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Abstract

This work analyses the implications, in terms of determinacy and E-stability of equilibrium, of a policy rule that responds to private sector expectations in forward looking models. In the literature, this type of policy has been both recommended and criticized. We try to understand the reasons for such different conclusions and shed some light on the desirability of this type of policy rules.

Key words: Monetary policy, expectations, learning, determinacy, E-stability.

JEL classification: E52, C62, D84.

1. Introduction

In the standard forward-looking New Keynesian model, as presented, e.g., in Clarida, Gali and Gertler (1999), the optimal fundamentals-based policy rule generates both indeterminacy of rational expectations equilibria and instability of learning dynamics, as shown in Evans and Honkapohja (2003). In the same framework, these authors show, a policy rule that responds also to private sector expectations

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is able to amend these shortcomings: the ensuing recommendation for policymakers is to use this type of rules. On the contrary, previous works had argued against the same type of policies. In particular, Bernanke and Woodford (1997) found that conditioning the policy rule on private expectations increases the possibility of generating indeterminacy, and thus advised policymakers not to respond to private expectations. The purpose of this paper is to shed some light on the reasons why such different results have been obtained, in order to understand whether or not a policymaker should respond to private expectations. We find that the criticism put forward by Bernanke and Woodford (1997) doesn’t survive to a more careful specification of what an expectations-based policy should respond to, provided the policy parameters are optimally tuned.

2. New Keynesian models

2.1. With current expectations

The model used in Evans and Honkapohja (2003) (hereafter E&H) is the standard forward-looking New Keynesian framework, as presented, e.g., in Clarida, Gali and Gertler (1999). The two relevant equations are:

\[ y_t = -\varphi (i_t - E_t \pi_{t+1}) + E_t y_{t+1} + g_t \]  
\[ \pi_t = \lambda y_t + \beta E_t \pi_{t+1} + u_t. \]  

Equation (1) is a forward-looking IS equation and (2) an expectations-augmented Phillips curve; \( y_t \) is the output gap, \( i_t \) the interest rate, here taken to be the policy instrument, and \( \pi_t \) is the inflation rate. \( E \) indicates expectations. The two exogenous shocks \( g_t \) and \( u_t \) follow AR(1) processes with damping coefficients \( \mu, \rho \in [0, 1) \).

2.2. With past expectation

The system used by Bernanke and Woodford (1997) (B&W) is described by the following equations:

\[ \ddot{y}_t = E_t \ddot{y}_{t+1} - \sigma (i_t - E_t \pi_{t+1} - \rho_t) \]  
\[ \pi_t = \beta E_{t-1} \pi_{t+1} + k E_{t-1} (\ddot{y}_t - \theta_t), \]  

where \( \ddot{y}_t \) represents real output. In this model it is assumed that any price change chosen at time \( t \) takes place at time \( t + 1 \), and this is the reason for the different timing of expectations in equation (4) compared to equation (2), where it
is assumed that price changes take place in the same period they are decided. Therefore in B&W inflation at time \( t \) depends only on information available at time \( t - 1 \), when pricing decisions were taken. The two exogenous disturbances \( \rho_t \) and \( \theta_t \) follow AR(1) processes with damping coefficients \( \lambda, \delta \in [0, 1) \).

2.3. Policy rule

2.3.1. Fundamentals-based policy rule

The policy problem is to minimize expected deviations of output gap and inflation from their target levels over the infinite future horizon. E&H show that an optimal fundamental-based policy rule\(^1\) in their setting leads to indeterminacy of equilibria, i.e., to multiple stable RE solutions; moreover, the minimum state variable (MSV) solution is E-unstable, which implies that it can not be learned by agents through recursive least square (RLS) procedures.\(^2\) When we derive an optimal discretionary fundamentals-based policy in the B&W framework, we find that the system is indeterminate and the fundamental equilibrium is E-unstable for values of the autoregressive parameter \( \delta \) close to 1.

2.3.2. Expectations-based policy rule

To overcome the negative results outlined above, E&H propose an alternative policy that responds also to private sector expectations. The optimal expectations-based discretionary policy they derive\(^3\) takes the form

\[
i_t = \left(1 + \frac{\lambda \beta}{\varphi(\alpha + \lambda^2)}\right) E_t \pi_{t+1} + \frac{1}{\varphi} E_t y_{t+1} + \frac{\lambda}{\varphi(\alpha + \lambda^2)} u_{t+1} + \frac{1}{\varphi} g_t.
\] (5)

Equations (1), (2) and (5) represent the new system describing the evolution of the economy given private sector expectations. E&H show that the MSV REE of this model is both determinate and E-stable (see their Proposition 3). Note that in equation (5) the Taylor principle is satisfied, since the nominal interest rate responds more than proportionally to changes in inflation expectations.\(^4\)

\(^1\)This is a policy rule that responds only to fundamentals of the economy, in particular here to the two exogenous shocks.

\(^2\)For the relation between E-stability and learnability, see Evans and Honkapohja (2001).

\(^3\)Here we assume that the target values for both inflation and output gap are zero.

\(^4\)Note also that policy rule (5) complies with Proposition 4 in Bullard and Mitra (2002), which indicates necessary and sufficient conditions for a policy rule to induce determinacy in
But here is the puzzle: B&W showed that when a policy rule of the form

\[ i_t = \phi_\pi \pi^f_{t+1} + \phi_y \bar{y}_t, \]  

where \( \pi^f_{t+1} \) is the forecast of time \( t + 1 \) inflation announced by private forecasters at time \( t \), is used, the problem of indeterminacy found under the fundamentals-based policy is likely to increase instead of decreasing. The ensuing advice for the policymaker is not to condition its policy on private expectations.

The policymaker is assumed here to observe all the past history of inflation and output, up to time \( t \), but not the value at time \( t \) of the fundamental shocks. He thus responds to current output and private sector inflation expectations, both of which he can observe.

We extend now B&W analysis and study E-stability of the MSV solution in their original setting, in order to compare results with E&H. Rewriting the system in the standard form

\[ x_t = AE_t x_{t+1} + B v_t, \]

with

\[ A = \begin{pmatrix} \frac{\beta}{\sigma(1-\phi_\pi)} & k \\ \frac{1}{1+\sigma y} & \frac{1}{1+\sigma y} \end{pmatrix}, \]

we find the actual law of motion (ALM) for the economy and derive the map from the perceived law of motion (PLM) of agents to the ALM, which governs the E-stability properties of the system.\(^5\) It turns out that the eigenvalues governing E-stability are the eigenvalues of the A matrix (8) that govern determinacy, multiplied by the two AR(1) coefficients for the shocks. Therefore, if the system is determinate, then the unique solution is also E-stable. If instead we are in the indeterminacy region, E-stability depends on the value of the autoregressive parameters \( \lambda \) and \( \delta \). We find that for some sensible parameter values of the structural and policy parameters, at least one eigenvalue lies outside the unit circle. Therefore, both indeterminacy and E-instability may plague this economy.

\( ^5\) The complete analysis is available from the author upon request.
3. Why such different results?

We want to understand here why such different conclusions were reached about the opportunity to base a policy rule on private expectations.

Comparing the E&H model with the B&W one, both closed with an expectations-based discretionary policy rule, we can see that equation (3) is qualitatively the same as equation (1). Differences come from the Phillips curve and the policy rule. The Phillips curve (4) differs from (2) for the timing of expectations: while in equation (2) the relevant variables are current output gap and expected future inflation, in equation (4) current inflation depends on past expectations of future inflation and on past expectations of current output gap.

Policy rule (6) differs from policy rule (5) in a more substantial way: equation (5) includes expectations of both output gap and inflation, in addition to fundamental shocks. On the contrary, equation (6) responds only to inflation forecasts, and to current output. It does not respond to the exogenous shocks, which are not supposed to be known to by policymaker. But if we look carefully, we can see that a policy rule of the form (6) does not actually respond to private expectations. In fact, when the system is put into the standard form (7), the Phillips curve (4) is moved forward: this implies that the current variables are time $t$ output and time $t+1$ inflation, exactly those the policymaker responds to, while expected variables in the Phillips curve, $E_t \pi_{t+2}$ and $E_t \tilde{y}_{t+1}$, do not enter into the policy rule. Therefore, the authority can not offset private expectations in an effective way and can not prevent them from introducing self-fulfilling elements in the system.

To better grasp the differences in the two models and their impact on the relevant equilibrium properties, we take progressive steps from the B&W’s towards the E&H’s setting.

In moving from B&W to E&H the first step is to change the timing in the Phillips curve. We suppose that inflation is no longer determined one period in advance, but within the period, as in most models. Equation (4) becomes

$$\pi_t = \beta E_t \pi_{t+1} + k(\tilde{y}_t - \theta_t).$$

(9)

Using equation (3) and policy rule (6), and writing the system in the standard

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6Since in the model it is assumed that prices are predetermined, that private agents have all the relevant information at time $t$ and that they make their forecasts by minimizing the variance of their forecast error, it follows that $\pi_{t+1}' = E_t \pi_{t+1} = \pi_{t+1}$.
form, the $A$ matrix governing determinacy is now

$$A = \begin{pmatrix} \beta + \frac{k\sigma(1-\phi_\pi)}{1+\alpha\phi_\pi} & \frac{k}{1+\alpha\phi_y} \\ \frac{\sigma(1-\phi_\pi)}{1+\alpha\phi_y} & \frac{1}{1+\alpha\phi_y} \end{pmatrix} \tag{10}$$

whose two eigenvalues depend, among other things, on the value of the policy parameters, and at least one is found to lie outside the unit circle when $\phi_\pi$ is positive and greater than one and $\phi_y$ is a negative small number.

By changing the timing in the Phillips curve, we have made inflation a non-predetermined variable: using a policy rule of the form (6), the policymaker is now actually responding to expectation of a current endogenous variable. But he is not responding yet to expectations of output, which are still free to float and affect the outcome of the system. The authority is also not responding to fundamental shocks, but this feature clearly doesn’t affect equilibrium determinacy and E-stability, which only depend on the values of the entries in matrix $A$.

We make now the policymaker respond to expected future output instead of to current output. The system, thus, is now the same as the one analyzed in E&H, except that here the policy parameters are left general and are not derived through optimality conditions. The new policy rule is (neglecting the terms pertaining to the shocks, which we have just seen are irrelevant for determinacy and E-stability analysis)

$$i_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t \bar{y}_{t+1} \tag{11}$$

and matrix $A$ becomes

$$A = \begin{pmatrix} \beta + \frac{k\sigma(1-\phi_\pi)}{1+\alpha\phi_\pi} & k(1-\phi_y) \\ \sigma(1-\phi_\pi) & 1-\alpha\phi_y \end{pmatrix}; \tag{12}$$

again, at least one of its eigenvalue is found to lie outside the unit circle for sensible values of the policy coefficients and structural parameters.

But if we derive the policy coefficients through optimality conditions, then both eigenvalues are constrained to lie inside the unit circle for any choice of the parameter values, and therefore both determinacy and E-stability obtain. In particular, an optimal policy is able to completely offset the effect of expected

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\(^7\)Another difference is, of course, that here output, as opposed to output gap, enters the policy rule; but this doesn’t affect our argument.

\(^8\)The two eigenvalues are 0 and $\alpha \beta/(k^2 + \alpha)$, with the second always less than one for the restrictions on structural parameters: $0 < \beta < 1$, $\alpha > 0$, $k > 0$. 

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future output on current output and on inflation. Mathematically, this makes the right column of the A matrix equal to zero, leading to a zero eigenvalue.\footnote{The optimal value for $\phi_y$ is $\frac{1}{\sigma}$, while that for $\phi_x$ is $1 + \frac{k\beta}{\sigma(\alpha+k\gamma)}$.}

We also find that even in the original setting of B&W, as represented by equations (3) and (4), the expectations-based discretionary policy generates both determinacy and E-stability when it is optimally tuned. But to obtain these results, the policy rule must respond to the variables that make the system forward-looking; thus, in B&W setting, to $E_t\bar{y}_{t+1}$ and $E_t\bar{\pi}_{t+2}$.

We want also to emphasize that if the central bank cannot respond to fundamental shocks, as assumed in B&W, it still can induce determinacy and E-stability of equilibrium (which in this case will not be the one with fully stabilized inflation and output gap) by responding solely to expectations in the optimal discretionary way. Stabilizing the economy w.r.t. exogenous fundamental shocks is a completely different task from that of inducing a unique learnable equilibrium: while the first task requires a policy response to the shocks, the second calls for a response to expectations.

The economic intuition behind the determinacy and E-stability results outlined above goes as follows: a policy rule that responds optimally to private sector expectations is able to offset their impact on current variables and can thus prevent them from becoming self-fulfilling and generating a multiplicity of equilibria; in addition, if expectations fail to be rational, the policy response guides them back towards rationality, thus assuring E-stability.

4. Conclusions

We have shown that different results present in the literature about the desirability of an expectations-based policy rule can be attributed to two reasons. First, given the timing in their setting, the type of policy presented by Bernanke and Woodford (1997) as one that targets private forecasts doesn’ t really respond to private expectations but to current values of the endogenous variables. Secondly, the policy parameters need to be optimally tuned in order to effectively offset the destabilizing influence of expectations on current variables.
References


