driven business cycle can arise in neo-classical models when one allows for a sufficiently rich description of inter-sector production technology. In particular, the key assumption giving rise to the Pigou cycle is that non-durable goods and durable goods exhibit enough complementarities in the production of the final goods.

Beaudry and Portier's work arouses researchers' interest in finding the possibility of

generating expectation driven business cycles in one sector models. Christiano, Motto and Rostagno (2006) and Jaimovich and Rebelo (2006) succeed in generating booms and busts of consumptions, investments and outputs as defined in Pigou's cycles by adding investment adjustment cost, variable utilization of capital, habit persistence in preference into a standard one sector model. However, it is not that straightforward to get a correct booms and busts of asset prices in their frameworks. The asset prices unexpectedly slump during the booms when all the other variables rise as expected. To solve this problem, Christiano, Motto and Rostagno (2006) extend their model by adding sticky prices, sticky wages and standard Taylor-rule monetary policies. They argued that monetary policy plays an important role in generating a boom-bust cycle in asset prices.

In this paper, we formulate a dynamic general equilibrium model with two sectors that produce durable and non-durable goods respectively. The model incorporates nominal price rigidity in each sector. We study the expectation driven business cycles under Ramsey optimal policy and simple policy rules. The comparison of impulse responses indicates that Ramsey optimal policy can be approximated by a simple policy rule targeting inflation rates in both sectors with appropriate weights, and the weights are determined by the degree of nominal rigidities, depreciation rate of durable goods, and relative shares of durable goods output to non-durable goods output. We also conduct sensitivity analysis to show that this result is robust to various complementarities between non-durable goods and durable goods. Another interesting result is that central bank does not need to detect whether a boom is caused by a real technology improvement or just an

expectation of improvement in technology. It is not necessary for central bank to detect whether an expectation will realize or not either. A simple policy rule targeting inflation in both sectors with appropriate weights can mimic the Ramsey policy rule closely under all circumstances.

This structure also allows us to study the following problem: is complementarities between non-durable goods and durable goods still a necessary condition to generate expectation driven business cycles when monetary policy exists in the model? Our finding indicates that monetary policy plays an important role in generating Pigou's cycles. In particular, a weak inflation targeting policy rule helps generate Pigou's cycles without assuming complementarities between non-durable goods and durable goods.

The framework in this paper is closely related to the recent development of two sector models with nominal rigidities. Aoki (2001) studies optimal monetary policy responses to relative-price changes in a two-sector framework with a flexible-price sector and a sticky-price sector. Benigno (2004) evaluate monetary policy in a currency area where price rigidities may differ between countries. Barsky, House and Kimball (2004) and Erceg and Levin (2006) introduce durable goods into otherwise conventional sticky price models. They highlight the distinction between non-durable and durable sector in that the durable goods sector is much more interest-sensitive than the non-durable sector. Monacelli (2006a,b) introduces collateral constraints into a two-sector model with non-durable and durable goods to study the comovements in these two sectors in response to monetary policy shocks and optimal monetary issues.

The remainder of this paper is organized as follows: section 2 outlines the dynamic general equilibrium model; section 3 describes the parameter calibration and solution methods; section 4 defines the Ramsey optimal monetary policy and compares the impulse responses under Ramsey policy with the impulse responses under simple monetary policy rules; section 5 conducts some sensitivity analysis; section 6 concludes.

## **2. The Model:**

The economy is composed of two sectors: a non-durable goods sector and a durable goods sector. There are two types of firms in each sector: final goods firms produce final goods using intermediate goods; intermediate goods firms are monopolistic competitors that each produces a differentiated product using labor. These intermediate goods firms determine their prices following a Calvo-type staggered price adjustment. Households supply labor to both sectors and derive utility from consumption of non-durable final goods and services of durable final goods. The central bank conducts monetary policy.

### **Households**

Household derives its utility from the consumption of a combination of non-durable goods and durable goods. Following Beaudry and Portier (2004) and Monacelli (2006a,b), this combination index  $X_t$  is defined as a CES composite of non-durable goods  $C_t$  and durable goods  $D_t$ .

$$X_t = \left[ (1-\alpha)^{\frac{1}{\eta}} (C_t)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (D_t)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1)$$

where  $\alpha$  is the share of durable goods in the composite consumption index.  $\eta > 0$  is the elasticity of substitution between non-durable goods and durable goods. In the case  $\eta \rightarrow 0$ , non-durable goods and durable goods are perfect complements; whereas if  $\eta \rightarrow \infty$ , the two goods are perfect substitutes.

The household maximizes the following expected utility

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \right\} \quad (2)$$

$$\text{Where } U(X_t, N_t) = \log(X_t) + v \log(1 - N_t)$$

Subject to the budget constraint (in nominal terms):

$$P_{c,t} C_t + P_{d,t} (D_t - (1-\delta)D_{t-1}) + B_t = R_{t-1} B_{t-1} + W_t N_t \quad (3)$$

where  $B_t$  is end-of-period  $t$  nominal bond;  $R_{t-1}$  is the nominal interest rate on the bond stipulated at period  $t-1$ ;  $W_t$  is the nominal wage;  $N_t$  is total labor supply. Labor is assumed to be perfectly mobile across sectors, implying that the nominal wage rate is common across sectors.

In real terms (measured in units of non-durable consumption), (3) reads

$$C_t + q_t (D_t - (1-\delta)D_{t-1}) + b_t = R_{t-1} \frac{b_{t-1}}{\pi_{c,t}} + w_t N_t \quad (4)$$

where  $q_t = \frac{P_{d,t}}{P_{c,t}}$  is the relative price of the durable good;  $b_t = \frac{B_t}{P_{c,t}}$  is the real bond;

$w_t = \frac{W_t}{P_{c,t}}$  is the real wage.

Household chooses  $\{N_t, b_t, D_t, C_t\}$  to maximize (2) subject to (4). By defining  $\lambda_t$  as the Lagrangian multiplier, and  $U_{k,t}$  ( $k = C, N, D$ ) as the marginal utility of respective variable, first order conditions for household's decision problem read:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_{c,t}} \quad (5)$$

$$U_{c,t} = \lambda_t \quad (6)$$

$$q_t U_{c,t} = U_{d,t} + \beta(1 - \delta)E_t(U_{c,t+1}q_{t+1}) \quad (7)$$

$$1 = \beta E_t \left( R_t \frac{U_{c,t+1}}{U_{c,t}} \frac{P_{c,t}}{P_{c,t+1}} \right) \quad (8)$$

Equation (5) links the real wage to household's marginal rate of substitution between consumption and leisure. Equation (7) requires the household to equate the marginal utility of current non-durable consumption to the marginal gain of durable services. The marginal gain of durable services includes two parts: (i) the direct utility gain of additional unit of durable; (ii) the expected utility stemming from the consumption of the resale value of the durable purchased in previous period. Equation (8) is a standard Euler condition. The term  $\beta \frac{U_{c,t+1}}{U_{c,t}}$  is defined as stochastic discount factor  $\Delta_t$ .

Combining (7) and (8), (7) can be rewritten as

$$\frac{\lambda_t}{P_{c,t}} \{ P_{d,t} - (1 - \delta) \frac{P_{d,t+1}}{R_t} \} = U_{d,t} \quad (8a)$$

The term in the brace is defined as user cost of durable goods. For one unit of



durable goods, user purchases it at the price  $P_{d,t}$ . Next period, after the depreciation, the left durable goods stock  $1 - \delta$  can be sold at price  $P_{d,t+1}$ . Then, user cost is the purchasing price of durable goods minus the present value of resale revenue.

### Final Good Producers

In each sector  $j$  ( $j = c, d$   $c$  denotes non-durable sector and  $d$  denotes non-durable sector), a perfectly competitive final good producer purchases  $Y_{j,t}(i)$  units of intermediate good  $i$ . The production function that transforms intermediate goods into final good is given by

$$Y_{j,t} = \left[ \int_0^1 Y_{j,t}(i)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di \right]^{\frac{\varepsilon_j}{\varepsilon_j - 1}} \quad (\varepsilon_j > 1, j = c, d) \quad (9)$$

where  $\varepsilon_j$  is the elasticity of substitution between differentiated intermediate goods in sector  $j$ .

Demand functions for intermediate good  $i$  in sector  $j$  can be derived from the following cost minimization problem: choose  $Y_{j,t}(i)$  to minimize

$$\int_0^1 P_{j,t}(i) Y_{j,t}(i) di \quad (10)$$

subject to

$$\left[ \int_0^1 Y_{j,t}(i)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di \right]^{\frac{\varepsilon_j}{\varepsilon_j - 1}} \geq Y_{j,t} \quad (11)$$

It is straightforward to show that demand function for intermediate good  $i$  in sector  $j$  is

$$Y_{j,t} = \left( \frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t} \quad (j = c, d) \quad (12)$$

where  $P_{j,t} = \left( \int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}}$  is the price of final good  $j$

### Intermediate goods producers

In both of the non-durable and durable sectors, there is a continuum of monopolistically competitive intermediate goods producers indexed by  $i \in [0,1]$ . Each intermediate goods producer faces the demand curve (12) for its product. It uses only labor to produce output according to following technology

$$Y_{j,t}(i) = A_{j,t} N_{j,t}(i) \quad (13)$$

where  $A_{j,t}$  is the technology in sector  $j$ .  $N_{j,t}(i)$  is the labor hired by firm  $i$  in sector  $j$ .

Intermediate goods producers set nominal prices on a staggered basis. Following Calvo (1983), we assume that firms adjust their prices infrequently and that opportunities to adjust arrived as an exogenous Poisson process. Each period, there is a constant probability  $1 - \omega_j$  that the firm can adjust its price, the remaining  $\omega_j$  fraction keep their prices fixed.

When a firm gets a chance to adjust its price, it sets the price  $p_{j,t}^*$  to maximize the following expected discounted profit

$$E_t \sum_{i=0}^{\infty} \omega_j^i \Delta_{t,t+i} \left[ \left( \frac{p_{j,t}^*}{P_{j,t+i}} \right) Y_{j,t+i} - MC_{j,t+i} Y_{j,t+i} \right] \quad (14)$$

where  $\Delta_{t,t+i} = \prod_{s=t}^{t+i} \Delta_s$ ,  $\Delta_t$  is the stochastic discount factor defined in household's

decision problem;  $MC_{j,t+i} = \frac{W_t}{A_{j,t}P_{j,t}}$  is the real marginal cost. When a firm gets a chance to adjust price at period  $t$ , it has to take into account that this new price will keep unchanged until period  $t+i$  with a probability  $\omega_j^i$ . The term in the square bracket denotes the firm's profit (in real terms) at period  $t+i$  if it does not get a chance to adjust its price. Note that each firm adjusting its price at period  $t$  faces the same profit maximization problem in (14), so all firms will set the same price  $p_{j,t}^*$ .

Using the definition of  $\Delta_{t+i}$  and demand curve in (12), it is straightforward to derive  $p_{j,t}^*$  from the maximization of (14)

$$\left(\frac{p_{j,t}^*}{P_{j,t}}\right) = \left(\frac{\varepsilon_j}{\varepsilon_j - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \omega_j^i \beta^i \lambda_{j,t+i} MC_{j,t+i} \left(\frac{P_{j,t+i}}{P_{j,t}}\right)^{\varepsilon_j}}{E_t \sum_{i=0}^{\infty} \omega_j^i \beta^i \lambda_{j,t+i} \left(\frac{P_{j,t+i}}{P_{j,t}}\right)^{\varepsilon_j - 1}} \quad (15)$$

The sector  $j$ 's price index satisfies

$$P_{j,t} = \left[ (1 - \omega_j)(p_{j,t}^*)^{1 - \varepsilon_j} + \omega_j P_{j,t-1}^{1 - \varepsilon_j} \right]^{\frac{1}{1 - \varepsilon_j}} \quad (16)$$

The sectoral inflation rate  $\pi_{j,t} = \frac{P_{j,t}}{P_{j,t-1}}$ .

### News shocks

We model the news shocks following Christiano, Motto and Rostagno (2006). Up until period  $t$ , the economy is at a steady state. In period  $t$ , a signal arrives that suggests technology in sector  $j$  will improve in period  $t+p$ . Then, in period  $t+p$ , the expected rise in technology in fact does not occur. A time series representation for productivity which captures this unrealized optimistic expectation is:

$$\log(A_{j,t}) = \rho_j \log(A_{j,t-1}) + \varepsilon_{j,t-p} + \zeta_{j,t} \quad (17)$$

With  $0 < \rho_j < 1$ ,  $\varepsilon_{j,t}$  and  $\zeta_{j,t}$  are uncorrelated over time and with each other. To see this setup can capture the unrealized optimistic expectation, suppose a signal arrives at period  $t - p$  indicating that productivity will increase at period  $t$ , that is,  $\varepsilon_{j,t-p}$  has a high value. This shifts up the expected value of  $\log(A_{j,t})$ , which reflects an optimistic expectation. However, at period  $t$ , if the realization of  $\zeta_{j,t} = -\varepsilon_{j,t-p}$ , then the high expected value of  $\log(A_{j,t})$  does not materialize. In this case,  $\varepsilon_{j,t-p}$  turns out to be a misleading signal. If the realization of  $\zeta_{j,t}$  is zero, then the high expected value of  $\log(A_{j,t})$  does materialize. In this case, the signal  $\varepsilon_{j,t-p}$  is perfectly informative.

For purpose of solving and simulating the model, it is useful to formulate  $\log(A_{j,t})$  in the following formulation:

$$\begin{bmatrix} \log(A_{j,t}) \\ \varepsilon_{j,t} \\ \varepsilon_{j,t-1} \\ \vdots \\ \varepsilon_{j,t-p+1} \end{bmatrix} = \begin{bmatrix} \rho_j & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ & & \cdots & & \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \log(A_{j,t-1}) \\ \varepsilon_{j,t-1} \\ \varepsilon_{j,t-2} \\ \vdots \\ \varepsilon_{j,t-p} \end{bmatrix} + \begin{bmatrix} \zeta_{j,t} \\ \varepsilon_{j,t} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (18)$$

## Equilibrium

$$\text{Equilibrium in non-durable goods market: } Y_{c,t} = C_t \quad (19)$$

$$\text{Equilibrium in durable goods market: } Y_{d,t} = D_t - (1 - \delta)D_{t-1} \quad (20)$$

$$\text{where } Y_{j,t} = \int_0^1 Y_{j,t}(i) di = A_{j,t} \int_0^1 N_{j,t}(i) di = A_{j,t} N_{j,t} \quad (j = c, d)$$

$$\text{Equilibrium in labor market: } N_t = N_{c,t} + N_{d,t} \quad (21)$$

An equilibrium allocation, with sticky prices in both sectors, is a sequence for  $N_t$ ,  $N_{j,t}$ ,  $C_t$ ,  $D_t$ ,  $X_t$ ,  $R_t$ ,  $q_t$ ,  $MC_{j,t}$ ,  $W_t$ ,  $\pi_{j,t}$ ,  $\lambda_t$  satisfying (1), (4), (5)-(8), (15)-(16) and (19)-(21). A monetary policy is still needed to complete the model. The details of monetary policy will be discussed later.

### 3. Calibration and Solution

The model is calibrated at a quarterly frequency. Some parameter values are typical in the business cycle literature. The discount factor  $\beta$  is set to be 0.99, consistent with a steady state annualized real interest rate of about 4%. The quarterly depreciation rate of the durable stock  $\delta$  is set to 0.025, implying an annual depreciation rate of 10%.  $\alpha$ , the parameter in the composite consumption index  $X_t$ , can be chosen so that the steady state share of durable goods output in total output is 0.2. The parameter  $\nu$  in the utility function is set so that steady-state labor supply is 1/3. The autoregressive coefficient in the productivity process  $\rho_j$  is set to 0.95.

Following Beaudry and Portier (2004), the elasticity of substitution between non-durable goods and durable goods,  $\eta$ , is set to be 0.2 in the baseline case, implying a strong complementarity between the non-durable goods and durable goods.

The parameter  $\omega_j$  determines how long a price contact will last<sup>1</sup>. The empirical evidence surveyed by Taylor (1999) suggests that nominal price contracts on average last for a year, implying  $\omega_j=0.75$ . Bils and Klenow (2004) argue that the observed frequency

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<sup>1</sup> The expected time between price adjustments is  $1/(1-\omega_j)$

of price adjustment in the U.S. is much higher, and in the order of two quarters, implying  $\omega_j = 0.5$ . In recent literatures of two-sector models, Erceg and Levin (2006) assumed symmetric price rigidities in non-durable and durable sectors. Barsky, House and Kimball (2004) and Monacelli (2006b) studied the cases with asymmetric price rigidities. In extreme case, price in one sector is assumed to be flexible, implying  $\omega_j = 0$ . We set  $\omega_j = 0.75$  for both sectors in the baseline calibration.

The parameters  $\varepsilon_j$  ( $j = c, d$ ) measure the elasticity of substitution between differentiated intermediate goods. Following Monacelli (2006b), we set both parameters to 8, which yields a steady state mark-up of 15% for intermediate goods producers.

The values of all the baseline parameters are summarized in table 1.

The model is solved by taking a log-linear approximation of the equilibrium conditions in the neighborhood of the steady state.

## **4. Ramsey Optimal Monetary Policy and Simple Policy Rules**

### **4.1 Ramsey Optimal Monetary Policy**

The research on Ramsey policy begins in the field of public finance. Recently, researchers are interested in finding Ramsey optimal monetary policy in models with nominal rigidities. For example, Levin, Onatski, Williams and Williams (2005) investigate the design of monetary policy when the central bank faces uncertainty about the true structure of the economy. They find the optimal policy regime that maximizes

household welfare using Ramsey approach and then evaluate the performance of alternative simple policy rules relative to this benchmark.

The Ramsey optimal policy under commitment can be computed by formulating an infinite horizon Lagrangian problem, in which the central bank maximizes conditional expected social welfare subject to the full set of non-linear constraints implied by the private sector's behavioral equations and the market-clearing conditions of the model economy.

In the case of our model, central bank's Lagrangian problem can be described as

$$\max E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(X_t, N_t) \right\}$$

Subject to (1), (4), (5)-(8), (15)-(16), (19)-(21)

Let  $\lambda_{k,t}$  ( $k=1,2,\dots,11$ ) represent Lagrangian Multipliers on these 11 constraints.

Solving this problem generates another 12 first order conditions<sup>2</sup>, which characterize the Ramsey optimal policy. To analyze the behavior of the economy under this Ramsey policy, we combine central bank's 12 first order conditions with 11 first order conditions characterizing private sector's behaviors. So, there are totally 23 equations for 23 unknown variables.<sup>3</sup>

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<sup>2</sup> Take derivative of the Lagrangian equation with respect to 11 endogenous variables and  $\lambda_t$ , the multiplier on budget constraint (4). Matlab procedures developed by Levin and Lopez-Salido (2004) are used to derive these first order conditions.

<sup>3</sup> Includes 11 endogenous variables, the multiplier on budget constraint (4)  $\lambda_t$  and 11  $\lambda_{k,t}$  multipliers in central bank's Ramsey problem. Software package Dynare has been used to solve the model. For details of Dynare, refer to <http://www.cepremap.cnrs.fr/dynare/>.

The limitation of this Ramsey approach is that we cannot solve for the closed-form policy reaction function. However, the optimal response of the policy variable to various shocks can be revealed in impulse response functions (IRFs).

The following simple experiment illustrates how Ramsey optimal monetary policy reacts to a news shock in our model with baseline parameters. We assume that agents receive a signal at period  $t$  suggesting technology in sector  $j$ ,  $A_j$ , will be increased by 1% at period  $t+4$ , that is, a high  $\varepsilon_{j,1}$  leads to an upward revision in the expectation of  $\log(A_{j,5})$ . In period 1, agents will act on that expectation. In period 5, the realization of  $\log(A_{j,5})$  is determined by the realization of  $\zeta_{j,5}$ . If it happens that  $\zeta_{j,5} = -\varepsilon_{j,1}$ , then the expected positive move in  $\log(A_j)$  does not occur.

Following Beaudry and Portier (2004), we first assume that agents receive a signal suggesting technology increase in non-durable sector. Figure 1 shows the impulse responses of chosen Marco-variables and policy instrument, nominal interest rate, to a news shock when central bank employs the Ramsey optimal policy. First, output in non-durable sector, output in durable sector (investment), stock of durable goods (capital stock), labor supply in both sectors experience a boom when agents expect a rise of technology in non-durable sector at period 1 and a bust when the optimistic expectation does not materialize at period 5. Second, the volatility of output in durable sector (investment) is much greater than that of output in non-durable sector. Third, relative price of durable goods to non-durable goods behaves rises in the boom period and slumps in the bust period. Fourth, if central bank follows the Ramsey optimal policy, annualized



nominal interest rate should be raised to about 4.35% (steady state 4%) in the boom periods and immediately drops to about 3.9% once agents realize that their expectations do not realize.

Next, we assume that agents receive a signal suggesting technology rise in durable sector. The Impulse responses on Figure 2 are consistent with Beaudry and Portier's result: macro aggregates move together downwards when agents expect a rise of technology in durable sector. This observation verifies that the assumption that agents receive signals about improvements in non-durable sector is essential to generate Pigou's cycle.

#### **4.2 Simple Policy Rules**

Following Taylor (1993), numerous researches have been done to estimate or evaluate all kinds of simple monetary policy rules. The policies are simple in the sense that they involve only a few observable macroeconomic variables. In terms of maximizing agents' welfare, the performances of simple rules are inferior to that of Ramsey optimal policy since Ramsey policy can react to all endogenous variables while simple rules only react to several observable variables. However, usually we cannot find a closed solution for Ramsey policy. What we can do is to find a simple policy rule that responds to shocks in the similar way that Ramsey policy does. This simple rule can tell central bank how to respond to a specific shock.

In this section, we study three series of simple policy rules: nominal interest rate reacts to (1) non-durable sector inflation only; (2) durable sector inflation only; (3) inflation rates of both sectors.

#### 4.21 Interest rate reacts to non-durable sector inflation only

First, we assume that nominal interest rate reacts to non-durable sector inflation only.

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) \quad (22)$$

where the steady state nominal interest rate  $R^* = 1/\beta$ , and target inflation rate  $\pi_c^* = 1$ . A unique stationary equilibrium exists as long as  $a > 1$ . We consider both strong inflation targeting case ( $a = 2$ ) and weak inflation targeting case ( $a = 1.1$ ). First, if central bank only reacts to non-durable inflation alone, Figure 3 shows that neither strong inflation targeting nor weak inflation targeting can closely mimic the impulse responses under the Ramsey policy. Strong inflation targeting is superior to weak targeting in that it generates much less volatility in inflations, outputs and investments. Second, investments and durable goods stocks slump during the boom period in the case of strong targeting even through strong complementarity between non-durable goods and durable goods is assumed. A further analysis unravels the reason: the rise of user cost, the determinant of investment in durable goods, almost doubles the corresponding rise of user cost in Ramsey policy case, which causes the drop in investment and durable goods stocks. This also explains that a drop in user cost causes a tremendous increase of investment and durable goods stocks under the circumstance of weak inflation targeting.

To see the difference of user cost in the case of strong inflation targeting and weak targeting, we rewrite equation (8a) as:

$$\lambda_t q_t \left\{ 1 - (1 - \delta) \frac{q_{t+1}}{q_t} \frac{1}{R_t / \pi_{c,t+1}} \right\} = \lambda_t u_{c,t} = U_{d,t} \quad (23)$$

The user cost  $uc_t$  is determined by three factors: the relative price of durable goods  $q_t$ , the growth rate of relative price, and the real interest rate  $R_t/\pi_{c,t+1}$ . Figure 3 reveals that monetary policy has no impact on either the relative price level or growth rate the relative price. The only difference between strong targeting and weak targeting that can cause a difference in user cost is the real interest rate. From Euler equation (8), marginal utility of consumption of non-durable goods  $\lambda_t$  is also determined by the real interest rate. A little difference in real interest rate can cause a great difference in user cost and marginal utility of non-durable goods consumption. From (23), a combination of the effects of real interest rate on user cost and  $\lambda_t$  determines the marginal utility of durable goods consumption, and further determines the desired durable goods stocks and investment. Finally, labor supply, investment and durable goods stock fall during the boom when central bank strongly targets non-durable sector inflation, which is not consistent with responses in Pigou's cycle. The conclusion is that even a strong complementarity between non-durable goods and durable goods is not a sufficient condition to produce Pigou's cycle when central bank targets inflation strongly.

This result arouses the interest to study the question: is complementarity between non-durable goods and durable goods still a necessary condition to generate expectation driven business cycles when central bank employs a weak inflation targeting? In the following experiment, we let the elasticity of substitution between non-durable goods and durable goods  $\eta = 2$ . Figure 4 shows that a strong complementarity between non-durable goods and durable goods is not necessary to generate Pigou's cycle any more provided

central bank employs a weak inflation targeting. Booms and busts of non-durable outputs, investments and stocks in durable sector, labor supplies in both sectors are consistent with the definition of Pigou's cycles, even though we do not assume a strong complementarity between non-durable goods and durable goods.

#### 4.22 Interest rate reacts to durable sector inflation only

In this experiment, we assume central bank targets durable sector inflation rate only.

$$\log(R_t) - \log(R^*) = a(\log(\pi_{d,t}) - \log(\pi_d^*)) \quad (24)$$

where the steady state nominal interest rate  $R^* = 1/\beta$ , and target inflation rate  $\pi_d^* = 1$ . A unique stationary equilibrium exists as long as  $a > 1$ . We consider both strong inflation target case ( $a = 2$ ) and weak inflation target case ( $a = 1.1$ ). The IRFs are shown in figure 6. First, Pigou's cycle can arise in both cases. Second, durable sector inflation targeting is superior to non-durable sector inflation targeting since IRFs under durable sector inflation targeting approximate IRFs under Ramsey policy better. However, it is difficult to identify the additional gain from targeting durable sector inflation more strongly.

#### 4.23 Interest rate reacts to inflation rates in both sectors

In this experiment, we allow central bank to react to inflation rates in both sectors.

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*)) \quad (25)$$

The problem is that it is not that obvious to decide the targeting weights  $a$  and  $b$ . However, intuition tells us that more weights should be given to durable sector since the IRFs under durable sector inflation targeting approximate IRFs under Ramsey policy better. Starting from targeting durable sector inflation only ( $a = 0, b = 1$ ), we increase the

weights given to non-durable sector little by little to see if targeting non-durable sector inflation can bring additional benefit. Figure 6 shows that a simple policy rule targeting inflation rates in both sectors is superior to the rule targeting only durable sector inflation. A small weight given to non-durable inflation ( $b = 0.2$ ) can lead to a significant improvement of the approximation of the IRFs with those under Ramsey policy.

### **4.3 Determinants of weights in the simple inflation targeting rule**

Through the above experiments, we observe that a simple rule targeting inflation rates in both sectors is superior to those rules targeting inflation in one sector only. The problem is how to determine the appropriate weights of inflation in each sector in the interest rate reaction function. We find that the weights are determined by the degrees of nominal rigidities, the depreciation rate and the relative share of durable goods output to non-durable goods output.

#### **4.31 Degrees of nominal rigidities**

In baseline calibration, we assume symmetric nominal rigidity in two sectors. Since we are not certain about the degree of nominal rigidities in both sectors, we will do an experiment to see how optimal monetary policy will change when the frequencies to reset the prices ( $\omega_j, j = c, d$ ) are different between non-durable sector and durable sector. This experiment also shows how central bank decides the weights given to each sector when the optimal policy is to target the inflation rates of both sectors. In this experiment, depreciation rate is assumed to be 100 percent and relative share of durable goods output

to non-durable goods output is set to 1 so that the only asymmetry between these two sectors is degree of nominal rigidity.

Figure 7 shows the impulse responses under Ramsey optimal policy, simple policy rules strictly targeting durable sector inflation exclusively and targeting non-durable sector inflation exclusively on the same graph. In the case that nominal rigidity in non-durable sector dominates the nominal rigidity in durable sector, simple rules targeting non-durable sector approximate Ramsey policy closely. When nominal rigidity in non-durable sector decreases and rigidity in durable sector increases, IRFs of Ramsey policy shifts away from the IRFs of non-durable-inflation targeting towards IRFs of durable-inflation targeting. When there is nominal rigidity in only one sector, central bank's task is to remove the inflation in that sector and allow the inflation in another sector fluctuate freely. When there are rigidities in both sectors, central bank has to face the tradeoff since a policy change may reduce the inflation in one sector while induce the inflation in another sector. The conclusion is that central bank should give greater weights to the sector with greater nominal rigidities.

#### **4.32 Depreciation rates**

In this experiment, we assume symmetric nominal rigidities and same output in two sectors so that the only difference between two sectors is that durable goods are depreciated over time. Erceg and Levin (2006) highlight two factors that are particular to durable goods. First, behavior of durable goods is particularly sensitive to the real interest rate change. From equation (23) (rewritten here for convenience),

$$\lambda_t q_t \left\{ 1 - (1 - \delta) \frac{q_{t+1}}{q_t} \frac{1}{R_t / \pi_{c,t+1}} \right\} = \lambda_t u c_t = U_{d,t}$$

User cost is determined by not only the relative price of durable goods, but also the discounted capital gains. For example, an expectation of capital gain can partially offset the rise in relative price and lower the user cost. Real interest rate plays a significant role here since capital gain has to be discounted to present value. However, all these could happen only if the depreciation is not 100 percent. The smaller is the depreciation rate, the greater is the effect of capital gain and real interest rate on user cost and durable goods demand.

Second, from (23), any changes in user cost or marginal utility of non-durable consumption lead to change in marginal utility of durable goods and demand of durable goods stock. Notice that if depreciation rate is small, durable goods stock is much greater than the durable goods production at each period. A small fluctuation in durable goods stock translates into a much greater fluctuation in durable goods output (investment) and labor supply in this sector.

Due to these two factors, a small deviation of the real interest rate from the level required to keep durable output at potential could induce a great output gap in durable sector and welfare loss. The conclusion is that central bank should give greater weights to the inflation in durable goods sector in the simple inflation targeting rule.

The result in figure 8 verifies this conclusion: IRFs of Ramsey policy are closely approximated by the IRFs of durable-sector-inflation targeting when we assume a normal

rate of depreciation rate 0.025. While in figure 7(c), IRFs of Ramsey policy is in the middle of IRFs of two simple rules when we assume 100 percent depreciation.

#### **4.33 Relative share of durable sector output to non-durable sector output**

Obviously, central bank should give more weights to the sector that account for more shares in total output. Comparing figure 9 and figure 8, we find that IRFs of Ramsey policy shift away from IRFs of durable-sector-inflation targeting towards IRFs of non-durable-sector inflation targeting when we increase the relative share of non-durable sector output to durable sector output from 1 to 4.

#### **4.4 Relative importance of three determinants**

Of three determinants of targeting weights, nominal rigidity dominates the other two. This can be seen through the following experiment: reduce the parameter measuring nominal rigidity in durable sector  $\omega_d$  from 0.75 to 0.4 and keep all other parameters unchanged. The results in previous experiments suggest that central bank should give more weights to durable sector in the case of symmetric rigidity since durable sector is more sensitive to real interest rate change. In the case of asymmetric rigidity, the sector with more nominal rigidity has dominant weights in the simple targeting rule. Figure 10 shows that simple rule giving dominant weights to non-durable sector inflation mimic the Ramsey policy closely when non-durable sector has a greater degree of rigidity. If central bank believes that durable goods sector has less nominal rigidity than non-durable goods sector, then a simple rule mainly targeting inflation in non-durable sector could be an optimal policy.



## **5. Sensitivity Analysis**

### 5.1 Various complementarities

Figure 11 indicates that a simple rule targeting inflation rates in both sectors can still mimic the Ramsey policy closely when the assumption of strong complementarity is released.

### 5.2 Technology shocks or News shocks

Figure 12 shows that central bank's optimal response to news shocks is also close to the optimal response to conventional technology shocks.

### 5.3 Unrealized expectation or realized expectation

In previous experiments, signals received by agents are assumed to be totally uninformative. Agents' expectations about the technology in the future do not realize.

Figure 13 shows that even agents' expectation are realized, the responses under the simple rule targeting inflation rates in both sectors remains close to those under the Ramsey policy.

The conclusion is that even central bank lacks the ability to detect whether the economy is hit by a conventional technology shock or a news shock or whether an expectation can be realized or not, as long as it employs a simple policy rule targeting inflations in both sectors with appropriate weights, the responses of macroeconomic aggregates are close to the responses under the Ramsey optimal policy.

## **Conclusion**

In this paper, we explored the expectation driven business cycles (Pigou's cycle) and optimal monetary policy in a two-sector economy with nominal rigidities in both non-durable goods and durable goods sectors.

Monetary policy plays an important role in generating Pigou's cycles. Complementarity between non-durable goods and durable goods is neither necessary nor sufficient to produce Pigou's cycles after we consider various monetary policies.

Simple policy rules targeting the inflation rates in both sectors with appropriate weights can approximate the Ramsey optimal policy closely. The weights assigned to each sector are determined by the degree of nominal rigidities, depreciation rate of durable goods and relative shares of output. Degree of nominal rigidities dominates the other two factors.

Both the conventional technology shock and news shock can cause the boom in the economy, since simple policy rules targeting inflation rates in both sectors work well under both circumstances, central bank's ability of detecting the news shock is not necessary.

In this paper, simple policy rules that can mimic the IRFs of Ramsey policy closely is said to be optimal. However, we did not define the measurement of closeness. A possible extension is to define an appropriate measurement of closeness so that different policy rules can be compared accurately. An alternative way to find optimal simple rule is to select policy rule coefficients within the set of implementable rules so as to maximize the level of welfare (or minimize the welfare loss) associated with the resulting competitive equilibrium.

## Appendix:

Table 1 : Baseline Parameter Values

$\beta$	$\alpha$	$\delta$	$\eta$	$\nu$	$\varepsilon_d$	$\varepsilon_c$	$\alpha_c$	$\alpha_d$	$\omega_c$	$\omega_d$	$\rho_c$	$\rho_d$
0.99	0.84	0.025	0.2	1.62	8	8	0.75	0.75	0.75	0.75	0.95	0.95

Figure 1

Impulse responses under Ramsey policy. Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5.

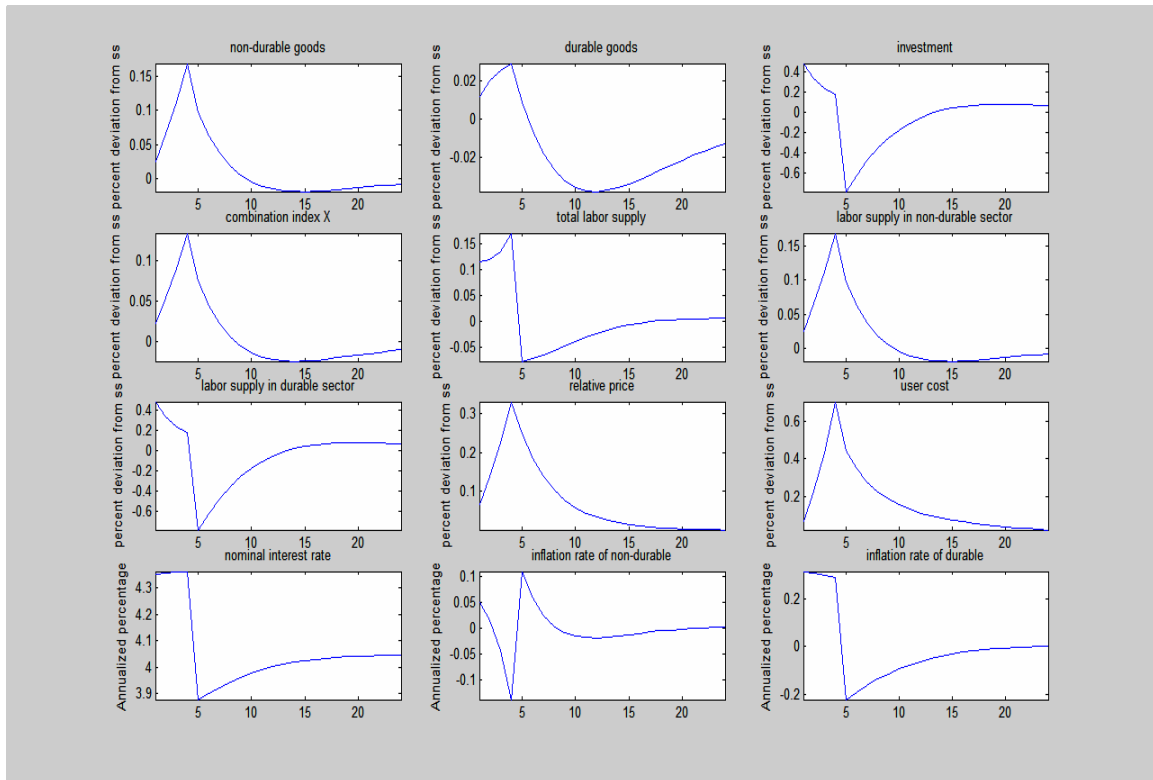


Figure 2

Impulse responses under Ramsey policy. Productivity in durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5.

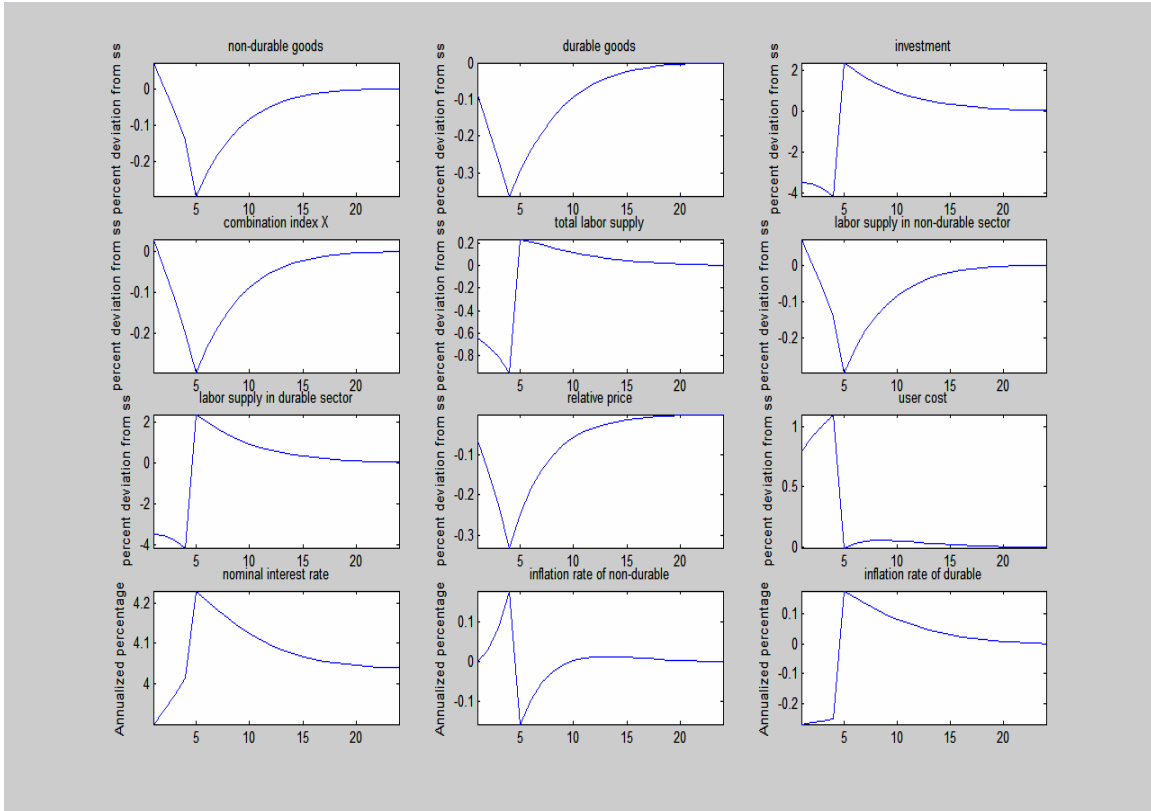


Figure 3

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy strongly reacting to non-durable sector inflation (solid) and weakly reacting to non-durable sector inflation (dashed), compared with impulse responses under Ramsey policy (lines with stars).

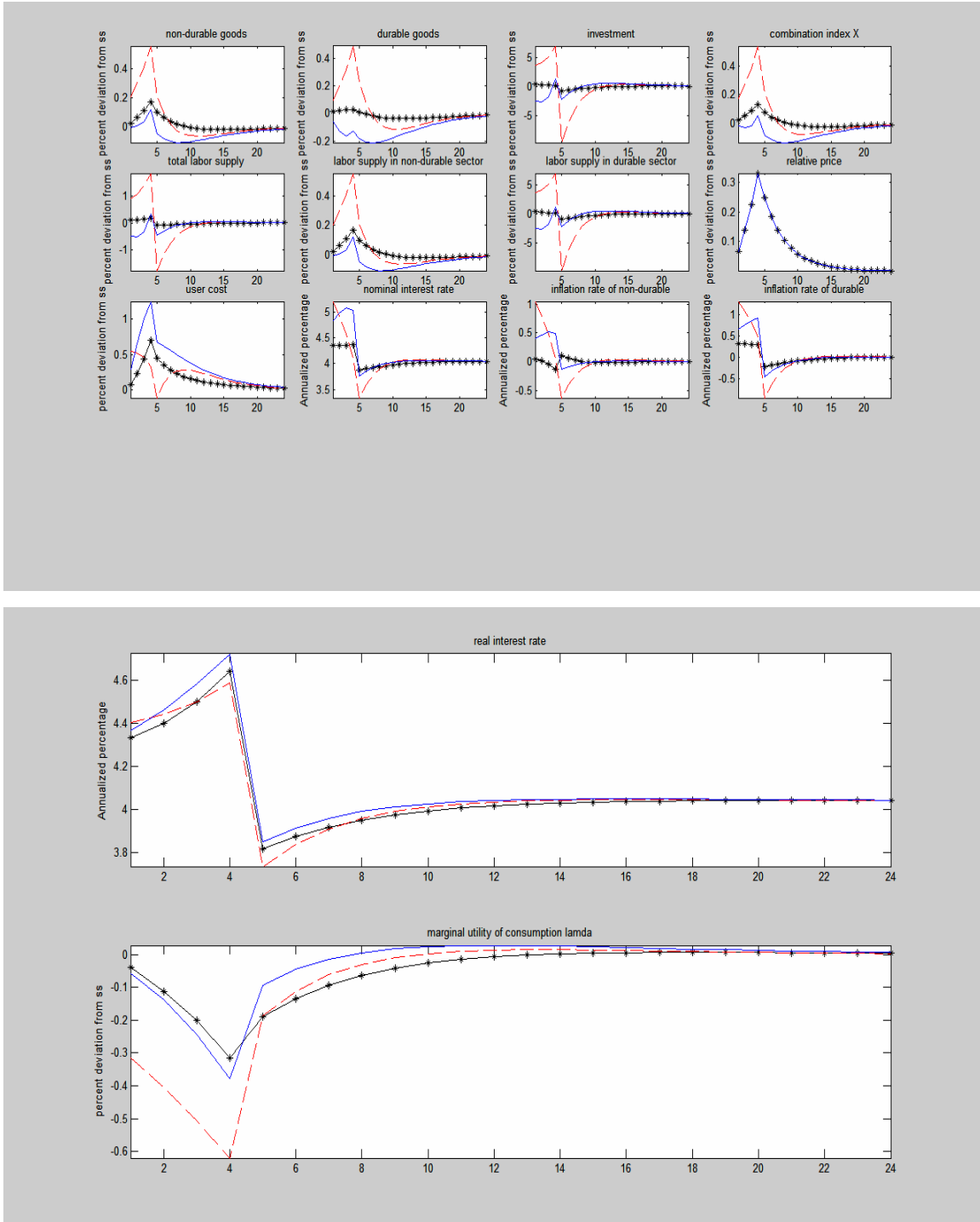


Figure 4

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy weakly reacting to non-durable sector inflation only. The elasticity of substitution between non-durable goods and durable goods  $\eta = 2$ .

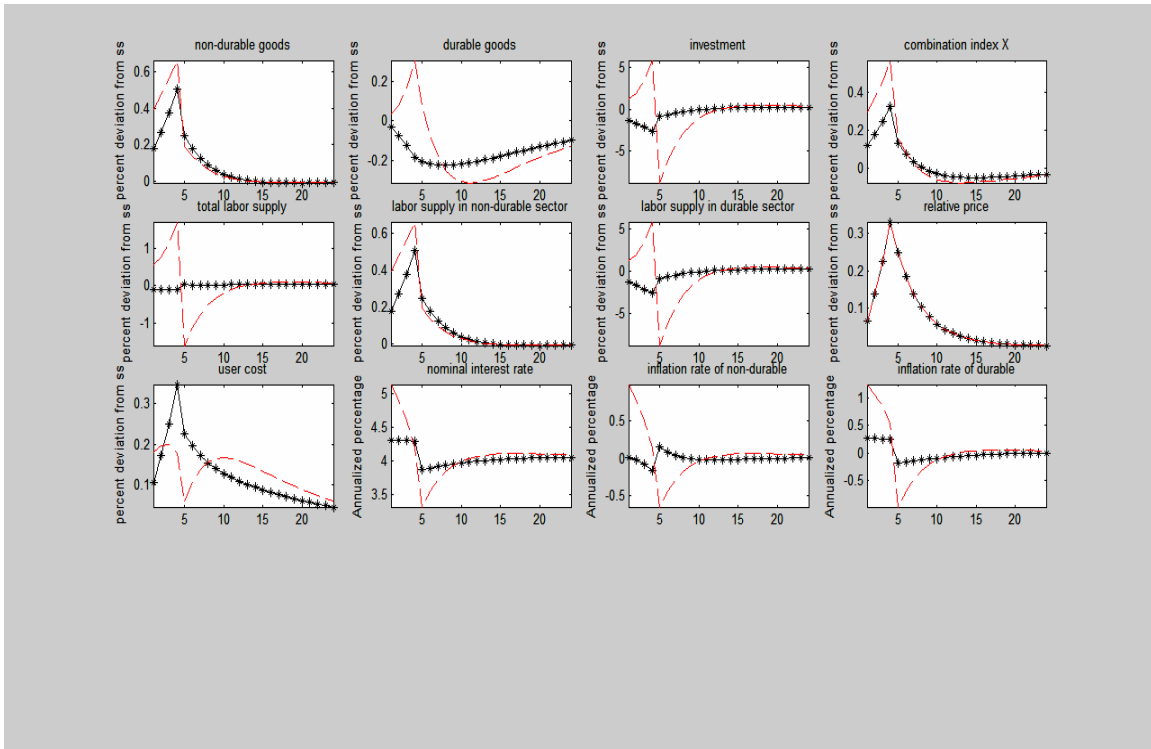


Figure 5

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy strongly reacting to durable sector inflation only (solid lines) and weakly reacting to durable sector inflation (dashed), compared with impulse responses under Ramsey policy (lines with stars).

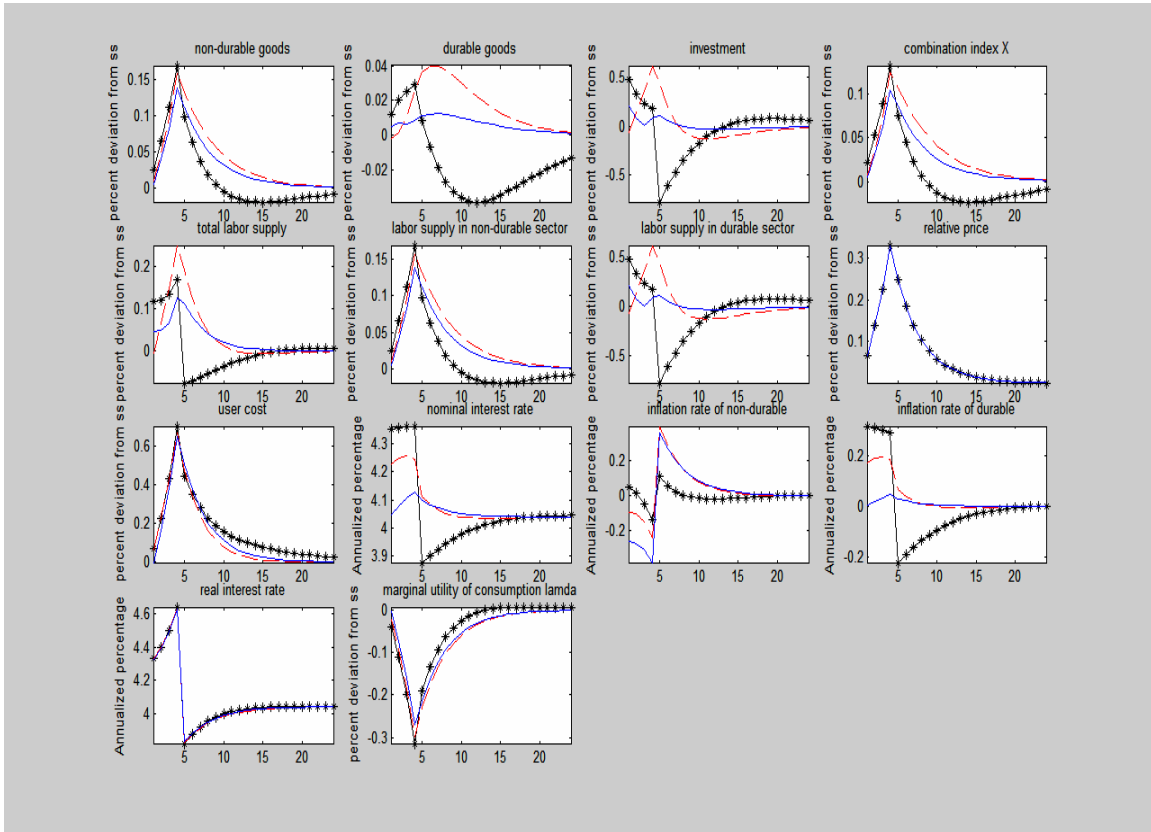


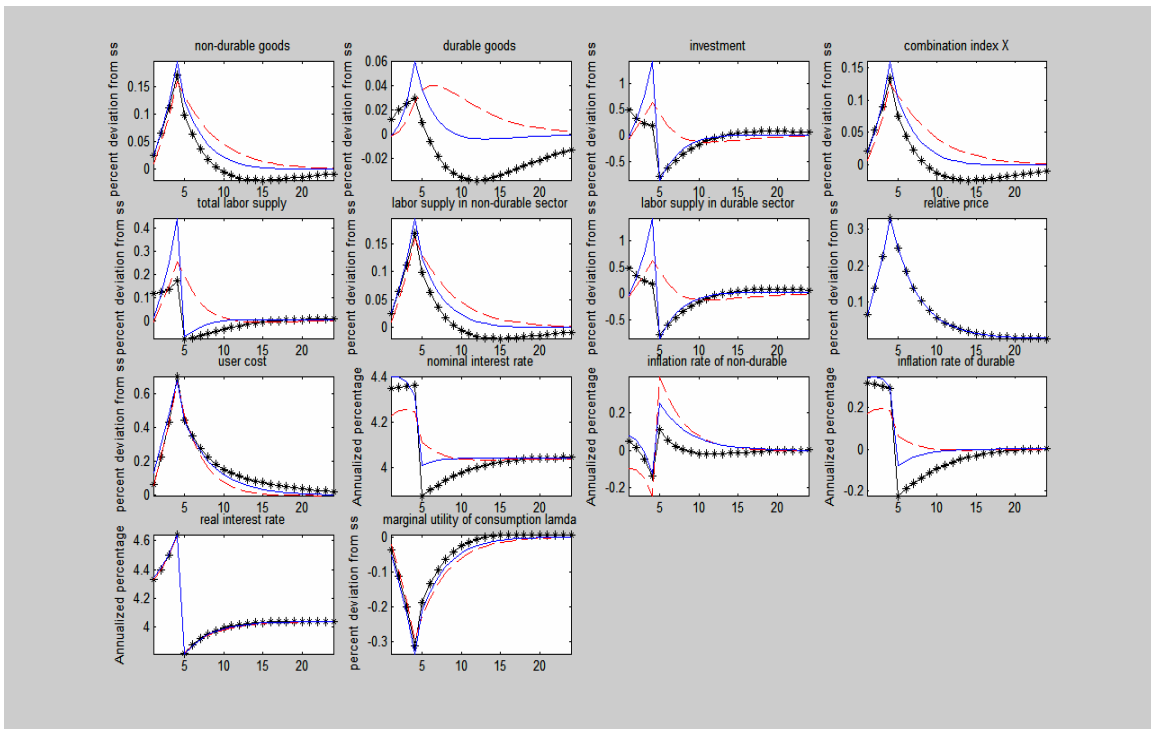
Figure 6

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy reacting to both durable sector inflation and non-durable sector inflation (solid lines) and reacting weakly to durable sector inflation only (dashed), compared with impulse responses under Ramsey policy (lines with stars).

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*))$$

Solid line:  $(a = 1.0; b = 0.2)$

Dashed line:  $(a = 1.1; b = 0)$

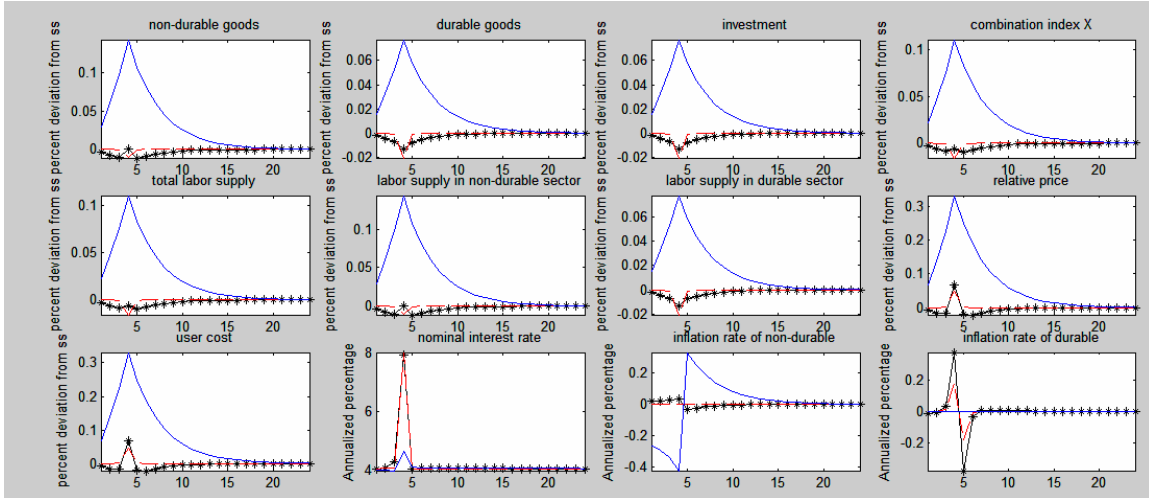




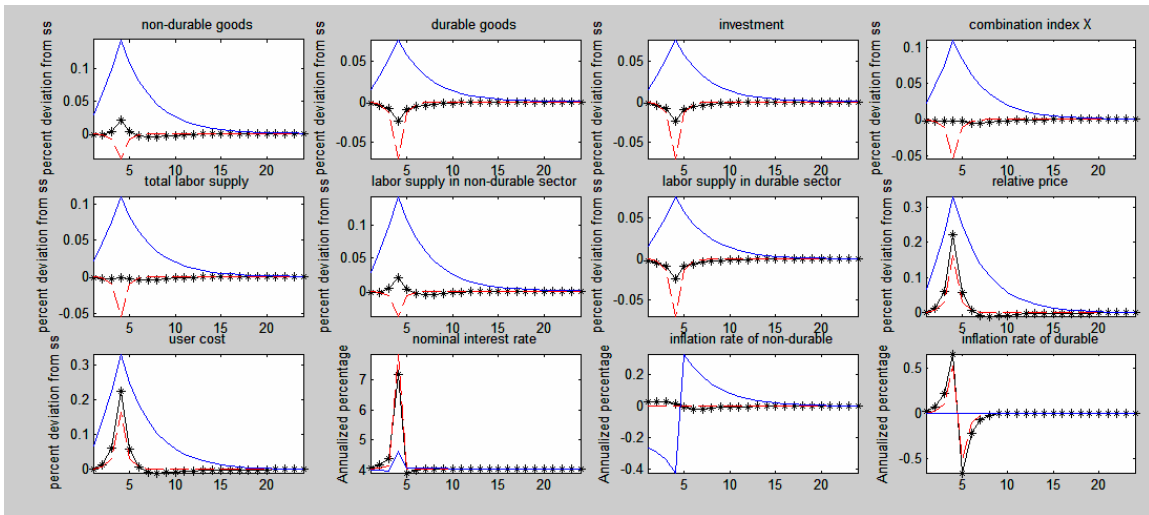
**Figure 7**

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy strongly reacting to durable sector inflation (solid lines) and strongly reacting to non-durable sector inflation only (dashed), compared with impulse responses under Ramsey policy (lines with stars).

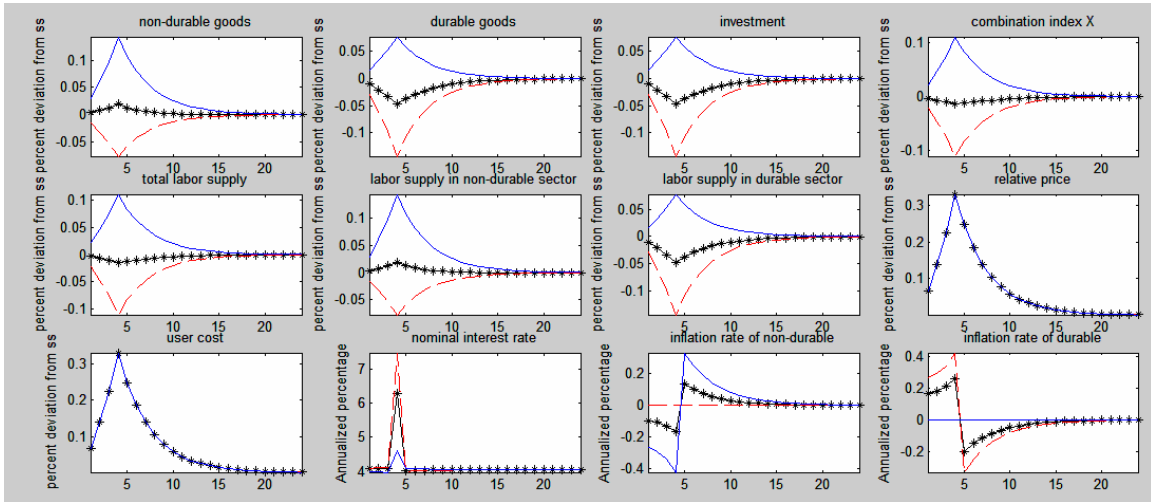
(a)  $\omega_c = 0.1; \omega_d = 0.75; \delta = 1; Y_c / Y_d = 1$



(b)  $\omega_c = 0.3; \omega_d = 0.75; \delta = 1; Y_c / Y_d = 1$

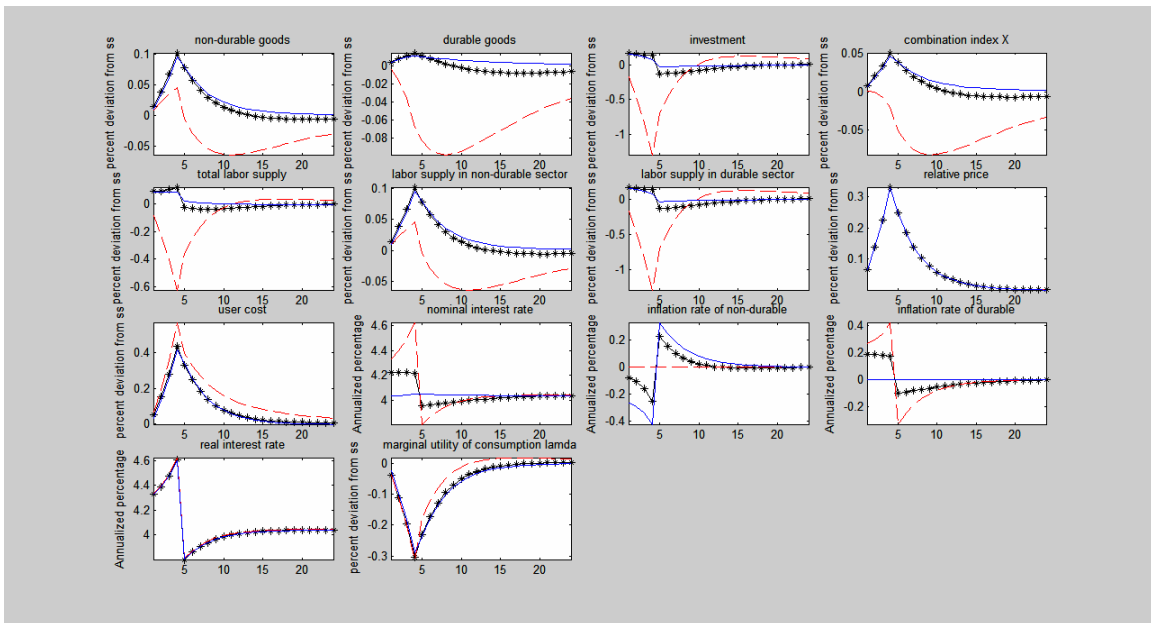


(C)  $\omega_c = 0.75; \omega_d = 0.75; \delta = 1; Y_c / Y_d = 1$



**Figure 8**

$\omega_c = 0.75; \omega_d = 0.75; \delta = 0.025; Y_c / Y_d = 1$



**Figure 9**

$\omega_c = 0.75; \omega_d = 0.75; \delta = 0.025; Y_c / Y_d = 4$

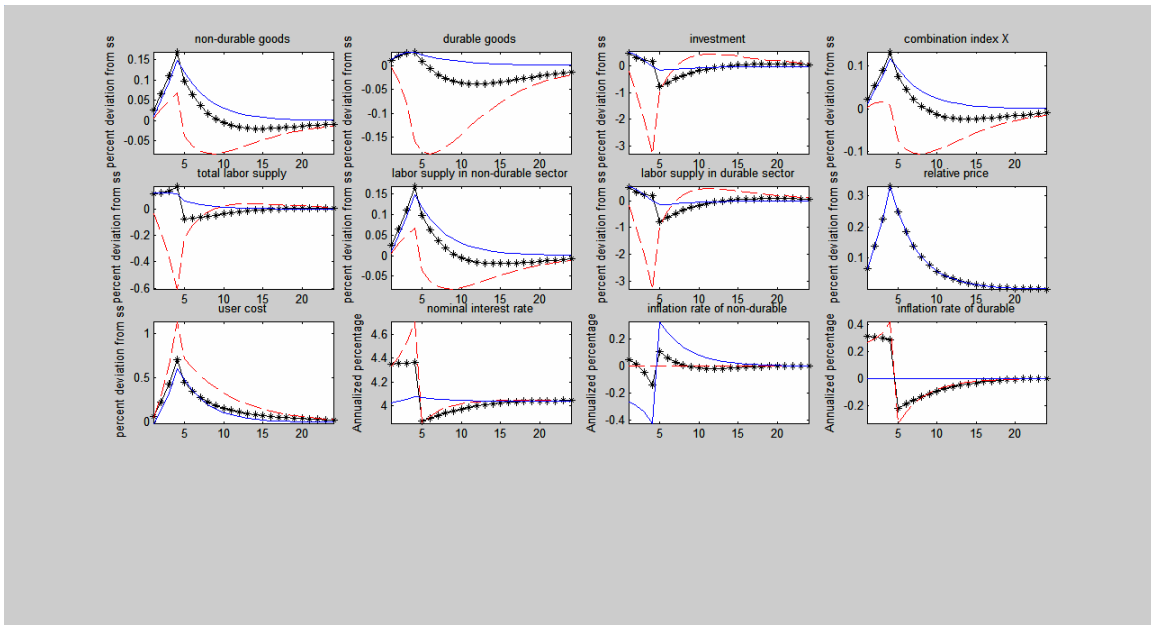
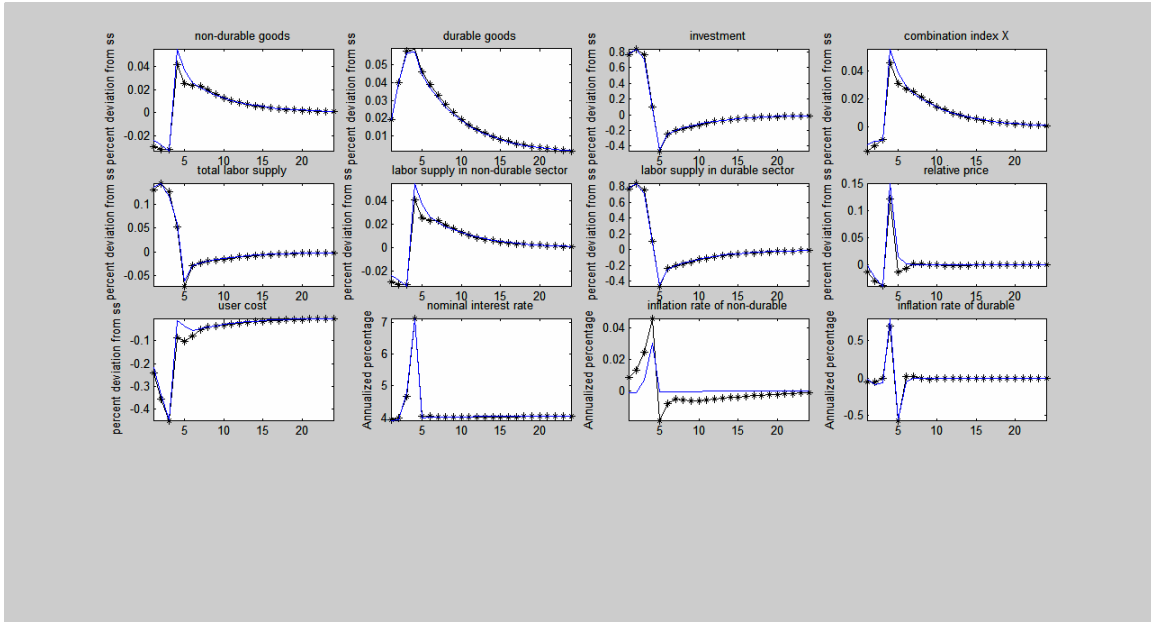


Figure 10 (Asymmetric nominal rigidity)

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy reacting to both durable sector inflation and non-durable sector inflation (solid lines), compared with impulse responses under Ramsey policy (lines with stars).

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*))$$

$$(a=100; b=0; \eta = 0.2; \varepsilon_c = 0.75; \varepsilon_d = 0.1)$$



$$(a=10; b=2; \eta = 0.2; \varepsilon_c = 0.75; \varepsilon_d = 0.4)$$

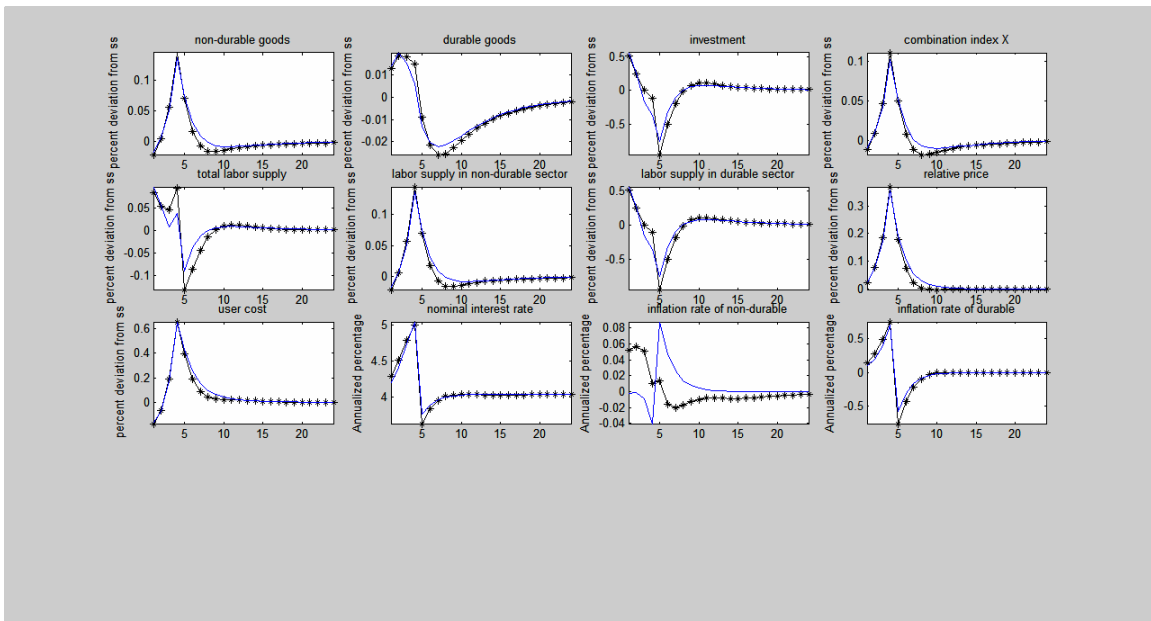


Figure 11

Productivity in non-durable sector is expected to increase by 1% at time 1 and no realization of that shock at time 5. Impulse responses under simple rule policy reacting to both durable sector inflation and non-durable sector inflation (solid lines), compared with impulse responses under Ramsey policy (lines with stars).

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*))$$

(a=1 ; b=0.2;  $\eta = 2$ )

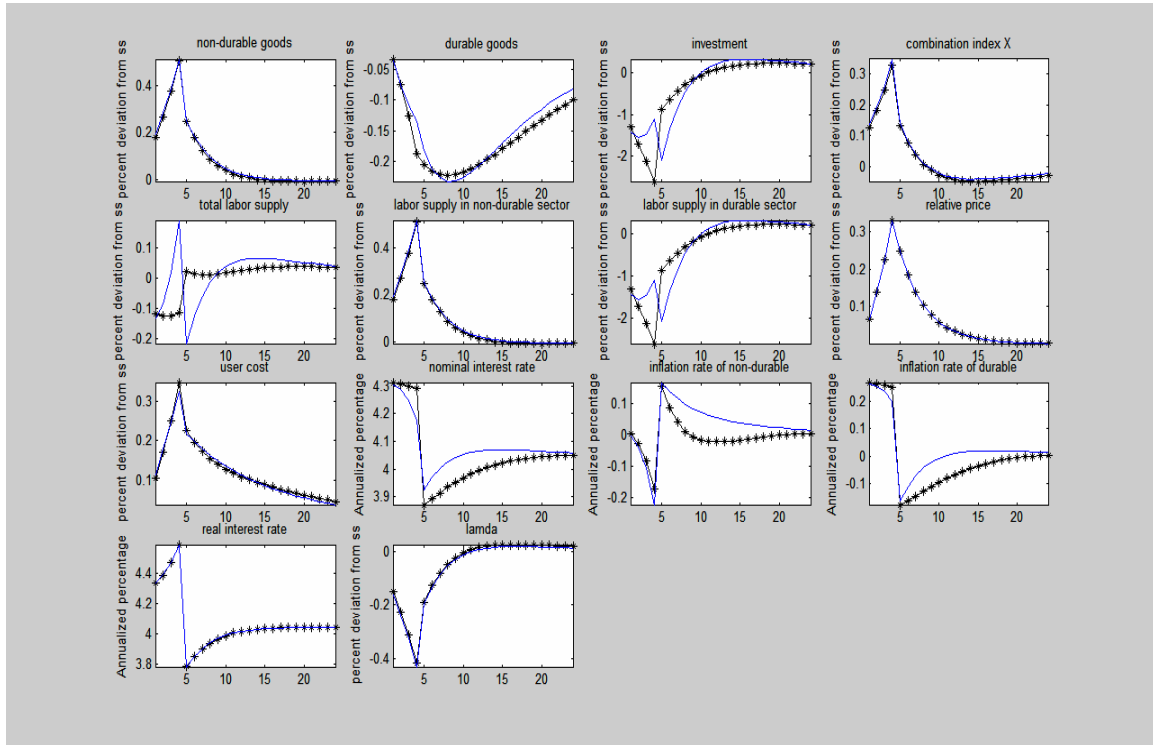


Figure 12 (Technology shocks)

Productivity in non-durable sector increases by 1% at time 1. Impulse responses under simple rule policy reacting to both durable sector inflation and non-durable sector inflation (solid lines), compared with impulse responses under Ramsey policy (lines with stars).

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*))$$

(a=1 ; b=0.2;  $\eta = 0.2$ )

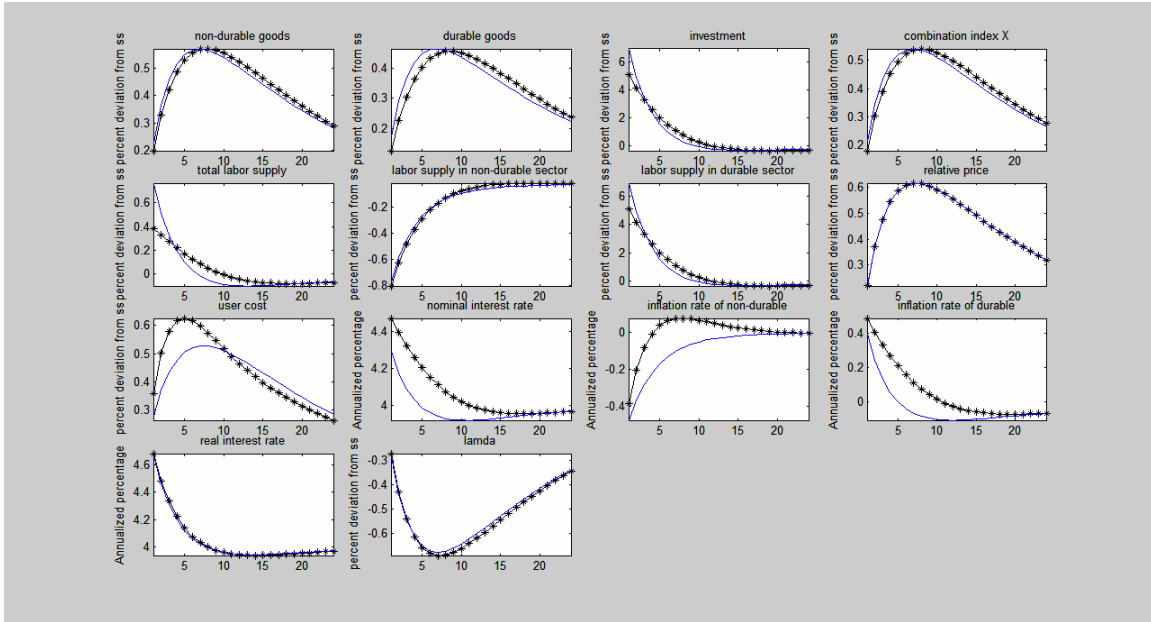
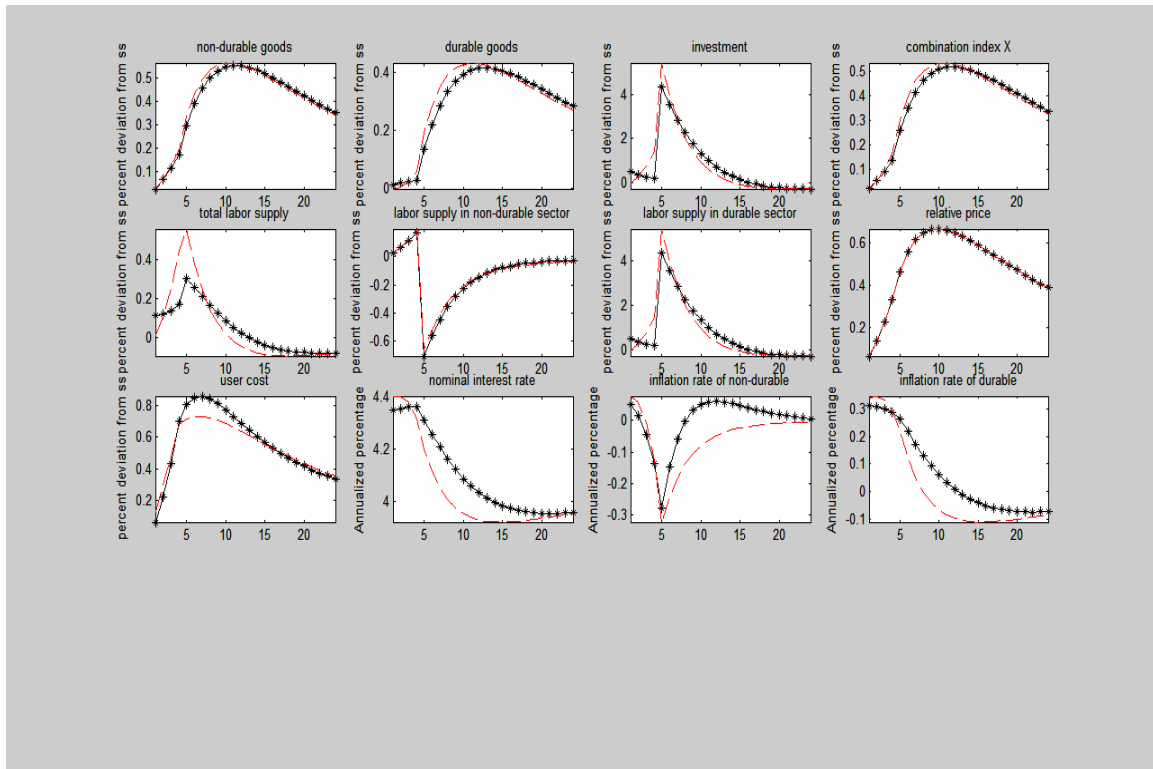


Figure 13 (Realized expectation)

Productivity in non-durable sector is expected to increase by 1% at time 1 and realized at time 5. Impulse responses under simple rule policy reacting to both durable sector inflation and non-durable sector inflation (solid lines), compared with impulse responses under Ramsey policy (lines with stars).

$$\log(R_t) - \log(R^*) = a(\log(\pi_{c,t}) - \log(\pi_c^*)) + b(\log(\pi_{d,t}) - \log(\pi_d^*))$$

(a=1 ; b=0.2;  $\eta = 0.2$ )



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