Ruptures in the probability scale? Calculation of ruptures' dimensions

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15 December 2009

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The article raises the question of possible existence of ruptures, gaps in the probability scale which are caused by noises, uncertainties. A hypothesis of existence of such ruptures may be used to solve a number of problems of, e.g., utility theory in economics. The calculations give the dimensions of ruptures can be more than 1/3 of the standard deviation for the standard probability distributions.

Contents

Introduction

This paper presents in English the results of Harin (2009, 2009-2), based on Harin (2005 and 2007).

Until recently, insufficient attention was paid to noises and uncertainties near the bounds, borders of the scale of probability. This article analyzes the possibility of existence of ruptures, gaps in the scale of probability, which are caused by such noises and uncertainties. Calculations of ruptures' dimensions for standard distributions are performed.

1. The idea

1.1. A far analogy. Vibrations near a rigid wall

Suppose an electro-drill or any similar device, e.g., sewing-machine, vibrosieve, machine-gun, electric hammer etc. which (when working) can vibrate quickly. Presume the device has rigid flank sides and vibrates with the amplitude of, say, 1 mm.

Can we approach a flank side of the non-working drill (or of the device) to a rigid wall or ledge:

A) as close as at the distance, say, 0.1 mm;

B) tightly?

Certainly. Both A) and B).

And now turn the drill (the device) on. What will be the distance from the rigid wall to the working drill? Vibrations will repulse, shift the drill from the wall.

Due to the vibrations:

A) the distance from the drill to the wall will be more than 0.1 mm;

B) the gap, rupture will arise between the drill and the wall.

1.2. An example. Aiming firing at a target

General conditions

Suppose a hypothetic transportable testing stand, arrangement for testing the quality of rifles, guns, cartridges etc. To avoid human errors, the arrangement is made in the form of a standing man, a rifle is fasten onto the arrangement and the aiming is performed automatically. Suppose firing errors are minimized and are much less than one point of the target.

Suppose the arrangement is placed near a railway or Metro. The vibrations of the ground increase firing errors up to, say, 2 points. For the sake of simplicity, assume the target is strongly elongated in one of directions. So, the consideration is reduced to one-dimensional and uniform (without effects of curvature) case. Suppose the points are located in the scale from "0" to "10": "9", "8", "7" etc. are located after "10". Before "0" there is the blank space which is equivalent to "0".

Suppose following dispersion takes place: one shot =exact; one shot =+2 points; one shot =-2 points.

If the aiming is performed at, say, "7", the mean result is the same as the aiming value. The result is $(7+9+5)/3=7$.

A) The shift from the borders to the middle of the target scale

If the aiming is performed at "9", one bullet should hit beyond the border "10" at "11", but really hits at "9". The result is *(9+9+7)/3=25/3=8⅓*. One bullet, instead of "11", hits "9", i.e. 2 less than the aiming value. The mean result is shifted from the border (from "10") to the middle (to ~"5") of the scale by 2/3 points.

If the aiming is performed at "1", one bullet should hit beyond the border "0" at "-1", but really hits at the blank space which is equivalent to "0". The result is *(1+3+0)/3=1⅓*. One bullet, instead of "-1", hits "0", i.e. 1 more than the aiming value. The mean result is shifted from the border (from "0") to the middle (to ~"5") of the scale by 1/3 points.

A) The dispersion causes the shifts of the mean results from the borders to the middle of the target scale.

B) The ruptures in the target scale

If the aiming is performed at the border of the target scale "10", one bullet should hit beyond the border "10" at "12", but really hits at "8". The result is *(10+8+8)/3=26/3=8⅔*. One bullet, instead of "12", hits "8", i.e. 4 less than the aiming value. The rupture between the mean result and the border "10" of the scale is 1⅓ points.

If the aiming is performed at the border of the scale "0", one bullet should hit beyond the border "0" at "-2", but really hits at the blank space which is equivalent to "0". The result is *(0+2+0)/3=2/3*. One bullet, instead of "-2", hits "0", i.e. 2 more than the aiming value. The rupture between the mean result and the border "0" of the scale is 2/3 points.

B) The dispersion causes the ruptures near the borders of the target scale.

1.3. The idea

The original idea is to determine how close to the border of the scale of probability can be the probability estimation of an event, if the distribution of this estimation has the non-zero dispersion. In other words, to define the minimum and maximum values of the probability estimation, which has a nonzero distribution dispersion.

If the minimum value of the probability estimation of an event is strictly greater than 0, then we may say the probability estimation of such event cannot accept values between 0 and the minimum value.

If the maximum value of the probability estimation of an event is strictly less than 1, then we may say the probability estimation of such event cannot accept values between this maximum value and 1.

That is, we may say that, near the borders of the scale of probability, for the probability estimation, whose probability distribution has a nonzero distribution dispersion, as though there are ruptures, which are strictly greater than 0.

It should be emphasized that, unlike most frequently considered examples, we speak here about the probability estimation distributions only for one of the values of any parameter. For example, we speak about the probability estimation distribution from lottery winning of $1 million, or about the probability estimation distribution when shooting in "8" of the target.

One cannot use the conclusions of this article to usually considered examples of probability estimation distributions of all the values of a parameter. For example, one cannot use the conclusions of this article to the probability estimation distribution of all the values from winning the lottery, the probability estimation distribution of all the values in target shooting hits, etc.

2. Uncertainty in probabilities' measurements

2.1. Background noises, measurement errors …

Real measurements of probability are almost always performed in the environment of external interference, background noises, disturbances etc. This leads to the finite non-zero external uncertainty of measurements.

In addition to such external interference, the measurements can be influenced by the internal interference. This leads to the finite non-zero internal uncertainty of measurements.

2.2. The total uncertainty

Thus, in almost any real case a finite non-zero degree of uncertainty is inherent in real measurements of probability. The total magnitude of this uncertainty can be both negligible and high relatively to useful signal, but it is finite and non-zero (it does not tend to zero).

This leads to the finite non-zero dispersion for all such cases.

3. The assumptions and procedure

3.1. Probability distributions near the border of the scale

Probability cannot be less than 0 or greater than 1. How does behave a distribution of probability estimation near the border of the scale of probability? When approaching the border of the probability the distribution can:

1) be deformed from the border:
   - a) be deformed from the border;
   - b) be reflected from the border;
2) remain the same (the part of the distribution that goes abroad is invalidated without impact on the rest of the distribution) and
   - a) be not included in total normalization;
   - b) be stored in total normalization;
3) be deformed to the border:
   - a) be deformed to the border;
   - b) be accumulated on the border:
     - ba) be partially accumulated on the border;
     - bb) be fully accumulated on the border.

3.1.1. The possibilities of reducing of the rupture's dimension

In case of (3b) where a portion of the probability distribution, going abroad the probability scale, is completely or partially accumulated at the border, the dimension of the rupture decreases. In case of full accumulation (3bb) the dimension of the rupture decreases in twice.

In the case (2) when a portion of distribution, beyond the borders of the scale of probability, is annulled, both directly and at general normalization (2b), the dimension of the rupture is also decreased twice, but due to normalization.

3.2. The procedure for the accounting of uncertainty

How close to the border of the probability scale can be a probability estimation? Take the maximum approach of a probability estimation to the border such approach, the probability estimation exactly coincide with that border when zero dispersion. Than the rupture's dimension $R_{rupture}$ is not less then a half of the mathematical expectation of the half of distribution $M_{1/2}$.

This situation is real, e.g., for cases where the level of uncertainty was so small that the dispersion could be considered equal to zero, but then uncertainty scale scores (e.g., appeared or increased noise), leading to increase the dispersion.

4. Calculation of ruptures' dimensions for standard distributions

4.1. Rupture's dimension for the uniform distribution

For the uniform distribution we have rupture's dimension $R_{rupture}$

$$D(p)=\int_{-l}^{+l} p^2\frac{1}{2l}dp=\frac{1}{2l}\frac{p^3}{3}\bigg|_{-l}^{+l}=\frac{1}{2l}\frac{l^3}{3}+\frac{1}{2l}\frac{l^3}{3}=\frac{l^2}{3}$$

$$R_{rupture}(p)\equiv\frac{1}{2}M_{1/2}(p)=\frac{1}{2}\int_0^l p\frac{1}{l}dp=\frac{1}{2}\frac{1}{l}\frac{p^2}{2}\bigg|_0^{+l}=\frac{1}{l}\frac{l^2}{4}=\frac{l}{4}$$

$$\frac{R_{rupture}(p)}{\sqrt{D(p)}}=\frac{l}{4}\frac{\sqrt{3}}{l}=\frac{\sqrt{3}}{4}\approx 0.433>\frac{1}{3}$$

4.2. Rupture's dimension for the normal distribution

For the normal distribution we have rupture's dimension $R_{rupture}$

$$D(p)=\sigma^2$$

$$R_{rupture}(p)\equiv\frac{1}{2}M_{1/2}(p)=\frac{1}{2}\int_0^{+\infty}2p\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{p^2}{2\sigma^2}}dp=$$

$$=\frac{\sigma}{\sqrt{2\pi}}\int_0^{+\infty}e^{-y}dy=\frac{\sigma}{\sqrt{2\pi}}e^{-y}\bigg|_0^{+\infty}=\frac{\sigma}{\sqrt{2\pi}}$$

$$\frac{R_{rupture}(p)}{\sqrt{D(p)}}=\frac{\sigma}{\sqrt{2\pi}}\frac{1}{\sigma}=\frac{1}{\sqrt{2\pi}}\approx 0.399>\frac{1}{3}$$

4.3. Rupture's dimension for Laplace's distribution

For Laplace's distribution we have rupture's dimension $R_{rupture}$

$$D(p)=\int_{-\infty}^{+\infty}p^2\frac{\lambda}{2}e^{-\lambda|p|}dp=-p^2\frac{1}{2}e^{-\lambda|p|}\Big|_{-\infty}^{+\infty}-\int_{-\infty}^{+\infty}2p\frac{1}{2}e^{-\lambda|p|}dp=$$

$$=-0+2p\frac{1}{2\lambda}e^{-\lambda|p|}\Big|_{-\infty}^{+\infty}+\int_{-\infty}^{+\infty}2\frac{1}{2\lambda}e^{-\lambda|p|}dp=\frac{1}{\lambda^2}+\frac{1}{\lambda^2}=\frac{2}{\lambda^2}$$

$$R_{rupture}(p)\equiv\frac{1}{2}M_{1/2}(p)=\frac{1}{2}\int_0^{+\infty}p\lambda e^{-\lambda|p|}dp=\frac{1}{2}pe^{-\lambda|p|}\Big|_0^{+\infty}-\frac{1}{2}\int_0^{+\infty}e^{-\lambda|p|}dp=$$

$$=0+\frac{1}{2\lambda}e^{-\lambda|p|}\Big|_0^{+\infty}=\frac{1}{2\lambda}$$

$$\frac{R_{ruptue}(p)}{\sqrt{D(p)}}=\frac{1}{2\lambda}\frac{\lambda}{\sqrt{2}}=\frac{1}{2\sqrt{2}}\approx 0.354>\frac{1}{3}$$

4.4. Rupture's dimension for the edge distribution

For the edge distribution (in limit - two Delta function on the edges) we have rupture's dimension $R_{rupture}$

$$D(p)=2\int_{L-l}^{L}p^2\frac{1}{2l}dp=\frac{1}{l}\frac{p^3}{3}\Big|_{L-l}^{L}=\frac{1}{3l}(L^3-(L-l)^3)=$$

$$=\frac{1}{3l}(L^3-L^3+3L^2l-3Ll^2+l^3)=\frac{1}{3l}(3L^2l-3Ll^2+l^3)=$$

$$=\frac{1}{3}(3L^2-3Ll+l^2)=L^2(1-\frac{l}{L}+\frac{l^2}{3L^2})\xrightarrow[l\to 0]{}L^2$$

$$R_{rupture}(p)\equiv\frac{1}{2}M_{1/2}(p)=\frac{1}{2}\int_{L-l}^{L}p\frac{1}{l}dp=\frac{1}{2}\frac{1}{l}\frac{p^2}{2}\Big|_{L-l}^{L}=\frac{1}{4l}(L^2-(L-l)^2)=$$

$$=\frac{1}{4l}(L^2-L^2+2Ll-l^2)=\frac{l}{4l}(2L-l)=\frac{L}{2}(1-\frac{l}{2L})\xrightarrow[l\to 0]{}\frac{L}{2}$$

$$\frac{R_{ruptue}(p)}{\sqrt{D(p)}}\xrightarrow[l\to 0]{}\frac{L}{2}\frac{1}{L}=\frac{1}{2}>\frac{1}{3}$$

5. General results

5.1. General evaluation of ruptures' dimensions

Calculations gave the ratio of the minimal dimension of the rupture to the dimension of the standard deviation:

for edge distribution (two Delta functions on the edges) = 0.5;
for uniform distribution ≈ 0,433;
for the normal distribution ≈ 0,399;
for the Laplace distribution ≈ 0,354.

Note, as the dominance of the central region on the lateral areas is increased, this relationship is reduced from 0.5 to 0.35.

Thus it can be stated:

1) Ruptures' dimensions for considered standard distributions are $O(\Delta P)$ of magnitudes of standard deviations ΔP.

2) For standard distributions whose central area dominates over the lateral areas not more than in the distribution of Laplace, ruptures' dimensions exceed 1/3 of the standard deviation. In the absence of the effect of accumulation, they exceed 2/3 of the standard deviation.

5.2. Consequences of ruptures existence in the probability scale. Applications to economic theory, forecasting, …

The principle of the uncertain future can be considered as the consequence of the existence of ruptures in the scale of the probability for probability estimations (actually, the development of the hypotheses of the existence of gaps in the scale of probability took place after the development of the principle of uncertain future). As the consequences of the principle of the uncertain future, we can specify, including the following:

In economic theory, a uniform solution is found of: Allais' (see, e.g., Allais 1953) and Ellsberg's (see, e.g., Ellsberg 1961) paradoxes, the problem of risk aversion, risk premiums, equity premium puzzle, small probabilities exaggeration and large probabilities discount, "four-fold-pattern" paradox, etc. (see Harin 2007).

In forecasting, a general correcting formula for long-term-use forecasts (see, e.g., Harin 2008) is developed.

In logic, the second consequence of the principle may convert one current event in an infinite number of events in the future. The same will happen for the denial of this event. Thus, the direct application of the law of excluded third for future events may be inadequate within 2-digits logic.

In the theory of complex systems, the application of the second consequence of the principle may lead to possible violation of subdivision on the groups of inconsistent events for future events (see Karassev 2007).

Conclusions

In the article, the possible existence of ruptures, gaps in the scale of the probability for probability estimations is illustrated on standard examples. Under the accepted assumptions and procedures, the calculations of ruptures' dimensions are performed for standard and limit edge distributions.

For the wide class of standard distributions, their ruptures' dimensions exceed 1/3 of standard deviations' magnitudes.

References

Allais, M. (1953) "Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine" Econometrica 21, 503-46.

Ellsberg, D. (1961) Risk, Ambiguity and the Savage Axioms. Quarterly Journal of Economics, 75, 643-669.

Harin, A. (2009-2) "Ruptures in the probability scale. Calculation of ruptures' values" MPRA, 16663. (in Russian)

Harin, A. (2009) "About existance of ruptures in the probability scale. Calculation of ruptures' values" Ninth International Scientific School "Modelling and Analysis of Safety and Risk in complex systems" 2009. (in Russian)

Harin, A. (2008) "To development of a general formula of forecasting" Proceedings of the 51-th scientific conference of MIPT – 2008 "Modern problems of fundamental and applied sciences".

Harin, A. (2007) "Principle of uncertain future, examples of its application in economics, potentials of its applications in theories of complex systems, in set theory, probability theory and logic" Seventh International Scientific School "Modelling and Analysis of Safety and Risk in complex systems" 2007. (in Russian)

Harin, A. (2005) "A new approach to solve old problems" Game Theory and Information from Economics Working Paper Archive at WUSTL, 0505005.

Karassev, V. (2007) private communication.